Brief Announcement: Space-Optimal Silent Self-Stabilizing Spanning Tree Constructions Inspired by Proof-Labeling Schemes

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Abstract. We present a general roadmap for the design of space-optimal polynomial-time silent self-stabilizing spanning tree constructions. Our roadmap is based on sequential greedy algorithms inspired from the design of proof-labeling schemes.

Context and objective. One desirable property for a self-stabilizing algorithm is to be silent, that is, to keep the individual state of each process unchanged once a legal state has been reached. Silentness is a desirable property as it guarantees that self-stabilization does not burden the system with extra traffic between processes whenever the system is in a legal state. Designing silent algorithms is difficult because one must insure that the processes are able to collectively decide locally of the legality of a global state of the system, based solely on their own individual states, and on the individual states of their neighbors. This difficulty becomes prominent when one takes into account an important complexity measure for self-stabilizing algorithms: space complexity. Keeping the memory space limited at each process reduces the potential corruption of the memory, and enables to maintain several redundant copies of variables (e.g., for fault-tolerance) without hurting the efficiency of the system.

Our objective is to compute some spanning tree $T$ of $G$. Typically, the tree $T$ is rooted at some node $r$, and it is distributedly encoded at each node $v$ by the identity of $v$’s parent $p(v)$ in $T$. (The root $r$ has $p(r) = \bot$). We are interested in all kinds of spanning trees, but will mostly focus our attention to two specific kinds of spanning trees: minimum-weight spanning trees (MST), and minimum-degree spanning trees (MDST). Constructing MSTs is a classical problem in the distributed computing setting. In the case of MDSTs, we aim at designing an algorithm which, for any given (connected) graph $G$, constructs a spanning tree $T$ of $G$ whose degree is minimum among all spanning trees of $G$. Our interest in MDSTs is motivated by resolving issues arising in the design of MAC protocols for sensor networks under the 802.15.4 specification. It is also worth pointing out that MDSTs arise in many other contexts, including electrical circuits, communication networks, as well as in many other areas. Since HAMILTONIAN-PATH is NP-hard, we actually slightly relax our task, by focussing on the construction

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of spanning trees whose degree is within +1 from the minimum degree \( \text{OPT} \) of any spanning tree in the given graph.

It is known that, for every task with a proof-labeling scheme on \( k \)-bit labels, there is a silent self-stabilizing algorithm for that task using registers on \( O(k + \log n) \) bits in \( n \)-node networks \([2]\). However, the convergence time of the generic algorithm in \([2]\) may be exponential.

**Our results.** We present a general roadmap for the design of *space-optimal polynomial-time* silent self-stabilizing constructions of spanning trees optimizing different kinds of criteria, under the state model\(^3\). Following our roadmap, we were able to design space-optimal algorithms for both MST and MDST constructions. Our MST algorithm uses registers of size \( O(\log^2 n) \) bits in \( n \)-node networks, which is known to be optimal. While there exists more compact MST algorithms, these algorithms designed for minimizing the size of the memory are not silent. Our MDST algorithm is an additive approximation algorithm. It returns a spanning tree with degree at most \( \text{OPT} + 1 \). It uses registers of \( O(\log n) \) bits, which is known to be optimal. It exponentially improves the previous best known \( (\text{OPT} + 1) \)-approximation algorithm, which is not silent, yet is using \( O(n \log n) \) bits of memory per node, and is converging in the same number of rounds. Both our algorithms converge in a number of rounds polynomial in \( n \), and perform polynomial-time computation at each node. In fact, our MDST algorithm constructs a special kind of trees, named *FR-trees* after Fürer and Raghavachari. Indeed, we show that verifying whether a given tree is an arbitrary trees of degree \( \leq \text{OPT} + 1 \) cannot be done in polynomial time, unless \( \text{NP} = \text{co-NP} \). Instead, we show that there is a proof-labeling scheme for FR-trees using labels on \( O(\log n) \) bits.

Our roadmap relies on a collection of ingredients. The first ingredient is the design of sequential *greedy* algorithms guided by *proof-labeling schemes*. The second ingredient is a *redundant* proof-labeling scheme for spanning trees, enabling the design of silent *loop-free* self-stabilizing algorithms for permuting tree edges with non-tree edges. The third ingredient is the design of a silent algorithm for the construction of the \( O(\log n) \)-bit label informative-labeling scheme for nearest common ancestor (NCA) from the literature, in order to identify the fundamental cycles. The two latter ingredients are used for implementing the sequential algorithms of the first ingredient in a distributed silent self-stabilizing manner.

More details are available in \([1,2]\).

**References**


\(^3\) Recall that, in the state model, every node has read/write access to its own public variables, and read-only access to the public variables of its neighbors in the network \( G \) connecting the node.