TAS scheme cannot satisfy the target BER for most of the SNR region, whereas the BER of the suboptimal scheme is still close to the scheme with an ideal PA. As SNR increases, the BER of the unconstrained TAS scheme becomes worse, because high-level QAM is more vulnerable to the nonlinearity of the PA, whereas the BER of the suboptimal TAS scheme is still reliable, regardless of the SNR region, compared with the target BER.

VI. CONCLUSION

In this paper, we have proposed optimal and suboptimal TAS schemes under a power-balancing constraint when the system exploits the transmit diversity gain over subcarriers to improve the reliability for TAS-OFDM systems. To overcome the high computational complexity of the optimal method, the outage subcarriers are considered only for diversity combining in the suboptimal scheme. Sequential search of the suboptimal method can reduce the heavy burden required to find the groups of combined subcarriers by exhaustive search in the optimal method. We showed, through the selected numerical examples, that the suboptimal method offers the spectral efficiency close to the optimal method while maintaining much lower complexity. Moreover, it outperforms the optimal TAS without diversity combining for a low-SNR region. We note that the suboptimal method can equally be deployed for the case of multiple receive antennas, which are used for the receiver diversity combining.

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CM-Based Blind Equalization of Time-Varying SIMO-FIR Channel With Single Pulsation Estimation

Dezhong Peng, Member, IEEE,
Yong Xiang, Zhang Yi, Senior Member, IEEE, and Shui Yu, Member, IEEE

Abstract—It is known that the constant modulus (CM) property of the source signal can be exploited to blindly equalize time-invariant single-input–multiple-output (SIMO) and finite-impulse-response (FIR) channels. However, the time-invariance assumption about the channel cannot be satisfied in several practical applications, e.g., mobile communication. In this paper, we show that, under some mild conditions, the CM criterion can be extended to the blind equalization of a time-varying channel that is described by the complex exponential basis expansion model (CE-BEM). Although several existing blind equalization methods that are based on the CE-BEM have to employ higher order statistics to estimate all nonzero channel pulsations, the CM-based method only needs to estimate one pulsation using second-order statistics, which yields better estimation results. It also relaxes the restriction on the source signal and is applicable to some classes of signals with which the existing methods cannot deal.

Index Terms—Blind channel equalization, complex exponential basis expansion model (CE-BEM), constant modulus (CM), time-varying (TV) channel.

I. INTRODUCTION

Time-varying (TV) finite-impulse-response (FIR) channels are often encountered in several practical communication systems, e.g., mobile communication [1]–[4]. The complex exponential basis expansion
model (CE-BEM) has widely been used to describe the TV FIR channels [5]–[9]. This model is particularly useful when the receivers are constantly moving and the transmitter and a few dominant reflectors are stationary [5], [8], [9]. Hilly and suburban terrain environments are some typical examples of such a scenario [8]. It is justified in [5] and [6] that the CE-BEM is effective in tracking the variations in land mobile channels.

Based on the CE-BEM, several algorithms are developed for the blind equalization of TV FIR channels with single input [5]–[9], among which the algorithms in [5]–[8] depend on the accurate knowledge of all nonzero channel pulsations. Although channel pulsations can be obtained using the estimation methods in [5] and [8], these estimation methods rely on the computation of the higher order statistics (HOS) of the channel output and require the source signal to have nonzero fourth-order moments. As a result, they are costly in computation to get good estimation results and are not applicable to some classes of signals, e.g., 8 phase-shift keying (8-PSK) signals [5]. Recently, a second-order statistics (SOS)-based approach has been proposed to estimate the pulsations [1]. However, its computational complexity dramatically rises as the number of pulsations increases. In [9], Bai et al. propose a blind channel equalization algorithm based on decision feedback techniques. However, this algorithm can only equalize a TV single-input–multiple-output (SIMO) FIR channel that is driven by a differential phase-shift keying (DPSK) signal.

This paper deals with the direct blind equalization of a TV SIMO-FIR channel that is driven by a signal with a constant modulus (CM) constellation, which is more general than the DPSK signal in [9]. It is well known that the CM property of digitally modulated signals can be exploited to recover source signals from their instantaneous or convolutive mixtures. Since Sato’s pioneering work on this topic [11], a large number of CM algorithms have been proposed for blind channel equalization [11]–[18]. For example, the CM approach has recently been applied to the blind detection of code-division multiple access (CDMA) signals in multipath channels [12]–[17] and the blind equalization of nonirreducible channels [18]. Although the methods in [15]–[17] are applicable to TV channels based on constrained optimization techniques, they can only cope with a particular kind of source signals, i.e., CDMA signals, and rely on the information of the signature sequences or the spreading codes of communication users.

In this paper, we show that, based on the CE-BEM, the zero-forcing (ZF) equalization can be achieved using the CM criterion. Compared with the existing methods in [5]–[9] and [12]–[17], the proposed CM-based equalization method only needs to estimate one pulsation using SOS, resulting in a more accurate estimation for pulsations, and can be applied to a wider range of source signals due to the relaxation of constraints on the source signal.

II. CHANNEL MODEL

Let us consider a TV SIMO-FIR channel system with M outputs, i.e.,

$$\mathbf{x}(k) = \sum_{l=0}^{L} \mathbf{h}(k;l) s(k-l) + \mathbf{v}(k)$$

where $\mathbf{x}(k) = [x_1(k), x_2(k), \ldots, x_M(k)]^T$ denotes the output vector, $s(k)$ is the source signal with CM constellation, $\mathbf{v}(k) = [v_1(k), v_2(k), \ldots, v_M(k)]^T$ is the vector with M additive white Gaussian noise signals, independent of $s(k)$, $\mathbf{h}(k;l) = [h_1(k;l), h_2(k;l), \ldots, h_M(k;l)]^T$ is the TV impulse response vector, and the superscript $^T$ stands for the transpose operation. In the CE-BEM, the TV impulse response $h_m(k;l)$ can be described as a linear combination of some complex exponential basis functions $\{e^{j\omega_1 k}, e^{j\omega_2 k}, \ldots, e^{j\omega_Q k}\}$ [7], i.e.,

$$h_m(k;l) = \sum_{q=1}^{Q} h_{m,q}(l)e^{j\omega_q k}$$

$$l = 0, 1, \ldots , L \quad m = 1, 2, \ldots, M$$

where $h_{m,q}(l)$ are some time-invariant coefficients, and $\omega_1, \omega_2, \ldots, \omega_Q$ are distinct real numbers called pulsations (or basis frequencies). Stacking $K + 1$ successive observations of the channel outputs yields

$$\mathbf{x}(k) = \left[ x^T(k), x^T(k-1), \ldots, x^T(k-K) \right]^T.$$

To proceed, we define the $M \times Q$ matrices $\mathbf{H}_l (l = 0, 1, \ldots, L)$, the $(M(K + 1) \times Q + 1)$ matrix $\mathbf{H}$, and the $(Q(P+1) \times Q(P+1))$ diagonal matrix $\mathbf{C}(k)$ as follows:

$$\mathbf{H}_l = \begin{bmatrix} e^{j\omega_1 l h_{1,1}}(l) & e^{j\omega_1 l h_{1,2}}(l) & \cdots & e^{j\omega_1 l h_{1,Q}}(l) \\ e^{j\omega_2 l h_{2,1}}(l) & e^{j\omega_2 l h_{2,2}}(l) & \cdots & e^{j\omega_2 l h_{2,Q}}(l) \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\omega_Q l h_{Q,1}}(l) & e^{j\omega_Q l h_{Q,2}}(l) & \cdots & e^{j\omega_Q l h_{Q,Q}}(l) \end{bmatrix}$$

$$ \mathbf{H}_l = \begin{bmatrix} \mathbf{H}_0 & \ldots & \mathbf{H}_L & \mathbf{0}_{M \times Q} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{0}_{M \times Q} & \ldots & \mathbf{H}_0 & \mathbf{H}_L \end{bmatrix}$$

$$\mathbf{C}(k) = \text{diag} \left( e^{j\omega_1 k}, e^{j\omega_2 k}, \ldots, e^{j\omega_Q k} \right)$$

$$\left( \begin{array}{c} \ldots, e^{j\omega_1 (k-P)}, \ldots, e^{j\omega_Q (k-P)} \end{array} \right)$$

where $P = K + L$. In the noiseless case, based on (1)–(5), it follows that

$$\tilde{x}(k) = \mathbf{H} \mathbf{C}(k) \tilde{s}(k)$$

where $\tilde{s}(k) = [s(k), \ldots, s(k), \ldots, s(k-P), \ldots, s(k-P)]^T$.

We will show in the next section that the CM criterion can be used to blindly equalize the TV SIMO-FIR channel in (1) under the following three assumptions.

A. The source signal $s(k)$ is temporally white and has $S = 2^R$ constellation points, which are uniformly distributed on the unit circle, where $R \geq 2$ is a positive integer.

B. The pulsations satisfy the relationship $0 = \omega_1 < \omega_2 < \ldots < \omega_Q$, and all possible differences of any two different pulsations in the set $\{\omega_1, \omega_2, \ldots, \omega_Q\}$ are distinct, i.e., $\omega_{q_2} - \omega_{q_1} \neq \omega_{q_3} - \omega_{q_4}$ if $q_1 \neq q_3$ or $q_2 \neq q_4$.

C. At least one column in the matrix $\mathbf{H}$ defined in (4) is not a linear combination of any other columns.

Assumption A is reasonable and in line with the CM signals used in practical communication systems. This assumption implies that the constellation points of the source signal $s(k)$ are from the following set:

$$\Omega = \left\{ e^{j\theta}, e^{j(2\pi/S + \theta)}, \ldots, e^{j(2(S-1)\pi/S + \theta)} \right\}$$

where $\theta$ is a positive rotation angle. Assumption B is easy to satisfy in a practical communication system and is also used in [5], [7], and [8]. Assumption C is another mild constraint on the channels.
III. Blind Equalization Using the Constant Modulus Criterion

Let \( g \) be an \( M(K + 1) \)-dimensional equalization vector and define

\[
y(k) = g^T \hat{x}(k).
\]

(8)

Our objective is to develop a ZF equalizer with the equalization vector \( \hat{g} \) that satisfies

\[
g^T H = [0, \ldots, 0, \zeta, 0, \ldots, 0] \quad \text{and} \quad \zeta \neq 0.
\]

(9)

Clearly, assumption C ensures the existence of the aforementioned optimum equalization vector \( \hat{g} \). To find the desired equalization vector \( \hat{g} \), the well-known CM criterion is shown here [11] as

\[
J(g) = \frac{1}{4} E \left\{ |y(k)|^2 - 1 \right\}^2.
\]

(10)

By minimizing the function \( J(g) \), a stochastic gradient algorithm has been developed as [11]

\[
g(k + 1) = g(k) - \mu \left( |y(k)|^2 - 1 \right) y(k) \hat{x}^*(k)
\]

(11)

where \( \mu > 0 \) is the step size, and the superscript * denotes the complex conjugate operation. Convergence analysis about the aforementioned CM algorithm has been well established in [19] and [20], which have proven that the vector \( g(k) \) in the algorithm (11) will converge to the minimum points set \( \hat{g} = \{g||y(k)| = 1\} \) of the cost function \( J(g) \). However, it is still an open question whether any point in the set \( \mathcal{G} \) can be taken as the ZF equalization vector \( \hat{g} \) for the TV SIMO-FIR system (1).

To find the answer to this question, we need the following lemma.

**Lemma 1:** If, for any \( P \) angles \( \theta_1, \theta_2, \ldots, \theta_P \) from \( \{\theta, 2\pi/S + \theta, \ldots, 2(S-1)\pi/S + \theta\} \), we have

\[
\sum_{p_1, p_2=0}^{P} d_{p_1, p_2} \bar{e}^{j\theta_{p_1} - \theta_{p_2}} = \rho
\]

(12)

where \( \rho \) and \( d_{p_1, p_2} \) are some constants, then \( d_{p_1, p_2} = 0 \) for any \( p_1 \neq p_2 \).

**Proof:** According to assumption A, we have \( \{\theta, \pi/2 + \theta, \pi + \theta\} \subset \{\theta, 2\pi/S + \theta, \ldots, 2(S-1)\pi/S + \theta\} \). Thus, we can assume that the values of \( \theta_0, \theta_1, \ldots, \theta_P \) are taken from the aforementioned seven cases.

- **Case 1:** \( \theta_k = \theta (k = 0, \ldots, P) \).
- **Case 2:** \( \theta_k = \pi + \theta \) and \( \theta_k = \theta (k \neq u) \).
- **Case 3:** \( \theta_k = \pi/2 + \theta \) and \( \theta_k = \theta (k \neq v) \).
- **Case 4:** \( \theta_k = \theta \) and \( \theta_k = \theta (k \neq u, v) \).
- **Case 5:** \( \theta_k = \pi + \theta \) and \( \theta_k = \theta (k \neq v) \).
- **Case 6:** \( \theta_k = \theta + \pi \) and \( \theta_k = \theta (k \neq u, v) \).
- **Case 7:** \( \theta_k = \pi/2 + \theta \), \( \theta_k = \pi + \theta \), and \( \theta_k = \theta (k \neq u, v) \),

where \( u < v \).

Substituting the values of \( \theta_0, \theta_1, \ldots, \theta_P \) in the aforementioned seven cases into (12), we obtain the following matrix equation:

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & -1 & 1 & 1 \\
-j & j & 1 & 1 & j & 1 & -j \\
1 & 1 & 1 & j & j & j & -j \\
-1 & -1 & -1 & -1 & 1 & -1 & -1 \\
1 & 1 & 1 & -1 & -1 & -1 & 1 \\
-j & j & 1 & j & 1 & j & 1 \\
\end{bmatrix}
\begin{bmatrix}
d_{u,v} \\
d_{u,v} \\
\xi_1 - \rho \\
\xi_1 \\
\xi_2 \\
\xi_3 \\
\xi_4 \\
\end{bmatrix}
= \mathbf{0}.
\]

(13)

Here, \( \xi_0 = \sum_{p_1, p_2=0}^{P} d_{p_1, p_2} + d_{u,v} + d_{u,v} \), \( \xi_1 = \sum_{p_2=0}^{P} d_{u,v} \), \( \xi_2 = \sum_{p_1, p_2=0}^{P} d_{u,v} \), \( \xi_3 = \sum_{p_1=0}^{P} d_{p_1, u,v} \), and \( \xi_4 = \sum_{p_1, p_2=0}^{P} d_{p_1, u,v} e^{j(\theta_{p_1} - \theta_{p_2})} \). It is easy to verify that the left matrix in (13) is of full column rank. This case yields \([d_{u,v}, d_{u,v}, \xi_0 - \rho, \xi_1, \xi_2, \xi_3, \xi_4]^{T} = 0_{7 \times 1} \), which leads to \( d_{u,v} = d_{v,u} = 0 \). This result completes the proof.

Based on Lemma 1, we propose the following theorem.

**Theorem 1:** If assumptions A and B hold, then any vector \( g \in \mathcal{G} = \{g||y(k)| = 1\} \) ensures that \( g^T H \) has only one nonzero entry.

**Proof:** See the Appendix.

The aforementioned theorem clearly shows that any vector in the set \( \mathcal{G} \) can be taken as the ZF equalization vector \( \hat{g} \). After obtaining \( \hat{g} \), the equalizer output can be computed by \( y(k) = g^T \hat{x}(k) \). Based on (5), (6) and (9), it can be further expressed as

\[
y(k) = g^T H C \tilde{x}(k) = c_s(k - \tau_0) e^{j\omega_q(k - \tau_0)}
\]

(14)

where \( \omega_q \in \{\omega_1, \omega_2, \ldots, \omega_Q\} \), and \( 0 \leq \tau_0 \leq L + K \). Obviously, \( y(k) \) is not the estimate of the source signal \( s(k) \). The remaining task is to estimate the unknown pulsation \( \omega_q \) and then remove \( e^{j\omega_q k} \) from \( y(k) \) to recover the source signal \( s(k) \).

We propose the following scheme to estimate \( \omega_q \). Let us consider any given channel output \( x_m(k)(1 \leq m \leq M) \). Because the source signal \( s(k) \) is temporally white and independent of the noise signal \( v_m(k) \), it holds, based on (1), (2) and (14), that

\[
E \left[ y(k) \cdot \sum_{\tau=0}^{K} \bar{z}_m(k - \tau) \right] = E \left[ \left\{ \sum_{r=0}^{K} \sum_{l=0}^{L} \sum_{q=1}^{Q} \bar{c}_m(k - r) \bar{c}_m(k - l) e^{j(\omega_q(k - r) - \omega_q(k - l))} \cdot s(k - \tau_0) s(k - \tau_0) e^{j(\omega_q(k - r) - \omega_q(k - l))} \right\} \right]
\]

\[
= E \left[ \sum_{r=0}^{K} \sum_{q=1}^{Q} \bar{c}_m(k - r) e^{j(\omega_q(k - r) - \omega_q(k - l))} \right] = \sigma_q^2 \sum_{q=1}^{Q} \beta_q \cdot e^{j(\omega_q - \omega_q) k}
\]

(15)

where \( \beta_q = \sum_{r=0}^{K} \bar{c}_m(k - r) e^{j(\omega_q(k - r) - \omega_q(k - l))} (q = 1, 2, \ldots, Q) \) and \( \sigma_q^2 = E[s(k) s^*(k)] \) are some constants. Based on (15), we can determine the frequency set \( \{\omega_q = \omega_q^1, \ldots, \omega_q^Q\} \) from the significant peaks in the Fourier series of the TV SOS \( E[y(k) \cdot \sum_{r=0}^{K} \bar{z}_m(k - \tau)] \), which can be computed by using the sample estimates of cyclic moments [10]. Because \( \omega_0 = 0 \) according to assumption B, the largest element in the set \( \{\omega_q - \omega_0\} \) is \( \omega_q = \omega_1 \) in theory and is the estimate of \( \omega_q \) in practice. Finally, the estimate of the source signal \( s(k) \) can be obtained by \( \hat{s}(k) = y(k) \cdot e^{-j\omega_1 k} \). Based on (14), we have \( s(k) = \xi_0 - \rho, s(k - \tau_0), s(k - \tau_0) e^{j\omega_q k} \), which means that the source signal \( s(k) \) is recovered up to a constant scalar and a fixed delay.

In summary, the CM-based blind equalization algorithm for TV SIMO-FIR channels is formulated as follows.

1. **Step 1.** Find the ZF equalization vector \( \hat{g} \) that satisfies (9) through algorithm (11), and compute \( y(k) = g^T \hat{x}(k) \).
2. **Step 2.** Compute the Fourier series of \( E[y(k) \cdot \sum_{r=0}^{K} \bar{z}_m(k - \tau)] \) by using the estimation method in [10] and then determine the set \( \{\omega_q - \omega_q^1\} \) from the significant peaks of the obtained Fourier series.
In [5] and [6], these channel pulsations are estimated by computing the fourth-order moment $E[x_m^4(k)]$ of any given channel output $x_m(k)$. In contrast, the proposed blind equalization method only estimates one pulse $\omega_q$ based on the SOS, $E[y(k) \cdot \sum_{r=0}^{K} x_m^r(k - \tau)]$. Table I compares the computational complexities of the proposed method and the methods in [5] and [6] in estimating the channel pulsations. Here, $N_1$ and $N_2$ are the numbers of samples that were used by the methods in [5] and [6] and the proposed method, respectively.

It is known that, because SOS generally has lower variance than HOS with the same number of data samples, the SOS-based approaches usually require much less samples to get accurate estimation results than the HOS-based approaches. For example, let us consider a first-order TV channel with four outputs and the following three pulsations: 1) $\omega_1 = 0$; 2) $\omega_1 = 2\pi/60$; and 3) $\omega_2 = 2\pi/20$. The channel is driven by a 4 quadrature-amplitude modulation (4-QAM) signal, and we choose $K = 2$. The top and bottom of Fig. 1 shows the Fourier series of $E[y(k) \cdot \sum_{r=0}^{K} x_m^r(k - \tau)]$ and $E[x_m^4(k)]$, respectively, where 1200 samples are used. In this figure, we mark the frequencies that are needed in estimating pulsations by squares. It is easy to see that, with the same number of samples, our method possesses higher accuracy than the approaches in [5] and [6]. For example, the rightmost square in the top of Fig. 1, stands for $\omega_2 - \omega_3 = \omega_2$, which can easily be identified to estimate $\omega_2$. On the contrary, the rightmost square in the bottom of Fig. 1, stands for $4\omega_2$, which can give the estimation for the pulsation $\omega_2$. However, it is overwhelmed by clutters.

In other words, the methods in [5] and [6] need much more samples than our method to achieve the same estimation accuracy, i.e., $N_1 \gg N_2$ in Table I. As a result, the proposed method needs much less mathematical operations (particularly multiplications) in pulsation estimation than the methods in [5] and [6].

### IV. Numerical Simulations

**Example 1:** In the simulation, we consider the TV SIMO-FIR channel system that is of order 1 and has six outputs, i.e., $L = 1$ and $M = 6$. The time-invariant channel coefficients are randomly generated, and we choose $K = 1$. For the proposed CM-based algorithm, the initial equalization vector $g(0)$ is randomly chosen at each run, and the step size is $\mu = 0.021$. We examine the performance of the proposed algorithm with different numbers of channel pulsations. The 8-PSK signal is used as the source signal $s(k)$. Because $E[s^4(k)] = 0$ for the 8-PSK signal, the blind equalization methods in [5] and [6] cannot cope with this type of signal. At the same time, the blind equalization methods in [9] and [15]–[17] do not work either, because they are only applicable to the DPSK and CDMA signals, respectively. Dissimilarly, the CM-based algorithm can effectively deal with the aforementioned signal and is used to perform blind equalization.

![Fig. 2. BER with different numbers of pulsations.](image)

**Scenario 1.** A mobile receiver moves at a speed of 100 km/h, the carrier frequency is 900 MHz, and the bit rate is 20 kb/s.

**Scenario 2.** A mobile receiver moves at a speed of 115 km/h, the carrier frequency is 1.8 GHz, and the bit rate is 20 kb/s.

**Scenario 3.** A mobile receiver moves at a speed of 70 km/h, the carrier frequency is 1.8 GHz, and the bit rate is 40 kb/s.

### Table I: Comparison of Computational Complexities in Pulsation Estimation

<table>
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<tbody>
<tr>
<td>Multiplications</td>
<td>$4N_1^2 + N_1$</td>
</tr>
<tr>
<td>Additions</td>
<td>$N_1^2$</td>
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![Image](image)
equalization performance of the CM-based algorithm to the existing approaches.

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</table>

**Proof of Theorem 1**

For any vector \( \mathbf{g} \in \mathcal{G} \), let us define a \( Q(P + 1) \)-dimensional row vector \( \mathbf{c} \) as

\[
\mathbf{c} = \mathbf{g}^T \mathbf{H} = \begin{bmatrix} c_{1,0} & \cdots & c_{1,P} & \cdots & c_{Q,0} & \cdots & c_{Q,P} \end{bmatrix}
\]

(16)

where \( P = K + L \). To prove Theorem 1 by contradiction, we assume that \( \mathbf{c} = \mathbf{g}^T \mathbf{H} \) is a zero vector or has at least two nonzero entries \( c_{a,b} \) and \( c_{c,f} \). Clearly, this assumption holds if and only if one of the following four propositions is satisfied.

1. **Proposition 1**: \( c_{q,p} = 0 \) for any \( 1 \leq q \leq Q \) and \( 0 \leq p \leq P \).
2. **Proposition 2**: \( c_{a,b} \neq 0, c_{c,f} \neq 0, a \neq c, \) and \( b \neq f \).
3. **Proposition 3**: \( c_{a,b} \neq 0, c_{a,f} \neq 0, a \neq c, \) and \( b = f \).
4. **Proposition 4**: \( c_{a,b} \neq 0, c_{a,f} \neq 0, a = c, \) and \( b \neq f \).

Next, we will consider these propositions in order.

First, based on (6), (8) and (16), we have \( y(k) = \mathbf{g}^T \mathbf{H} \mathbf{s}(k) = \mathbf{c} \mathbf{s}(k) \), where \( \mathbf{s}(k) = \mathbf{C}(k) \mathbf{s}(k) \). Because \( \mathbf{g} \in \mathcal{G} \), \( |y(k)| = |\mathbf{c} \mathbf{s}(k)| = 1 \). Clearly, \( \mathbf{c} \) is not a zero vector, and thus, Proposition 1 is false.

Second, it results from assumption A and (7) that, for any \( \tilde{k} > P \), the following event occurs with a nonzero probability: \( s(k), s(\tilde{k} - 1), \ldots, s(\tilde{k} - P) = [e^{j\theta_0}, e^{j\theta_1}, \ldots, e^{j\theta_P}], \) where \( \theta_0, \theta_1, \ldots, \theta_P \) are any \( P \) angles from the set \{0, 2\pi/S + \theta, 2(2 - 1)\pi/S + \theta\}. Based on (6), (8), and (16), it follows that, for any \( \tilde{k} > P \), \( y(k) = \mathbf{g}^T \mathbf{H} \mathbf{s}(\tilde{k}) = \mathbf{c} \mathbf{s}(\tilde{k}) = \sum_{q=1}^{Q} \sum_{p=0}^{P} c_{q,p} e^{j\theta_p} e^{j\omega_q (\tilde{k} - p)} \). In addition, because \( \mathbf{g} \in \mathcal{G} \), we can obtain

\[
|y(k)|^2 = |\mathbf{g}^T \mathbf{H} \mathbf{s}(\tilde{k})|^2 = \gamma_0 + \sum_{q_1,q_2=1}^{Q} \gamma_{q_1,q_2} e^{j(\omega_{q_1} - \omega_{q_2})\tilde{k}} = 1
\]

(17)

for any \( \tilde{k} > P \),

\[
\gamma_0 = \sum_{q=1}^{Q} \left( \sum_{p_1,p_2=0}^{P} c_{q,p_1}^* c_{q,p_2} e^{(-\omega_{p_1} + \omega_{p_2})} e^{j(\theta_{p_1} - \theta_{p_2})} \right)
\]

and

\[
\gamma_{q_1,q_2} = \sum_{p_1,p_2=0}^{P} c_{q_1,p_1}^* c_{q_2,p_2} e^{(-\omega_{p_1} + \omega_{p_2})} e^{j(\theta_{p_1} - \theta_{p_2})}.
\]

(18)

Considering \( S \) indexes \( \tilde{k}_n = P + n(n = 1, 2, \ldots, S) \), it results, based on (17), that

\[
\gamma_0 - 1 + \sum_{q_1,q_2=1}^{Q} \gamma_{q_1,q_2} e^{j(\omega_{q_1} - \omega_{q_2})} = 0 \quad n = 1, 2, \ldots, S
\]

where \( S \geq Q^2 - Q + 1 \), and \( \omega_{q_1,q_2} = \omega_{q_1} - \omega_{q_2} \). The aforementioned \( S \) equations can be expressed in matrix form in (21), shown at the top of the next page, where \( q_1 \neq q_2 \), and

\[
\mathbf{v} = \begin{bmatrix} e^{j(\omega_{q_1} - \omega_{q_2})} & e^{j(\omega_{q_1} - \omega_{q_2})} P & e^{j(\omega_{q_1} - \omega_{q_2})} (P+1) & \cdots & e^{j(\omega_{q_1} - \omega_{q_2})} (P+Q-1) & e^{j(\omega_{q_1} - \omega_{q_2})} (P+Q-2) \\
\end{bmatrix}^T.
\]

Fig. 4. BER in different Doppler scenarios.

V. CONCLUSION

In this paper, we have investigated the feasibility of applying the CM criterion to the blind equalization of TV SIMO-FIR channels. A CM-based algorithm is proposed to equalize the TV SIMO-FIR channels that are described by the CE-BEM and to recover the source signal. Compared with the existing blind equalization methods, the proposed CM-based method relaxes the restriction on the source signal and only requires the source signal to have the CM property. As a result, it can deal with a wider range of applications. Furthermore, unlike several existing pulsation estimation methods, which use the higher order moments of the channel output to estimate all nonzero channel pulsations, the proposed algorithm only employs the SOS of the channel output to estimate one channel pulsation. This approach results in a more accurate estimation result. Simulation examples illustrate the superior
Based on assumption B, it can easily be shown that there are no equal elements in the set \( \Phi = \{1, \ldots, e^{j\omega_0 q_1 q_2}, e^{j\omega_2 q_1 q_2}, \ldots, e^{j\omega_{Q-1} q_1 q_2}, e^{j\omega_{Q-1} q_1 q_2} \} \). Because \( S \geq Q^2 - Q + 1 \), the left matrix in (21) is a Vandermonde matrix with a full column rank. Then, it yields that the vector \( v \) in (21) is all zero, i.e.,

\[
\gamma_0 = 1 \quad \text{and} \quad \gamma_{q_1 q_2} = 0 \quad \text{for any} \quad q_1 \neq q_2.
\]  

(19)

Based on (18) and (19), it follows that, for any \( q_1 \neq q_2 \), we have

\[
\gamma_{q_1 q_2} = \sum_{p_1, p_2=0}^{P} c_{q_1, p_1} c_{q_2, p_2} e^{j(\omega_{Q-1} q_1 q_2)p_1 + \omega_{Q-1} q_2 p_2} e^{j(\theta_{p_1} - \theta_{p_2})} = 0.
\]  

(20)

Note that (20) holds for any \( P \) angles \( \theta_0, \theta_1, \ldots, \theta_P \) from \( \{0, 2\pi/S + \theta, \ldots, 2(S-1)\pi/S + \theta \} \). Thus, using Lemma 1, we have \( c_{q_1, p_1} c_{q_2, p_2} = 0 \) for any \( p_1 \neq p_2 \) and \( q_1 \neq q_2 \), which contradicts Proposition 2. Therefore, Proposition 2 does not hold. Following the aforementioned analysis procedure, we can also prove that Proposition 3 does not hold.

Finally, because both Propositions 2 and 3 are false, we can deduce that, if \( c_{a, b} \neq 0 \), then \( c_{q, p} = 0 \) for any \( q \neq a \) and \( 0 \leq p \leq P \). Furthermore, based on (18) and (19), it yields

\[
\gamma_0 = 1 \quad \text{and} \quad \gamma_{q_1 q_2} = 0 \quad \text{for any} \quad q_1 \neq q_2.
\]  

Based on the aforementioned equation, it follows that

\[
\sum_{p_1, p_2=0}^{P} c_{a, p_1} c_{a, p_2} e^{j(\omega_{Q-1} q_1 q_2)p_1 + \omega_{Q-1} q_2 p_2} e^{j(\theta_{p_1} - \theta_{p_2})} = 1
\]

holds for any \( P \) angles \( \theta_0, \theta_1, \ldots, \theta_P \) from \( \{0, 2\pi/S + \theta, \ldots, 2(S-1)\pi/S + \theta \} \). Based on Lemma 1, we obtain \( c_{a, p_1} c_{a, p_2} = 0 \) for any \( p_1 \neq p_2 \). Because \( b \neq f \) and \( a = e \) as assumed in Proposition 4, it holds that \( c_{a, b} c_{e, f} = 0 \), which contradicts Proposition 4.

By proving that Propositions 1–4 are false, we can conclude that the previous assumption that \( \epsilon = g^H H \) is a zero vector or has at least two nonzero entries does not hold. This case completes the proof.

REFERENCES


