Cooperative Dual-Hop Relaying Systems with Beamforming over Nakagami-\textit{m} Fading Channels

Daniel Benevides da Costa, Member, IEEE, and Sonia Aïssa, Senior Member, IEEE

Abstract—In this paper, we investigate the end-to-end performance of dual-hop relaying systems with beamforming over Nakagami-\textit{m} fading channels. Our analysis considers semi-blind (fixed-gain) relays with single antennas, and source and destination nodes equipped with multiple antennas. Closed-form expressions for the outage probability (OP), moment generating function (MGF), and generalized moments of the end-to-end signal-to-noise ratio (SNR) are derived. The proposed expressions apply to general operating scenarios with distinct Nakagami-\textit{m} fading parameters and average SNRs between the hops. The influence of the power imbalance, fading parameters, and antenna configurations on the overall system performance are analyzed and discussed through representative numerical examples. Furthermore, the exactness of our formulations is validated by means of Monte Carlo simulations.

Index Terms—Beamforming, dual-hop transmissions, Nakagami-\textit{m} fading, performance analysis, semi-blind relays.

I. INTRODUCTION

Dual-hop relaying techniques [1]–[5] have recently gained great interest in the context of cooperative wireless networks, due to their numerous benefits over single-hop relaying such as increasing the signal reliability and extending the radio coverage. By inserting beamforming techniques (also called maximal-ratio strategies) in these transmission schemes, such advantages become even more pronounced, which renders them very promising for future wireless networks.

In this respect, beamforming in cooperative networks has been investigated in several works. In [6], the effects of transmit beamforming on the ergodic capacities of multiple relay networks has been studied. In [7], beamforming in dual-hop relay systems with variable-gain relays and undergoing Rayleigh fading was investigated. In [8], distributed beamforming with single antenna at each relay was studied and, in [9], these results were extended to the multi-antenna case. Finally, in [10], transmit beamforming has been explored and optimal beamforming weights were derived taking into account not only the relay nodes but also all the links of the system.

In the open literature, works considering beamforming in fixed-gain relaying systems are not as rich as for variable-gain relaying or decode-and-forward relaying techniques. Moreover, although most of these studies rely on a Rayleigh fading assumption, the Nakagami-\textit{m} fading distribution, which covers a broad variety of fading scenarios, has received less attention.

Manuscript received October 10, 2008; revised February 26, 2009; accepted May 19, 2009. The associate editor coordinating the review of this letter and approving it for publication was M. Uysal.

The authors are with INRS-EMT, University of Quebec, Montreal, QC, Canada (e-mail: {costa, aissa}@emt.inrs.ca).

This work was supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

Digital Object Identifier 10.1109/TWC.2009.081353

In this work, we investigate the end-to-end performance of dual-hop cooperative networks with fixed-gain relays over Nakagami-\textit{m} fading channels. Our analysis accounts for beamforming applied at both the source and destination nodes. Closed-form expressions for the outage probability (OP), probability density function (PDF), moment generating function (MGF), and generalized moments of the end-to-end signal-to-noise ratio (SNR) are derived. Such expressions apply to general operating scenarios with distinct Nakagami-\textit{m} fading parameters and average SNRs between the hops. Our analytical results are validated by means of Monte Carlo simulation, and the effects of the fading severity, power imbalance and antenna configurations, on the overall system performance, are discussed through some representative numerical results and comparisons.

Mathematical Notations and Functions: Vectors are shown with bold letters. \(E(\cdot)\) denotes expectation. \(\|\cdot\|_F\) is the Frobenius norm, \((\cdot)^\dagger\) is the conjugate transpose operator. \(\Pr[\cdot]\) symbolizes probability, \(I_\nu\) is the \(\nu \times 1\) identity vector, \(f_X(\cdot)\) and \(F_X(\cdot)\) represent the PDF and cumulative distribution function (CDF) of random variable \(X\), respectively. \(\Gamma(\cdot,\cdot)\) and \(\Gamma(\cdot,\cdot,\cdot)\) indicate the Gamma function [11, Eq. 8.310.1] and the incomplete Gamma function [11, Eq. 8.350.2], respectively. \(W_{a,b}(\cdot)\) stands for the Whittaker function [12, Eq. 13.1.33], \(G_{a,b}^{c,d}(\cdot)\) designates the Meijer’s G-function [11, Eq. 9.301], and \(K_{\nu}(\cdot)\) denotes the \(\nu\)-order modified Bessel function of second kind [12, Eq. 9.6.22].

II. SYSTEM AND CHANNEL MODELS

Consider a dual-hop wireless system where a source node \(S\) communicates with a destination node \(D\) through the help of a relay \(R\), as shown in Fig. 1. Nodes \(S\) and \(D\) are equipped with \(N_S\) and \(N_D\) antennas, respectively, whereas the relay has a single antenna. We assume that due to the unsatisfactory quality of the channel between nodes \(S\) and \(D\), there is no direct link between them and the communication can be performed only through the relay. This latter introduces fixed gain on the received signal regardless of the amplitude on the first hop, resulting hence in an output signal with variable power. Systems with fixed-gain relays are less complex to implement and are cost-effective compared to those with variable-gain relays.

The channel state information (CSI) is supposed to be available at nodes \(S\) and \(D\). Also, the channels of the links \(S-R\) and \(R-D\) are assumed to experience independent block fading, so that they remain constant during two consecutive time slots. In the first time slot, the source signal, \(x\), is weighted with a \(N_t \times 1\) transmit beamforming vector \(w_t\) to form the transmit signal vector. Assuming that \(x\) has an average power normalized to unity, the received scalar signal...
at the relay, \( y_{\mathcal{R}} \), is then given by the product of the transmitted signal vector and the channel vector, added to the noise signal; that is,

\[
y_{\mathcal{R}} = h_{S\mathcal{R}}^\dagger w_i x + n_{S\mathcal{R}},
\]

where \( h_{S\mathcal{R}} \) is the \( N_1 \times 1 \) channel vector between \( S \) and \( \mathcal{R} \) with Nakagami-\( m \) fading entries, and \( n_{S\mathcal{R}} \) is the additive white Gaussian noise (AWGN) signal with mean power \( N_{S\mathcal{R}} \). Based on [13], we consider \( \mathbf{w}_i = \frac{h_{S\mathcal{R}}}{\|h_{S\mathcal{R}}\|_F} \). In the second time slot, the output signal at the relay is multiplied by the gain of the relay, \( G \), and then retransmitted to the destination. As such, the \( N_r \times 1 \) received vector signal at the destination can be written as

\[
y_D = h_{\mathcal{R}D} G(h_{S\mathcal{R}}^\dagger w_i x + n_{S\mathcal{R}}) + n_{\mathcal{R}D},
\]

where \( h_{\mathcal{R}D} \) stands for the \( N_r \times 1 \) channel vector between \( \mathcal{R} \) and \( D \) with Nakagami-\( m \) fading entries, and \( n_{\mathcal{R}D} \) denotes the \( N_r \times 1 \) noise vector with mean power \( N_{\mathcal{R}D} \). In order to maximize the received SNR at node \( D \), the received signal \( y_D \) is multiplied by a \( 1 \times N_r \) receive beamforming vector \( \mathbf{w}_r \), resulting in

\[
z_D = \mathbf{w}_r h_{\mathcal{R}D} G(h_{S\mathcal{R}}^\dagger w_i x + n_{S\mathcal{R}}) + \mathbf{w}_r n_{\mathcal{R}D},
\]

where \( \mathbf{w}_r = \frac{h_{\mathcal{R}D}}{||h_{\mathcal{R}D}||_F} \). Hereafter, to alleviate the notation, we define \( h_1 \triangleq h_{S\mathcal{R}}, h_2 \triangleq h_{\mathcal{R}D}, N_{S\mathcal{R}} \triangleq N_1, \) and \( N_{\mathcal{R}D} \triangleq N_2 \). Then, by performing some algebraic manipulations in (3), the end-to-end SNR can be expressed as

\[
\gamma_{\text{end}} = \frac{(\alpha(h_1)/N_0)(\alpha(h_2)/N_0)}{(\alpha(h_2)/N_0) + (1/G^2 N_0)}.
\]

where \( \alpha(k) = \|k\|_F^2 \). When fixed-gain relays are used, the gain \( G \) established in the connection is given by \( G^2 = 1/(CN_0) \), whereby \( C \) is a constant. Plugging the gain’s expression into (4), it follows that

\[
\gamma_{\text{end}} = \frac{\gamma_1 \gamma_2}{C + \gamma_2},
\]

where \( \gamma_i = \alpha(h_i)/N_0 \) represents the instantaneous SNR of the \( i \)-th hop \((i = 1, 2) \). The PDF and CDF of \( \gamma_i \) are respectively given by

\[
f_{\gamma_i}(\gamma_i) = \frac{m_i^{N_i m_i} \gamma_i^{N_i m_i - 1}}{\Gamma(N_i m_i) \gamma_i^{N_i m_i}} \exp\left(-\frac{m_i \gamma_i}{\gamma_i}\right),
\]

\[
F_{\gamma_i}(\gamma_i) = 1 - \frac{\Gamma(N_i m_i, m_i \gamma_i/\gamma_i)}{\Gamma(N_i m_i)},
\]

whereby \( \gamma_i = E(\gamma_i) \), \( \gamma_i = \gamma_1 \) when \( m_i = m_1 \) and \( N_i = N_1 \), or \( \gamma_i = \gamma_2 \) when \( m_i = m_2 \) and \( N_i = N_2 \). The factor \( m_i \) denotes the Nakagami-\( m \) parameter of the \( i \)-th hop and is related to the number of multipath clusters that arrive at the receiver. As the value of \( m_i \) increases, the severity of the fading decreases. When \( m_i = 1 \), the hops experience Rayleigh fading. For integer values of \( N_i, m_i \), (7) can be rewritten based on [11, Eq. 8.352.2] as

\[
F_{\gamma_i}(\gamma_i) = 1 - \exp\left(-\frac{m_i \gamma_i}{\gamma_i}\right) \sum_{x=0}^{N_i m_i - 1} \frac{1}{x!} \left(\frac{m_i \gamma_i}{\gamma_i}\right)^x.
\]

In the sequel, considering dual-hop fixed-gain relaying systems with beamforming and operation over Nakagami-\( m \) fading channels, closed-form expressions for the outage probability, probability density function, moment generating function, and generalized moments of the end-to-end SNR are derived.

### III. END-TO-END PERFORMANCE

#### A. Generalized Moments of the End-to-End SNR

The generalized moments of the end-to-end-SNR can be expressed as

\[
E(\gamma_{\text{end}}^n) = \frac{\Gamma(n + N_1 m_1) C^{N_1 m_2} N_2^{m_2}}{\Gamma(N_1 m_1) \Gamma(N_2 m_2)} \left(\frac{\gamma_2}{C m_2}\right)^n (n + N_1 m_2).
\]

**Proof:** The proof is given in Appendix I.

#### B. Outage Probability

An important performance measure in noise-limited systems is the outage probability (OP), which is defined as the probability that the received signal falls below a certain threshold, \( \gamma_b \). This threshold is a protection value for the SNR, above which the quality-of-service is deemed satisfactory. In the system under study, the OP can be expressed in closed-form according to

\[
F_{\gamma_{\text{end}}}(\gamma_b) = 1 - \frac{2 m_1^{N_2 m_2} e^{-\frac{m_1 \gamma_b}{\gamma_1}}}{\gamma_2 m_1 \gamma_1} \sum_{x=0}^{N_1 m_1 - 1} \frac{m_1^{x \gamma_b}}{\gamma_1^x} \sum_{l=0}^{x} \binom{x}{l} \left(\frac{C m_2^{l \gamma_2}}{\gamma_2^{l \gamma_2}}\right).
\]

**Proof:** The proof is provided in Appendix II.

#### C. Average Bit Error Rate

The well-known MGF-based approach [14] is a simple and efficient method to evaluate the average BER in fading scenarios. Thus, in order to provide a powerful tool for the calculation of the average BER, a closed-form expression for
Given that the latter changes slowly (relative to the entries of \( h \)) due to their low complexity and easy of deployment as assisted relaying schemes, making therefore the use of semi-blind relays attractive from a practical standpoint, especially due to their low complexity and easy of deployment as compared to CSI-assisted relays\(^1\). Accordingly, knowing that the entries of \( h \) undergo Nakagami-\( m \) fading, the parameter \( C \) can be obtained after performing the required statistical averaging in (15) as

\[
C = \bar{\gamma}_1^{N_t} \left( m_1 N_t m_1 \gamma_1 \right) \left( 1 - N_t m_1, m_1 \gamma_1 \right)^{-1}. \tag{16}
\]

In the graphics shown next, we use (16) to compute the relay gain.

**IV. NUMERICAL EXAMPLES AND DISCUSSIONS**

In this section, we present illustrative numerical examples for the performances metrics obtained previously. Monte Carlo simulation results are also provided in order to validate our analysis. Importantly, a very good agreement between the analytical and simulated curves is attested in all the cases.

Fig. 2 depicts the average end-to-end SNR, obtained from (9) by setting \( n = 1 \), against the average SNR per hop \( (\bar{\gamma}_2 = \bar{\gamma}_1) \). The influence of the Nakagami-\( m \) fading parameters on the performance is analyzed considering the \( N_t = N_r = 3 \) antenna configuration. It can be observed that when one of the hops experiences Rayleigh fading and the other undergoes better fading conditions, higher average end-to-end SNRs are attained when Rayleigh fading characterizes the link \( R - D \). This is because the relay gain applied to the received signal, which is obtained from the inverse of (16), is higher when \( m_1 = 2 \) and \( m_2 = 1 \) compared to the case when \( m_1 = 1 \) and \( m_2 = 2 \). Note that (16) depends on the statistics of the first hop only. Interestingly, when both links have fading conditions better than Rayleigh, \( \bar{\gamma}_{\text{end}} \) is approximately the same for high values of \( \bar{\gamma}_1 = \bar{\gamma}_2 \), independently on the values of the fading parameters.

Fig. 3 plots the OP, given in (10), against \( \bar{\gamma}_1 \) for a threshold value \( \gamma_{th} = -10 \) dB. The number of antennas is set to \( N_t = N_r = 3 \) and the hops have fading parameters given by \( m_1 = m_2 = 2 \). Here, the power imbalance between the hops is investigated. Note that such imbalance may be either beneficial or harmf ul for the overall system performance. Indeed, for \( \bar{\gamma}_2 > \bar{\gamma}_1 \), it is advantageous as compared to the balanced case, otherwise, it is detrimental. Other threshold values were also analyzed and the same conclusions are drawn.

Fig. 4 and 5 sketch the average BER of binary DPSK modulation, obtained using (11) according to \( 1/2M_{\text{end}}(1) \), against the average SNR per hop \( (\bar{\gamma}_2 = \bar{\gamma}_1) \). In the curves, we consider \( m_1 = m_2 = 2 \) (Fig. 4), \( m_1 = 1, m_2 = 1.5 \) (Fig. 5), and analyze the influence of the antenna configurations on the system performance. As expected, a substantial improvement in performance can be attained from the increase of the number of antennas at the source and destination nodes. The other two performance metrics, namely, OP and average end-to-end SNR, were also analyzed and the same conclusions were drawn.

\(^1\)Note that semi-blind relays do not require access to instantaneous CSI of the first hop. Indeed, only statistical CSI about the first hop is needed for the gain’s evaluation, namely knowledge of the average fading power \( E[|h_1|^2] \). Given that the latter changes slowly (relative to \( h \)), systems with semi-blind relays do not require continuous monitoring of the channel as it is the case in CSI-assisted relaying.
Fig. 2. Average end-to-end SNR against the average SNR per hop ($\bar{\gamma}_2 = \gamma_1$) of a beamforming based dual-hop semi-blind relaying system with $N_t = N_r = 3$.

Fig. 3. Outage probability against $\gamma_1$ for a threshold $\gamma_t = -10$ dB of a beamforming based dual-hop semi-blind relaying system with balanced or unbalanced links ($N_t = N_r = 3$, $m_1 = m_2 = 2$).

Fig. 4. Average BER against the average SNR per hop ($\bar{\gamma}_2 = \gamma_1$) for binary DPSK in a beamforming based dual-hop semi-blind relaying system with $m_1 = m_2 = 2$.

Fig. 5. Average BER against the average SNR per hop ($\bar{\gamma}_2 = \gamma_1$) for binary DPSK in a beamforming based dual-hop semi-blind relaying system with $m_1 = 1$, $m_2 = 1.5$.

APPENDIX I

By definition, the $n$-th moment of $\gamma_{\text{end}}$ in (5) is given by

$$E(\gamma_{\text{end}}^n) = \int_0^\infty \int_0^\infty \left( \frac{\gamma_1 \gamma_2}{C + \gamma_2} \right)^n f_{\gamma_1}(\gamma_1) f_{\gamma_2}(\gamma_2) d\gamma_1 d\gamma_2.$$  \hspace{1cm} (17)

Then, by substituting appropriately (6) in (17), $E(\gamma_{\text{end}}^n)$ can be written as

$$E(\gamma_{\text{end}}^n) = \frac{m_1^{N_t} m_2^{N_r} m_1^{N_t} m_2^{N_r}}{\Gamma(N_t m_1) \Gamma(N_r m_2)} \frac{\gamma_1^{N_t} \gamma_2^{N_r}}{\gamma_2^{N_t + N_r - 1}} \exp \left( -\frac{m_2 \gamma_2}{\gamma_2} \right) \frac{\gamma_1^{N_t} \gamma_2^{N_r}}{\gamma_1^{N_t} \gamma_2^{N_r - 1}} \exp \left( -\frac{m_1 \gamma_1}{\gamma_1} \right) d\gamma_1.$$  \hspace{1cm} (18)

The first integral in (18), i.e., with respect to $\gamma_2$ and denoted $I_2$, can be solved by expressing its integrands in terms of Meijer’s G-functions $^2$ \cite[Eqns. 10 and 11]{15}, namely, using (1 + $C / \gamma_2$) $^{-n}$ = \(1 / \Gamma(n)\) $G_{1,1}^{1,1}[\gamma_2 / C]_{n}^{1}$ and $\exp(-m_2 \gamma_2 / \gamma_2)$ $= G_{0,1}^{1,0}[m_2 \gamma_2 / \gamma_2]_{0}^{1}$, thus yielding

$$I_2 = \int_0^\infty \frac{\gamma_2^{N_r - m_2 - 1}}{\Gamma(n)} G_{1,1}^{1,1} \left[ \frac{\gamma_2}{C} \middle| \frac{1}{n} \right] G_{0,1}^{1,0} \left[ \frac{m_2 \gamma_2}{\gamma_2} \middle| 0 \right] d\gamma_2.$$  \hspace{1cm} (19)

Now, knowing that the integral of the product of a power term and two Meijer’s G-functions is also a Meijer’s G-function \cite[Eq. 21]{15}, (19) can be expressed as

$$I_2 = \frac{C^{N_r, m_2}}{\Gamma(n)} G_{2,1}^{1,2} \left[ \frac{\gamma_2}{C m_2} \middle| 1, 1 + N_r m_2 \right].$$  \hspace{1cm} (21)

The second integral in (18), i.e., with respect to $\gamma_1$ and denoted $I_1$, can be easily solved using \cite[Eq. 2.33]{11} according to

$$I_1 = \Gamma(n + N_t m_1) (\frac{\gamma_1}{m_1})^{N_t + N_r m_1}.$$  \hspace{1cm} (22)

Thus, performing the product of (21) with (22), as required into (18), the generalized moments of the end-to-end SNR can be obtained as shown in (9).

APPENDIX II

From our system model, the OP can be calculated as

$$F_{\gamma_{\text{end}}} (\gamma_t) = \int_0^\infty \Pr \left[ \frac{\gamma_1 \gamma_2}{\gamma_1 + C} \leq \gamma_t \right] f_{\gamma_2}(\gamma_2) d\gamma_2.$$
(20) and with some algebraic manipulations, the MGF can be obtained by differentiating (10) with respect to \( \gamma_1 \) and using 
\begin{align}
\frac{d}{d\gamma_1} \mathcal{F}_{\gamma_{th}}(\gamma_1) = \frac{\gamma_1}{\gamma_2} \mathcal{F}_{\gamma_{th}}(\gamma_1) \cdot \frac{d}{d\gamma_1} \mathcal{F}_{\gamma_{th}}(\gamma_1) \cdot \frac{d}{d\gamma_2} \mathcal{F}_{\gamma_{th}}(\gamma_1) \cdot \frac{d}{d\gamma_2} \mathcal{F}_{\gamma_{th}}(\gamma_1)
\end{align}

Then, by substituting the CDF of \( \gamma_1 \), obtained from (8), and the PDF of \( \gamma_2 \), expressed using (6), into (23), and performing some algebraic manipulations, it follows that
\begin{align}
F_{\gamma_{th}}(\gamma_{th}) = 1 - \frac{m_2 N_r x}{x! \gamma_1} \sum_{x=0}^{N_r-1} \frac{x! \gamma_2}{\gamma_2} \int_0^\infty \left( 1 + \frac{C}{\gamma_2} \right) x \gamma_2 \exp \left( \frac{m_2 \gamma_2}{\gamma_2} - \frac{m_1 \gamma_1}{\gamma_1} \left( 1 + \frac{C}{\gamma_2} \right) \right) d\gamma_2.
\end{align}

Then, expressing the integrand \((1 + C/\gamma_2)^x\) in terms of finite sums, based on the binomial theorem [12, Eq. 3.1.1], and with the help of [11, Eq. 3.471.9], a closed-form expression for the OP can be derived as shown in (10).

APPENDIX III

The MGF can be defined as \( M_{\gamma_{th}}(s) = E(\exp(-s\gamma_{th})) \). For performing the statistical average over \( \gamma_{th} \), note that the PDF of the end-to-end SNR is required. This latter can be obtained by differentiating (10) with respect to \( \gamma_{th} \), which results in (20) shown on the top of this page. For the single-antenna case (\( N_r = 1 \)) with Rayleigh fading (\( m_1 = m_2 = 1 \)), (20) reduces exactly to [3, Eq. 10]. From (20) and with some algebraic manipulations, the MGF can be formulated as in (11), where
\begin{align}
I_1 = \int_0^\infty \frac{2 \gamma + m_2 - 1}{\gamma_1} \exp \left( -\gamma \left( s + \frac{m_1}{\gamma_1} \right) \right) d\gamma,
\end{align}

\begin{align}
I_2 = \int_0^\infty \frac{2 \gamma + m_2 - 1}{\gamma_1} \exp \left( -\gamma \left( s + \frac{m_1}{\gamma_1} \right) \right) d\gamma.
\end{align}

\begin{align}
I_3 = \int_0^\infty \frac{2 \gamma + m_2 - 1}{\gamma_1} \exp \left( -\gamma \left( s + \frac{m_1}{\gamma_1} \right) \right) d\gamma.
\end{align}

Now, using the identity given in [11, Eq. 6.643.3] for solving the three integrals above, Eqs. (12), (13) and (14) can be obtained.

REFERENCES