Hamiltonian graphs involving neighborhood conditions

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October 5, 2006

Abstract

Let $G$ be a graph on $n$ vertices, $\delta$ and $\alpha$ be the minimum degree and independence number of $G$, respectively. In this paper we prove that if $G$ is a 2-connected graph and $|N(x) \cup N(y)| \geq n - \delta - 1$ for each pair of nonadjacent vertices $x, y$ with $1 \leq |N(x) \cap N(y)| \leq \alpha - 1$, then $G$ is hamiltonian or $G \in \{G_{n-1}^{\ast}, K_{n+1}/2, K_2^{\ast} \vee 3K_{n-2}/4\}$ where $K_2^{\ast}$ and $G_{n-1}^{\ast}$ are subgraphs on 2 and $\frac{n-1}{2}$ vertices respectively. As a corollary, if $G$ is a 2-connected graph and $|N(x) \cup N(y)| \geq n - \delta$ for each pair of nonadjacent vertices $x, y$ with $1 \leq |N(x) \cap N(y)| \leq \alpha - 1$, then $G$ is hamiltonian. It extends the following two theorems by Faudree et al and Yin, respectively.

If $G$ is 2-connected graph and $|N(u) \cup N(v)| \geq n - \delta$ for each pair of nonadjacent vertices $u, v \in V(G)$, then $G$ is hamiltonian.

If $G$ is 2-connected graph and $|N(u) \cup N(v)| \geq n - \delta$ for each pair of nonadjacent vertices $u, v$ with $d(u, v) = 2$, then $G$ is hamiltonian.