A DYNAMIC PROGRAMMING APPROACH TO OPTIMIZATION FOR PAVEMENT MANAGEMENT SYSTEMS

FEIGHAN, Kieran J.
Technical Research Assistant
School of Civil Engineering
Purdue University
W. Lafayette, Indiana

SHAHIN, Mohamed Y.
Research Engineer
United States Army
Construction Engineering Research Laboratory
Champaign, Illinois

SINHA, Kumares C.
Professor of Civil Engineering
Purdue University
W. Lafayette, Indiana

TRB Committee AFD10 on Pavement Management Systems is providing the information contained herein for use by individual practitioners in state and local transportation agencies, researchers in academic institutions, and other members of the transportation research community. The information in this paper was taken directly from the submission of the author(s).
A DYNAMIC PROGRAMMING APPROACH TO OPTIMIZATION
FOR PAVEMENT MANAGEMENT SYSTEMS

Feighan, K.J., Shahin, M.Y., Sinha, K.C.

ABSTRACT

Dynamic programming is employed in conjunction with a Markov chain probability-based prediction model to obtain minimum cost maintenance strategies over a given life-cycle analysis period. Dynamic programming is an "approach" to optimization, based on the principle of taking a single, complex problem and breaking it into a number of simpler and more easily solvable problems. An extremely important advantage that dynamic programming has over almost all classical optimization techniques is that it will, if set up correctly, determine absolute optima rather than local optima.

Pavement sections are split into families on the basis of characteristics such as surface type, distress modes and traffic levels. Prediction curves are fitted using Markov chain theory to obtain transition probability values that define future performance in terms of states, where each state is defined to be a 10 point PCI range, e.g. 90-100 PCI is state 1. Various maintenance alternatives are considered, and the initial cost of applying each alternative in each state for each family is input as are the probability values associated with the predicted performance of the maintenance alternatives in the future. Any life-cycle period can be specified, and any interest rate can be used.

The dynamic programming algorithm then takes these inputs and, simultaneously for every state in every family, outputs the desired maintenance alternative that will minimize the total expected cost over a specified life-cycle subject to keeping all sections above pre-defined performance standards. The algorithm is very simple, extremely fast and produces guaranteed global optimal solutions.

Results obtained thus far are very reasonable with a routine or preventive maintenance option being chosen by the dynamic programming algorithm in the upper states, surface treatments and overlays in the lower states.

KEY WORDS: Dynamic Programming, Life-cycle Analysis, Optimization Cost Effectiveness, Maintenance Option Selection
INTRODUCTION

In the present day, when almost all pavement managers are working in budget-constrained situations, it is imperative that the techniques used to aid the managers in their decision process are as useful and accurate as possible. Specifically, the condition prediction methods and optimization tools used should be capable of responding to a wide range of potential network set-ups in a quick and efficient way.

In this paper, dynamic programming is employed in conjunction with a Markov chain probability-based prediction model to obtain minimum cost maintenance strategies over a given life-cycle analysis period. Because of the efficient structure of dynamic programming, the executable program is short and simple, and executes very quickly on microcomputers.

BACKGROUND

The present research is part of an ongoing effort to improve the prediction and optimization capabilities of the PAVER and Micro PAVER pavement management systems. The PAVER system has been well documented elsewhere (1). It is founded upon the Pavement Condition Index (PCI), an index reflecting the current condition of the pavement section with a range of 0 to 100.

The PAVER databases were used in the development of prediction models. There is a separate database for each installation. Generally, each pavement section stored in the database can be identified by location, pavement type, pavement use and pavement rank. In taking all of the available condition data in a network, very large variation is expected from section to section, even at a particular age. In order to reduce the variation in the data to obtain more confidence in the predicted performance over time, it is necessary to group the data using common variable characteristics (2).

Currently, the variables used to define groups, or 'families', of data are pavement surface type, pavement use and pavement rank. It is possible that the primary source of pavement distress may also be included subsequently (3).

There has been extensive research performed in attempting to establish the relationship between pavement condition and cost to repair and maintain the pavement. From a search of available literature, it appears that very few currently operating PMS's have access to, or use, this type of information in arriving at the most desirable and cost-effective solutions. However, this data is of critical importance in weighing up the tradeoffs between repairing now or repairing later, both at project level and at network level.

Cost information for the PCI is available from the results of an ongoing study into the relationship between PCI and cost being carried out by Purdue University for USA-CERL (4). The study identifies the cost of applying various maintenance alternatives on different surface as a function of the condition of the surface. This information is readily incorporated into a cost optimization framework.
MARKOV PREDICTION MODEL

It is not possible to describe the functioning of the dynamic programming algorithm without first describing the prediction model used, as the two are inextricably linked. The following is a necessarily abbreviated and generalized description of the model; a more detailed and comprehensive description has already been published elsewhere (3,5). Much of the terminology used in this description also is used in dynamic programming.

The PCI range of 0 to 100 is divided into 10 states, each state being 10 PCI points wide. A pavement is modelled to begin its life in near-perfect condition, and to deteriorate as it is subjected to a sequence of duty-cycles. A duty-cycle is defined as one year's imposition of the effects of weather and traffic. A state vector indicates the probability of a pavement section being in each of the 10 states in any given year. Figure 1 shows the schematic representation of state, state vector and duty cycle.

The sections are grouped into families, and all of the sections of a family are categorized into one of the ten states at any age. It is assumed that all of the pavement sections are in state 1 (PCI of 90 to 100) at an age of 0 years.

To model the way in which the pavement deteriorates with time, it is necessary to identify the Markov probability matrix. The assumption made is that the pavement condition will not drop by more than one state in a single year. Thus, the pavement will either stay in its current state or transit to the next lowest state in one year. The probability transition matrix has a diagonal structure as shown in Figure 2. The entry of 1 in the last row of the transition matrix indicates a trapping state. The pavement condition cannot transit from this state unless repair action is performed.

The state vector for any duty cycle $t$, $S(t)$, is obtained by multiplying the initial state vector $S(0)$ by the transition matrix $P$ raised to the power of $t$. Thus

$$ S(1) = S(0)*P $$
$$ S(2) = S(1)*P = S(0)*P^2 $$
$$ \vdots $$
$$ S(t) = S(t-1)*P = S(0)*P^t $$

If the transition matrix probabilities can be estimated, the future condition of the road at any duty cycle (time) $t$ can be predicted.

The probabilities are estimated using a non-linear programming approach which has its objective function the minimization of the absolute distance between the actual PCI versus age data points, and the expected (predicted) PCI for the corresponding age generated by the Markov chain using the nine probability parameters. It has been found that this approach can accurately model the pavement deterioration over time. A sample output of this program is shown in Figure 3.
TRB Committee AFD10 on Pavement Management Systems is providing the information contained herein for use by individual practitioners in state and local transportation agencies, researchers in academic institutions, and other members of the transportation research community. The information in this paper was taken directly from the submission of the author(s).
INTRODUCTION TO DYNAMIC PROGRAMMING

Dynamic programming is an "approach" to optimization. It is not based upon a single, well-defined procedure such as the simplex algorithm of linear programming. Rather, it seeks to take a single, complicated problem and break it down into a number of simpler, more easily solvable problems. The solution to these problems is generally much faster and requires much less computational effort.

The principle of optimality, the foundation of dynamic programming, states that "every optimal policy consists only of optimal subpolicies" (6). Instead of examining all possible combinations of decisions, dynamic programming examines a small, carefully chosen subset of the combinations, rejecting those combinations that cannot possibly lead to an optimal solution. If performed correctly, the subset examined is guaranteed to contain the optimal solution.

Dynamic programming has the advantage over almost all classical optimization techniques that it will yield the absolute or global maxima or minima rather than local optima. Dynamic programming is also extremely robust in that it can handle integrality, negativity and discreteness of variables and other constraints very easily. It also, by its nature, produces the optimal solution to all of its constituent subproblems. These solutions may be of interest to the pavement manager, especially in time-varying problems such as pavement performance.

The major drawback with dynamic programming is that the proposed problem to be solved may not be formulatable in a manner capable of using the power of dynamic programming effectively. However, if it is possible to set up the problem in an effective form, dynamic programming provides an outstanding optimization tool.

Structure of dynamic programming: The basic components of dynamic programming are states, stages, decision variables, returns and transformation or transition functions (6). A physical system is considered to progress through a series of consecutive stages. In pavement performance, each year is viewed as a stage.

At each stage, the system must be capable of being fully described by the state variables or state vector. In the present case, as described earlier, each state is a 10 PCI bracket for every pavement family and the condition of the pavement at any year (stage) can be defined as being in one of the ten states.

At each stage, for every possible state, there must be a set of allowable decisions. The decisions being made in the dynamic programming model are what repair alternative to implement in each state at every stage.

Finally, there is the transformation or transition function. If a process is in a given state, and a feasible decision is made, there must be a function that determines to which state the process moves. In general, dynamic programming transformation functions can be deterministic or stochastic. In
In this particular case, the transition function is defined by the Markov probability matrix derived in the curve-fitting process described earlier and hence is a stochastic process.

In summary, the problem set-up for this dynamic programming formulation is:

MINIMIZE: Expected Cost over a specified life-cycle length subject to keeping all sections above a defined performance standard.

The dynamic programming parameters are:
STATES: Each bracket of 10 PCI points in a family.
STAGES: Each year in the analysis period.
DECISION VARIABLES: Which maintenance treatment to apply.
TRANSFORMATION FUNCTION: The Markov Transition Probability Matrix defines the transformation.
RETURN: Expected cost if a particular decision is made in each state at each stage.

INPUTS REQUIRED FOR THE DYNAMIC PROGRAMMING ALGORITHM

The inputs required for the dynamic programming algorithm are:

1. Markov Transition Probabilities for state i of matrix j  
   \[ p_{ij} \] ; \( i = 1, \ldots, 10 \) states \( j = 1, \ldots, m \) families

2. Cost of applying treatment k to family j in state i  
   \[ C_{ijk} \] ; \( k = 1, \ldots, n \) maintenance alternatives
   Routine maintenance is always designated as \( k = 1 \).
   The cost is entered on a dollar per square yard basis.

3. Feasibility Indicator for alternative k when in state i of family j  
   \[ R_{ijk} = 1 \] if maintenance alternative is feasible  
   \[ = 0 \] if maintenance alternative is infeasible

4. Number of Years in the life-cycle analysis; \( N \)

5. Interest Rate ; \( r \)

6. Inflation Rate ; \( f \)

7. Rate of Increase in Funding ; \( q \)

8. The associated benefit over one year of being in state i  
   \[ B_i = 95, 85, \ldots, 5 \] for \( i = 1, 2, \ldots, 10 \)
   The benefit is taken to be the area under the PCI curve over a period of one year.

9. The minimum allowable state for each family, i.e. the lowest state that the network manager will allow a particular family to deteriorate to before performing some major maintenance. This is designated by \( S_j \).
10. The transformations that define the new family to move to if treatment k is applied in family j: \((j,k) \rightarrow (j^1, l)\).

**OPERATION OF DYNAMIC PROGRAMMING**

The dynamic programming process starts at year \(N\), the final year of the life-cycle analysis. In dynamic programming terms, this is stage 0. Effectively, the life-cycle cost analysis is being performed over 0 years at this stage.

The first step is to calculate the routine maintenance cost for each state in every family in year \(N\). Routine maintenance is not feasible if (i) \(R_{ijk} = 0\), or (ii) \(S_1 > 1\) for family \(j\). For all feasible states, the cost of routine maintenance is obtained from \(c_{ijl,N} = C_{ijl}\) and these values are stored. If routine maintenance is not feasible, a very large value is added to the cost to ensure that it will not be chosen as the cheapest alternative.

All other feasible alternative costs are also calculated for all states in each family from \(c_{ijk,N-n} = c_{ijk}\).

The optimum repair strategy for each state in year \(N\) is then given by:

\[
C^*_{ij,N} = \min\{c_{ijl,N}, c_{ijk,N}\} \quad \text{for all } i,j.
\]

In general, the decision process can be described for year \(N-n\), or equivalently for stage \(n\). As before, routine maintenance is examined for feasibility. If found to be feasible, the following expression is used to calculate the total present worth of applying routine maintenance now when the analysis period is \(n\) years long:

\[
c_{ijl,N-n} = c_{ijl} + \left[ P_{ijl} C^*_{ij,N-n-1} + (1-P_{ijl}) C^*_{i+1,j,N-n-1} \right] * 1/(1+i^*)
\]

(i) \(\text{This expression is composed of two parts. The part indicated by}\)

(ii) \(\text{is the immediate cost of routine maintenance in year } n, \text{while}\)

\(\text{is the total expected cost to be incurred in the remaining}\)

\(\text{years as a consequence of applying routine maintenance. As shown}\)

\(\text{Figure 4, this expected cost is obtained by identifying the}\)

\(\text{probability of remaining in a given state and multiplying this}\)

\(\text{probability by the expected cost of that state and then finding}\)

\(\text{the associated probability of dropping a state if routine}\)

\(\text{maintenance is applied and multiplying this by the expected}\)

\(\text{sum is then discounted by the effective interest rate, } i^*, \text{to bring the total into present worth dollars}\)

\(\text{in the year } N-n.\)

Similarly, the cost of all other feasible maintenance alternatives can be calculated. The expression used is:

\[
c_{ijk,N-n} = c_{ijk} + \left[ P_{ij}^1 C^*_{ij1,N-n+1} + (1-P_{ij}^1) C^*_{2j1,N-n+1} \right] * 1/(1+i^*)
\]

This expression differs from the routine maintenance one in that it is known that the pavement condition will return to state 1
TRB Committee AFD10 on Pavement Management Systems is providing the information contained herein for use by individual practitioners in state and local transportation agencies, researchers in academic institutions, and other members of the transportation research community. The information in this paper was taken directly from the submission of the author(s).
after the repair alternative is carried out. The family that the pavement moves to, \( j \), as a result of having this alternative performed is defined in the input transformation matrix. For example, if a thin overlay is performed on an AC pavement section, that section will move to the thin overlay family for performance prediction after the overlay is placed.

The optimal cost is then given by:

\[
C_{ij,N-n} = \min(C_{ij1,N-n}, C_{ijk,N-n}) \quad \text{for all } k
\]

with the associated optimal maintenance alternative to be performed for this \((i,j)\) family/state combination in year \(N-n\) being the choice of \(k\) that minimizes the above expression.

This backward recursion is performed for every successive year of the analysis period until the analysis for year 0, or stage \(N\), is reached.

Dynamic Programming Output

The output from the dynamic programming program consists of:

(i) a file containing the optimal maintenance alternative in every year for every family/state combination.

(ii) the discounted present worth costs expected to be accrued over the life-cycle specified if the optimal decisions are implemented.

(iii) the expected effectiveness accrued as a result of following the optimal decisions is calculated for every family/state combination.

(iv) the effectiveness/cost ratio for every family/state combination is calculated.

Thus, all that is necessary is to define which family/state combination any particular section belongs to, and the optimal maintenance alternative and associated cost and effectiveness are readily obtained. As the intention of the research is to produce updated programs for both the PAVER and Micro PAVER systems, it was imperative that all of the programs be executable on a microcomputer. The dynamic programming program executes extremely quickly at this level.

A very short example follows to illustrate the operation of the program. Performance curves were developed for the Great Lakes Naval Center based on PCI surveys performed there. Families were defined on the basis of branch use and surface type. For the branch use of "roadway", there were four families defined: asphalt concrete, surface treated, thin overlay and structural overlay.

There were four maintenance alternatives considered; routine maintenance, surface treatment, thin overlay and structural overlay. Dollar cost as a function of PCI was defined for both initial repair cost and subsequent routine maintenance cost in the Purdue University research [4]. Markov probability calculations for each family were performed, and the probability transition matrices obtained.
The dynamic programming program was then run on a Compaq 286 microcomputer with an 80287 math co-processor installed. An effective interest rate of 5% was input, and a life-cycle analysis of 30 years was specified. The program took 45 seconds to produce the entire output for every year of the 30 years.

Figure 5 illustrates a small part of the output. This is the output in year 1, the present time, for family 4. This is the family composed of sections with structural overlays. As can be seen, routine maintenance was selected as the optimal treatment to apply in the upper states, surface treatment in the middle states and structural overlay in the lower states. Obviously, this result is completely dependent upon the cost and performance relationships defined, but the overall trend of the optimal decisions is certainly reasonable.

FUTURE DEVELOPMENTS

There are a number of potential areas for further use of this approach in pavement management. It should be possible to explore the impact of deferred maintenance by limiting the maintenance options available to just routine maintenance for a given number of years, and noting the detrimental effect that this policy has on the cost and E/C ratios for the life-cycle period.

The output from dynamic programming fits easily into a network prioritization framework. The flowchart describing the overall intended process is shown in Figure 6. The details of the prioritization package will be described in the future. It is also intended to validate the dynamic programming outputs for a number of different databases by comparing its output to that obtained from a more traditional, deterministic approach to life-cycle effectiveness and cost calculation.

REFERENCES


### PAVER DATABASE RAW DATA

**INPUT:** Common Characteristics to classify sections into families (surface type, traffic, primary source of distress).

**OUTPUT:** Families of pavement sections with PCI versus Age data.

### MARKOV PREDICTION PROCESS

**INPUT:** Families with PCI versus Age data.

**OUTPUT:** Markov Transition Probabilities for each family and maintenance alternative.

### DYNAMIC PROGRAMMING PROGRAM

**INPUT:** Markov Transition Probabilities, costs by state and family for each alternative, planning horizon, interest rates, performance standards by family, benefit by state.

**OUTPUT:** Optimal maintenance decision (on the basis of minimized cost or maximized B/C ratio) for each state of each family; associated cost and benefit.

### PRIORITIZATION PROGRAM

**INPUT:** B/C ratio for each section, weights by family (and possibly by state), Actual Budget, any necessary (user-defined) sections that must be repaired, even if sub-optimal.

**OUTPUT:** Ranked list of sections that can be repaired within budget limitations.

### PROJECT LEVEL ANALYSIS