Hamilton Cycles in Regular 2-Connected Graphs

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We shall prove the following result.

THEOREM 1. Every 2-connected, k-regular graph on at most 3k vertices is hamiltonian.

This result is best possible for \( k = 3 \) since the Petersen graph is a non-hamiltonian, 2-connected, 3-regular graph on 10 vertices. It is essentially best possible for \( k \geq 4 \) since there exist non-hamiltonian, 2-connected, \( k \)-regular graphs on \( 3k + 4 \) vertices for \( k \) even, and \( 3k + 5 \) vertices for all \( k \). Examples of such graphs are given in [1, 3]. The problem of determining the values of \( k \) for which all 2-connected, \( k \)-regular graphs on \( n \) vertices are hamiltonian was first suggested by G. Szekeres. Erdős and Hobbs [3] proved that such graphs are hamiltonian if \( n < 2k + ck^{1/2} \), where \( c \) is a positive constant. Subsequently, Bollobás and Hobbs [1] showed that \( G \) is hamiltonian if \( n < \frac{3}{4}k \).

We shall in fact prove a result slightly stronger than Theorem 1.

THEOREM 2. Let \( G \) be a 2-connected graph on \( n \) vertices with minimum degree \( k \). Suppose that \( n < 3k \) and

\[
\sum_{v \in V(G)} (d(v) - k) \leq k - 1.
\]

Then \( G \) is hamiltonian.

Thus the regularity condition of Theorem 1 may be relaxed somewhat. The upper bound for \( \sum_{v \in V(G)} (d(v) - k) \) cannot be increased since \( K_{k+1,k} \) is a non-hamiltonian, 2-connected graph on \( 2k + 1 \) vertices, with minimum degree \( k \), and

\[
\sum_{v \in V(K_{k+1,k})} (d(v) - k) = k.
\]