The Shortest Route Cut and Fill Problem in Linear Topological Structure

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ABSTRACT
In this paper, we study the Shortest Route Cut and Fill (SRCF) problem where a fleet of vehicles with fixed capacities travel between cut and fill points to finish all cut and fill requirements so that the travel time of the fleet is minimized. We state and classify the SRCF problem and present a complete view of its computational complexities with NP-hardness proof for the basic linear topological structure. Moreover, a simple cycle-based optimization method is proposed to solve the SRCF in linear time complexity, for the version of single vehicle in linear structure without time window constraint.

Categories and Subject Descriptors
F.2.2 [Nonnumerical Algorithms and Problems]:

General Terms
Algorithms

Keywords
Cut and Fill, Optimization

1. INTRODUCTION
In this paper, we study the Shortest Route Cut and Fill (SRCF) problem where a fleet of vehicles with fixed capacities travel between cut and fill points to finish all cut and fill requirements so that the travel time of the fleet is minimized. We state and classify the SRCF problem and present a complete view of its computational complexities with NP-hardness proof for the basic linear topological structure. Moreover, a simple cycle-based optimization method is proposed to solve the SRCF in linear time complexity, for the version of single vehicle in linear structure without time window constraint.

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2. PROBLEM STATEMENT AND MAJOR RESULTS
There are $N$ cut points and $N$ fill points. In addition, we have $K$ vehicles to finish all the cut and fill requirements.

- $V = \{v_1, v_2, \ldots, v_K\}$: the set of vehicles’ initial positions (the depots of vehicles);
- $VC = \{vc_1, vc_2, \ldots, vc_K\}$: the set of the capacities of vehicles;
- $C = \{c_1, c_2, \ldots, c_N\}$: the set of cut points’ positions;
- $F = \{f_1, f_2, \ldots, f_N\}$: the set of fill points’ positions;
- $R = \{R^1, R^2, \ldots, R^K\}$: the set of routes for vehicles, where $R^i = ((r^i_1, l^i_1), (r^i_2, l^i_2), \ldots, (r^i_p, l^i_p))$. Here, $r^i_j$ is the $j$-th position of route $R^i$ for the $i$-th vehicle and $l^i_j$ is the corresponding working load (a positive number for cut and a negative number for fill) of that vehicle in the $j$-th position, where $r^i_1 = r^i_p = v_i, l^i_1 = l^i_p = 0$ and $r^i_j \in C \cup F$ and $l^i_j \neq 0, \forall 1 < j < p$, $\forall 1 \leq i \leq K$;
- $d(p, q)$: the distance between two positions $p$ and $q$, where $p, q \in V \cup C \cup F$;
- $D(R^i) = \sum_{j=1}^{p-1} d(r^i_j, r^i_{j+1})$: the total travel distances of the route $R^i$ served by the $i$-th vehicle.

The Shortest Route Cut and Fill (SRCF) problem is stated as: given the sets of cut points and fill points, $C$ and $F$, how
to arrange the routes $R$ of a fleet of vehicles at their initial locations $V$ with capacities $VC$ to serve all the cut and fill requirements. In the meanwhile, make the longest route length among all vehicles is the shortest.

The SRCF problem can be classified by the following four items:
1. the number of vehicles - $K$;
2. the capacity of vehicles - $VC$;
3. the time windows constraints of serving cut and fill points - "withTW" for with time windows or "withoutTW" for without time windows;
4. the topological structure of the positions distribution - "Linear", "Tree" or "Normal".

Based on this classification, variations of the SRCF problem can be denoted by SRCF($K;VC;\text{with}/\text{without}\text{TW};\text{linear}/\text{tree}/\text{normal}$).

Our results of the computational complexities on various scenarios are illustrated in Table 1 for linear structure.

<table>
<thead>
<tr>
<th>$K$ = 1</th>
<th>$K$ = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>without time window</td>
<td>with time window</td>
</tr>
<tr>
<td>P</td>
<td>NP-hard</td>
</tr>
<tr>
<td>P</td>
<td>NP-hard in the strong sense</td>
</tr>
</tbody>
</table>

### Table 1: Computational complexity

3. **SRCF(1;1;withoutTW;linear)**

**Definition 1. Leg:** the segmentation split by two neighborhood points despite of whether they are cut or fill points. In other words, we sort all positions $u$ in set $C$ and $F$, i.e., $u \in C \cup F$ such that $u_i \leq u_{i+1}, 1 \leq i < M + N$. We have $M + N - 1$ legs, i.e. $leg_i = (u_i, u_{i+1}), 1 \leq i < M + N$. We denote $d(leg_i) = d(u_i, u_{i+1})$ as the length of $leg_i$.

**Definition 2. Flow of Leg:** the loads that the vehicle must finished by transferring via the leg, i.e. $flow(leg_i) = \sum_{j<i}A(u_j \in C) + \sum_{j>i}A(u_j \in F) (-1)$.

**Lemma 1.** In the optimal route of SRCF on single vehicle with unit capacity in linear topological structure (SRCF(1;1;withoutTW;linear)), for each leg $leg_i = (u_i, u_{i+1})$, the vehicle should travel via the leg $2 \cdot \max\{flow(leg_i), 1\}$ times. In other words, the sub-route "$u_i u_{i+1} \cdots u_{i+1} u_i$" should be repeated by $\max\{flow(leg_i), 1\}$ times.

**Definition 3. Segment:** a sequence of successive legs where all the flows of these legs have the same flow directions (in other words, their flow are either positive or negative).

**Lemma 2.** In each segment, the most left point and the most right point of the segment should be one cut point and one fill point separately. i.e. for segment $\{\text{leg}_j\}, a \leq i \leq b$, $u_a$ and $u_{b+1}$ should belong to different types of points, one cut and one fill.

An algorithm (illustrated in Algorithm 1) is presented to solve the SRCF(1;1;withoutTW;linear), which includes the following four steps:

Step 1: by Lemma 1, the necessary sub-cycle is generated for each leg.

Step 2: link cycles within a segment;
Step 3: link cycles between two consecutive segments;
Step 4: determine the initial direction of the vehicle by the position of its own depot.

#### Algorithm 1 Solving SRCF(1;;withoutTW;Linear)

**for all segment:** \{\(u_a, u_{a+1}, \ldots, u_b\)\}

- **if** $u_a$ is cut point **then**
  - index of the start point $s = a$; index of the end point $e = b$;
  - direction $= 1$;

- **else**
  - index of the start point $s = b$; index of the end point $e = a$;
  - direction $= -1$;

- **end if**

- **for** $i : a \leq i \leq b$ **do**

- **if** $u_i \in C$ **then**

- **end if**

- **else**

- **end if**

**end for**

**for** $i = s$; **LoadVehicle** = 0; **while** $i : a \leq i \leq b$ **do**

- **if** CASE 1: $\text{Load}[i] = 0$ **then**

- **end if**

- **else**

- **end if**

**end while**

**end for**

Link the cycles between two neighborhood segments
**if** $v$ is not in any segment **then**

- the vehicle can have either initial direction;

**else**

- the initial direction should be the same with the direction of that segment

**end if**

**end if**

**end if**

**end if**

**end if**

Theorem 1. Algorithm 1 can optimally solve SRCF(1;1;withoutTW;linear) (the SRCF problem on single unit capacity vehicle in linear structure) in polynomial time $O(N)$.

4. **REFERENCES**


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