CHAPTER III

Impact of visco-elasticity of MHD free-convective flow past a vertical surface embedded in a porous medium.

3.1 INTRODUCTION

Analytical solutions of simultaneous heat and mass transfer problems in convective flows in presence of magnetic effects have been analysed by many researchers due to their various applications viz industrial, geographical, astrophysical and similar kinds for the last few decades. When a magnetic field acts in fluid flow past a flat plate, then due to the interaction of the magnetic and velocity fields, Lorentz force is created and acts on a resistive force in the direction opposite to that of the fluid velocity. Due to this, the skin friction at the plate is subdued and hence the transformation from laminar to the turbulent flow may be prohibited and as a result the boundary layer flow may be restricted. Also, porous media are considered to be useful in diminishing the natural free-convection which would otherwise occur intensely on a vertical heated surface.

In order to make heat insulation of surface more competent it is essential to study the free-convention flow through a porous medium and to assess its effect in heat and mass transfer.

Magnetohydrodynamic free-convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux has been investigated by Raptis and konfousius (1982). Bejan and Khair (1985) have analyzed heat and mass transfer in a porous medium. Heat and mass transfer through a porous medium in presence of buoyancy effect has been studied by Trevisan and Bejan (1985). Singh and Singh (2000) have studied MHD effects on heat and mass transfer in flow of a viscous fluid past a flat plate. Magnetic field effects on the free-convention and mass transfer flow through

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porous medium with constant suction and constant heat flux has been presented by Acharya *et al.*(2000). Soundalgekar *et al.* (2004) have studied the transient free-convective flow of dissipative fluid past an infinite vertical porous plate. MHD free convection and mass transfer flow past a flat plate has been interpreted by Singh *et al.*(2007), Das (2011) has investigated the effects of heat and mass transfer on MHD free convection flow near a moving vertical plate of a radiating and chemically reacting fluid. Unsteady heat and mass transfer in MHD flow over an oscillating stretching surface with Soret and Dufour effects has been considered by Zheng *et al.* (2013).

In the above mentioned works, the Newtonian viscous fluid is considered but nowadays the study of non-Newtonian fluids has gained utmost interest due to its numerous technological applications, including performance of lubricants, manufacturing of plastic sheets and movement of biological fluids etc. Visco-elastic fluid is a subclass of non-Newtonian model where the viscosity signifies the physics of the energy dissipated during the flow and its elasticity represents the energy stored during the flow. The contribution of many authors are found in this field in literature but few of them are cited here viz. Vajravelu and Rollins (2004) Cortell (2007), Hayat *et al.(*2008), Reza and Gupta(2008), Choudhury and Dey (2010), Sekhar and Reddy (2012), Choudhury and Das (2014), Choudhury and Dhar (2014) etc.

They have analysed the flow patterns of visco-elastic fluids and studied their behaviour under different valid and suitable boundary conditions with different geometries. The objective of this study is to analyse the influence of visco-elastic of MHD fluid flow past a plate embedded in a porous medium.

3.2 **MATHEMATICAL FORMULATION**

Let us consider a two-dimensional, steady, laminar free-convective flow of visco-elastic incompressible electrically conducting fluid over a semi infinite flat plate in space bounded by a vertical infinite surface in presence of uniform magnetic field, which is applied normal to the direction of flow. Medium of the fluid is considered porous and the effect of induced magnetic field is neglected. Also, we consider that the magnetic Reynolds number is small. The x-axis taken along the surface which is in vertically upwards direction and y-axis normal to it. Dimension of bounding surface is infinite, so all the variables are functions of the variable y. As the magnetic field is not that much strong so Joulean heat dissipation is not considered. Using boundary layer approximation, the governing equations are:

Equation of continuity: $\frac{\partial v}{\partial y} = 0$ i.e. $v = -v_o$ (constant) (3.2.1)

Equation of motion: $v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - k_o v\frac{\partial^3 u}{\partial y^3} + g\beta(T - T_\alpha) + g\beta^*(C - C_\alpha)$

$-\frac{\sigma B_0^2}{\rho}u - \frac{v}{k}u$ (3.2.2)

Energy equation: $v\frac{\partial T}{\partial y} = \frac{\lambda}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{k_o}{2C_p}v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}$ (3.2.3)

The concentration equation: $v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}$ (3.2.4)

The relevant boundary conditions are:

$$y = 0; u = 0, \frac{\partial T}{\partial y} = -\frac{q}{\lambda}, \frac{\partial C}{\partial y} = -\frac{m}{D}$$

$$y \to \infty; u = 0, T = T_\infty, C = C_\infty$$

We introduce the following non-dimensional parameters:

$f(\eta) = \frac{u}{v_o}$ (velocity),

$\eta = \frac{v_o y}{v}$ (distance),

$P_r = \frac{\mu C_p}{\lambda}$ (Prandlt number),

$S_c = \frac{v}{D}$ (Schmidt number),

$\theta = \frac{(T - T_\alpha)v_o\lambda}{qv}$ (Temperature),

$C_1 = \frac{(C - C_\alpha)v_o D}{mv}$ (Species concentration),

$\alpha = \frac{v_o^2 k}{v^2}$ (Permeability parameter),

$G_r = \frac{g\beta q v^2}{v_o^4\lambda}$ (Thermal Grashof number),

$G_m = \frac{g\beta^* m v^2}{v_o^4 D}$ (Solutal Grashof number),

$M = \frac{\sigma B_o^2 v}{\rho v_o^2}$ (Magnetic Parameter),

$E = \frac{\lambda v_o^3}{q v C_p}$ (Eckert number),

where q is the heat flux per unit area and m is the mass flux per unit area.

The non-dimensional forms of the equations (3.2.2) to (3.2.4) are given as follows:

$$\frac{d^2 f}{d\eta^2} + \frac{df}{d\eta} + k_1 \frac{d^3 f}{d\eta^3} + G_r\theta + G_m C_1 - f(M + \alpha^{-1}) = 0 \tag{3.2.5}$$

$$-P_r \frac{d\theta}{d\eta} = \frac{d^2\theta}{d\eta^2} + P_r\left(\frac{df}{d\eta}\right)^2 E - k_1 E P_r \frac{df}{d\eta}\frac{d^2 f}{d\eta^2} \tag{3.2.6}$$

$$-S_c \frac{dC_1}{d\eta} = \frac{d^2 C_1}{d\eta^2} \tag{3.2.7}$$

where $k_1 = \frac{k_o v_o^2}{v^2}$ is the visco-elastic parameter.

The transformed boundary conditions are:

$$\left.\begin{array}{l} \eta = 0;\ f = 0, \theta' = -1, C_1' = -1 \\ \eta \to \infty;\ f \to \infty, \theta \to 0, C_1 \to 0 \end{array}\right\} \tag{3.2.8}$$

where prime denotes differentiation w.r.t. η.

3.3 SOLUTION OF THE PROBLEM

The solution of (3.2.7) subject to boundary conditions (3.2.8) is represented by

$$C_1 = a_2 e^{-s_c \eta} \tag{3.3.1}$$

To get the solution of the coupled non-linear equations (3.2.5) and (3.2.6), we expand velocity and temperature in powers of Eckert number as for incompressible fluid flows E<<1. We write

$$\left.\begin{array}{l} f(\eta) = f_o(\eta) + E f_1(\eta) + O(E^2) \\ \theta(\eta) = \theta_o(\eta) + E\theta_1(\eta) + O(E^2) \end{array}\right\} \tag{3.3.2}$$

Introducing (3.3.2) into the equations (3.2.5) and (3.2.6) and equating the like powers of E with the neglect of $O(E^2)$ and higher order terms, we get the following set of equations:

Zeroth-order equations:

$$f_o'' + f_o' + k_1 f_o''' - f_o(M+\alpha^{-1}) = -G_r\theta_o - G_m a_2 e^{-S_c\eta} \tag{3.3.3}$$

$$-P_r\theta_o' = \theta_o'' \tag{3.3.4}$$

First-order equations:

$$f_1'' + f_1' + k_1 f_1''' - f_1(M+\alpha^{-1}) = -G_r\theta_1 \tag{3.3.5}$$

$$-P_r\theta_1' = \theta_1'' + P_r f_o^{12} - k_1 P_r f_o' f_o'' \tag{3.3.6}$$

with the corresponding boundary conditions

$$\eta = 0; f_o = 0, f_1 = 0, \theta_o' = -1, \theta_1' = 0 \tag{3.3.7}$$

$$\eta \to \infty;\ f_1 \to 0, \theta_o \to 0 \tag{3.3.8}$$

The solution of equation (3.3.4) subject to boundary conditions (3.3.7) and (3.3.8) is given by

$$\theta_0 = \frac{1}{P_r} e^{-P_r\eta} \tag{3.3.9}$$

To solve equations (3.3.3), (3.3.5) and (3.3.6) subject to boundary conditions (3.3.7) and (3.3.8), we use multiparameter perturbation scheme for k_1 following Nowinski and Ismail (1965) as $k_1 << 1$ for small shear rate.

Thus,

$$f_0 = f_{oo} + k_1 f_{01} + O(k_1^2), \qquad f_1 = f_{1o} + k_1 f_{11} + O(k_1^2),$$
$$\theta_1 = \theta_{1o} + k_1\theta_{11} + O(k_1^2) \tag{3.3.10}$$

Using (3.3.10) into the equations (3.3.3), (3.3.5) and (3.3.6), equating the like powers of k_1 and neglecting the co-efficients of $O(k_1^2)$ and higher order terms we get,

Zeroth-order equations:

$$f_{oo}'' + f_{oo}' - (M+\alpha^{-1}) f_{oo} = -G_r a_4 e^{-P_r\eta} - G_m a_2 e^{-S_c\eta} \tag{3.3.11}$$

$$f_{10}'' + f_{10}' - (M+\alpha^{-1}) f_{1o} = -G_r\theta_{10} \tag{3.3.12}$$

$$-P_r\theta_{10}' = \theta_{10}'' + P_r f_{00}'^2 \tag{3.3.13}$$

First-order equations:

$$f_{oo}''' + f_{o1}'' + f_{01}' - (M + \alpha^{-1})f_{o1} = 0 \tag{3.3.14}$$

$$f_{11}'' + f_{11}' + f_{10}''' - (M + \alpha^{-1})f_{11} = -G_r\theta_{11} \tag{3.3.15}$$

$$-P_r\theta_{11}' = \theta_{11}'' + 2P_r f_{00}' f_{01}' - P_r f_{00}' f_{00}'' \tag{3.3.16}$$

The relevant boundary conditions are:

$$\eta = 0;\ f_{oo} = 0, f_{01} = 0, f_{10} = 0, f_{11} = 0, \theta_{oo}' = -1, \theta_{01}' = 0, \theta_{10}' = 0, \theta_{11}' = 0$$

$$\eta \to \infty;\ f_{oo} \to 0, f_{o1} \to 0, f_{1o} \to 0, f_{11} \to 0, \theta_{oo} \to 0, \theta_{o1} \to 0, \theta_{10} \to 0, \theta_{11} \to 0 \tag{3.3.17}$$

Solving the equations (3.3.11) to (3.3.16) subject to boundary conditions (3.3.17) we get the following expressions:

$$f_{oo} = a_7 e^{a_5\eta} + a_8 e^{-a_6\eta} + a_9 e^{-P_r\eta} + a_{10} e^{-S_c\eta} \tag{3.3.18}$$

$$\theta_{10} = a_{12} e^{-P_r\eta} + a_{13} e^{-2a_6\eta} + a_{14} e^{-2P_r\eta} + a_{15} e^{-2S_c\eta} + a_{16} e^{-(a_6+P_r)\eta} + a_{17} e^{-(a_6+S_c)\eta} + a_{18} e^{-(P_r+S_c)\eta} \tag{3.3.19}$$

$$f_{10} = a_{20} e^{-a_6\eta} + a_{21} e^{-P_r\eta} + a_{22} e^{-2a_6\eta} + a_{23} e^{-2P_r\eta} + a_{24} e^{-2S_c\eta} + a_{25} e^{-(a_6+P_r)\eta} + a_{26} e^{-(a_6+S_c)\eta} + a_{27} e^{-(P_r+S_c)\eta} \tag{3.3.20}$$

$$f_{01} = a_{32} e^{-a_6\eta} + a_{30} e^{-P_r\eta} + a_{31} e^{-S_c\eta} \tag{3.3.21}$$

$$\theta_{11} = a_{40} e^{-P_r\eta} + a_{41} e^{-2a_6\eta} + a_{42} e^{-2P_r\eta} + a_{43} e^{-2S_c\eta} + e^{-(a_6+P_r)\eta} a_{45} + e^{-(S_c+P_r)\eta} \tag{3.3.22}$$

$$f_{11} = a_{64} e^{-a_6\eta} + a_{57} e^{-P_r\eta} + a_{58} e^{-2a_6\eta} + a_{59} e^{-2P_r\eta} + a_{60} e^{-2S_c\eta} - (a_6 P_r)\eta - (a_6 S_c)\eta \tag{3.3.23}$$

With the use of above solutions, we find the velocity, temperature expressions as follows:

$$f = a_{83} e^{-a_6\eta} + a_{84} e^{-P_r\eta} + a_{67} e^{-S_c\eta} + a_{85} e^{-a_6\eta} + a_{86} e^{-2P_r\eta} + a_{87} e^{-S_c\eta} + a_{88} e^{-(a_6+P_r)\eta} + a_{89} e^{-(a_6+S_c)\eta} + a_{90} e^{-(P_r+S_c)\eta} \tag{3.3.24}$$

and

$$\theta = a_4 e^{-P_r\eta} + E\{a_{76} e^{-P_r\eta} + a_{77} e^{-2a_6\eta} + a_{88} e^{-2P_r\eta} + a_{79} e^{-2S_c\eta} + a_{80} e^{-(a_6+P_r)\eta} + a_{81} e^{-(a_6+S_c)\eta} + a_{82} e^{-(P_r+S_c)\eta} \tag{3.3.25}$$

The constants of the solutions are obtained as follows:

$$a_1 = 0, \qquad a_2 = \frac{1}{s_c} = A_1,$$

$$a_3 = 0, \qquad a_4 = \frac{1}{p_r} = A_2.$$

$$a_5 = \frac{-1+\sqrt{1+4\left(M+\alpha^{-1}\right)}}{2}, \qquad a_6 = \frac{-1+\sqrt{1+4\left(M+\alpha^{-1}\right)}}{2},$$

$$a_7 = 0, \qquad a_8 = -a_9 - a_{10},$$

$$a_9 = -\frac{G_r a_4}{p_r^2 - p_r - \left(M+\alpha^{-1}\right)}, \qquad a_{10} = -s\frac{1}{s_c^2 - s_c - \left(M+\alpha^{-1}\right)}, \qquad a_{11} = 0,$$

$$a_{12} = \frac{1}{p_r}\left(-2a_{13}a_6 - 2a_{14}p_r - 2a_{15}s_c - a_6\left(a_6 + p_r\right) - a_{17}\left(a_6 + s_c\right) - a_{18}\left(p_r + s_c\right)\right),$$

$$a_{13} = -\frac{p_r\left(a_8^2 a_6\right)}{4a_6 - 2p_r}, \qquad a_{14} = -\frac{p_r a_9^2}{2},$$

$$a_{15} = \frac{-p_r a_{10}^2 s_c}{4s_c - 2p_r}, \qquad a_{16} = \frac{-2p_r^2 a_6 a_8 a_9}{\left(a_6 + p_r\right)^2 - p_r\left(a_6 + p_x\right)},$$

$$a_{17} = \frac{-2p_r a_6 a_8 a_{10} s_c}{\left(a_6 + s_c\right)^2 - p_r\left(a_6 + s_c\right)}, \qquad a_{18} = \frac{-2p_r a_9 a_{10} p_r s_c}{\left(p_r + s_c\right)^2 - p_r\left(p_r + s_c\right)},$$

$$a_{19} = 0, \qquad a_{20} = -\left(a_{21} + a_{22} + a_{23} + a_{24} + a_{25} + a_{26} + a_{27}\right),$$

$$a_{21} = \frac{-G_r a_{12}}{p_r^2 - p_r - \left(M+\alpha^{-1}\right)}, \qquad a_{22} = \frac{-G_r a_{13}}{4a_6^2 - 2a_6 - \left(M+\alpha^{-1}\right)},$$

$$a_{23} = \frac{-G_r a_{14}}{4p_r^2 - 2p_r - \left(M+\alpha^{-1}\right)}, \qquad a_{24} = \frac{-G_r a_{15}}{4s_c^2 - 2s_c - \left(M+\alpha^{-1}\right)},$$

$$a_{25} = \frac{-G_e a_{16}}{\left(a_6 + p_r\right)^2 - \left(a_6 + p_r\right) - \left(M+\alpha^{-1}\right)}, \qquad a_{26} = \frac{-G_r a_{17}}{\left(a_6 + s_c\right)^2 - \left(a_6 + s_c\right) - \left(M+\alpha^{-1}\right)},$$

$$a_{27} = \frac{-G_r a_{18}}{\left(p_r + s_c\right)^2 - \left(p_r + s_c\right) - \left(M+\alpha^{-1}\right)}, \qquad a_{27} = 0,$$

$$a_{28} = -\left(a_{29} + a_{30} + a_{31}\right), \qquad a_{29} = \frac{a_8 a_6^3}{a_6^2 - a_6 - \left(M+\alpha^{-1}\right)},$$

$$a_{30} = \frac{a_9 p_r^3}{p_r^2 - p_r - \left(M+\alpha^{-1}\right)}, \qquad a_{31} = \frac{a_{10} s_c^3}{s_c^2 - s_c - \left(M+\alpha^{-1}\right)},$$

$$a_{32} = a_{28} + a_{29}, \qquad a_{33} = -p_r a_8 a_6^2\left(a_8 a_6 + 2a_{32}\right),$$

$a_{34} = -p_r^3 a_9 \left(a_9 p_r + 2a_{30}\right)$, $\qquad a_{35} = -p_r a_{10} s_c^2 \left(a_{10} s_c + 2a_{31}\right)$,

$a_{36} = -\left\{p_r a_8 a_9 \left(a_9 p_r^2 + 2a_{38} p_r\right) + p_r^2 a_9 \left(a_8 a_6^2 + 2a_{32} a_6\right)\right\}$,

$a_{37} = -\left\{p_r a_8 a_6 \left(a_{10} s_c^2 + 2a_{31} s_c\right) + p_r a_{10} s_c \left(a_8 a_6^2 + 2a_{32} a_6\right)\right\}$,

$a_{38} = -\left\{p_r^2 a_9 \left(a_{10} s_c^2 + 2a_{31} s_c\right) + p_r a_{10} s_c \left(a_9 p_r^2 + 2a_{30} p_r\right)\right\}$, $a_{39} = 0$,

$a_{40} = \dfrac{1}{p_r}\left[-2a_{41} a_6 - 2a_{42} p_r - 2a_{43} s_c - (a_6 + p_r) a_{44} - (a_6 + s_c) a_{45} - (s_c + p_r) a_{46}\right]$,

$a_{41} = \dfrac{a_{33}}{4a_6^2 - 2p_r a_6}$, $\qquad a_{42} = \dfrac{a_{34}}{2p_r^2}$,

$a_{43} = \dfrac{a_{35}}{4s_c^2 - 2p_r s_c}$, $\qquad a_{44} = \dfrac{a_{36}}{(a_6 + p_r)^2 - p_r(a_6 + p_r)}$,

$a_{45} = \dfrac{a_{37}}{(a_6 + s_c)^2 - p_r(a_6 + s_c)}$, $\qquad a_{46} = \dfrac{a_{38}}{(s_c + p_r)^2 - p_r(s_c + p_r)}$,

$a_{47} = a_{21} - G_r a_{40}$, $\qquad a_{48} = a_{22} - G_r a_{41}$,

$a_{49} = a_{25} - G_r a_{42}$, $\qquad a_{50} = a_{24} - G_r a_{43}$,

$a_{51} = a_{25} - G_r a_{44}$, $\qquad a_{52} = a_{26} - G_r a_{45}$,

$a_{53} = a_{27} - G_r a_{46}$, $\qquad a_{54} = 0$,

$a_{55} = -\left(a_{56} + a_{57} + a_{58} + a_{59} + a_{60} + a_{61}\right)$, $\qquad a_{56} = \dfrac{a_{20}}{a_6^2 - a_6 + \left(M + \alpha^{-1}\right)}$,

$a_{57} = \dfrac{a_{47}}{p_r^2 - p_r - \left(M + \alpha^{-1}\right)}$, $\qquad a_{58} = \dfrac{a_{48}}{4a_6^2 - 2a_6 - \left(M + \alpha^{-1}\right)}$,

$a_{59} = \dfrac{a_{49}}{4p_r^2 - 2p_r - \left(M + \alpha^{-1}\right)}$, $\qquad a_{60} = \dfrac{a_{50}}{4s_c^2 - 2s_c - \left(M + \alpha^{-1}\right)}$,

$a_{61} = \dfrac{a_{51}}{(a_6 + p_r)^2 - (a_6 + p_r) - \left(M + \alpha^{-1}\right)}$, $\qquad a_{62} = \dfrac{a_{52}}{(a_6 + s_c)^2 - (a_6 + s_c) - \left(M + \alpha^{-1}\right)}$,

$a_{63} = \dfrac{a_{52}}{(p_r + s_c)^2 - (p_r + s_c) - \left(M + \alpha^{-1}\right)}$, $\qquad a_{64} = a_{55} + a_{56}$,

$a_{65} = a_8 + k_1 a_{32}$, $\qquad a_{66} = a_9 + k a_{30}$,

$a_{67} = a_{10} + k_1 a_{31}$, $\qquad a_{68} = a_{20} + k_1 a_{64}$,

$a_{69} = a_{21} + k_1 a_{57}$, $\qquad a_{70} = a_{22} + k_1 a_{58}$,

$a_{71} = a_{23} + k_1 a_{59}$, $a_{72} = a_{24} + k_1 a_{60}$,

$a_{73} = a_{25} + k_1 a_{61}$, $a_{74} = a_{26} + k_1 a_{62}$,

$a_{75} = a_{27} + k_1 a_{63}$, $a_{76} = a_{12} + k_1 a_{40}$,

$a_{77} = a_{13} + k_1 a_{41}$, $a_{78} = a_{14} + k_1 a_{42}$,

$a_{79} = a_{15} + k_1 a_{43}$, $a_{80} = a_{16} + k_1 a_{44}$,

$a_{81} = a_{17} + k_1 a_{45}$, $a_{82} = a_{18} + k_1 a_{46}$,

$a_{83} = a_{65} + E a_{68}$, $a_{84} = a_{66} + E a_{69}$,

$a_{85} = E a_{70}$, $a_{86} = E a_{71}$,

$a_{87} = E a_{72}$, $a_{88} = E a_{73}$,

$a_{89} = E a_{74}$, $a_{90} = E a_{75}$,

$a_{91} = E a_{76}$, $a_{92} = E a_{77}$,

$a_{93} = E a_{78}$, $a_{94} = E a_{79}$,

$a_{95} = E a_{80}$, $a_{96} = E a_{81}$,

$a_{97} = E a_{82}$.

For practical purposes, the quantities of physical interest viz. shearing stress, Nusselt number, Sherwood number can be defined as follows:

The dimensionless shearing stress at the plate (y=0) is obtained as

$$\sigma = \left(\frac{\sigma_{xy}}{\rho v_0^2}\right)_{y=0} = f'(0) + \alpha_1 f''(0)$$

The dimensionless Nusselt number or wall heat transfer coefficient is defined by

$$N_u = \left(\frac{d\theta}{d\eta}\right)_{n=0}$$

The Sherwood number or wall deposition flux is represented by

$$S_u = \left(\frac{dc}{d\eta}\right)_{n=0}$$

3.4 **RESULTS AND DISCUSSION**

The present work comprises the MHD visco-elastic free-convective flow past a vertical plate embedded in a porous medium. The non-zero value of the visco-elastic parameter k_1 characterizes the non-Newtonian fluid and its zero value represents the phenomenon of Newtonian fluid motion. All the values of the flow parameters are chosen arbitrarily. In numerical computations, with variation of physical parameter we have considered fixed values of $\alpha = 0.9$ and E=0.001. Figures 3.1 to 3.6 depict the fluid velocity f against y for various values of the flow parameters involved in the problem. From the figures it is noticed that the fluid velocity accelerates rapidly near the plate but then starts to deccelerates considerably away from the plate for both Newtonian and non-Newtonian cases. In all the cases, the visco-elasticity diminishes the speed of the fluid velocity in comparison with Newtonian fluid. The effect of Schmidt number S_c is studied from the figures 3.1 and 3.2. The Schmidt number signifies the momentum diffusivity to concentration diffusitivity. The rising trend of the Schmidt number shows an enhancement in fluid velocity for Newtonian case but reverse pattern is exhibited for visco-elastic fluid. Again, Prandtl number plays an important role in heat transfer problems. It is a material property and defines the rate of the kinematic viscosity to thermal diffusivity. The growth of Prandtl number exhibits a downfall in fluid velocity (figures 3.1 and 3.3) in both Newtonian and non-Newtonian fluid flow phenomenon.

The thermal Grashof number characterizes the free-convention which means flows where the motion of the fluid is caused by the effect of gravity on heated fluids of variable density. In this problem, we have considered $G_r > 0$ which corresponds to an externally cooled plate. It is shown that the rising value of G_r stimulates the fluid velocity in both types of fluids (figures 3.1 and 3.4). The solutal Grashof number Gm characterizes the free-convection for mass transfer. Figures 3.1 and 3.5 reveal that the increasing value of G_m experiences an enhanced trend in the speed of fluid velocity in both types of fluids. The growth of the magnetic parameter M diminishes the fluid velocity and it clearly agrees with the physical situation (figures 3.1 and 3.6) that the fluid velocity is retarded for the formation of Lorentz force.

For practical purposes, it is very necessary to calculate the viscous drag or resistive force. The resistive force or viscous drag on the surface of a body due to the motion of a fluid is called the shearing stress. Figures 3.7 to 3.11 demonstrate the variations of shearing stress against the pertinent flow parameters Prandtl number P_r, Schmidt number S_c, the magnetic parameter M, Thermal Grashof number G_r, Solutal Grashof number G_m respectively.

It is inferred from the figures that the shearing stress declines due to the enhancement of Prandtl number, Schmidt number, the magnetic parameter but reverse behaviour occurs in case of thermal Grashof number and solutal Grashof number. In all the cases, the visco-elasticity decelerates the viscous drag formed in the fluid. Figures 3.12 and 3.13 infer that the visco-elastic parameter has no significant affects on the temperature and concentration fields though for Newtonian fluid they are eloquent.

3.5 **CONCLUSION**

Based on the entire study, the key features are highlighted below:

- The fluid velocity accelerates near the plate but decelerates far away from the plate in both Newtonian and non-Newtonian cases.
- Growth in visco-elasticity slows down the speed of fluid flow.
- The fluctuation of shearing stress with the enhancement of significant flow parameters are prominent in both Newtonian and non-Newtonian fluid flow phenomenon.
- The effect of visco-elasticity on the shearing stress diminishes with the increase of Prandtl number, Schmidt number, Grashof number for mass transfer, Magnetic parameter and Grashof number for heat transfer.
- The temperature and concentration profiles are not significantly affected by the visco-elastic parameter due to restraining effect played by the elasticity of the fluid.
- The Nusselt number and the Sherwood number are not considerably affected by visco-elastic parameter.

Figures:

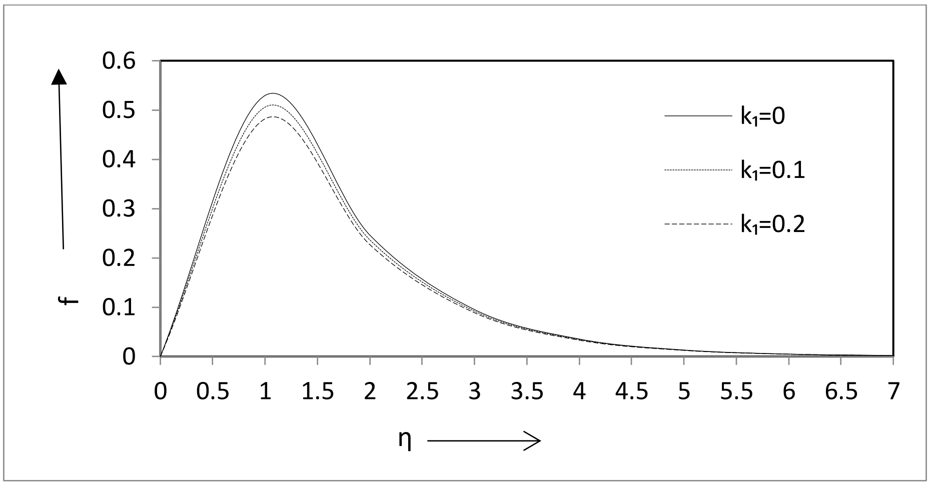

Figure 3.1: The fluid velocity f versus η for P_r=5, S_c=1, G_m=6, Gr=3, M=2

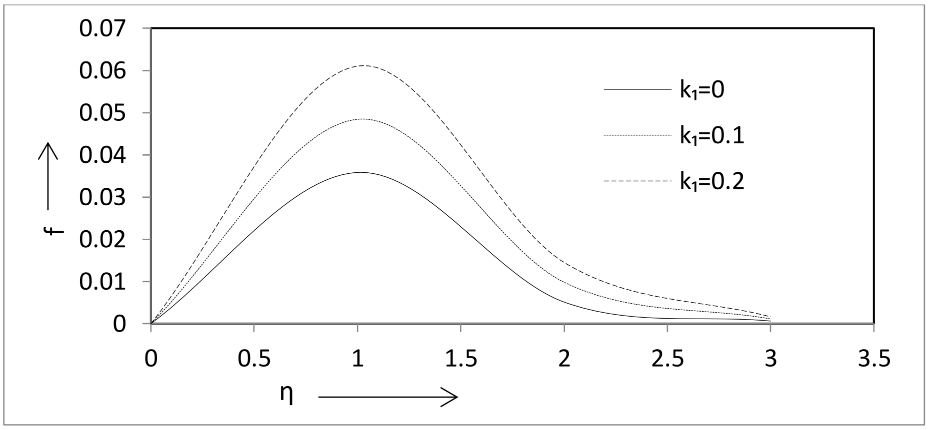

Figure 3.2: The fluid velocity f versus η for P_r=5, S_c=3, G_m=6, Gr=3, M=2

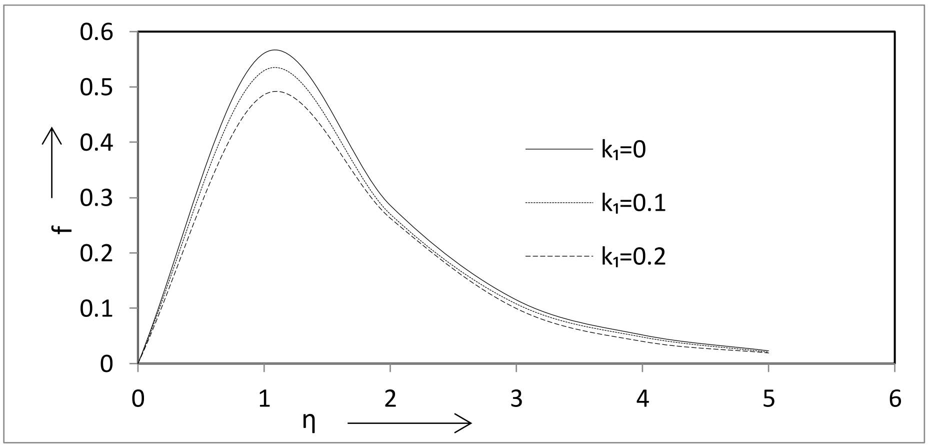

Figure 3.3: The fluid velocity f versus η for P_r=7.5, S_c=1, G_m=6, Gr=3, M=2

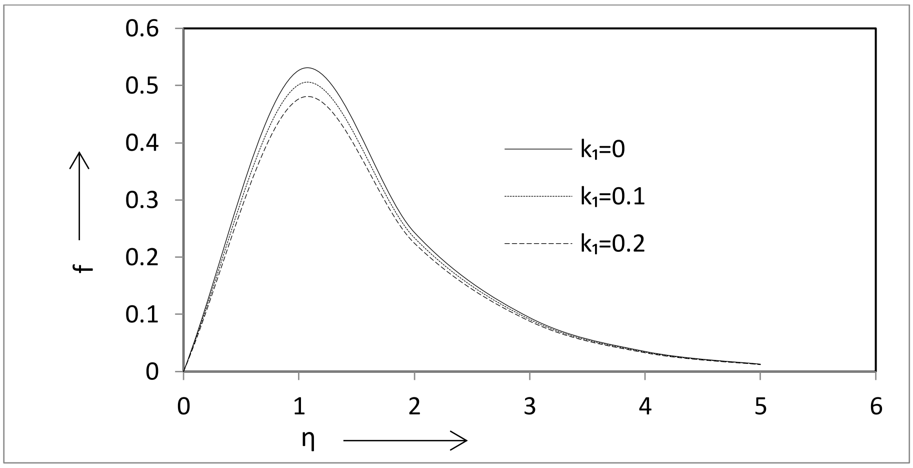

Figure 3.4: The fluid velocity f versus η for P_r=5, S_c=1, G_m=6, Gr=4, M=2

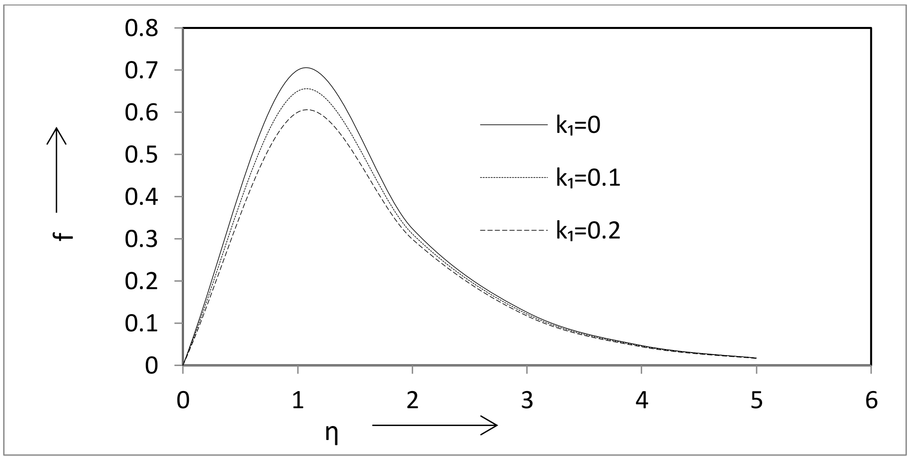

Figure 3.5: The fluid velocity f versus η for P_r=5, S_c=1, G_m=8, Gr=3, M=2

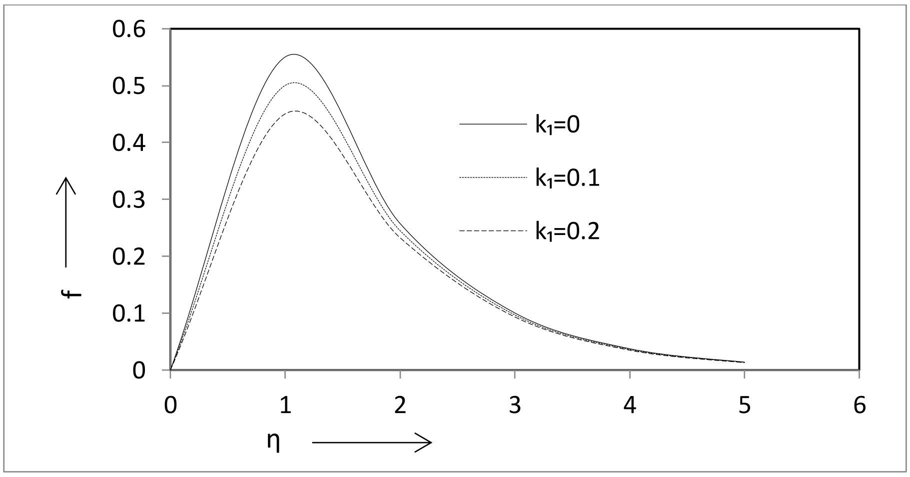

Figure 3.6: The fluid velocity f versus η for P_r=5, S_c=1, G_m=6, Gr=3, M=1.8

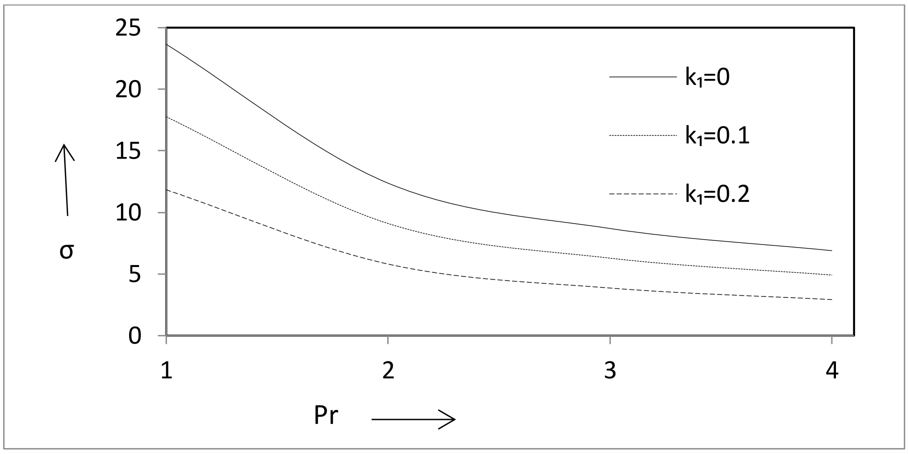

Figure 3.7: Effects of P_r on shearing stress for S_c=1, G_m=6, Gr=3, M=2

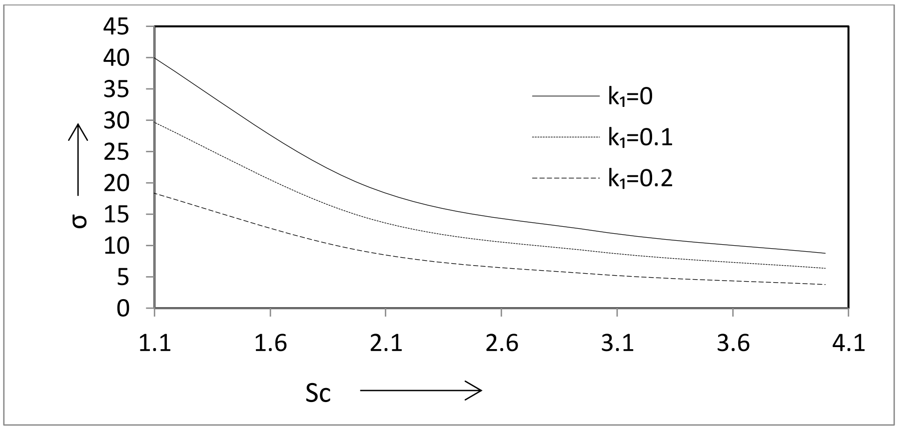

Figure 3.8: Effects of Sc on shearing stress for P_r=5, G_m=6, Gr=3, M=2

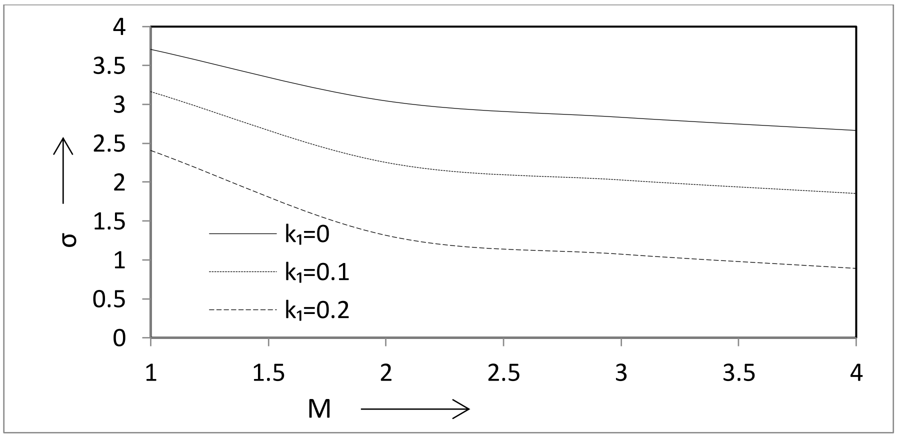

Figure 3.9: Effects of M on shearing stress for P_r =5, S_c=1, G_m=6, Gr=3

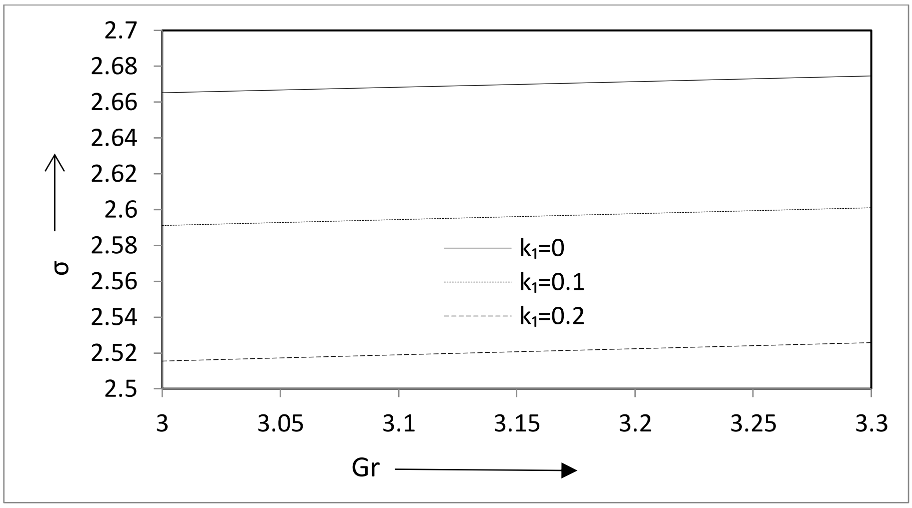

Figure 3.10: Effects of G_r on shearing stress for Pr=5, S_c=1, G_m=6, M=2

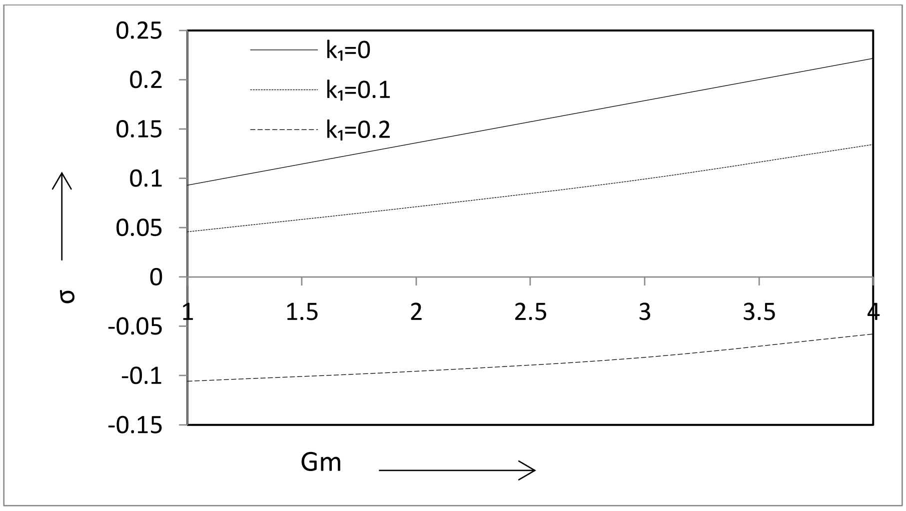

Figure 3.11: Effects of Gm on shearing stress for Pr=5, S_c=1, G_r=3, M=2

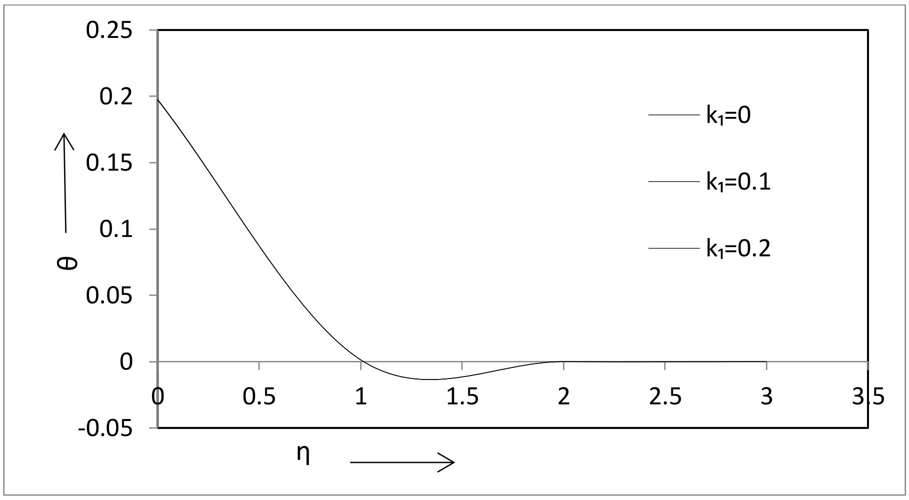

Figure 3.12: Variation of θ against η for Pr=5, S_c=1, G_m=6, G_r=3, M=2

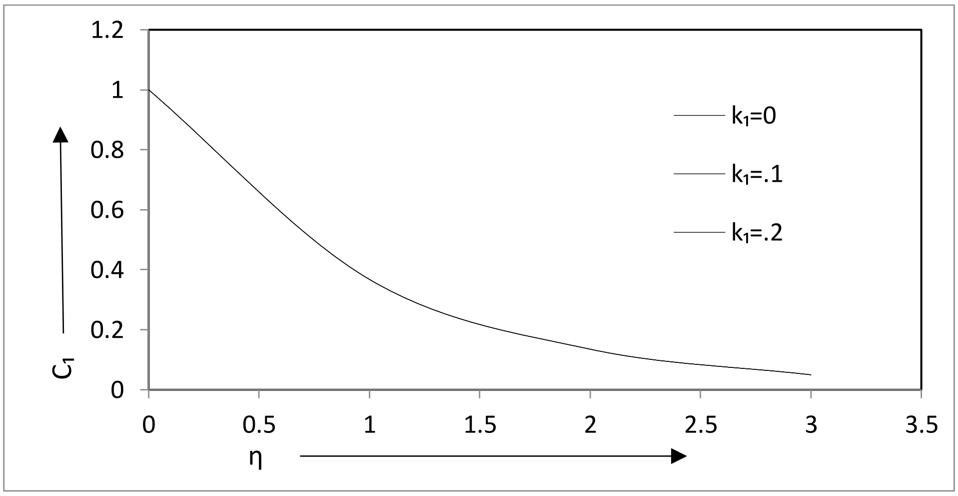

Figure 3.13: Variation of C_1 against η for P_r=5, S_c=1, G_m=6, Gr=3, M=2