A Limited Feedback Technique for Beamspace MIMO Systems with Single RF Front-end

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Abstract—Recently, a novel beamspace multiple input multiple output (BS-MIMO) transmission technique has appeared, which increases the spectral efficiency of open-loop communication systems while using compact antenna structures with a single radio-frequency (RF) front-end at the transmitter. In this paper, we extend the aforementioned architecture proposing an efficient limited feedback technique for capacity optimization, considering electronically steerable parasitic array radiator (ESPAR) antennas at the transmitter. The performance of the resulting closed-loop BS-MIMO is evaluated against equivalent traditional MIMO systems that require a much larger number of active antenna elements to operate. The results are very promising, paving the way for integrating the proposed system in cost and size sensitive wireless handheld devices such as mobile terminals and mobile personal digital assistants.

Keywords Antenna arrays, MIMO systems, Beamspace, Spatial multiplexing, ESPAR antennas, limited feedback

I. INTRODUCTION

In conventional MIMO systems, the antenna elements are typically driven by uncorrelated signals, implying that MIMO transmitters include as many separate radio frequency (RF) front-ends as the total number of antennas used. Moreover, in order to ensure uncorrelated fading characteristics of the wireless channel, antenna elements should have a substantial spacing of at least half a wavelength, leading to large antenna array implementations [1]. Therefore, the cost of implementing multiple antenna structures with multiple RF chains and large inter-element distances required to ensure the signals’ orthogonality, makes MIMO systems technically difficult to be integrated on cost and size sensitive applications such as mobile telephony and mobile computing. On the other hand, the need to investigate alternative architectures which enable the integration of MIMO benefits on mobile handsets has appeared, satisfying several restrictions such as size, energy and cost limitations.

Recently, a new methodology, labeled as “beam-space MIMO” (BS-MIMO) [2], [3], [4] and built upon the theoretical framework introduced in [5] and [6] has been presented for implementing MIMO systems using a single active element and compact antenna structures at the transmitter. In this approach, instead of sending different symbol streams in different elements of the antenna array as in the traditional case, symbols are mapped directly into the beamspace domain of the transmit antenna, i.e. towards different angles of departure (AoDs). This approach, achieves performance characteristics comparable to traditional MIMO (T-MIMO) multiplexing systems, while using only a single RF front-end at the transmitter. It should be noticed, that the beamspace approach is usually considered in multiuser MIMO systems with spatially distributed users. Although until now only open-loop BS-MIMO systems were examined, it is well known that the performance of MIMO systems increases when channel state information (CSI) is present at the transmitter. On the other hand, full channel knowledge requires high amount of feedback information which can be gathered in the expense of channel throughput and is often inaccurate in time varying and frequency selective channels.

In this paper, we examine the performance of closed-loop BS-MIMO systems, proposing an efficient limited feedback technique. In particular, the paper is organized as follows: In Section II the beamspace system model is reviewed, along with a description of the beamspace transmitter functionality. In section III we present the system setup used. Moreover, the feedback functionality and the corresponding scenarios are depicted in detail. In section IV simulation results are discussed and the paper concludes with a summary of the findings.

II. BS-MIMO ARCHITECTURE

A. Beamspace System Model

In order to present the BS-MIMO architecture, a parametric physical model that considers the geometry of the scattering environment is required. Such models have extensively been studied in the literature [7], [8], where each path $i=1,...,K$ connecting the area of the transmitter to the area of the receiver has a single angle of departure (AoD) $\theta_{d,i}$, a single angle of arrival (AoA) $\theta_{r,i}$, and a path gain.

In this paper we consider a compact antenna structure with a single RF front-end at the transmitter, able to send diverse symbol streams towards different AoDs, and a conventional MIMO receiver equipped with a uniform linear array (ULA) with $M_r$ elements. Therefore, the received signal vector corresponding to one symbol period is expressed as [4]:
\[
\mathbf{y} = \mathbf{H}_{\text{BS}} \mathbf{x}_{\text{BS}} + \mathbf{n}
\]  

(1)

where \( \mathbf{x}_{\text{BS}} \) is the system input vector. The elements of matrix \( \mathbf{H}_{\text{BS}} \) represent the coupling between the transmit antennas and the receive antenna elements. We denote the direction vectors of the AoAs and AoDs by \( \hat{\theta}_R \) and \( \hat{\theta}_T \) respectively, whereas \( \mathbf{A}_R(\hat{\theta}_R) \) is the \( (M_r \times K) \) receive steering matrix and \( \mathbf{H}_b \) is a diagonal \( (K \times K) \) matrix whose entries are the complex gain of each path from the transmitter to the receiver. Then, (1) may be written equivalently as [4]:

\[
\mathbf{y} = \mathbf{A}_R(\hat{\theta}_R) \mathbf{H}_b \mathbf{B}_T \mathbf{x}_{\text{BS}} + \mathbf{n}
\]  

(2)

where the matrix \( \mathbf{B}_T \) contains \( M_r \) column vectors of length \( K \), representing the vector functions of a basis towards the scatterers, i.e.

\[
\mathbf{B}_T = \begin{bmatrix} B_1(\hat{\theta}_T) & B_2(\hat{\theta}_T) & \ldots & B_{M_r}(\hat{\theta}_T) \end{bmatrix}
\]  

(3)

The product \( (\mathbf{B}_T \mathbf{x}_{\text{BS}}) \) in (2), represents the actual radiation pattern created at the transmitter at every symbol period, and is a linear combination of the basis patterns with the input vector.

B. BS-Transmitter Functionality

The key factor of the proposed technique is that the set of required radiation patterns can be produced using compact antennas at the transmitter. In particular, the transmitter is equipped with only a single RF front end and an appropriate ESPAR antenna structure [9], [10]. ESPAR antennas are smart antenna systems implemented using a single active antenna element and a number of parasitic elements placed on a circle around the active element. The parasitic elements are short-circuited and loaded with variable reactors, i.e. varactors, that control the imaginary part of the parasitic elements’ input impedances. Due to the presence of mutual coupling among the antenna elements, by adjusting the varactors’ response, i.e. the reactance value, we are able to control the complex currents running on the parasitic elements, thus controlling the radiation pattern of the ESPAR antenna.

Although BS-MIMO systems have been already evaluated for BPSK and QPSK modulation schemes, the proposed closed loop BS-MIMO architecture is demonstrated using a 2x2 system example, where the symbols are modulated in BPSK format. Moreover, the transmitter is equipped with a 5-element ESPAR antenna, with inter-element distance equal to \( \lambda/16 \). The selected ESPAR antenna configuration is able to map BPSK modulated symbols onto orthogonal basis patterns given by:

\[
B_1(\hat{\theta}_T) = 1 + \cos(\hat{\theta}_T)
\]

\[
B_2(\hat{\theta}_T) = 1 - \cos(\hat{\theta}_T)
\]  

(4)

Furthermore, we represent the input vector as \( \mathbf{x}_{\text{BS}} = [x_0 \ x_x] \) to denote that the diverse BPSK symbol streams will be transmitted towards the virtual channel angles \([0 \ \pi]\). Based on the aforementioned, the possible radiation patterns are:

\[
\mathbf{B}_T [1 \ 1]^T = B_1(\hat{\theta}_T) + B_2(\hat{\theta}_T)
\]

\[
\mathbf{B}_T [1 \ -1]^T = B_1(\hat{\theta}_T) - B_2(\hat{\theta}_T)
\]

\[
\mathbf{B}_T [-1 \ 1]^T = -B_1(\hat{\theta}_T) + B_2(\hat{\theta}_T)
\]

\[
\mathbf{B}_T [-1 \ -1]^T = -B_1(\hat{\theta}_T) - B_2(\hat{\theta}_T)
\]  

(5)

The set of possible radiation patterns given by (5) may be produced in the following way: we feed the active element with the symbol \( x_0 \), and adjust the varactors’ values appropriately, enabling the ESPAR antenna to produce the radiation patterns given by (6), at a symbol period rate, depending on the symbol’s ratio value:

\[
P_{(i)}(\hat{\theta}_T) = B_1(\hat{\theta}_T) + B_2(\hat{\theta}_T)
\]

\[
P_{(-i)}(\hat{\theta}_T) = B_1(\hat{\theta}_T) - B_2(\hat{\theta}_T)
\]  

(6)

Therefore, a more compact representation of the produced patterns is:

\[
\mathbf{B}_T \mathbf{x}_{\text{BS}} = P_{(i)}(\hat{\theta}_T)x_0
\]  

(7)

III. SYSTEM SETUP AND LIMITED FEEDBACK

A. Training Procedure

Regarding the BS channel matrix \( \mathbf{H}_{\text{BS}} \), the receiver acquires the necessary feedback information, through the training procedure described herein. In particular, we define a beamspace-time training matrix \( \mathbf{W} \) as:

\[
\mathbf{W} = \begin{bmatrix} 1 & -1 \end{bmatrix} \downarrow \text{Time}
\]

(8)

The structure of the matrix \( \mathbf{W} \) implies that both patterns in (6) will be used sequentially during the training procedure.
Denoting the system response to the matrix $\mathbf{W}$ by $\mathbf{R}$, the estimated BS-MIMO channel matrix becomes:

$$
\mathbf{H}_{\text{BS}} = \mathbf{R}\mathbf{W}^{-1}
$$

where each element $h_{ij}^{\text{BS}}$ denotes the amplitude gain between $j$-th basis pattern and $i$-th receive antenna.

**B. Feedback Functionality**

The core idea of the proposed limited feedback technique is that the feedback information sent by the receiver participates in the beamforming operation at the transmitter, allocating different power to each basis pattern, thus producing a radiation pattern which is adaptively matched to channel conditions. Therefore, an optimal power allocation policy is followed at the beamspace and not at the antenna domain. In particular, the basis patterns participating in the linear combination $(\mathbf{B}_r \mathbf{x}_{\text{BS}})$ are weighted by the square root of the power allocation coefficients computed at the receiver side. Therefore, (6) for closed-loop systems becomes:

$$
P_{(i)}(\mathbf{\theta}_r) = \sqrt{\gamma_{\text{BS},i}} B_i(\mathbf{\theta}_r) + \sqrt{\gamma_{\text{BS},2}} B_2(\mathbf{\theta}_r)
$$

and the compact representation of the produced radiation patterns is:

$$
\mathbf{B}_r \left( \mathbf{x}_{\text{BS}} \odot \sqrt{\gamma_{\text{BS}}} \right) = P_{\left( \mathbf{\theta}_r \right)} \mathbf{x}_0
$$

where $\odot$ denotes the Hadamard product. Fig.1 depicts example radiation patterns given in (10) when power is allocated equally to each basis pattern, i.e. $\gamma_{\text{BS}} = [1,1]^T$ and for a non-equal power allocation of $\gamma_{\text{BS}} = [0.6666, 1.3334]^T$ when feedback is present.

**C. Feedback Scenarios**

The examined feedback scenarios in this paper are:

- Full Feedback
- Limited Feedback (Partial CSI at the Transmitter)

In case of Full Feedback scenario, the receiver performs Singular Value Decomposition (SVD) of the modified virtual channel matrix and executes the waterfilling algorithm [11]. Then, it sends back to the transmitter the matrix with the right singular vectors, as well as the optimal power allocation coefficients which correspond to the optimal amount of power at every transmit direction. These coefficients are denoted as $\gamma_{\text{BS},i}$, $i = 1, \ldots, r$, where $r$ is the rank of the BS-channel matrix, which in our system example equals to 2.

Although full feedback is inefficient due to the large amount of information needed to be sent back to the transmitter, it is presented as reference for comparison purposes. Since the transmitter has full channel knowledge, the ergodic capacity in the BS-MIMO case is calculated by [11], [12]:

$$
C = E \left\{ \sum_{i=1}^{N} \log_2 \left( 1 + \frac{\text{SNR}}{N} \gamma_{\text{BS},i} \right) \right\}
$$

where $N$ is the number of the basis patterns, in our case $N = 2$, and $\lambda_{\text{BS}}$ are the eigenvalues of the matrix $\mathbf{H}_{\text{BS}}^H\mathbf{H}_{\text{BS}}$.

On the other hand, regarding the proposed limited feedback technique, the receiver instead of sending back the full channel information, it confines its feedback sequence to only the outputs of the waterfilling algorithm $\gamma_{\text{BS},i}$, $i = 1, \ldots, r$, while the order of precedence in the feedback sequence denotes the corresponding basis function to be weighted. Therefore, the proposed feedback sequence in this case is confined to $r$ real elements. Regarding our 2x2 system example, $r = 2$ and the feedback sequence is given by:

$$
\text{LF} = \{ \gamma_{\text{BS},1}, \gamma_{\text{BS},2} \}
$$

The effective channel matrix with the proposed limited feedback approach now is given by:

$$
\mathbf{H}_{\text{BS}}^{\text{OF}} = \begin{cases}
\sqrt{\gamma_{\text{BS},1}} h_{1,1} \mathbf{B}_1 & \sqrt{\gamma_{\text{BS},2}} h_{2,1} \mathbf{B}_2, \\
\sqrt{\gamma_{\text{BS},1}} h_{1,2} \mathbf{B}_1 & \sqrt{\gamma_{\text{BS},2}} h_{2,2} \mathbf{B}_2, \\
\sqrt{\gamma_{\text{BS},1}} h_{1,2} \mathbf{B}_1 & \sqrt{\gamma_{\text{BS},2}} h_{2,1} \mathbf{B}_2,
\end{cases}
$$

if $\|h_1\|_F > \|h_2\|_F$

$$
\begin{cases}
\sqrt{\gamma_{\text{BS},1}} h_{1,1} \mathbf{B}_1 & \sqrt{\gamma_{\text{BS},2}} h_{2,2} \mathbf{B}_2, \\
\sqrt{\gamma_{\text{BS},1}} h_{1,2} \mathbf{B}_1 & \sqrt{\gamma_{\text{BS},2}} h_{2,1} \mathbf{B}_2, \\
\sqrt{\gamma_{\text{BS},1}} h_{1,2} \mathbf{B}_1 & \sqrt{\gamma_{\text{BS},2}} h_{2,1} \mathbf{B}_2,
\end{cases}
$$

if $\|h_1\|_F < \|h_2\|_F$
where \( h_j \), \( j = 1, 2 \) is the \( j \)-th column of the BS channel matrix \( H_{BS} \) given by (9). The selection of the proper branch in (14) is done on a per snapshot basis (i.e. at every channel realization). In this case the ergodic capacity is calculated by [11]:

\[
C = E \left\{ \log_2 \det \left( I_z + \frac{SNR_{CE}}{N} (H_{BS}^{new})^H H_{BS}^{new} \right) \right\} \tag{15}
\]

D. Simulation Environment

Recall that since in BS-MIMO systems the transmit radiation pattern is not omni-directional, a geometric based approximation (e.g. [7], [8]) is the appropriate choice for the beamspace channel modeling. In our approach, we consider a transmitter located at the centre of a circle with radius \( R_{m} = 100 \) and surrounded by 100 scatterers, while the receiver is located in free space at a distance of 200m from the transmitter and at random directions in range \([0 \text{ to } 2\pi]\). The path length from the phase centre of the transmitter to the \( k \)-th scatterer and finally to the \( i \)-th receive element is denoted by \( d_{ik}^{(k)} \). The corresponding amplitude losses are given by:

\[
\mathbf{A}_K \left( \mathbf{H}_b \right) = \begin{bmatrix} \sqrt{(PL)_{11}} & \cdots & \sqrt{(PL)_{1K}} \\ \sqrt{(PL)_{21}} & \cdots & \sqrt{(PL)_{2K}} \end{bmatrix}
\]

(16)

where:

\[
\sqrt{(PL)_{ik}} = \sqrt{\left( \frac{\lambda}{4\pi} \right)^2 G_S \frac{1}{d_{ik}^{(k)}}} e^{j\phi_i}
\]

(17)

The angles \( \phi_i \in [0, 2\pi] \) are uniformly distributed random variables, \( G_S = \frac{1}{2\pi} \) is the power gain of each scatterer and \( n \) is the path loss factor.

IV. Simulation Results

Up to now we have considered an unlimited number of bits for the power allocation coefficients, which constitute the limited feedback sequence. Since in practical systems this is not feasible, we examine the effect of quantization of the power allocation coefficients on the capacity. In particular, the real value \( \gamma_{BS} \) can be uniformly quantized into a set of discrete states \( S = [S_0, \ldots, S_{Q-1}] \) using \( \log_2 Q \) bits. Fig. 2 presents the capacity CDF at 15dB SNR in case of a 2x2 closed-loop BS-MIMO system with limited feedback (partial CSI at the transmitter) for different quantization levels. One may observe that using 2 bits/ \( \gamma_{BS} \) the performance degradation is negligible, while for lower SNR values the corresponding curves coincide. Obviously, the most appropriate quantization level is 2 bits per \( \gamma_{BS} \). Consequently, a look-up table is considered (i.e. Table 1), for the efficient formation of the transmit radiation pattern according to (10). The optimal reactive weights presented in Table 1, were calculated by an exhaustive search method. However, it should be noted that the control of the reactive weights guarantee the formation of the amplitude pattern produced by the ESPAR antenna, while the corresponding phase pattern is not achieved adequately in all cases. Since we are concerned about the phase pattern too, the phase adjustment \( \phi \) is the necessary correction applied to the phase pattern of the transmitting antenna. Therefore, the

![Figure 2. Quantization effect of the power allocation coefficients on capacity](image)

TABLE I. LOOK-UP TABLE CORRESPONDING TO 2 BITS QUANTIZATION LEVEL

<table>
<thead>
<tr>
<th>Symbol ratio: ( x_c/x_c = -1 )</th>
<th>( \gamma_c )</th>
<th>( \gamma_c )</th>
<th>Reactive weights (( \Omega ))</th>
<th>Phase adj. ( \phi ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>[80 10 70 70]</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1.3334</td>
<td>0.6666</td>
<td>[40 50 70 70]</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>0.6666</td>
<td>1.3334</td>
<td>[−50 40 70 70]</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>[−10 80 −70 −70]</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol ratio: ( x_c/x_c = 1 )</th>
<th>( \gamma_c )</th>
<th>( \gamma_c )</th>
<th>Reactive weights (( \Omega ))</th>
<th>Phase adj. ( \phi ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>[80 10 70 70]</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1.3334</td>
<td>0.6666</td>
<td>[−90 −30 −50 −50]</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>0.6666</td>
<td>1.3334</td>
<td>[−30 −90 −50 −50]</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>[−10 80 −70 −70]</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
In this paper we evaluate the performance of a novel beamspace MIMO transmission architecture, combined with an efficient limited feedback technique that includes power loading of the basis patterns at the transmitter. The performance of the proposed system is evaluated using the capacity as a performance metric, against traditional open and closed-loop MIMO systems, which require a much larger number of active antenna elements to operate. The results show that the performance of traditional MIMO systems and the proposed BS-MIMO systems is comparable in case of open-loop and full feedback modes. Moreover, the proposed limited feedback technique introduces a gain of 1 dB in the low SNR regime, with respect to the open-loop case, at the cost of only 4 feedback bits.

V. CONCLUSIONS

REFERENCES