The minimum lung pressure to sustain vocal fold oscillation

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In previous experimental studies it has been observed that the minimum lung pressure to sustain vocal fold oscillation after its onset is lower than the threshold pressure needed to initiate it. This phenomenon is studied analytically using a previous body-cover model of the vocal folds and applying the describing function method to the general case of large amplitude oscillations. It is shown that the phenomenon is a consequence of the nonlinear characteristic of the effective aerodynamic damping introduced by the air pressure acting on the vocal folds. The results predict a value for minimum sustaining pressure equal to half the threshold pressure for a rectangular prephonatory glottis, which is in the order of experimental results. © *1995 Acoustical Society of America.*

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INTRODUCTION

It is known that a minimum positive value of the lung air pressure, called the oscillation threshold pressure, is required to initiate the vocal fold oscillation. Using a simplified body-cover model of the vocal folds, Titze (1988) showed analytically that for lung pressure above the threshold value, the energy transferred from the airflow to the vocal folds overcomes the energy lost in the tissues by dissipation, permitting the generation of their oscillation. In experimental studies on excised canine larynges, Baer (1975) also observed that the minimum pressure required to sustain the oscillation after its onset is lower than the threshold value; i.e., the pressure has to be lowered a certain amount below the threshold to stop the oscillation. He measured values in the range of 0.3–0.8 kPa for the threshold pressure, and 0.2–0.4 kPa for the minimum sustaining pressure. More recently, Titze *et al.* (1994) obtained similar results on a physical model of the vocal folds, with 0.37–0.65 and 0.33–0.55 kPa for the same pressures.

This paper presents an analytical study of this phenomenon to explain the lower value of the minimum sustaining pressure with respect to the threshold. A previous attempt was presented elsewhere by Lucero and Gotoh (1993) based on a variation of the two-mass model. Here, Titze's body-cover model will be adopted, which incorporates the layered tissue structure of the vocal folds and is thus closer to their physiology. In his analysis, small amplitude oscillations were assumed to linearize the equations of motion and determine the threshold conditions to initiate the oscillations. At threshold, the oscillation amplitude is zero and the small amplitude restriction is valid. Since we are interested in the conditions to maintain the oscillation after it has started, i.e., when the oscillation amplitude has some value, general large amplitude oscillations will be considered in the present study.

A better knowledge of the minimum sustaining pressure would find some application to phonation theory. For example, in a recent work, Titze (1992) included the phonation threshold pressure to derive aerodynamic laws relating the lung pressure and glottal flow, based on the fact that a finite pressure is required to establish an infinitesimal oscillatory flow. Those laws might be improved by replacing the threshold pressure by the minimum sustaining pressure, since the oscillatory flow created by the threshold pressure takes a finite value after the oscillation has started.

I. VOCAL FOLD MODEL

The body-cover model (Titze, 1988) is schematically shown in Fig. 1. There, the body of the vocal fold is stationary, and the cover propagates a surface wave along the glottis in the direction of the airflow. The vocal folds are assumed to be symmetric with respect to the vertical midline. For clarity of the present analysis, let us recall briefly Titze's derivation of the equation of motion.

The glottal area along the glottis is

$$a(z,t)=2L[\xi_0(z)+\xi_1(z,t)], \tag{1}$$

where z is the distance from the midpoint of the glottis in the direction of the airflow, L is the length of the vocal folds, $\xi_0(z)$ is the prephonatory glottal half-width, and $\xi_1(z,t)$ is the displacement of the cover due to the surface wave. The general expression of the surface wave displacement is

$$\xi_1(z,t)=\xi_1(t-z/c), \tag{2}$$

which is the solution of the one-dimensional wave equation with wave velocity c. This expression is approximated expanding it in a Taylor series around $z=0$, and keeping the linear terms

$$\xi_1(t-z/c)\approx\xi-\left(\frac{z}{c}\right)\frac{d\xi}{dt}, \tag{3}$$

where $\xi=\xi_1(0,t)$ is the displacement of the cover at the midpoint. This approximation implies the assumption of a low value for the relation

$$\tau=(z/c), \tag{4}$$

which is the time delay of the surface wave in movement from the lower edge of the vocal fold to the midpoint, or

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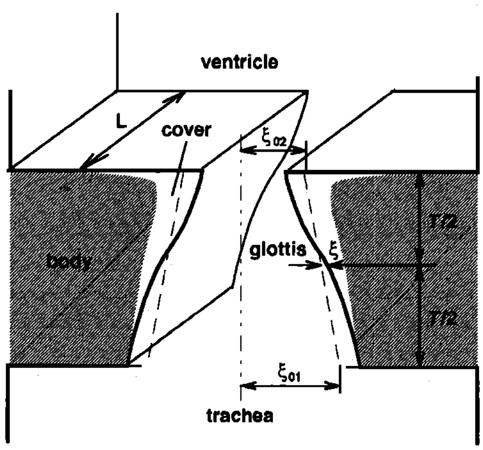

FIG. 1. Diagram of the body-cover model of the vocal folds. Broken line: prephonatory position. T: vocal fold thickness. ξ_{01} and ξ_{02}: prephonatory glottal half-widths at the lower and upper edges of the vocal folds. ξ: displacement of the cover at the midpoint of the vocal fold (Titze, 1988).

from the midpoint to the upper edge. To test the validity of this assumption, Titze considered the solution $\xi_1(z,t)=\sin\omega(t-z/c)$. In this case, the approximation in Eq. (3) is equivalent to the small-angle approximation $\sin(\omega z/c)\approx(\omega z/c)$, or

$$\omega\tau\ll\pi/2. \tag{5}$$

Typical values obtained experimentally for the phase delay $\omega\tau$ are in the range of 30°–45° [60°–90° for the phase delay between the lower and upper edges of the vocal folds (Baer, 1975)]. The above approximation is then a bit crude for the vocal fold oscillation, but it may be adopted to facilitate the analytical treatment.

The glottal pressure P_g acting on the vocal folds is calculated as the mean glottal pressure

$$P_g=\frac{1}{T}\int_{-T/2}^{+T/2}P(z)dz, \tag{6}$$

where $P(z)$ is the glottal pressure distribution along the glottis and T is the half-thickness of the vocal fold. $P(z)$ is derived from the Bernoulli energy equation

$$P(z)=P_2+P_{k2}\left[1-\frac{a_2^2}{a^2(z)}\right], \tag{7}$$

where P_2 is the exit pressure, $P_{k2}=(\rho/2)|u|u/a_2^2$ is the kinetic pressure, ρ is the air density, u is the exit airflow, and a_2 is the glottal area at exit. The integral in Eq. (6) can be evaluated considering the glottal area gradient along the glottis, from Eq. (1),

$$\frac{\partial a}{\partial z}=2L\left[\frac{d\xi_0}{dz}+\frac{\partial\xi_1}{\partial z}\right]. \tag{8}$$

Using next the linear approximation in Eq. (3) to calculate $\partial\xi_1/\partial z$, we obtain

$$\frac{\partial a}{\partial z}=2L\left[\frac{d\xi_0}{dz}-\frac{1}{c}\frac{d\xi}{dt}\right]. \tag{9}$$

The second term between the brackets in this equation is independent of z; hence the variation of the glottal area gradient with z depends only on the first term, related to the prephonatory shape. For simplicity, we assume a linear variation of the prephonatory glottal area along the glottis, i.e.,

$$\xi_0(z)=\frac{(\xi_{01}+\xi_{02})}{2}-\frac{(\xi_{01}-\xi_{02})z}{T}, \tag{10}$$

where ξ_{01} and ξ_{02} are the prephonatory glottal half-widths at the lower and upper edges of the vocal folds, respectively. Substituting $d\xi_0/dz$ in Eq. (9), we obtain

$$\frac{\partial a}{\partial z}=2L\left[\frac{\xi_{02}-\xi_{01}}{T}-\frac{1}{c}\frac{d\xi}{dt}\right], \tag{11}$$

which is independent of z. This means that under the assumption of a linear variation of the prephonatory glottal area along the glottis, the approximation of Eq. (3) implies a linear variation along the glottis also for the time-varying glottal area, even in the general case of large amplitude oscillations. Using this result, the integral in Eq. (6) yields

$$P_g=P_2+P_{k2}\left(1-\frac{a_2}{a_1}\right), \tag{12}$$

where a_1 is the glottal area at entry.

We will consider the simple case in which (1) the subglottal pressure is constant during the oscillation and equal to the lung pressure, (2) the vocal tract input pressure is equal to the atmospheric pressure, and (3) the supraglottal area is large compared with the glottal area at the upper edge of the vocal folds. The first two assumptions correspond to an excised larynx, and the last one is a typical condition for the vocal folds. In this case, the glottal pressure becomes

$$P_g=\left(\frac{P_L}{k_t}\right)\frac{\xi_{01}-\xi_{02}+2\tau(d\xi/dt)}{\xi_{01}+\xi+\tau(d\xi/dt)}, \tag{13}$$

where P_L is the lung pressure, and k_t is an empirical coefficient related to the pressure losses due to turbulent flow and glottal viscous resistance.

The mechanical properties of the vocal fold tissue are next lumped at the midpoint of the glottis, which yields the equation of motion

$$M\frac{d^2\xi}{dt^2}+B\frac{d\xi}{dt}+K\xi=P_g, \tag{14}$$

where M, B, and K are the lumped effective mass, damping, and stiffness per unit area of the cover. The details on the derivation of the above equations can be found in Titze's paper (Titze, 1988); in particular, the above equation (13) is identical to Titze's Eq. (22).

It is also important to note that the above equations correspond to oscillations in an open glottis, i.e., the glottal closure is not included in the model. This restricts the oscillation amplitude to values smaller than the one at which the glottal closure occurs. However, we remark that the equation of motion was derived without any assumption of infinitesimally small oscillation amplitude; therefore, the equation of motion is valid for finite (large) values of the oscillation

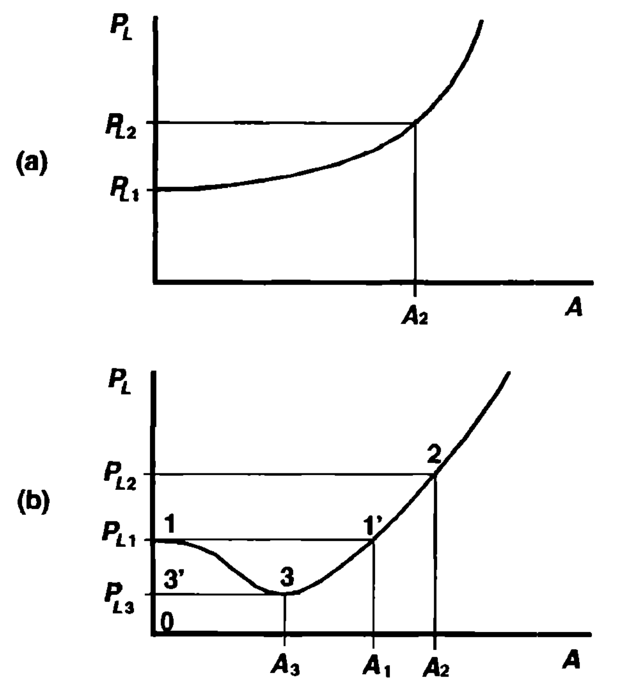

FIG. 2. Possible laws for variation of the lung pressure P_L to sustain the oscillation at an amplitude A.

amplitude, although within the open glottis condition, and may be used to study the oscillation after its onset.

II. ANALYSIS OF THE AIR PRESSURE

A. Relation between the lung pressure and the oscillation amplitude

Before the analytical treatment, let us discuss the relation between the lung pressure and the oscillation amplitude, to clarify concepts. Suppose that we measure the oscillation amplitude A at various fixed lung pressures P_L, and plot the results.

Figure 2 shows two possible shapes for the resultant curve $P_L(A)$. It would seem more natural to consider the oscillation amplitude as a function of the lung pressure, but we have choosen the inverse for comparison with the analytical results of next sections. In Fig. 2(a), the lung pressure increases monotonously with the oscillation amplitude, i.e., higher pressures are necessary to produce oscillation at higher amplitudes. The oscillation threshold pressure is the pressure at zero amplitude (P_{L1}), since a pressure higher than this value is required to produce the oscillation. If the pressure has a value $P_{L2} \geqslant P_{L1}$, the oscillation amplitude will grow from zero until the value A_2, shown in Fig. 2, after which it will continue with this constant amplitude.

In Fig. 2(b), the lung pressure first decreases with the oscillation amplitude, and then increases at large amplitudes. Let us consider what will happen when the lung pressure is gradually increased from 0 to a value such as P_{L2}, and then decreased back to 0. When reaching P_{L1} (the oscillation threshold), the oscillation will start and its amplitude will grow from zero (point 1) until the steady amplitude A_1 (point 1'). In the plot it would seem that the amplitude will jump suddenly from 0 to A_2. However, remember that we have plotted the steady-state values of the amplitude; the actual growth of the amplitude will be gradual. The amplitude will then grow until A_2, as the lung pressure reaches P_{L2} (point 2). When the pressure is decreased back, the amplitude will also decrease until A_3 at pressure P_{L3} (point 3). We can see that the oscillation will continue even though the lung pressure has a value lower than the oscillation threshold pressure ($P_{L3}<P_{L1}$). When the pressure is decreased below this point, the amplitude will decrease to 0 (point 3') and the oscillation will stop. The minimum lung pressure to sustain the oscillation is then P_{L3}, i.e., the minimum of the curve. Note that a hysteresis loop appears, given by points 1-1'-3-3'.

In both cases the lung pressure increases to infinity at large values of the oscillation amplitude. This is a physical necessity to limit the oscillation amplitude; otherwise it would grow unbounded for finite values of the lung pressure. Hence, the curve of the lung pressure must have a minimum, at zero amplitude as in Fig. 2(a), or at a larger amplitude as in Fig. 2(b).

In the following analysis we will see that the vocal fold oscillation corresponds to the second case, which explains the experimental measurements.

B. Describing function for the glottal pressure

In his analysis, Titze linearized the equation of the glottal pressure assuming small amplitude oscillations, and expressed its action in terms of an effective aerodynamic stiffness and an effective aerodynamic damping. He then obtained the threshold pressure as the pressure required for a zero value of the total damping, equal to the aerodynamic damping plus the cover damping. This approach is extended here to the case of large amplitude oscillations, to derive expressions for the aerodynamic terms as functions of the oscillation amplitude.

When the vocal folds are oscillating with a constant amplitude, the tissue displacement ξ is some periodic function of time $\xi(t)$, which must be a solution of Eq. (14). We will apply the describing function method (Siljak, 1969) to determine the conditions for this periodic solution to exist.

First, we assume that the periodic solution may be approximated by the sinusoidal function

$$\xi=\bar{\xi}+\tilde{\xi}, \tag{15}$$

where $\bar{\xi}$ is a static displacement and $\tilde{\xi}$ is an oscillatory component, given by

$$\tilde{\xi}=A \sin \omega t, \tag{16}$$

where A is the oscillation amplitude. The numerical solution of Eq. (14) shows that the oscillation is close to a sinusoidal function (Titze, 1988), which justifies the approximation. As noted before, the amplitude A may have any finite value, with the restriction that it must be smaller than the value to cause the glottal closure. Due to the assumption of a linear prephonatory glottal shape, in the case of a convergent prephonatory glottis ($\xi_{01}<\xi_{02}$) the range of A is $0 \leqslant A<\xi_{02}+\bar{\xi}$.

and in the case of a divergent prephonatory glottis ($\xi_{01}>h\xi_{02}$) it is $0\leq A<\xi_{01}+\bar{\xi}$.

The only nonlinear term in Eq. (14) is P_g. This term can be quasilinearized through its describing function, as follows. Introducing Eq. (15) into Eq. (13), we obtain

$$P_g=\left(\frac{P_L}{k_t}\right)\frac{\xi_{01}-\xi_{02}+2\omega\tau A\cos\omega t}{\xi_{01}+\bar{\xi}+A\sin\omega t+\omega\tau A\cos\omega t}. \qquad (17)$$

This equation is next expanded into a Fourier series,

$$P_g=N_0+N_1\sin\omega t+N_2\cos\omega t+\ldots, \qquad (18)$$

where $N_0, N_1, N_2,\ldots$, are the Fourier coefficients. Since we have assumed that the oscillation is close to a sinusoidal function, then the higher harmonics terms in Eq. (18) may be considered small and neglected. Hence, keeping only the constant and first harmonic terms, and using also Eqs. (15) and (16), we obtain

$$P_g\approx N_0-K'\tilde{\xi}+B'\frac{d\tilde{\xi}}{dt}, \qquad (19)$$

where $K'=N_1/A$ is the effective aerodynamic stiffness and $B'=N_2/(\omega A)$ is the effective aerodynamic damping (the signs are chosen to obtain positive values for K' and B', equivalent to Titze's notation). Introducing Eqs. (15) and (19) into the equation of motion (14), this becomes

$$M\frac{d^2\tilde{\xi}}{dt^2}+(B-B')\frac{d\tilde{\xi}}{dt}+(K+K')\tilde{\xi}+K\bar{\xi}=N_0. \qquad (20)$$

The coefficients N_0, K', and B' are calculated through the relations

$$N_0=\frac{1}{2\pi}\int_0^{2\pi}P_g(\theta)d\theta, \qquad (21)$$

$$K'=-\frac{1}{\pi A}\int_0^{2\pi}P_g(\theta)\sin\theta\, d\theta, \qquad (22)$$

$$B'=\frac{1}{\pi\omega A}\int_0^{2\pi}P_g(\theta)\cos\theta\, d\theta, \qquad (23)$$

where $\theta=\omega t$. Since the equation of motion was derived assuming a small value for $\omega\tau$, the last term in the denominator of Eq. (17) may be neglected, which facilitates the evaluation of the above integrals.

Considering first N_0, we have

$$N_0=\frac{P_L}{2\pi k_t}\int_0^{2\pi}\frac{\xi_{01}-\xi_{02}+2\omega\tau A\cos\theta}{\xi_{01}+\bar{\xi}+A\sin\theta}\,d\theta. \qquad (24)$$

The limits of integration may be changed using the relations

$$\int_0^{2\pi}f(\sin\theta,\cos\theta)d\theta=\int_0^{\pi}[f(\sin\theta,\cos\theta)+f(-\sin\theta,\cos\theta)]d\theta, \qquad (25)$$

$$\int_0^{\pi}f(\sin\theta,\cos\theta)d\theta=\int_0^{\pi/2}[f(\sin\theta,\cos\theta)+f(\sin\theta,-\cos\theta)]d\theta, \qquad (26)$$

with the result

$$N_0=\frac{2P_L(\xi_{01}-\xi_{02})}{\pi k_t(\xi_{01}+\bar{\xi})}\int_0^{\pi/2}\frac{d\theta}{1-a^2\sin^2\theta}, \qquad (27)$$

where

$$a=A/(\xi_{01}+\bar{\xi}) \qquad (28)$$

is the normalized oscillation amplitude. This integral may then be calculated introducing the change of variable $t=\tan\theta$, which yields

$$N_0=\frac{2P_L(\xi_{01}-\xi_{02})}{\pi k_t(\xi_{01}+\bar{\xi})}\int_0^{\infty}\frac{dt}{1+(1-a^2)t^2}$$

$$=\frac{2P_L(\xi_{01}-\xi_{02})}{\pi k_t(\xi_{01}+\bar{\xi})}\left[\frac{\tan^{-1}(\sqrt{1-a^2}t)}{\sqrt{1-a^2}}\right]_0^{\infty}$$

$$=\frac{P_L(\xi_{01}-\xi_{02})}{k_t(\xi_{01}+\bar{\xi})}\left(\frac{1}{\sqrt{1-a^2}}\right). \qquad (29)$$

The integrals for K' and B' may be calculated through the same steps as above, with the final results

$$K'=\frac{P_L(\xi_{01}-\xi_{02})}{k_t(\xi_{01}+\bar{\xi})^2}\left(\frac{2}{1-a^2+\sqrt{1-a^2}}\right), \qquad (30)$$

$$B'=\frac{2\tau P_L}{k_t(\xi_{01}+\bar{\xi})}\left(\frac{2}{1+\sqrt{1-a^2}}\right). \qquad (31)$$

Finally, the static displacement $\bar{\xi}$ is obtained setting to zero $\tilde{\xi}$ and its derivatives in Eq. (20), obtaining

$$K\bar{\xi}=\frac{P_L(\xi_{01}-\xi_{02})}{k_t(\xi_{01}+\bar{\xi})}\left(\frac{1}{\sqrt{1-a^2}}\right). \qquad (32)$$

Equations (30)–(32) are similar to those obtained by Titze for K', B', and $\bar{\xi}$. In the case of Eqs. (30) and (32), the only difference is the introduction of the last factors containing the normalized amplitude a, which expresses the dependence of K' and $\bar{\xi}$ on the oscillation amplitude. Equation (31), besides the presence of the last factor in a, is slightly different from Titze's equation for B', as a consequence of the neglect of the last term in the denominator of Eq. (17).

Note in Eq. (20) that the aerodynamic damping B' subtracts from the tissue damping B, indicating a transfer of energy from the airflow to the vocal folds. The oscillation is generated when the energy transferred to the vocal folds overcomes the energy dissipated in the tissues. The aerodynamic damping (and the energy transferred to the vocal folds) has its minimum at an oscillation amplitude $A=0$, i.e., at the start of the oscillation, and increases with A. This nonlinear characteristic of the aerodynamic damping is responsible for the lower value of the minimum pressure to sustain the oscillation, as explained in the next section.

C. Minimum pressure to sustain the oscillation

From Eq. (20), we have the following conditions for a sustained oscillation of the vocal folds. First, the static displacement $\bar{\xi}$, given by the solution of Eq. (32), must exist; otherwise, the assumed solution of the equation of motion, Eq. (15), would not be valid. Next, the roots of the charac-

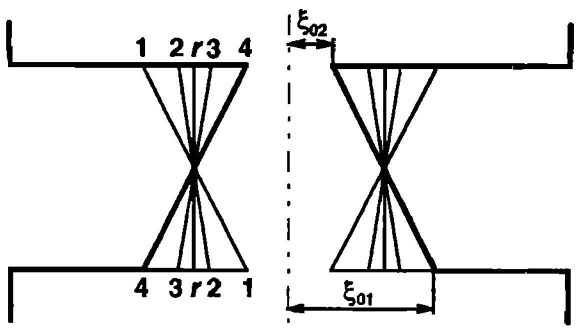

FIG. 3. Prephonatory glottal configurations. 1: $\xi_{01}=0.04$ cm, $\xi_{02}=0.16$ cm. 2: $\xi_{01}=0.08$ cm, $\xi_{02}=0.12$ cm. r: $\xi_{01}=\xi_{02}=0.1$ cm. 3: $\xi_{01}=0.12$ cm, $\xi_{02}=0.08$ cm. 4: $\xi_{01}=0.16$ cm, $\xi_{02}=0.04$ cm.

teristic equation of Eq. (20) must be purely imaginary, which implies the conditions $B-B'=0$ and $K+K'>0$. Replacing τ in Eq. (31) by $T/(2c)$, and solving for the lung pressure, we obtain

$$P_L=\frac{k_t Bc(\xi_{01}+\bar{\xi})}{T}\left(\frac{1+\sqrt{1-a^2}}{2}\right), \tag{33}$$

which is the lung pressure necessary to sustain the oscillation at a given normalized amplitude a. This equation contains the static displacement $\bar{\xi}$, which according to Eq. (32) is also a function of the lung pressure P_L. Combining Eqs. (32) and (33) to eliminate P_L, and solving for $\bar{\xi}$, we obtain

$$\bar{\xi}=\frac{Bc(\xi_{01}-\xi_{02})}{2KT}\left(\frac{1+\sqrt{1-a^2}}{\sqrt{1-a^2}}\right), \tag{34}$$

which is the static displacement at a sustained oscillation of normalized amplitude a (i.e., at a lung pressure to sustain the oscillation of amplitude a). Similarly, combining Eqs. (30) and (33), we obtain the effective aerodynamic stiffness at the same oscillation amplitude

$$K'=\frac{Bc(\xi_{01}-\xi_{02})}{T(\xi_{01}+\bar{\xi})}\left(\frac{1}{\sqrt{1-a^2}}\right). \tag{35}$$

At the start of the oscillation, $a=0$. In this case, the value of P_L given by Eq. (33) is the threshold pressure

$$P_L=\frac{k_t Bc(\xi_{01}+\bar{\xi})}{T}. \tag{36}$$

The static displacement at the threshold pressure reduces to

$$\bar{\xi}=\frac{Bc(\xi_{01}-\xi_{02})}{KT}. \tag{37}$$

To examine the behavior of P_L for oscillation amplitudes $a>0$, we will consider the same four prephonatory glottal configurations studied by Titze (1988), shown in Fig. 3, plus the rectangular configuration. The glottal convergence angle is increased from configuration 1 to 4 keeping the mean glottal half-width constant at 0.1 cm, and varying ξ_{01} and ξ_{02} from 0.04 to 0.16 cm in opposite directions. Configurations 1 and 2 are divergent, and 3 and 4 are convergent. The rectangular configuration is denoted as configuration r,

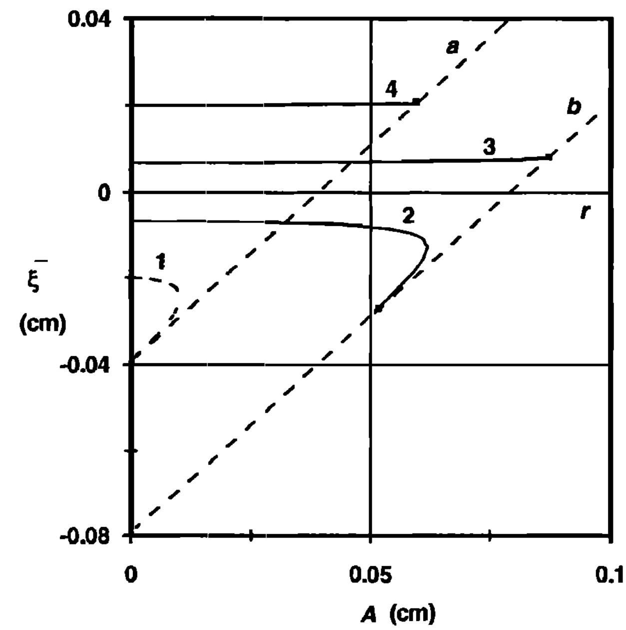

FIG. 4. Static displacement $\bar{\xi}$ vs the oscillation amplitude A. Lines a and b: glottal closure for configurations 1 and 4, and 2 and 3, respectively. Curves in broken lines: $K+K'\leq 0$.

with $\xi_{01}=\xi_{02}=0.1$ cm. The other parameters are given the values $T=0.3$ cm, $Bc=1$ kPa, $K=200$ kdyn/cm^3, and $k_t=1.2$.

Figure 4 shows the static displacement $\bar{\xi}$ versus the oscillation amplitude A, obtained from Eqs. (28) and (34). Line a has the equation $\bar{\xi}=A-0.04$, which is the glottal closure condition for configurations 1 ($\bar{\xi}=A-\xi_{01}$) and 4 ($\bar{\xi}=A-\xi_{02}$), while line b has the equation $\bar{\xi}=A-0.08$, which is the glottal closure condition for configurations 2 and 3. In the divergent configurations 1 and 2, $\bar{\xi}$ tends asymptotically to these lines. On the curves in broken lines, i.e., the entire curve for configuration 1 and part of the curve for configuration 2, the total stiffness is $K+K'\leq 0$, and hence the condition of positive total stiffness for the existence of the oscillatory solution is not satisfied. Note that configuration 1 does not satisfy this condition at any amplitude. Also, note that in the divergent configuration 2 there is a region with two possible values of the static displacement for a given oscillation amplitude, and that there is a maximum value for the oscillation amplitude. The implications of these features for the divergent configurations are beyond the scope of this analysis, and hence are left for future studies.

Figure 5 shows the lung pressure P_L to sustain the oscillation at an amplitude A, obtained from Eqs. (28) and (33), and using the values of the static displacement calculated previously. Only the curves which satisfy the conditions for the oscillatory solution are shown; configuration 1 has been left out altogether and the curve for configuration 2 stops at the zero total stiffness condition. Curves for configurations 3, 4, and r stop at the glottal closure. Note that in all the configurations the values of the lung pressure decrease with the oscillation amplitude after the oscillation starts. In the cases of the divergent configuration 2, the curve shows that the oscillation amplitude first grows until a maximum value, af-

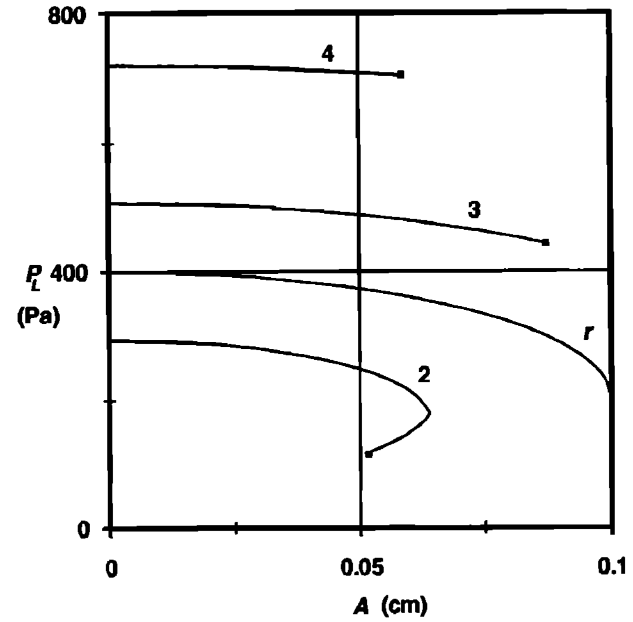

FIG. 5. Lung pressure P_L to sustain the oscillation at an amplitude A.

ter which it decreases as the static displacement becomes closer to the glottal closure (compare with Fig. 4). However, along all this variation of the oscillation amplitude the minimum sustaining pressure decreases continuously. The pressure values increase with the prephonatory convergence angle, i.e., more pressure is required to sustain oscillation in a convergent glottis than in a divergent one. This follows from Eqs. (33) and (34): Note that P_L is proportional to $\xi_{01}+\bar{\xi}$, and also that $\bar{\xi}$ is positive in a convergent glottis and negative in a divergent one.

The relation between the lung pressure and the oscillation amplitude correspond then to the case shown schematically in Fig. 2(b). The curves in Fig. 5 do not show the increase of the lung pressure at large oscillation amplitudes, since they stop at the limits of validity of the model (i.e., the glottal closure or the zero stiffness condition). The minimum pressure to sustain the oscillation is then the minimum value of P_L. In Fig. 5 this minimum occurs at the end of the curves; the model does not permit one to tell then whether this is the actual minimum, since this could be located beyond these limits of validity (as discussed previously, a minimum must exist; otherwise the oscillation amplitude would grow unbounded). However, we may calculate this minimum value as reference. In the case of the rectangular configuration, the minimum occurs at $a=1$; from Eq. (33) we obtain for this value

$$P_L=\frac{k_t Bc(\xi_{01}+\bar{\xi})}{2T}. \tag{38}$$

Comparing with Eq. (36), we have that the ratio between the threshold pressure and the minimum sustaining pressure is 1/2. In the case of Fig. 5, the threshold and minimum sustaining values are 400 and 200 Pa, respectively. We can compare these values with those obtained by Titze *et al.* (1994). For a prephonatory glottal half-width of 0.1 cm, they measured values of 370–590 and 330–510 Pa for the threshold and minimum sustaining pressures, respectively, with a ratio between both pressures around 0.87. The analytical value for the minimum sustaining pressure is lower than the experimental values, but they are in the same order. Considering the simplifying assumptions introduced in the analysis, we may say that the results agree with the experimental measurements. In the cases of configurations 2–4 the threshold pressures are 293, 507, and 720 Pa, respectively, and the minimum sustaining pressures are 117, 443, and 703 Pa, respectively. The ratios between both pressures are 0.40, 0.87, and 0.98, for the respective configurations.

III. CONCLUSION

The observed lower value of the minimum pressure to sustain the vocal fold oscillation after its onset, compared with the threshold value, has been analytically examined using a body-cover model of the vocal folds. It has been shown that this phenomenon is a consequence of the nonlinearity of the effective aerodynamic damping introduced by the glottal pressure on the vocal folds. The results are in agreement with previous experimental measurements, which validates the analysis.

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