4.1 Background

Probability of intercept (POI) is a key performance feature of EW surveillance and reconnaissance systems; it relates to the probability of time coincidence of two or more parametric “window functions,” such as scanning antennas, sweeping or stepping receivers, and frequency agile emitters.

Many publications and examples that examine POI and its associated intercept time statistics invariably quote examples involving crystal video (wide open) receivers (CVRs), scanning superheterodyne (SSH) receivers, and the implications of scanning, directional antennae. However, over the last 20 years, significant developments in technology have occurred as have several armed conflicts. These have dictated radical changes in requirements, force modus operandi, and other issues.

In previous chapters, the impact of receiver developments, the impact of more recent conflicts (such as the Gulf War), and the emergence of a number of new threats such as the low probability of intercept (LPI) emitter have been examined. Clearly, there is a significant POI issue associated with such emitters in the presence of the littoral electromagnetic environment. Finally, as shown in the USS Cole incident, the new threat (now referred to as the “asymmetric threat”) from fast attack craft (FAC), for example, is significant and must be accounted for in tasks, combat strategies, weapons, sensors, and so on, for fast reaction and adequate firepower.

So today’s EW systems have to deal with increasing complexity and density of the electromagnetic environment; comprising multimode emitters, high and low ERPs, increased signal agilities (RF, PRI, and so forth), complex scans, and fleeting emissions.

EW system architectures are now more complex and, with more capable weapon systems (hard kill and soft kill), result in a more sophisticated modus operandi involving integration not only within their own platform or service but with other platforms and force structures. EW receiving systems have become EW suites which provide situational awareness, protection, and ELINT (fine grain analysis) capabilities. Note that some of the latest ESM systems are offering a specific emitter identification (SEI) capability—typically through amplitude and/or phase digitization on a per-pulse basis. Given the complex mission environments, and thus the required EW system architectures, ESM is now a likely, and necessary, precursor to ELINT.
In summary, there is now far less distinction between ESM and ELINT. Emphasis is turning to EW systems designs with both a tactical (ESM) and strategic (ELINT) capability. Systems designers need new tools to assist them, now that POI is now a more complex problem.

4.2 Developments in the Theory Behind POI

4.2.1 Intercept Description

Table 4.1 summarizes a range of typical kinds of intercept [1]; these can be subdivided into spatial, frequency, and time domains.

- **Spatial domain:** The beam-on-beam example where both transmitting (Tx) and receiving (Rx) systems use rotating, directional antennas and where the type of intercept is related to detectability (main beam or a percent of sidelobes).
- **Frequency domain:** Typically a scanning or sweeping receiver (e.g., a SSH, against a frequency agile transmitter).
- **Time domain:** Here, a number of distinct types of intercept are relevant. For example, the concept of a minimum number of pulses (or intercept duration) is important since it is required in ESM systems in order to differentiate between real signals and noise, as well as for subsequent signal sorting processes. Emission control policies will dictate when a transmitter is on and when it is not—thus introducing the scenario element to the intercept time arena. The time for an ESM system to process initially detected pulses will, in certain processing architectures, dictate whether or not subsequent pulses from the same emitter are tracked continuously or lost due to insufficient processing bandwidth/capability.
- **Others:** Propagation fluctuation aspects can also impact the time to intercept. These can be subdivided into two principal effects, namely, coherent (specular) scattering and tropospheric scattering. Coherent scattering is important for within-the-radio-horizon scenarios and low elevation threats (particularly sea-skimming missiles or high altitude platforms near the radio horizon). Tropospheric scattering relates to scattering of electromagnetic waves off the Earth’s troposphere as witnessed by rapidly varying signal amplitude and fading. The intercept probability of the ESM system will be determined by the mean period of simultaneous overlap between the random pulse train representing such propagation fluctuations and the regular pulse trains representing the cyclic properties of the emitter and ESM systems.

4.2.2 Implications of Today’s Environments/Operations on Intercept Time

The following are the types of intercepts shown in Table 4.1:

- In the spatial domain, phased array technology allows scanning space in a pseudo-random manner and such radar systems often perform multiple
Table 4.1 Types of Intercepts

<table>
<thead>
<tr>
<th>Spatial Domain</th>
<th>Receiver Activity</th>
<th>Transmitter Activity</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Scanning antenna</td>
<td>Scanning antenna</td>
<td>Beam-on-beam intercept</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Main beam to main beam</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Main beam to sidelobes (variable)</td>
</tr>
<tr>
<td>Frequency Domain</td>
<td>Stepping or scanning</td>
<td>Fixed RF, random jumping, or some other varying transmitted RF</td>
<td>Narrow instantaneous bandwidth (IBW) receiver which is stepping or sliding across a defined RF bandwidth looking for either a fixed or agile threat emitter RF. One window function if fixed RF else two.</td>
</tr>
<tr>
<td>Time Domain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Minimum intercept duration</td>
<td>Main beam</td>
<td>MB or SL intercept</td>
<td>Given a spatial intercept has occurred, how long is the coincidence; valid intercept if the number of received pulses is ≥ threshold</td>
</tr>
<tr>
<td>2. Emitter EMCON</td>
<td>Main beam</td>
<td>MB or SL intercept</td>
<td>For ECCM purposes, the emitter switches on for only short periods of time</td>
</tr>
<tr>
<td>3. Missing pulses</td>
<td></td>
<td></td>
<td>Environmental and emitter transmit effects could give rise to missing pulses in the received pulse train</td>
</tr>
<tr>
<td>4. ESM processing time</td>
<td>Receive and processing architecture</td>
<td>For nonparallel architectures and/or nonmultiple buffering data, the ESM could be blind to new data while it is processing the last received data</td>
<td></td>
</tr>
</tbody>
</table>

Others

| Tropospheric scattering effects | Scanning | Scanning | Rayleigh distributed amplitude versus time Window function description: $t =$ time above detection threshold; $(T - t) =$ time below detection threshold (nulls) |

Source: [1].

functions. The electronic scanning antenna allows eliminating mechanical inertia as a design limitation; however, radar functions performed often impose a structure on the motion of the beam that can be exploited by would-be interceptors. Radar systems using mechanically scanned antennae
have provided the scan characteristics of the intercept time problem. A change of mode (e.g., surveillance to tracking) or the switching on of a specific tracking radar afforded the EW designer further useful intelligence. Multifunction phased array radars reduce such possibilities, thus making intercept much more difficult.

- The time domain, including such operational aspects as emission control (EMCON), further increases the time to intercept. Operations in the littoral (i.e., close to shore) raise additional multipath and possibly missing pulse issues.

- In the frequency domain, the littoral electromagnetic environment makes signal search and identification that much more difficult in comparison to deep ocean environments. Clearly, more agility in the Tx signal parameters (be it RF or PRI) again directly impacts time to signal intercept. Additionally, low power frequency modulated continuous wave (FMCW) signals force the EW designer to long integration times, and thus, directly impacting time to intercept.

In summary, EW system designers are being pushed more and more towards architectures that deliver single pulse or pulse burst intercept capabilities.

4.2.3 Mathematical Models

The methodology of Self and Smith [2, 3] has been widely used due to its simplicity and ease of use, its applicability to scenarios involving more than two window functions, and its ability to predict intercept time accurately in a wide range of situations.

This section includes both the key formulae as well as several enhancements to the original work, including some specific, realistic examples with practical application. This is followed by a number of new and exciting theoretical developments involving some elegant mathematical theories on the subject of synchronism.

From [3], a convenient way of investigating the intercept problem is to represent the activities of the receiving and transmitting systems by window functions, as shown schematically in Figure 4.1. A key assumption is that the window functions are independent (or of “random-phase”). Such ‘window functions’ can also be described or represented as pulse trains—in this chapter, these descriptions are synonymous and interchangeable.

Key formulae are as summarized next.

For a number of coincidences, $M$, required, the equation for the mean period between coincidences, $T_0$, has the following explicit form:

$$T_0 = \prod_{j=1}^{M} \left( \frac{T_j}{\tau_j} \right)$$

and for $M = 1, 2, \text{and } 3$, this becomes:
4.2 Developments in the Theory Behind POI

**Figure 4.1** Basic window functions and their coincidences.

\[ M = 1 \quad T_0 = T_1 \]  
\[ M = 2 \quad T_0 = \frac{T_1 T_2}{(\tau_1 + \tau_2)} \]  
\[ M = 3 \quad T_0 = \frac{T_1 T_2 T_3}{(\tau_1 \tau_2 + \tau_1 \tau_3 + \tau_2 \tau_3)} \]

where \( T_1, T_2, T_3 \) are the periods of the individual window functions and \( \tau_1, \tau_2, \tau_3 \) are the durations of the individual window openings.

The probability of at least one intercept in time \( T \) is

\[ P(T) = 1 - Ke^{-(T/T_0)} \]  

where \( K = 1 - P(0) \), and \( P(0) \), the probability of an intercept occurring in the first instant, is given by

\[ P(0) = \prod_{j=1}^{M} \frac{\tau_j}{T_j} \]  

The mean duration \( \tau_0 \) of the simultaneous overlaps of all the \( M \) window functions is given by

\[ \frac{1}{\tau_0} = \sum_{j=1}^{M} \frac{1}{\tau_j} \]  

which, for \( M = 1, 2 \) and 3, becomes
\[ M = 1 \quad \tau_0 = \tau_1 \]  
\[ M = 2 \quad \tau_0 = \frac{1}{\tau_1 + \frac{1}{\tau_2}} \]  
\[ M = 3 \quad \tau_0 = \frac{1}{\tau_1 + \frac{1}{\tau_2} + \frac{1}{\tau_3}} \]

Turning to the issue of intercept \textit{duration}, Figure 4.2 shows a comparison [3] of computer simulation results for examples of three-window intercept scenarios. Results are expressed as cumulative probability versus intercept duration. Analytic forms for these results show the following (in each case \( \tau_1 \) denotes the smallest window and \( T \) denotes time into the actual intercept) (private communication with N. R. Burke, April 1983).

For the case of two windows:

\[ \tau_0 = \frac{1}{\tau_1 + \frac{1}{\tau_2}} \]

\textbf{Figure 4.2} Intercept duration.
4.2 Developments in the Theory Behind POI

\[ P(T) = \frac{2T}{\tau_1 + \tau_2} \quad 0 \leq T < \tau_1 \]
\[ = 1 \quad T \geq \tau_1 \]

(4.11)

For the case of three windows:

\[ P(T) = \frac{2T(\tau_1 + \tau_2 + \tau_3) - 3T^2}{(\tau_1 \tau_2 + \tau_2 \tau_3 + \tau_3 \tau_1)} \quad 0 \leq T < \tau_1 \]
\[ = 1 \quad T \geq \tau_1 \]

(4.12)

A number of developments of the theory of window functions with random phase have been made, and these are summarized here.

**Interception of at least \( m \) pulses** Declaration of an adequate intercept may require at least \( m \) pulses. Thus, if the PRI of the received signal is \( T_2 \), then

\[ d = mT_2 \]

(4.13)

\[ T_0 = \frac{\prod_{j=1}^{M} T_j}{\sum_{j=1}^{M} (\tau_j - d)} \quad \text{where } \tau_j > mT_2 \]

Using (4.3), Table 4.2 demonstrates the effect on \( T_0 \) of increasing the minimum intercept duration \( d \) from 0 to 25 ms for the cases of \( T_1 = 0.7 \) and 1.3 seconds (two pulse trains are \( \tau_1 = 0.058, \tau_2 = 0.078, T_1, T_2 = 6.0 \) seconds). If a minimum of 10 pulses is required for a valid signal detection, then a minimum intercept duration of 20 ms is required for a 600-Hz PRF radar (less than 5 ms for a 4-kHz radar), which, in turn, implies a mean time between intercepts of at least 40 seconds for a receiver scan period of 0.7 second; this is in comparison to a mean time between intercepts of only 31 seconds for the purely theoretical example where receiver integration aspects are not included.

**Effect of Auto Pause Mechanism** For a sweeping receiver, such a mechanism compromises intercept time. This can be deduced as an extension of the theory. If

<table>
<thead>
<tr>
<th>( d ) (ms)</th>
<th>( T_1 = 0.7 )</th>
<th>( T_1 = 1.3 )</th>
<th>( \text{Number of Pulses} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>31</td>
<td>57</td>
<td>PRF = 600 Hz</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
<td>62</td>
<td>PRF = 600 Hz</td>
</tr>
<tr>
<td>10</td>
<td>36</td>
<td>67</td>
<td>PRF = 4 kHz</td>
</tr>
<tr>
<td>15</td>
<td>40</td>
<td>74</td>
<td>PRF = 4 kHz</td>
</tr>
<tr>
<td>20</td>
<td>44</td>
<td>81</td>
<td>PRF = 4 kHz</td>
</tr>
<tr>
<td>25</td>
<td>49</td>
<td>91</td>
<td>PRF = 4 kHz</td>
</tr>
</tbody>
</table>

Source: [3].
the receiver interrupts its frequency sweep for a known amount of time every time a signal is encountered, then the effective sweep period of the receiver will be extended proportionately and thus the mean period between coincidences will depend both on the auto pause time and the average number of signals transmitting at any one time.

Propagation Fluctuations  The effects of propagation can be added in a simple manner to this theory [1]. Propagation fluctuations are assumed to be able to be subdivided into two types:

1. Long-term fading effects (affects the way in which average power received varies and hence will show variations in terms of hourly, daily changes);
2. Short-term fading effects (affects POI more directly).

Intercept of the radar signal from a distant transmitter will be possible when the strength of the fading signal rises above the threshold level of the receiver. The occurrence (in time) of bursts of the signal above the threshold may be represented by a random rectangular pulse train. The mean period of the pulse train $<T>$ is the reciprocal of the mean rate of upward crossings of the threshold level by the fluctuating signal amplitude. The mean duration of the pulse train $<\tau>$ is the mean duration of the signal amplitude above the threshold level.

The intercept probability will be determined by the overlaps between the random pulse train representing propagation fluctuations and the regular pulse trains representing the cyclic properties of the transmit and receive systems:

$$<T> = \left( \frac{1}{\pi a} \right)^{1/2} \left( \frac{1}{2f} \right) \cdot e^a \quad (4.14)$$

$$<\tau> = \left( \frac{1}{\pi a} \right)^{1/2} \left( \frac{1}{2f} \right) \quad (4.15)$$

where:

$a = L/b_0$ is a dimensionless threshold parameter

$b_0$ = mean power level of the fading signal

$f$ = rms bandwidth of amplitude fluctuations

$L$ = threshold level

For example, if $f = 1$ Hz at S-band and $a = 1$, then substituting into (4.14) and (4.15) gives $<T> = 0.77$ second and $<\tau> = 0.28$ second.

A Practical Example of Three Pulse Trains
Looking at a more realistic radar type example, let us derive the intercept time statistics [1]. Table 4.3 shows the assumed pulse train values (fading is neglected).
The receiver’s sweep is chosen to have an instantaneous bandwidth of 1%, corresponding to 20 MHz in a total sweep range of 2 GHz. The mean time between intercepts, $T_0$, is calculated as a function of the sweep period, $T_3$, which varies over a wide range, say, $10^{-4}$ seconds to $10^4$ seconds.

The correct expression for $T_0$ is to be chosen for any particular value of $T_3$, from among the equations that represent the mean period of overlaps of different combinations of the three overlapping pulse trains:

$$T_0 = T_j \text{ where } j = 1, 2, 3 \quad (4.16)$$

$$T_0 = \frac{T_j T_k}{(\tau_j + \tau_k)} \text{ where } j = 1, 2, 3 \text{ and } k \neq j \quad (4.17)$$

$$T_0 = \frac{T_1 T_2 T_3}{(\tau_1 \tau_2 + \tau_2 \tau_3 + \tau_1 \tau_3)} \quad (4.18)$$

In general, the above formula for mean time between intercepts does not discriminate against the possibility of multiple overlaps, and thus, it may, in certain circumstances, underestimate the mean time. Smith analyzes such a possibility and concludes that the correct formula is given by the one which has the greater numerical value, based on there being $2^M - 1$ formulae in the case of $M$ overlapping pulse trains (in the above case this gives seven distinct formulae) (private communication with B. G. Smith, 1983).

The curve of $T_0$ versus $T_3$ divides into four regions as shown in Figure 4.3.

- **Region #1**: Very large values of $T_3$. The duration of the sweep is much larger than the scan period; the pulse modulation period of the mean period of overlaps between these two trains. Therefore, for large $T_3$, the mean time between intercepts equals the sweep period [(4.16) with $j = 3$].

- **Region #2**: As $T_3$ decreases, the duration of the frequency sweep falls until it is less than the period of the scan cycle. Interception in a single sweep is no longer certain. The mean duration of the overlap between the scan and sweep cycles is long compared with the period of the pulse modulation, and so the mean time between intercepts is determined by the scan and sweep parameters in (4.17) with $j = 3$ and $k = 1$.

- **Region #3**: As $T_3$ falls further, the mean overlap between the scan and sweep cycles falls below the period of the pulse modulation and the mean period between intercepts is (4.18) (dependent on parameters of the three interacting pulse trains).
Region #4: Eventually, $T_3$ falls sufficiently that the mean period of overlaps of the sweep and pulse modulation cycles falls below the duration of the scan cycle. Interception within a single scan is certain, and the mean time between intercepts equals (4.16) with $j = 1$.

Specific Example of a Fast Step-Scan Receiver

From [2], consider the following example: a broad-beam receiving antenna so that it is not necessary to search in azimuth, but add the use of a fast step-scan receiver which searches a 2,000-MHz frequency band in 200 steps, each step being 10 MHz. (The receiver bandwidth should be about 20 MHz to provide overlapping coverage from one step to the next.) See Table 4.4.

Hence, from (4.3), the mean time between intercepts is now 17.74 seconds, and from (4.9) the mean duration of the intercepts is $4.93 \times 10^{-4}$ seconds, and the

<table>
<thead>
<tr>
<th>Case 1: The Result Using a Receiver</th>
<th>Scan Time of 100 ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>$\tau_2$</td>
</tr>
<tr>
<td>0.0005 sec</td>
<td>0.0333 sec</td>
</tr>
</tbody>
</table>
average coincidence fraction is $2.78 \times 10^{-5}$. This leads to a time for 90% probability of intercept of 40.85 seconds. See Table 4.5.

Now the mean time between intercepts is increased to 277.13 seconds, their mean duration to 7.69 ms, and the average coincidence fraction remains at $2.78 \times 10^{-5}$. The time required for a 90% probability of intercept is increased to 638.12 seconds or 10.64 minutes. Clearly, increasing the receiver sweep time results in a much longer time to intercept the signal, with an attendant increase in the expected duration of the intercept.

Next, consider three pulse trains by adding a rotating antenna. The receiver is sweeping in frequency as well as searching in angle, and the transmitter is also circularly scanning. See Table 4.6.

From (4.4), the mean time between intercepts is increased drastically to 5,423 seconds (about 1.5 hours), and from (4.10) the average duration of an intercept is $1.266 \times 10^{-6}$ seconds and to reach a probability of intercept of 90% requires 3.47 hours. See Table 4.7.

This increases the mean time between intercepts to 7.35 hours with an average intercept duration of $2.02 \times 10^{-5}$ seconds and a 90% probability of intercept time of 16.92 hours.

Table 4.8 summarizes the calculations for $T_0$ in the above examples from [3], and shows their relationship to POI over the interval 30% to 90%.

Cases 3 and 4 illustrate why it is usually necessary to eliminate one or more of the pulse trains. A highly sensitive receiver (or strong signal level at the receiver) allows us to drop the transmitter antenna scan pulse train from the analysis by making it possible to receive the signal of interest through the emitter antenna’s side or back lobes. The best sensitivity is achieved economically with a narrowband swept receiver and a narrowbeam receiving antenna. Thus, by introducing two search processes at the receiver, it may be possible to eliminate the transmitter’s scan from the probability of intercept calculation.

A limitation of this approach is that it does not address detailed impacts of synchronism effects between pulse trains. The above examples clearly identify such

**Table 4.5** Case 2: The Same Situation as for Case 1, with the Receiver Sweep Slowed to 2 Seconds

<table>
<thead>
<tr>
<th>$\tau_1$</th>
<th>$\tau_2$</th>
<th>$T_1$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01 sec</td>
<td>0.0333 sec</td>
<td>2 sec</td>
<td>6 sec</td>
</tr>
</tbody>
</table>

**Table 4.6** Case 3: The Result of the Faster Receiver Sweep

<table>
<thead>
<tr>
<th>$\tau_1$</th>
<th>$\tau_2$</th>
<th>$\tau_3$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00278 sec</td>
<td>0.0333 sec</td>
<td>0.0005 sec</td>
<td>1.0 sec</td>
<td>6.0 sec</td>
<td>0.1 sec</td>
</tr>
</tbody>
</table>

**Table 4.7** Case 4: The Result Using the Slower Receiver Sweep

<table>
<thead>
<tr>
<th>$\tau_1$</th>
<th>$\tau_2$</th>
<th>$\tau_3$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00278 sec</td>
<td>0.0333 sec</td>
<td>0.01 sec</td>
<td>1.0 sec</td>
<td>6.0 sec</td>
<td>2.0 sec</td>
</tr>
</tbody>
</table>
Table 4.8 Predicted Results for POI and $T_0$ for Cases 1 to 4

<table>
<thead>
<tr>
<th>Case</th>
<th>Parameters (sec)</th>
<th>Mean Period of Simultaneous Overlaps, $T_0$ (sec)</th>
<th>Probability of Intercept P(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>$\tau_1 = 0.005$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_2 = 0.0333$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\tau_1 = 0.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_2 = 0.0333$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\tau_1 = 0.00278$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_2 = 0.0333$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_3 = 0.0005$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$\tau_1 = 0.00278$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_2 = 0.0333$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau_3 = 0.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_1 = 0.1$</td>
<td>18</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>$T_2 = 6.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_1 = 2.0$</td>
<td>277</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>$T_2 = 6.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_1 = 1.0$</td>
<td>5,423</td>
<td>1,934</td>
</tr>
<tr>
<td></td>
<td>$T_2 = 6.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_3 = 0.1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_1 = 1.0$</td>
<td>26,450</td>
<td>9,434</td>
</tr>
<tr>
<td></td>
<td>$T_2 = 6.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_3 = 2.0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: [3].

relationships between the values of $T_{[1,2,3]}$; thus, for a two pulse train example, if the period of one pulse train is harmonically related to the period of the other, then the two pulse trains will either be in synchronism or they will not. If the latter, then an intercept will not occur (note that $\tau_i \leq T_i$ in the radar case); if the former, then there will be synchronism and intercepts will occur at known, discrete times that can be deduced. Note that for regular pulse trains, there will always be a fixed relationship between the pulse trains once the first intercept has occurred. In such instances, an ESM system operating with a directional antenna may improve its intercept probability if its rotation rate is variable.

### 4.2.4 Recent Developments on POI

More recent work on intercept time and POI has focused on the underlying theory of pulse train coincidence where the pulse train phases are nonrandom. In other words, earlier references [1–3, 5], and private communications with N. R. Burke (April 1983), A. G. Self (1986), and B. G. Self (1983), examined noncorrelated pulse trains (herein now referred to as the random-phase model). However, there are instances (for example, with a stepping/scanning receiver or scanning transmit and receive antennae) where the resultant intercept pulse train becomes highly correlated and dependent on the input pulse train attributes. In this section, the latest results on synchronism effects [6] are summarized as well as the elegant application of number theory [4, 7] to the intercept problem domain.

#### 4.2.4.1 Synchronization Effects on POI

Historically, mechanical scanning transmit systems (especially long range surveillance systems) had rotation rates of typically 3, 6, 9, 10 (and beyond) rpm. Clearly, a receiving system with the same or sub/multiples of such rotation rates (such as 4:3 or 7:5) encounters synchronism issues, expressed as “the ratio of the periods.” In the worst case, coincidence (intercept) may never occur, or occur in such long time periods that fast interception requirements could not be met. Reference [6]
examines the problem of pulse synchronization with pairs of pulse trains, and
derives a theoretical approach supported by Monte Carlo simulations.

The theory [6] is constrained to the intercept probability of two pulse trains
with specific reference to a swept SSH Rx and a pulsed emitter. The approach is
based on the concept of pulse clusters overlapping in phase space (i.e., based on
the relative start times or phases between the various pulse trains). Based on this,
exact expressions are developed for the POI after a given number of sweeps,
averaged over all relative start phases of the transmitter’s pulse train and the
receiver’s pulse (or sweep) train.

Given the pulse train characteristics are \([T_1, \tau_1, T_2, \tau_2]\), the analysis [6] starts
by non-dimensionalizing this set by dividing by \(T_2\), which yields \((R, \tau_1/T_2, 1, D_2)\)
where \(R = T_1/T_2\) is the PRI ratio and \(D_2 = \tau_2/T_2\). When considering the POI, it
is only the distance from one pulse to the nearest pulse of the other train that is
of interest; the TOA of a pulse can then be considered as a dimensionless quantity
that can be viewed as being equivalent to wrapping or folding the TOA sequence
around a cylinder of circumference 1. The phase is chosen such that the initial
pulse from train 1 falls at zero; all pulses from train 2 will fall at the same point
given a fixed PRI (analysis considers this as a series of spikes) while pulses from
train 1 are distributed on the interval \([0, 1]\). After \(n\) pulses from train 1, the region
covered by pulses represents the set of positions of the train 2 spike, which leads
to interception within \(n\) pulses. The POI is the probability that the spike generated
by train 2 intersects a region covered by pulses from train 1. Regarding the folded
interval concept, then, this probability is equivalent to the ratio of the length
covered by pulses from train 1 to the length of the interval. As shown in [6], the
POI can be considered within four distinct regions:

\[
P_n = P_n^{(j)} \quad j = 1, 2, 3, 4
\]  

(4.19)

where the magnitude of \(j\) defines the region of interest dependent on the magnitude
of the sweep number \(n\) relative to certain critical values which may vary with the
PRI ratio and \(P_1 [T_1/T_0]\).

These four regions can be expressed as follows, where \(n_c = \text{number of clusters}:

\[
P_n^{(1)} = nP_1 \quad n \leq n_c
\]  

(4.20)

\[
P_n^{(2)} = P_n^{(1)} + (n - n_c)\Delta \quad n_c \leq n \leq n'\n\]  

(4.21)

\[
P_n^{(3)} = P_n^{(2)} + \frac{(n - n')}{n_c} (1 - P_n^{(2)}) \quad n' \leq n \leq n' + n_c
\]  

(4.22)

\[
P_n^{(4)} = 1 \quad n > n' + n_c
\]  

(4.23)

and \(\Delta\) represents an amount that each new pulse adds to the phase length of its
cluster; for an exact coincidence, \(\Delta = 1/2 - |n_c R \mod 1 - 1/2|\). From [6], Figure
4.4 shows the POI plotted for \(P_1 = 0.1\), with sweep numbers ranging from \(n = 1\)
to \(n = 12\) as a function of the PRI ratio \(R\). It shows a number of interesting features:

- A number of plateau regions of extended maxima—these are obtained when
  \(n \leq n_c\) and where (4.20) holds.
The extended maxima are separated by broad peaked minima which occur when \( n > n_c \) and (4.21) holds, or if \( n > n' \) and (4.22) is valid.

The width of each minima varies in proportion to \( P_1 \).

When \( n = 1 \), \( P_n \) is given by the random phase result; for \( n > 1 \), the random-phase theory generally underestimates the POI when it is in a plateau and overestimates the POI when it is approaching a minimum. As \( n \) increases, the density of the minima also increases.

Other minima of \( P_n(R) \) associated with \( P_m(R) \) for \( m < n \) are also present.

Figure 4.4 can be useful in selecting the optimal search parameters against a given transmitted PRI in order to maximize the POI.

However, this approach has limitations against unknown pulse train characteristics when using a uniform pulse train on the receiving system. Adding jitter to the search pulse train is suggested in [6] in order to smooth out regions of destructive synchronization (i.e., where no intercept will happen or not for very long periods). Adding a small amount of jitter in a cumulative fashion on each pulse is also suggested in [6]. Based on random walk theory, the position of each pulse then performs a random walk about a mean TOA position with a step distribution given by the distribution of jitter on each pulse. From [6], for a uniform step distribution ranging from \(-J/2\) to \(+J/2\), where \( J \) is defined as the jitter amplitude, the variance of the mean PRI of the pulse train is given by

\[
\sigma^2 = nJ^2/3 \quad (4.24)
\]

Since jitter is assumed cumulative, the variance on the TOA of the \( n \)th pulse increases linearly with \( n \), and so the pair of pulse trains are more likely to satisfy the requirements of the random phase assumption as the sweep number increases.
4.2 Developments in the Theory Behind POI

From [6] and using Monte Carlo simulations, Figure 4.5 shows POI as a function of the mean PRI ratio \( R \) and the jitter amplitude on \( T_1 \), measured in units of \( T_2 \), for \( n = 4 \) sweeps and \( P_1 = 0.1 \). Jitter was added to \( T \) so that the PRI varied over the range \( T_1 (1 - J/2) \) to \( T_1 (1 + J/2) \). Each point on the graph shows the mean POI in 4 sweeps. When the jitter amplitude is zero, it is seen that Figure 4.4 is derived; as the jitter amplitude increases, \( P_n(R) \) approaches the value given by the random-phase theory earlier. As [6] points out, quite small values of \( J \) can move the POI as derived from this cluster theory to that of the random-phase model as a function of \( P_1 \). Also, as expected, the required value for \( J \) decreases as the number of sweeps, \( n \), increases.

In summary, there now exists a more complete and accurate picture of POI for cases where the phases of pulse trains are nonrandom. As shown in [6], such occurrences give rise to sharp maxima and minima in POI space, and this can aid an EW system designer in optimizing search strategies. However, in real life, synchronization effects can work against the performance of receiving systems, and the authors propose introducing a jitter mechanism to smooth out such cases. This, in turn, is shown to link directly back to the random-phase case of earlier POI publications.

4.2.4.2 POI and Number Theory Application

In [4, 7] the mathematical background is developed even further for the two periodic pulse train cases and beyond. The applicability of number theory to a range of POI problems involving pulse trains is demonstrated in [7], where (1) phases are known and equal; (2) known and unequal; and (3) one or both are random variables. The problem becomes finding an algorithm for computing when the first intercept occurs and then when subsequent intercepts will occur. Key in this analysis is that the ratio of the PRIs (principal focus is for two pulse trains only) is not restricted to rational numbers. Through formulating the problem as a Diophantine approximation (i.e., based on only integer solutions), [7] demonstrates how number theory can directly assist in the intercept problem.

![Figure 4.5](image.png)

**Figure 4.5** Effect of adding jitter. POI as a function of PRI ratio and jitter amplitude for sweep number \( n = 4 \) on pulse train 1. *(From: [6]. © 1996 IEEE. Reprinted with permission.)*
While mathematically intensive, [7] contains an elegant theoretical basis for pulse train synchronism. Indeed, a recursive algorithm is outlined showing how POI is related to the Farey point notation, where the Farey series $F_N$ is the set of all fractions in lowest terms between 0 and 1 whose denominators do not exceed $N$, arranged in order of magnitude. For example, $F_6$ is given by [8]:

$$
\begin{align*}
0 & \frac{1}{6} \frac{1}{6} \frac{1}{5} \frac{1}{4} \frac{2}{5} \frac{1}{3} \\
\frac{1}{3} & \frac{2}{5} \frac{1}{2} \frac{3}{4} \frac{5}{6} \frac{5}{6}
\end{align*}
$$

The mean time to intercept for the discrete time case can be expressed as [7]:

$$
E(N) = \left\{ T_2 + q(\tau)A(\tau)^2 + a(\tau)Q(\tau)^2 + 2[a(\tau) + q(\tau) - \tau]A(\tau)Q(\tau) \right\} / 2T_2
$$

(4.25)

Using the above, plus a range of other theorems and definitions contained in detail in [4, 7], the POI can be determined over the interval

$$
T_1 \in [p'T_2/p, q'T_2/q]
$$

(4.26)

other parameters being held constant. Based on being given two adjacent Farey points and their left/right parents, Figure 4.6 shows the POI as a function of the number of pulses $N$ from the first pulse train, its PRI $T_1$ with $T_2 = 1$ and $\tau = 0.1$. The diagram shows the rise of POI from small values of $N$ to unity; also, at various PRIs, there are deep troughs which are centered around the Farey points (conversely, this is where the mean time to intercept approaches infinity). Figure 4.7 shows how Figure 4.6 can be interpreted as regions [7]. For any given $N$ and $T_1$, it shows

![Figure 4.6](image-url) Probability of intercept as a function of PRI ratio. (From: [7]. © 2004 Association of the Old Crows. Reprinted with permission.)
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Figure 4.7 POI regions as a function of PRI ratio. (From: [4]. © 1996 IEEE. Reprinted with permission.)

how the POI lies in the region of initial growth, single or double overlap or probability 1.

Taking a specific example [4], Figure 4.8 shows the intercept time between two pulse trains—the duty cycles for pulse trains 1 and 2 are 0.13/0.26, respectively. The PRI of train 1, \( T_1 \), is held constant with \( T_1 = 1 \) and \( T_2 \) allowed to vary between 0.1 and 4. Intercept time is shown as the solid line, and it can be seen how it goes to infinity at the synchronization ratios. The theoretical lower limit of intercept time given by

\[
T_0 = \frac{T_1 T_2}{(\tau_1 + \tau_2)}
\]

Figure 4.8 Intercept time as a function of PRI 2. (From: [4]. © 1996 IEEE. Reprinted with permission.)
is also shown as the dotted line where the predicted intercept time approaches $T_0$ at several points. Using such a formalism, it is possible to assist in deriving receiver characteristics such as sweep time that will minimize $T_0$ against known radar transmitter scan characteristics. Ongoing work into three window intercept cases is suggested at the conclusion of [4].

### 4.3 Summary

Since [2], there have been some significant developments on the important subject of POI. Several new publications have appeared indicating both developments of the (random-phase) window functions model and its application across a variety of Rx architectures. Tools now exist to support EW system designers beyond the earlier two or three window function scenarios such as scan-on-scan and stepping or scanning Rx. This wider understanding of POI has been critical not just to improve system designs but also to acknowledge that certain conventional window functions such as scan may diminish in today’s POI analyses (due to the proliferation of phased array radars), thus requiring more in-depth review and appreciation.

More recently, there have been a number of new publications addressing the real-life issue of synchronization; this has been described through some elegant mathematics. Current theory appears well matched to describing the two window case (such as with a scanning receiver system against a rotating transmitter antenna); future work is already examining three windows and beyond, together with a variety of techniques to aid EW system designers. EW practitioners are already considering practical solutions to correlation; for example, [8] looks at POI issues for a naval EW system against radar threats incorporating a wide instantaneous field of view (IFOV) receiver behind a high rotation speed, directional antenna. Detailed calculations are shown for a scan-on-scan scenario where synchronism between receive and transmit window functions is removed (thus enabling intercept) through a variable receiver scan rotation rate.

Overall, it is clear that there now exists strong and clear linkage between uncorrelated and correlated aspects of POI.

A scenario of a frequency agile emitter and a sweeping narrowband receiver is analyzed in some detail in Appendix C. The random frequency agility allows the use of conventional probability concepts to characterize the data provided at the receiver output.

### References


