Scientific Annals of Computer Science vol. 22 (1), 2012, pp. 201–251
DOI: 10.7561/SACS.2012.1.201

Overview of Thirty Semantic Formalisms for Reo

Sung-Shik T.Q. JONGMANS[1], Farhad ARBAB[2]

Abstract

Over the past decades, coordination languages have emerged for the specification and implementation of interaction protocols for communicating software components. This class of languages includes Reo, a platform for compositional construction of connectors. In recent years, many formalisms for describing the behavior of Reo connectors have emerged. In this paper, we give an overview of all these classes of semantic models. Furthermore, we investigate the expressiveness of two more prominent classes, constraint automata and coloring models, in detail.

Keywords: concurrency, coordination, Reo, semantics, constraint automata, connector coloring

1 Introduction

Over the past decades, *coordination languages* have emerged for the specification and implementation of interaction protocols for communicating software components. This class of languages includes Reo [5], a platform for compositional construction of *connectors*. Connectors in Reo serve as communication mediums through which components can interact with each other. Essentially, Reo connectors impose constraints on the order in which components can exchange *data items* with each other. Although ostensibly simple, Reo connectors can describe complex protocols (e.g., a solution to the Dining Philosophers problem [6]). Development tools for Reo exist as plug-ins for the Eclipse IDE, called the *Extensible Coordination Tools* (ECT),[3,4] including tools for simulation, animation, and verification of Reo connectors.

[1]CWI, Amsterdam, the Netherlands. Email: jongmans@cwi.nl
[2]CWI, Amsterdam, the Netherlands; LIACS, Leiden, the Netherlands
[3]http://reo.project.cwi.nl
[4]Formerly known as the Eclipse Coordination Tools.

Coalgebraic models (Section 3.1)	*Operational models* (Section 3.2)
Timed data streams	Constraint automata
Record streams	*Variants of constraint automata*
	Timed
Coloring models (Section 3.3)	Probabilistic
Two colors	Continuous-time
Three colors	Quantitative
Tile models	Resource-sensitive timed
	Transactional
Other models (Section 3.4)	*Context-sensitive automata*
Process algebra	Büchi automata
Constraints	Guarded automata
Petri nets & intuitionistic logic	Intentional automata
Unifying theories of programming	Action constraint automata
	Behavioral automata
	Structural operational semantics

Table 1: Semantic formalisms for describing the behavior of Reo connectors.

In recent years, many semantic formalisms for describing the behavior of Reo connectors have emerged, including coalgebraic models, operational models, and models based on graph-coloring. Table 1 shows a list of the classes of semantic models that currently (late 2011) exist for modeling Reo connectors. (We discuss the entries in this list in more detail in Section 3.) Although each of the classes in Table 1 serves its own purpose, we identify two burdens that their large quantity inflicts.

Issue 1 (Tracking them)**.** *Because there exist so many different semantic formalisms by now, keeping track of all of them becomes nontrivial and requires a significant amount of effort. As a result, in more than one publication, authors forget to mention relevant related work at the appropriate places in their own contributions. Extrapolated to the extreme, this can cause papers to become "forgotten" and scientist to redo the work.*

Issue 2 (Relating them)**.** *Questions about how the various classes of models relate to each other in terms of their expressiveness naturally arise. These questions, moreover, require answering for two reasons.* First*, from a theoretical point of view, such answers provide us with better and possibly new insights into the fundamentals of Reo.* Second*, from a more practical perspective, such answers broaden the applicability of tools—both existing and*

future—that assist developers in designing their connectors.

In this paper, we provide a snapshot that offers a solution to Issue 1 and take a step towards answering the questions mentioned in Issue 2.

Contributions We attribute two main contributions to this paper. In its first half, to resolve Issue 1, we provide a comprehensive overview of all existing classes of semantic models for describing the behavior of Reo connectors. Although our presentation remains mainly informal, a similarly detailed overview does, to our knowledge, not exist.

In the second half of this paper, to take a step towards resolving Issue 2, we investigate the relation between two of the most important classes of semantic models of Reo connectors: *(connector) coloring models* (with two colors) [29] and *constraint automata* [17]. We show how to transform the former to the latter and demonstrate bisimilarity between an original and its transformation. In the opposite direction, we show how to transform constraint automata to equivalent coloring models, prove that these transformations define each other's inverse, and again show bisimilarity. Additionally, we prove the compositionality of our transformation operators. To ensure that our transformation operators map one-to-one (instead of many-to-one), we extend coloring models with data-awareness.

A preliminary version of this paper, which excludes our comprehensive overview of semantic formalisms, appeared as [41].

Organization In Section 2, we give an informal description of Reo connectors; in Section 3, we provide an overview of semantic formalisms for describing their behavior. In Section 4, we extend coloring models with data-awareness. In Section 5, we discuss our transformation from coloring models to constraint automata; in Section 6, we discuss our transformation in the opposite direction. In Section 7, we discuss a potential application of our transformation operators. Section 8 concludes this paper.

2 Reo Connectors

A Reo connector consists of *nodes* (from the universe of nodes) through which data items (from the universe of data items) can *flow*.

Definition 1 (Universe of nodes)**.** $\mathbb{N}\text{ODE}$ *is the set of nodes.*

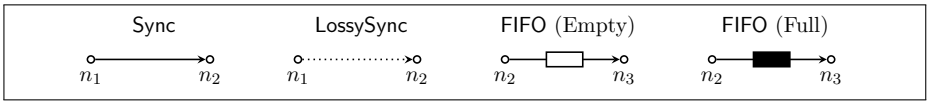


Figure 1: Pictorial representation of Sync, LossySync and FIFO.

Definition 2 (Universe of data items)**.** $\mathbb{D}\text{ATA}$ *is the finite set of data items.*

Within a connector, we distinguish three types of nodes: *input nodes* on which components can perform *write operations* for data items, *internal nodes* that a connector uses to internally route data items, and *output nodes* on which components can perform *take operations* for data items. We call input and output nodes collectively, the *boundary nodes* of a connector. Write and take operations, collectively called I/O-*operations*, remain *pending* on a boundary node until they *succeed*, in which case the respective nodes *fire*. We call connectors without internal nodes, which form the most elementary mediums between components, *primitives*.

Figure 1 shows pictorial representations of three common (binary) primitives. The Sync primitive consists of an input node and an output node. Data items flow through this primitive only if both its nodes have pending I/O-operations. The LossySync primitive behaves similarly, but loses a data item if its input node has a pending write operation, while its output node has no pending take operation. We call LossySync a *context-sensitive* connector: depending on the presence or absence of pending I/O-operations on its nodes, i.e., its *context*, LossySync behaves differently—LossySync must *never* lose a data item if its output node has a pending take operation.[5] Unfortunately, not all formalisms for modeling Reo connectors can describe context-sensitivity (at least, not directly); we address this topic in more detail in Section 3.

In contrast to the previous two primitives, connectors can have *buffers* to store data items in. Such connectors exhibit different states, while the internal configuration of Sync and LossySync always stays the same. For instance, the FIFO primitive consists of an input node, an output node, and a buffer of size 1. In its Empty state, a write operation on the input node of FIFO causes a data item to flow into its buffer—this buffer becomes full—while a take operation on its output node remains pending. Conversely, in its Full state, a write operation on its input node remains pending, while a take operation on its output node causes a data item to flow from the

[5]Bonsangue et al. formalize context-sensitivity in [20, 21].

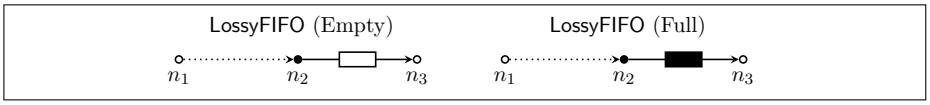


Figure 2: Pictorial representation of LossyFIFO.

buffer to this output node—the buffer becomes empty.

We interpret n_1, n_2, and n_3 in Figure 1 as *variables* over nodes in NODE: rather than depicting single connectors, this figure shows *classes* of connectors. For example, we can *instantiate* Sync by setting n_1 to a concrete node A and n_2 to a concrete node B. In text, we distinguish instances from classes by explicitly specifying the set of nodes an instance consists of. For example, Sync(A, B) denotes a connector instance, while Sync denotes a connector class. However, if no confusion can arise, we write "connector Conn" rather than "instance of connector Conn" for brevity.

We can construct complex connectors from simpler constituents by *joining* them. Informally, two connectors Conn_1 and Conn_2 can join iff, for each of their common nodes, this node serves as an input node in Conn_1 and as an output node in Conn_2 or vice versa.[6]

To illustrate joining, Figure 2 shows the pictorial representation of LossyFIFO, a connector composed of LossySync and FIFO. LossyFIFO consists of one input node, one internal node, and one output node. Similar to FIFO, LossyFIFO exhibits the states Empty and Full. Informally, in the Empty state, a write operation on the input node of LossyFIFO *always* causes a data item to flow into its buffer, while a take operation on its output node remains pending. In the Full state, a write operation on its input node *always* causes a data item to flow from its input node towards its buffer, but gets lost before reaching its internal node; a take operation on its output node causes a data item to flow from its buffer to this output node.

We call connectors that arise from joining smaller connectors *compos-*

[6]In this paper, we adopt the notion of nodes as introduced in [29]: at most two connectors can join on the same node. Alternatively [5], one can consider nodes entities on which an arbitrary number of connectors may coincide (rather than at most two). Under that interpretation, a node acts as a *pumping station*: at each execution step, it accepts a data item on one of its input connectors and replicates this data item to all its output connectors. Moreover, in that view, connectors can join on non-boundary nodes. We favor [29]-style nodes, however, because of their simplicity. This choice does not affect the generality of our results, because we can implement [5]-style nodes with [29]-style nodes by joining ternary primitives (Merger and Replicator) that emulate this pumping station behavior [29].

ites. By *hiding*—another operation on connectors—the internal nodes of a composite Conn, abstracting these nodes from the definition of Conn such that the outside world cannot observe and interact with them anymore, we transform Conn to a primitive.

3 Overview of Semantic Formalisms

One can trace the history of Reo back to [3]: in that paper, Arbab informally introduces the basic concepts of Reo, while leaving a formalization of its semantics for future work. Indeed, as shown in Table 1, many formalization arose! In this section, we give an overview of these classes of models. We aim at comprehensiveness: to the best of our knowledge, we discuss *all* classes of semantic models that researches have developed in the context of Reo.

While we skip most of the formal definitions, we consider two semantic formalisms in more detail: *constraint automata* in Section 3.2.1 and *coloring models* in Section 3.3. This has two reasons. First, these two classes influenced and formed the basis of many other classes of models as well as implemented tools. Thus, these formalisms have played a crucial role in the research on the semantics of Reo. Second, in the subsequent sections of this paper, we prove the correspondence between these two classes, for which we need their formal definitions.

We continue this section as follows (see also Table 1). First, in Section 3.1, we discuss two classes of coalgebraic models. Second, in Section 3.2, we summarize operational models of Reo connectors. Third, in Section 3.3, we treat coloring models. Finally, in Section 3.4, we discuss those models that do not fit the previous three categories.

3.1 Coalgebraic Models

In the literature, we find two classes of coalgebraic models for describing the behavior of Reo connectors: those based on *timed data streams* (TDS) and those based on *record streams* (RS). In both these classes, the coalgebraic notion of *streams* plays a prominent role. Informally, a stream s over a set S denotes an infinite sequence of elements from S. We denote the set of all the streams over S by S^ω. More formally, we define streams over S as total functions from the natural numbers to elements in S, i.e., $\mathsf{s} \in S^\omega$ IFF $\mathsf{s} : \mathbb{N} \to S$. We refer to the i-th element in a stream s with the notation $\mathsf{s}(i)$.

Timed data streams In [4, 5, 14, 62], Arbab and Rutten introduce TDS models as the first formalization of the semantics of Reo connectors. Informally, a TDS model of a connector describes for each of its nodes *which* and *when*—in dense time—data items flow through this node. It does so by associating each node with a *timed data stream* (TDS). We define a TDS as a pair of two streams over two different sets: a *data stream* d over $\mathbb{D}\text{ATA}$ (the data domain; see Definition 2) and a monotonically increasing *time stream* t over $\mathbb{R}_{\geq 0}$ (the positive real numbers including zero). Informally, a TDS of a node n conveys that data item $\mathsf{d}(i)$ passes through n at time $\mathsf{t}(i)$. By associating each node of a connector Conn with its own TDS in a TDS *tuple*, thus, we describe a single execution of Conn. To describe all possible executions of Conn, we associate Conn with a set of TDS tuples; we call such a set the TDS model of Conn. Commonly, however, we define such a TDS model as a predicate on TDSs that induces the set of admissible TDS tuples of Conn. (Enumerating all admissible TDS tuples of Conn becomes impossible because, not only each stream in a TDS itself is infinite, the set of admissible TDS tuples usually contains infinitely many elements.)

Record streams The second class of coalgebraic models of Reo connectors contains models that, to describe a single execution of a connector Conn, associate Conn with a single stream of *records* (cf., TDS models associate each node of Conn with a pair of streams). Izadi et al. introduce RS models in [38, 40]. Informally, records describe single execution steps of a connector: a record associates the nodes through which a data item flows (in the execution step it models) with those data items.

Definition 3 (Record [40])**.** *A record* r *is a partial function* $\mathsf{r} : \mathbb{N}\text{ODE} \rightharpoonup \mathbb{D}\text{ATA}$ *that maps a node* n *to a data item* $\mathsf{r}(n)$*. We denote the set of all records by* $\mathbb{R}\text{ECORD}$*.*

A *record steam* (RS) rs denotes a stream over the set $\mathbb{R}\text{ECORD}$. If the domain of every record in an RS rs includes only nodes from a connector Conn, rs potentially describes a single execution of Conn (cf., a TDS tuple): if so, record $\mathsf{rs}(i)$ describes which data items flow through which nodes at an abstract time instant i. To describe all executions of a connector Conn, we associate Conn with a set of RSs, which we call its RS model.

Compared to TDS models, RS models differ in their disregarding of the exact arrival times of data items at nodes: RS models capture only the order in which data items arrive. In [38, 40], Izadi et al. also define an operator

for transforming TDS tuples to RSs and an operator for transforming RSs to TDS tuples. Furthermore, Izadi et al. assert that the latter operator forms the inverse of the former operator (but not vice versa).

3.2 Operational Models

Although stream-based models provide an intuitive way of thinking about flow through nodes, they turned out difficult to derive implementations from and analyze (e.g., by means of model checking). To remedy this, researchers looked for other types of models for describing the behavior of Reo connectors, including operational models. In fact, many such classes of operational models came to existence, e.g., (numerous variants of) *constraint automata*, various automata for describing context-sensitive connectors, and a *structural operational semantics*.

3.2.1 Constraint automata

In [10, 17], Baier et al. introduce the first class of operational models for describing the behavior of Reo connectors: *constraint automata* (CA). Similar to how ordinary automata accept strings, CA accept TDS-tuples.

Informally, a CA consists of a (possibly singleton) set of states, which correspond one-to-one to the states of the connector whose behavior it models, and a set of transitions between them; in contrast to ordinary automata, however, CA do not have accepting states.[7] A transition of a CA, which describes a single execution step, carries a label that consists of two elements: a set of nodes and a *data constraint*. The former, called a *firing set*, describes which nodes synchronously fire in the state the transition leaves from; the latter specifies the conditions that the data items that flow through these firing nodes must satisfy.

Definition 4 (Universe of data constraints [17])**.** $\mathbb{DC}(N)$ *with* $N \subseteq \mathbb{NODE}$ *is the set of data constraints such that each* $dc \in \mathbb{DC}(N)$ *complies with the following grammar:*

$$dc ::= dc \wedge dc \mid \neg dc \mid \top \mid \#n = d \text{ with } n \in N \text{ and } d \in \mathbb{DATA}.$$

Informally, $\#n$ means "the data item that flows through n," while $\wedge$, $\neg$, and $\top$ have their usual meaning. We also allow their derived Boolean operators

[7] Because CA do not have accepting states, CA have only accepting runs of infinite length (similar to ω-automata [63]). This property makes CA suitable for accepting TDS-tuples (because every TDS in such a tuple has an infinite length).

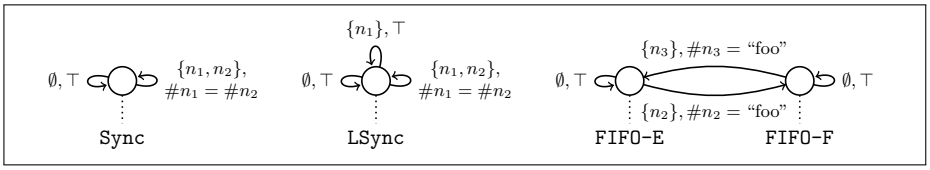


Figure 3: CA of Sync, LossySync, and FIFO for $\mathbb{D}\text{ATA} = \{\text{“foo”}\}$.

as syntactic sugar, e.g., $\vee$, and adopt $\#n_1 = \#n_2$ (with $n_1, n_2 \in \mathbb{N}\text{ODE}$) as an abbreviation of $\bigvee_{d \in \mathbb{D}\text{ATA}}(\#n_1 = d \wedge \#n_2 = d)$. Below, we define CA.

Definition 5 (Constraint automaton [17])**.** *A constraint automaton* CA *over* $[N \subseteq \mathbb{N}\text{ODE}, DC \subseteq \mathbb{D}\text{C}(N)]$ *is a triple* $\langle Q, \mathsf{T}, q_0 \rangle$ *with* Q *a set of states,* $\mathsf{T} \subseteq Q \times \wp(N) \times DC \times Q$ *a transition relation such that* $\langle q, F, dc, q' \rangle \in \mathsf{T}$ *implies* $dc \in \mathbb{D}\text{C}(F)$*, and* $q_0 \in Q$ *an initial state.*

The condition “$\langle q, F, dc, q' \rangle \in \mathsf{T}$ implies $dc \in \mathbb{D}\text{C}(F)$” asserts that a data constraint cannot constrain data through nodes that do not fire. Henceforth, we consider only CA whose transition relations satisfy the following condition:

$$\langle q, F_1, dc_1, q_1' \rangle, \langle q, F_2, dc_2, q_2' \rangle \in \mathsf{T} \text{ and } q_1' \neq q_2' \text{ implies } \langle F_1, dc_1 \rangle \neq \langle F_2, dc_2 \rangle$$

This condition ensures that we can cast transition relations into transition functions,[8] and it comes without loss of generality.[9]

To illustrate Definition 5, Figure 3 shows the CA of Sync, LossySync, and FIFO. For simplicity, we assume the universe of data items a singleton with the string “foo” as its sole element. In general, the CA of FIFO contains a distinct state for each data item in $\mathbb{D}\text{ATA}$ that may occupy its buffer. Note that rather than naming states symbolically (e.g., $q, p, q_0, q_1, \ldots$), we name states by meaningful *indexes* (below the states in `font`).[10]

[8] One may call CA satisfying this condition “syntactically deterministic.” This “syntactic determinism,” however, differs from determinism as originally defined for CA [17]: the latter takes into account logical equivalence between data constraints in transitions leaving the same state (instead of syntactic equivalence). This yields a stronger form of determinism. Thus, every “logically deterministic” CA is a “syntactically deterministic” CA. Conversely, every “syntactically nondeterministic” CA is a “logically nondeterministic” CA.

[9] For every “logically nondeterministic” CA, there exists a language-equivalent “logically deterministic” CA [17]. Consequently, for every “syntactically nondeterministic” CA, there exists a “syntactically deterministic” CA that models the behavior of the same connector (at the level of granularity of language equivalence).

[10] More precisely, Figure 3 shows *classes* of CA similar to how Figure 1 shows the pictorial representations of *classes* of connectors. An *instance* of a CA from Figure 3 contains actual nodes instead of the variables n_1 and n_2. Moreover, the indexes in such an instance convey which nodes this instance contains, e.g., $\texttt{Sync}(A, B)$.

Definition 6 (Universe of indexes)**.** $\mathbb{I}\text{NDEX}$ *is the set of indexes.*

Henceforth, without loss of generality, we assume $Q \subseteq \mathbb{I}\text{NDEX}$ for all CA $\langle Q, \mathcal{T}, q_0 \rangle$. Finally, unlike the original publications on CA (but similar to many later papers), we model the possibility of connectors to idle explicitly with the inclusion of self-transitions labeled by $\langle \emptyset, \top \rangle$.[11] This simplifies the definition of the join operator for CA, below.

When we join two connectors Conn_1 and Conn_2 with CA as their behavioral model, we can compute the CA of the resulting composite by joining the CA of Conn_1 and Conn_2: the join operator for CA takes the Cartesian product of the sets of states of its arguments, designates the pair of their initial states as the initial state of the new CA, and determines a new transition relation.

Definition 7 (Join of CA [17])**.** *Let* $\mathtt{CA}_1 = \langle Q_1, \mathcal{T}_1, q_0^1 \rangle$ *and* $\mathtt{CA}_2 = \langle Q_2, \mathcal{T}_2, q_0^2 \rangle$ *be* CA *over* $[N_1, DC_1]$ *and* $[N_2, DC_2]$*. Their join, denoted by* $\mathtt{CA}_1 \bowtie \mathtt{CA}_2$*, is a* CA *over* $[N_1 \cup N_2, DC_1 \wedge DC_2]$[12] *defined as:*

$$\mathtt{CA}_1 \bowtie \mathtt{CA}_2 = \langle Q_1 \times Q_2, \mathcal{T}, \langle q_0^1, q_0^2 \rangle \rangle$$

$$\text{with: } \mathcal{T} = \left\{ \left\langle \begin{array}{c} \langle q_1, q_2 \rangle, \\ F_1 \cup F_2, dc_1 \wedge dc_2, \\ \langle q_1', q_2' \rangle \end{array} \right\rangle \middle| \begin{array}{l} \langle q_1, F_1, dc_1, q_1' \rangle \in \mathcal{T}_1 \\ \text{and } \langle q_2, F_2, dc_2, q_2' \rangle \in \mathcal{T}_2 \\ \text{and } F_1 \cap N_2 = F_2 \cap N_1 \end{array} \right\}.$$

The condition $F_1 \cap N_2 = F_2 \cap N_1$ in the previous definition asserts the following: if transitions in $\mathtt{CA}_1$ and $\mathtt{CA}_2$ form a new transition in $\mathtt{CA}_1 \bowtie \mathtt{CA}_2$, those transitions agree on the firing of the common nodes of $\mathtt{CA}_1$ and $\mathtt{CA}_2$. Furthermore, we remark that because CA abstract from the direction of flow, Definition 7 does not take into account the prerequisite in Section 2 that "*connectors* Conn_1 *and* Conn_2 *can join iff, for each of their common nodes, this node serves as an input node in* Conn_1 *and as an output node in* Conn_2 *or vice versa.*"

To illustrate Definition 7, Figure 4 shows the CA of LossyFIFO, obtained by joining the CA of LossySync and FIFO (see Figure 3). Note that this CA does *not* model the intended semantics of LossyFIFO: its transition $\langle \mathtt{LFIFO\text{-}E}, \{n_1\}, \top, \mathtt{LFIFO\text{-}E} \rangle$ describes the inadmissible loss of data in the case of an empty buffer. This example shows that CA cannot model context-sensitive connectors directly; in Section 3.2.3, we discuss operational formalisms that can.

[11] Generally, transitions labeled by $\langle \emptyset, \top \rangle$ serve as τ-transitions, and they can occur also between two different states.

[12] Henceforth, we write $DC_1 \wedge DC_2$ for $\{ dc_1 \wedge dc_2 \mid dc_1 \in DC_1 \text{ and } dc_2 \in DC_2 \}$.

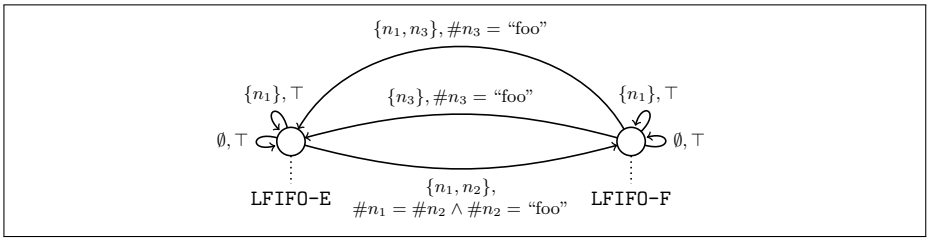


Figure 4: CA of LossyFIFO for $\mathbb{D}\text{ATA} = \{\text{“foo”}\}$. Let `LFIFO-E` denote $\langle \texttt{LSync}, \texttt{FIFO-E} \rangle$, and let `LFIFO-F` denote $\langle \texttt{LSync}, \texttt{FIFO-F} \rangle$.

For more examples of CA, including nondeterministic CA, we refer to [10, 17]. Additionally, in [10, 17], Baier et al. define (bi)simulation for CA, they present the hide operators for CA, and they prove the compositionality of these operators under (bi)simulation. Furthermore, in [25, 26], Clarke introduces the *forget* operator for CA to model reconfiguration of connectors.

3.2.2 Variants of CA

Several variations on ordinary CA came to existence in the past five years. We start with three less intricate variants.

Port automata In [45], Koehler and Clarke introduce *port automata* (PA) by abstracting data constraints from CA. This means that a transition of a PA carries only a firing set (or, equivalently, a firing set and $\top$).[13]

CA with state memory In [60], Pourvatan et al. introduce CA with *state memory* (CASM) by extending CA with a construct that enables data constraints to refer to the values of memory cells (e.g., the buffers of instances of FIFO). More formally, Pourvatan et al. associate a CASM not only with the usual ingredients of CA, but also with a set of *memory cells* and a *value function*. This latter function maps, for each state, memory cells to data items. Furthermore, Pourvatan et al. extend the syntax and semantics of data constraints with constructs for the formulation of propositions over memory cells.

Labeled CA In [44], Klüppelholz and Baier introduce *labeled* CA. In addition to the usual ingredients of CA, Klüppelholz and Baier associate each labeled CA with a set of propositions and a labeling function that maps each state to those propositions that hold in it.

[13] In fact, if $|\mathbb{D}\text{ATA}| = 1$, there exists a language equivalent PA for every CA.

We proceed with six more complex extensions: *timed* CA, *probabilistic* CA, *continuous-time* CA, *quantitative* CA, *resource-sensitive timed* CA, and *transactional* CA.

Timed CA In [8, 9], Arbab et al. introduce *timed* CA (TCA) for describing the behavior of *time-dependent* connectors, e.g., a variant of FIFO that loses a data item in its buffer after three time units.

In addition to the usual ingredients of CA, Arbab et al. include a set of *clocks* in each TCA: special variables used to register and evaluate constraints about the passage of time (cf., clocks in timed automata [2]). Similar to ordinary CA, the states of a TCA correspond one-to-one to the internal states of the connector it models. However, Arbab et al. associate each of those states also with a *clock constraint*, which asserts a condition on the values of the available clocks. Only as long as the clock constraint of a state holds, the TCA can remain in that state; otherwise, it *must* make a transition. The label on a transition in a TCA consists not only of a firing set and a data constraint, but also of a clock constraint cc and a *reset set* $Reset$. Informally, a transition $\langle q, F, dc, cc, Reset, q' \rangle$ in a TCA describes an execution step of a connector in a state q before which the clock constraint cc holds and wherein data items that satisfy dc flow through the nodes in F, bringing the connector in the successor state q' after resetting the clocks in $Reset$.

In [8, 9], Arbab et al. also define the join and hide operators for TCA and assert their compositionality under language equivalence.

Probabilistic CA In [15], Baier introduces *probabilistic* CA (PCA) for describing the behavior of *probabilistic* connectors, e.g., a probabilistic variant of LossySync or a randomized Sync.[14]

Similar to ordinary CA, the states of a PCA correspond one-to-one to the internal states of the connector it models. The transition relation of a PCA, however, contains pairs that consist of: a state and a *(discrete) probability distribution* $\Pi : \wp(N) \times \mathbb{R}\text{ECORD} \times Q \to [0, 1]$ over firing sets, records,[15] and states. Informally, a transition $\langle q, \Pi \rangle$ in a PCA describes an execution step of a connector in a state q wherein, with a probability $\Pi(F, \mathsf{r}, q')$, data items flow through the nodes in F according to r, bringing the connector in the

[14] A randomized Sync transforms data items flowing through its input node to other data items according to some probability distribution. See Example 4 in [15].

[15] In [15] (and in other papers on CA and its variants), Baier calls records *data assignments*. Here, we use the terminology and notation of records for uniformity.

successor state q'. In the same paper, Baier introduces an abstraction of PCA, called *simple probabilistic* CA (SPCA), whose transition relation contains quadruples that consist of: a state, a firing set, a data constraint, and a probability distribution $\pi : Q \to [0, 1]$ over states. Informally, a transition $\langle q, F, dc, \pi \rangle$ in an SPCA describes an execution step of a connector in a state q wherein data items that satisfy dc flow through the nodes in F, bringing the connector in the successor state q' with probability $\pi(q')$. Thus, PCA allow for a finer definition of the probability distribution than SPCA.

In [15], Baier also shows that one can transform any CA to an equivalent SPCA, and every SPCA to an equivalent PCA (and therefore, every CA to an equivalent PCA). Moreover, Baier defines the join and hide operators for SPCA and PCA, she defines bisimulation for PCA, and she asserts the compositionality of the join and hide operators for PCA under bisimulation.

Continuous-time CA In [18], Baier and Wolf introduce *continuous-time* CA (CCA) for describing both the behavior of connectors and their *time-dependent stochastic assumptions*, e.g., the stochastic waiting time of pending I/O-operations. This extension of CA enables a formal analysis of the performance of connectors. Although both TCA and CCA incorporate a notion of time, TCA model *functional* temporal aspects of connectors, while CCA model (some of) their *non-functional* temporal stochastic properties.

Similar to ordinary CA, the states of a CCA correspond one-to-one to the internal states of the connector it models. The transition relation of a CCA, however, has two partitions: one with ordinary CA transitions, called *interactive transitions* in the context of CCA, and a partition with *Markovian transitions*. This latter partition contains triples that consist of: a state q, a *rate* λ, and a successor state q'. The rate, a positive real number, denotes the rate parameter of an exponential distribution over time (as in continuous-time Markov chains). Informally, a Markovian transition $\langle q, \lambda, q' \rangle$ in a CCA describes the occurrence of a delay—whatever the source—of t or less time units in state q with probability $1 - e^{-\lambda t}$. A Markovian transition can fire only in the absence of enabled interactive transitions.

In [18], Baier and Wolf also define the join and hide operators for CCA. Moreover, Baier and Wolf define three variants of bisimulation for CCA—strong, weak, and very weak—and they assert the compositionality of the join and hide operators for CCA under strong and weak bisimulation.

Quantitative CA In [12, 53], Arbab et al. introduce *quantitative* CA (QCA) for describing both the behavior of connectors and the *quality of service* (QoS) guarantees they provide, e.g., reliability or shortest transmission time. From a conceptual perspective, QCA differ from TCA and PCA, because these latter two classes of models describe functional aspects of connectors, while QoS guarantees belong to the class of non-functional properties. The difference between QCA and CCA seems more subtle. On the one hand, QCA generalize CCA, because QCA can describe not only information about delays, but also about, e.g., reliability. On the other hand, CCA describe stochastic delays, which QCA seem unable to do.[16] Thus, the sets of connectors whose QoS guarantees QCA and CCA can describe overlap, but seem not coincident.

Similar to CA, the states of a QCA correspond one-to-one to the internal states of the connector it models. The label on a transition in a QCA, however, consists not only of a firing set and a data constraint, but also of a *cost* that represents a QoS metric (e.g., reliability). Informally, a transition $\langle q, F, dc, c, q' \rangle$ in a QCA describes an execution step of a connector in state q wherein data items that satisfy dc flow through the nodes in F, bringing the connector in the successor state q', while providing the QoS guarantees described by c. Formally, a cost c comes from the domain of a *Q-algebra*—an algebraic structure, introduced by Chothia and Kleijn in [24], for the compositional description of QoS properties.

In [12], Arbab et al. also define the join and hide operators for QCA. Moreover, Arbab et al. define four types of simulation for QCA—strong, weak, and (weak) quality improving—and they prove the compositionality of the join and hide operators for QCA under strong, weak, and quality improving simulation. One can use quality improving simulations to analyze, e.g., if an implementation of a connector provides a higher QoS than its specification.

Resource-sensitive timed CA Concurrently with QCA, in [51], Sun Meng and Arbab introduce *resource-sensitive timed* CA (RSTCA) for describing the behavior of time-dependent *resource-sensitive* connectors, e.g., a connector that requires a sufficiently large bandwidth and that times out if this resource remains unavailable during some interval. The class of RSTCA differs from the class of CCA, because CCA incorporate a stochastic notion of the passage of time, while RSTCA incorporate crisp values. Compared to TCA, RSTCA seem more restrictive in the sense that an RSTCA does not feature explicit

[16] To show that QCA *can* describe stochastic delays, one must show how to encode continuous probability distributions as *Q-algebras* [24].

clocks: instead, each state q and each transition t in an RSTCA has its own implicit clock, which this RSTCA resets to 0 once it reaches q or fires t.

In addition to the usual ingredients of CA, Sun Meng and Arbab include a set of *resources* in each RSTCA: special variables used to register information and evaluate constraints about resources (e.g., available bandwidth). Similar to ordinary CA, the states of an RSTCA correspond one-to-one to the internal states of the connector it models. The transition relation of an RSTCA, however, has two partitions: one with *interactive transitions* (different from the interactive transitions in CCA) and another with *timeout transitions*.

The former partition contains transitions with a label that consists not only of a firing set and a data constraint, but also of a *resource constraint* rc and a *duration function* df. Resource constraints assert a condition about the required resources, while duration functions map resources to those times at which a connector must have finished using them. Informally, an interactive transition $\langle q, F, dc, rc, \mathsf{df}, q' \rangle$ in an RSTCA describes an execution step of a connector in state q wherein data items that satisfy dc flow through the nodes in F, while utilizing a set of resources R that satisfy rc, bringing the connector in the successor state q' within time $\mathsf{df}(R)$.

The partition with timeout transitions contains triples that consist of: a state q, a *timeout* t, and a successor state q'. Informally, a timeout transition $\langle q, t, q' \rangle$ describes an execution step of a connector in state q wherein it transits to the successor state q', if no interactive transitions could fire for t time units. Timeout transitions in QCA resemble Markovian transitions in CCA, but carry a crisp maximum value rather than a stochastic parameter.

In [51], Sun Meng and Arbab also define the join and hide operators for RSTCA. Moreover, Sun Meng and Arbab define two types of simulation for RSTCA—functional and strong—and prove the compositionality of the join and hide operators for RSTCA under strong simulation.

Transactional CA In [54], Sun Meng and Arbab introduce *transactional* CA (TNCA) for describing the behavior of connectors whose execution involves *long-running transactions*. Such transactions often provide weaker guarantees than the traditional ACID guarantees for transactions [36] (atomicity, consistency, isolation, durability): for instance, because data involved in long-running transactions is not locked, such transactions typically lack isolation. Long-running transactions occur in service-oriented architectures, thus when using Reo in this application domain, support for long-running transactions becomes important.

Similar to CA, the states of a TNCA correspond one-to-one to the internal states of the connector it models. The label on a transition in a TNCA, however, consists of a set of nodes and *either* a data constraint *or* a transaction ψ. In the former case, we obtain a transition as in ordinary CA, called an *atomic transition* in the context of TNCA, and we interpret the set of nodes this transition carries as a firing set. In the latter case, in contrast, we have a *transaction transition*. Informally, a transaction transition $\langle q, F, \psi, q' \rangle$ in a TNCA describes an execution step of a connector in state q wherein the nodes in F (commit to) participate in the transaction ψ, bringing the connector in the successor state q'. As transactions themselves typically involve interactions and coordination among multiple parties, Sun Meng and Arbab model transactions as Reo connectors.

In [54], Sun Meng and Arbab also define the join operator for TNCA.

3.2.3 Context-Sensitive Automata

As mentioned during our discussion of LossyFIFO in Section 3.2.1, there exists a significant class of connectors whose behavior CA cannot describe satisfactorily: context-sensitive connectors.[17] We recall that the behavior of a context-sensitive connector depends not only on its own internal state, but also on the presence or absence of pending I/O-operations on its boundary nodes, e.g., a LossySync should lose a data item only if it has no pending take operation on its output node. Unfortunately, CA can directly model only a *nondeterministic* approximation of LossySync (as in Figure 3). Below, we discuss other classes of automaton-based models that can describe the behavior of context-sensitive connectors.

Büchi automata In [38, 40], Izadi et al. propose to use *Büchi automata of records* (BAR), i.e., automata on infinite words [63] over records (see Definition 3), for describing the behavior of connectors. Similar to how CA accept TDSs, BAR accept RSs, i.e., record streams. Moreover, the states of a BAR (in its simple form) correspond one-to-one to the internal states of the connector Conn it models, while its transitions, each labeled with a record, describe the execution steps of Conn. Because BAR feature (sets of) accepting states, called *(generalized) acceptance conditions*, BAR can model connectors with *fairness* constraints (e.g., if a node can fire from some instant onwards, it eventually does so), whose behavior CA cannot describe.

[17] Later in this paper, however, we demonstrate that CA actually *can* describe the behavior of context-sensitive connectors.

For describing the behavior of context-sensitive connectors, in [39, 40], Izadi et al. introduce *augmented* BAR (ABAR), whose states do no longer correspond one-to-one to the internal states of connectors, but *many*-to-one: each state of an ABAR does not only represent the internal state of the connector Conn it models, but also registers information about the presence or absence of I/O-operations on the nodes in Conn. While BAR accept infinite sequences of records, i.e., RSs, ABAR accept *alternating* infinite sequences of (i) sets of nodes that have pending I/O-operations, and (ii) records, describing what data items actually flow through which of those nodes.

In [38, 40], Izadi et al. also define the join and hide operators for BAR (the join operator for BAR differs from the usual product operator for Büchi automata). Moreover, Izadi et al. define an operator for transforming an arbitrary CA to a BAR and prove its compositionality (in [40]). Similarly, in [39, 40], Izadi et al. define the join and hide operators for ABAR.

Guarded automata In [20, 21], Bonsangue et al. define *guarded automata* (GA) for describing the behavior of (context-sensitive) connectors.[18] Similar to Izadi's ABAR, GA accept alternating infinite sequences of *guards* and firing sets. We can consider these guards generalizations of the sets of nodes that appear in the infinite words that ABAR accept—guards assert a condition about the presence or absence of I/O-operations on nodes—while we can consider these firing sets abstractions of the records that appear in the words that ABAR accept (because firing sets exclude information about data).

Similar to CA, but in contrast to ABAR, the states of a GA correspond one-to-one to the internal states of the connector it models: information about the presence or absence of I/O-operations does not appear in the states of a GA (as in ABAR), but as guards on its transitions. Transitions additionally carry a firing set, i.e., the nodes through which a data item flows if the corresponding guard holds and the transition fires. Unlike ABAR, GA do not have accepting states.

In [20, 21], Bonsangue et al. also define the join operator for GA (although indirectly, i.e., in terms of two other operators, namely product and synchronization). Moreover, Bonsangue et al. define bisimulation for GA, and assert the compositionality of the join operator under bisimulation. Also, in [20], Bonsangue et al. cast port automata (PA—see Section 3.2.2) and CA into GA, which yields context-sensitive variants of PA and CA.

[18] Bonsangue et al. use also the name *Reo automata* in [20, 21], but because Costa uses this name to refer to a different class of automata in [33], we write GA to avoid confusion.

In [56, 57], Moon et al. extend ordinary GA to *stochastic* GA (SGA) by (i) associating each node with an arrival rate that describes, stochastically, the rate at which I/O-operations arrive, and (ii) associating each transition t with information about the stochastic delay of flow through the nodes that participate in t (similar to the rate on a Markovian transition in a CCA). In [56, 57], Moon et al. also define the join operator for SGA (indirectly in terms of product and synchronization).

Intentional automata In [33], Costa introduces *intentional automata* (IA) for describing the behavior of (context-sensitive) connectors.[19]

Similar to Izadi's ABAR, the states of an IA correspond many-to-one to the internal states of the connector it models: each state of an IA not only captures such an internal state, but also registers the presence of I/O-operations. The classes of ABAR and IA differ, however, in the type of labels their transitions carry: in ABAR, transitions carry records, while in IA, transitions carry pairs that each consist of a firing set and *another* set of nodes. Such a second set of nodes on a transition t contains those nodes on which an I/O-operation becomes pending during the execution step t describes. Similar to GA, but unlike ABAR, IA do not have accepting states.

In [33], Costa also defines the join and hide operators for IA. Moreover, Costa defines (weak) bisimulation for IA, and proves the compositionality of the join and hide operators for IA under weak bisimulation.

In [13], Arbab et al. extend ordinary IA to *quantitative* IA (QIA) by extending the labels on transitions in ordinary IA with (i) a data constraint (as in CA) and (ii) information about the stochastic delay and arrival rates of I/O-operations and data items on the nodes that participate in a transition. In [13], Arbab et al. also define the join operator for QIA (although indirectly, i.e., in terms of two other operators, namely product and refinement).

Action CA In [46], Kokash et al. introduce *action* CA (ACA) for describing the behavior of (context-sensitive) connectors and their temporal QoS properties *without* incorporating an explicit notion of time. Instead, Kokash et al. model delays through the successive execution of distinct *actions* such as "block node n" and "unblock node n." Thus, despite their name, ACA differ significantly from ordinary CA and other CA-based models for describing the temporal QoS aspects of connectors (i.e., CCA, QCA, and RSTCA).

[19] Costa uses also the name *Reo automata* in [33], but because Bonsangue et al. use this name to refer to a different class of automata in [20, 21], we write IA to avoid confusion.

Similar to Izadi's ABAR and Costa's IA, the states of an ACA correspond many-to-one to the internal states of the connector it models and, among other information, can register the presence or absence of I/O-operations. The transition relation of an ACA contains quadruples that consist of: a state, a set of actions over a set of nodes N, a data constraint (as in ordinary CA), and a successor state. An action in the set of actions that a transition t carries denotes an operation (e.g., blocking/unblocking nodes from accepting I/O-operations or starting/stopping the propagation of data between nodes) that a connector performs in the execution step t describes. In the subclass of ACA wherein each action has the form "propagate a data item through node n," sets of actions reduce to firing sets, and ACA simplify to CA.

In [46], Kokash et al. also define the join and hide operators for ACA. Moreover, Kokash et al. encode the ACA of seven primitives into the process algebra `mCRL2`. Shortly, in Section 3.4, we discuss `mCRL2` models for Reo connectors in more detail.

Behavioral automata In [61], Proença introduces *behavioral automata* (BA) for modeling the behavior of connectors in a *step-wise* manner. Proença uses BA in [61] as the basis for an implementation of Reo and to justify the assumptions that underlie his Dreams framework, a distributed implementation of Reo. Because BA can embed other classes of semantic models—including GA—BA can describe the behavior of context-sensitive connectors.

Similar to CA and GA, the states of a BA correspond one-to-one to the internal states of the connector it models.

The transitions of a BA carry abstract *labels*, which map to quintuples $\langle N, F, I, O, data \rangle$ that describe atomic execution steps of a connector. In such a quintuple, N denotes a set of nodes, $F \subseteq N$ denotes a firing set, $I \subseteq F$ and $O \subseteq F$ denote the input and output nodes involved, and $data$ denotes a function from $I \cup O$ to $\mathbb{D}$ATA. Depending on the specific instantiation of a BA (Proença instantiates BA, among other classes of models, for CA and GA in [61]), some components in these quintuples remain unused. For instance, in the instantiation for GA, $I = O = data = \emptyset$.

Furthermore, Proença associates each state of a BA with a *concurrency predicate*. Essentially, a concurrency predicate in a BA `BA` for a state q describes a set of transitions *in a* BA *different from* `BA` that can execute concurrently while `BA` remains in q. More formally, a concurrency predicate C for a state q (in `BA`) denotes a set of labels such that $l \in C$ implies that transitions (outside `BA`) labeled by l can execute concurrently.

In [61], Proença also defines the join operator for BA.

3.2.4 Structural Operational Semantics

Finally, in [58], Mousavi et al. formalize the semantics of (some of the primitives in) Reo with a *structural operational semantics* (SOS) in Plotkin's style [59]. For each primitive,[20] Mousavi et al. define a set of transition rules that induce a transition system. The states in this transition system consist of pairs $\langle Sys, Val \rangle$ with Sys a *Reo system term* (RST) and Val a function that maps nodes to (possibly infinite) sequences of data items. An RST describes the structure of a Reo connector Conn, including the specific primitives that Conn consists of, while the sequences of data items to which Val maps a node n represent those data items that in some instant already have arrived at n. Supplemented with the *maximal progress* assertion of Khosravi et al. in [43], the SOS semantics can model context-sensitive connectors.

In [58], Mousavi et al. also define special transition rules for joining two connectors—these rules correspond to the join operators defined for other classes of semantic models of Reo connectors. Moreover, Mousavi et al. define (bi)simulation for the transition systems that their transition rules induce.

3.3 Coloring Models

We proceed with a third kind of semantic formalisms: *(connector) coloring models* (CM), introduced by Clarke et al. in [28, 29] and later extended by Costa in [33]. Coloring models work by marking the nodes of a connector with *colors* that specify whether data items flow through these nodes or not. Depending on the number of colors, different models with different levels of expressiveness arise. In Section 3.3.1, we discuss CMs with three colors. For now, we assume a total of two colors, which yields *2-coloring models* (2CM): the *flow color* —— (data items can flow through the nodes it marks) and the *no-flow color* - - - - (data items cannot flow through the nodes it marks).

Definition 8 (Colors [29])**.** $\mathbb{C}\text{OLOR} = \{$ ——, - - - - $\}$ *is a set of colors.*

To describe an execution step of a connector, CMs consist of *colorings*: maps from a set of nodes to a set of colors, which assign to each node in the

[20] Including Sync, LossySync, and FIFO.

set a color that indicates whether this node fires (or not) in the execution step the coloring describes. We collect all such possible execution steps of a connector in sets of colorings, called *coloring tables*.

Definition 9 (Coloring [29]). *A coloring* c *over* $N \subseteq \mathbb{N}\text{ODE}$ *is a function* $\mathsf{c} : N \to \mathbb{C}\text{OLOR}$ *that maps a node* n *to a color* $\mathsf{c}(n)$*. We denote the set of all colorings over* N *by* $\mathbb{C}\text{OL}(N)$*.*

Definition 10 (Coloring table [29]). *A coloring table* T *over* $N \subseteq \mathbb{N}\text{ODE}$ *is a set* $T \subseteq \mathbb{C}\text{OL}(N)$ *of colorings over* N*.*

To accommodate connectors that exhibit different behavior in different states (e.g., connectors with buffers), CMs feature *coloring table maps* (CTM): maps from a set of indexes (representing the states of a connector as in CA; see Definition 6) to a set of coloring tables (describing the admissible execution steps in these states).[21] To model the change of state a connector incurs when (some of) its nodes fire, CMs feature *next functions*. The next function of a connector maps an index i in the domain I of a CTM S and a coloring in the coloring table to which S maps i to an index in I (possibly the same i).

Definition 11 (CTM [33]). *A* CTM S *over* $[N \subseteq \mathbb{N}\text{ODE}, I \subseteq \mathbb{I}\text{NDEX}]$ *is a function* $S : I \to \wp(\mathbb{C}\text{OL}(N))$ *that maps an index* i *to a coloring table* $S(i)$ *over* N*.*

Definition 12 (Next function [33]). *Let* S *be a* CTM *over* $[N, I]$*. A next function* η *over* S *is a partial function* $I \times \mathbb{C}\text{OL}(N) \rightharpoonup I$ *that maps every [index, coloring]-pair* $\langle i, \mathsf{c} \rangle$ *such that* $\mathsf{c} \in S(i)$ *to an index* $\eta(i, \mathsf{c})$*.*

Finally, we define the CM of a connector Conn as a pair that consists of a next function (describing the behavior of Conn) and an index (representing its initial state).

Definition 13 (CM). *Let* S *be a* CTM *over* $[N, I]$*. A* CM $\mathtt{CM}$ *over* S *is a pair* $\mathtt{CM} = \langle \eta, i_0 \rangle$ *with* η *a next function over* S *and* $i_0 \in I$*.*

To illustrate the previous definitions, Figure 5 shows the 2CMs of Sync, LossySync, and FIFO, whose structures we depicted in Figure 1.

When we join two connectors Conn_1 and Conn_2 with CMs as their behavioral model, we can compute the CM of the resulting composite by joining the CMs of Conn_1 and Conn_2. We describe this joining process in a

[21] In [33], Costa calls CTMs *indexed sets of coloring tables*.

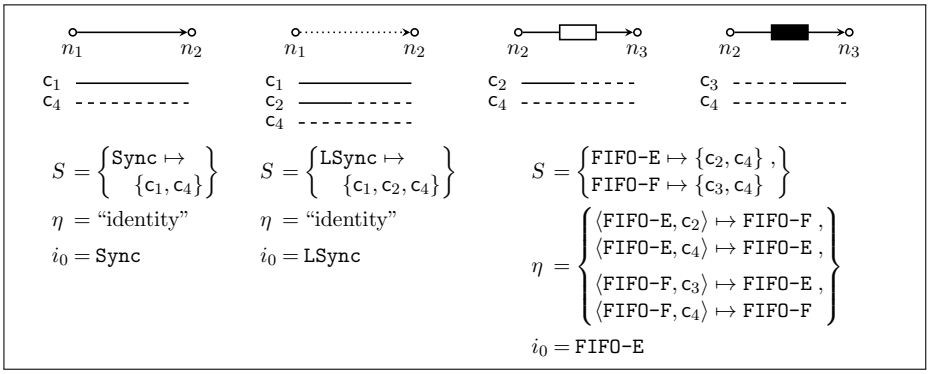


Figure 5: Colorings, CTMs, and next functions of Sync, LossySync and FIFO.

bottom-up fashion. First, to join two *compatible colorings*—colorings that assign the same colors to their common nodes—we merge the domains of these colorings and map each node n in the resulting set to the color that one of the colorings assigns to n. The join of two coloring tables comprises the computation of a new coloring table that contains the pairwise joins of the compatible colorings in the two individual coloring tables.

Definition 14 (Join of colorings [29])**.** *Let* c_1 *and* c_2 *be colorings over* N_1 *and* N_2 *such that* $\mathsf{c}_1(n) = \mathsf{c}_2(n)$ *for all* $n \in N_1 \cap N_2$*. Their join, denoted by* $\mathsf{c}_1 \cup \mathsf{c}_2$*, is a coloring over* $N_1 \cup N_2$ *defined as:*

$$\mathsf{c}_1 \cup \mathsf{c}_2 = \left\{ n \mapsto \kappa \;\middle|\; n \in N_1 \cup N_2 \text{ and } \kappa = \begin{pmatrix} \mathsf{c}_1(n) & \text{if } n \in N_1 \\ \mathsf{c}_2(n) & \text{otherwise} \end{pmatrix} \right\}.$$

Definition 15 (Join of coloring tables [29])**.** *Let* T_1 *and* T_2 *be coloring tables over* N_1 *and* N_2*. Their join, denoted by* $T_1 \cdot T_2$*, is a coloring table over* $N_1 \cup N_2$ *defined as:*

$$T_1 \cdot T_2 = \left\{ \mathsf{c}_1 \cup \mathsf{c}_2 \;\middle|\; \begin{array}{l} \mathsf{c}_1 \in T_1 \text{ and } \mathsf{c}_2 \in T_2 \text{ and} \\ \mathsf{c}_1(n) = \mathsf{c}_2(n) \text{ for all } n \in N_1 \cap N_2 \end{array} \right\}.$$

The join of two CTMs comprises the computation of a new CTM that maps each pair of indexes in the Cartesian product of the domains of the two individual CTMs to the join of the coloring tables to which these CTMs map the indexes in the pair. We define the join of two next functions in terms of the Cartesian product, the join of colorings, and the join of CTMs.

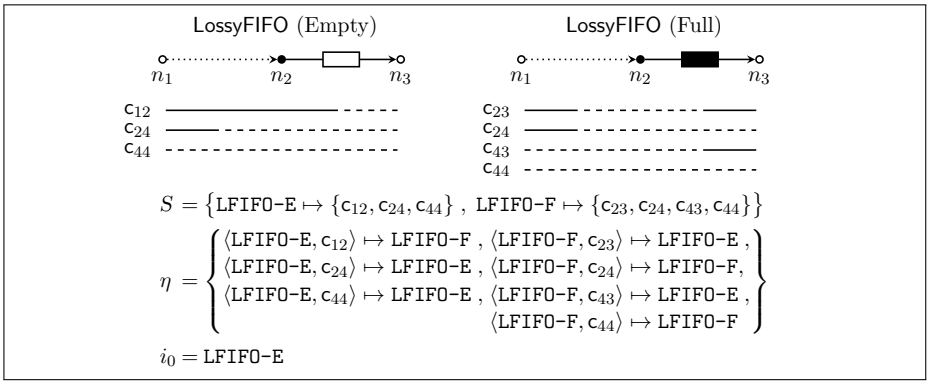


Figure 6: Colorings, CTM, and next function of LossyFIFO. Let `LFIFO-E` denote $\langle \texttt{LSync}, \texttt{FIFO-E}\rangle$, and let `LFIFO-F` denote $\langle \texttt{LSync}, \texttt{FIFO-F}\rangle$.

Definition 16 (Join of CTMs [33])**.** *Let S_1 and S_2 be* CTM*s over $[N_1, I_1]$ and $[N_2, I_2]$. Their join, denoted by $S_1 \odot S_2$, is a* CTM *over $[N_1 \cup N_2, I_1 \times I_2]$ defined as:*

$$S_1 \odot S_2 = \{\langle i_1, i_2\rangle \mapsto S_1(i_1) \cdot S_2(i_2) \,|\, i_1 \in I_1 \text{ and } i_2 \in I_2\}.$$

Definition 17 (Join of next functions [33])**.** *Let η_1 and η_2 be next functions over S_1 and S_2 defined over $[N_1, I_1]$ and $[N_2, I_2]$. Their join, denoted by $\eta_1 \otimes \eta_2$, is a next function over $S_1 \odot S_2$ defined as:*

$$\eta_1 \otimes \eta_2 = \left\{ \langle i_1, i_2\rangle, c_1 \cup c_2 \mapsto \langle \eta_1(i_1, c_1), \eta_2(i_2, c_2)\rangle \;\middle|\; \begin{array}{l} \langle i_1, i_2\rangle \in I_1 \times I_2 \text{ and} \\ c_1 \cup c_2 \in (S_1 \odot S_2)(\langle i_1, i_2\rangle) \end{array} \right\}.$$

Finally, we define the join of CMs in terms of the join of next functions and take the pair of the initial states as the initial state of the join.

Definition 18 (Join of CMs)**.** *Let $\texttt{CM}_1 = \langle \eta_1, i_0^1\rangle$ and $\texttt{CM}_2 = \langle \eta_2, i_0^2\rangle$ be* CM*s over S_1 and S_2. Their join, denoted by $\texttt{CM}_1 \bowtie \texttt{CM}_2$, is a* CM *over $S_1 \odot S_2$ defined as:*

$$\texttt{CM}_1 \bowtie \texttt{CM}_2 = \langle \eta_1 \otimes \eta_2, \langle i_0^1, i_0^2\rangle\rangle.$$

To illustrate the previous definitions, Figure 6 shows the 2CM of LossyFIFO, whose structure we depicted in Figure 2. In this figure, the index of a coloring specifies its origin: a coloring c_{ij} results from joining c_i (of LossySync) with c_j (of FIFO) in Figure 5. Note that this 2CM does not model the intended behavior of LossyFIFO: its coloring c_{24} describes the

inadmissible loss of data in the case of an empty buffer. Thus, as CA, 2CMs seem incapable of modeling context-sensitive connectors. Shortly, we revisit this topic.

The introduction of CMs for Reo stood at the basis of several new tools—now part of the ECT—for animation, simulation, and verification of Reo connectors.

3.3.1 Three colors

In the same publications [28, 29] as those in which Clarke et al. introduce 2CMs, they introduce CMs with three colors, i.e., *3-coloring models* (3CM). The difference between 2CMs and 3CMs lies in the instantiation of $\mathbb{C}$OLOR: when using 3CMs, we replace the no-flow color - - - - with the *two* different no-flow colors - -◁- - and - -▷- -. Although both these colors indicate that the nodes they mark have no flow, they also provide information about the *reason* for this absence: informally, we can think of the former color as stating that the nodes it marks *require* a reason, while we can think of the latter color as stating that the nodes it marks *provide* a reason (e.g., no pending I/O-operation). Apart from this different instantiation of $\mathbb{C}$OLOR, however, *nothing* changes: Definitions 9–13 and 14–18 remain the same.[22]

The availability of two no-flow colors allows 3CMs, in contrast to 2CMs, to properly model context-sensitive connectors. For instance, in the case of LossySync, its 3CM asserts that this connector can lose a data item only if its output node receives a reason for an absence of flow by means of the following coloring (which replaces c_2 in Figure 5):

$$\{n_1 \mapsto \text{——} \,,\, n_2 \mapsto \text{- -◁- -}\}.$$

The potential of CMs to satisfactorily model context-sensitivity formed the main reason for their investigation and introduction. At the time of the first publications on CMs, two colors seemed insufficient for this, while three appeared adequate. In [42], however, its authors present an information-preserving mapping from 3CMs to 2CMs, which demonstrates that 2CMs actually *can* describe the behavior of context-sensitive connectors. This works as follows.

Usually, in 2CMs and 3CMs, one represents every *conceptual* node—a node that one would draw in a Reo diagram—with a single concrete node

[22] One may, however, use an optimization called the *flip-rule* to collapse large coloring tables into smaller ones without loss of information. In that case, Definition 14 becomes slightly more complex. We refer to [29, 33] for details.

in colorings. With two colors, subsequently, one can attribute two different behavior alternatives to each conceptual node, e.g., flow or no-flow. However, if we represent every conceptual node with *two* concrete nodes in colorings, we can attribute 2^2 different behavior alternatives to each conceptual node. Thus, to encode a 3CM as a 2CM, one should double the number of concrete nodes in the 3CM and consistently map every color in the 3CM to a pair of colors in the 2CM. Interestingly, such an encoding of three colors using two concrete nodes and two colors also has an intuitive meaning [42]. More generally, one can attribute 2^k behavior alternatives to a conceptual node represented by k concrete nodes and using two colors.

Because we can transform any CM with $k \geq 3$ colors to a CM with only two colors, we consider only 2CMs in (most of) the rest of this paper.

3.3.2 Tile models

In [11] (based on [22]), Arbab et al. introduce *tile models* for describing the behavior of Reo connectors by casting CMs of connectors into *tiles*. Tiles, introduced in [34], extend Plotkin-style inference rules: they describe transitions from an *initial* configuration (if some *trigger* goes off) to a *final* configuration (producing some *effect*). Different from such inference rules, however, one can compose tiles in three different ways: horizontally (synchronization), vertically (composition in time), and in parallel (concurrency).

In [11], Arbab et al. consider Reo connectors hypergraphs and formalize these as morphisms in a symmetric monoidal category. Subsequently, these morphisms serve as configurations on which tiles operate: Arbab et al. describe the behavior of connectors by associating these morphisms with tiles, each of which describes one of its possible execution steps. More precisely, Arbab et al. first define a series of tiles for many common primitives based on their semantics in terms of 2CMs, prove the correctness of these tiles (with respect to the corresponding 2CMs), and assert the compositionality of these tiles with respect to horizontal and parallel composition. Second, Arbab et al. define a series of tiles for a subset of these common primitives based on their semantics in terms of 3CMs and assert the correctness (with respect to the corresponding 3CMs) and compositionality of these tiles. Note that this second series of tiles, i.e., those derived from 3CMs, can model context-sensitive connectors.

Although, as remarked in [61], tile models comprise one of the most complete semantic models of Reo connectors, no tools support them.

3.4 Other Models

Process algebra In [47, 48, 49], Kokash et al. define the semantics of Reo connectors in terms of *processes* of the process algebra `mCRL2` [35],[23] an algebra that extends the process algebra ACP [19] with data and time. Kokash et al. propose two translations: one based on CA (extended with a TCA-style notion of time in [48]) and another based on 3CMs. In [49], Kokash et al. prove the correctness and compositionality of their translation from CA to `mCRL2` specifications and assert similar results for their translation from 3CMs to `mCRL2`. The ECT includes an implementation of this work.

Because analysis tools, including a model checker, exist for `mCRL2` specifications, we can use the translation from Reo to `mCRL2` for verification of Reo connectors. In fact, the translation of a 3CM of a connector to its corresponding `mCRL2` specification formed the only way to verify context-sensitive connectors until recently.[24]

Constraints In [30, 31, 32], Clarke et al. define the semantics of Reo connectors in terms of propositional *constraints* (but different from the data constrains in CA). Initially, in [31], Clarke et al. define each connector in terms of four different kinds of constraints: *synchronization constraints*, *data flow constraints*, *state constraints*, and *external constraints*. Synchronization constraints describe which nodes can synchronize in some execution step, while data flow constraints describe what particular data items flow in such a step through which nodes. State constraints describe how the state of a connector evolves during its execution, while external constraints capture externally maintained state. In [32], Clarke et al. prove the correctness and compositionality of this constraint-based approach with respect to CA.

In [32], Clarke et al. extend their initial approach with *context constraints* for modeling context-sensitive connectors. Similar to how Kokash et al. take 3CMs as the basis for their encoding of context-sensitive connectors into `mCRL2`, Clarke et al. base their context constraints on colorings in 3CMs. Moreover, Clarke et al. prove the correctness of this context-sensitive constraint-based approach with respect to 3CMs.

In [61], Proença uses SAT-solvers to execute connectors based on their constraint model, and he shows that this approach significantly improves

[23] http://www.mcrl2.org

[24] Another model checker for Reo, called *Vereofy* [16], operates on CA, which many consider incapable of modeling context-sensitivity (see also Section 3.2.1). We discuss Vereofy and its ability to model check context-sensitive connectors in Section 7.

performance (as compared to the execution of a connector based on its CM).

Petri nets and intuitionistic temporal linear logic In [27], Clarke defines the semantics of nine Reo connectors (including common primitives such as Sync, LossySync, and FIFO) in terms of *zero-safe nets* [23], a special class of Petri nets. Different from ordinary Petri nets, zero-safe nets feature two types of places: *zero places* and *stable places*. When moving from one marking to the next, a zero-safe net fires until all tokens occupy only stable places. Such an *epoch* may consist of multiple firings, but those intermediate markings wherein some tokens occupy zero places remain unobservable.

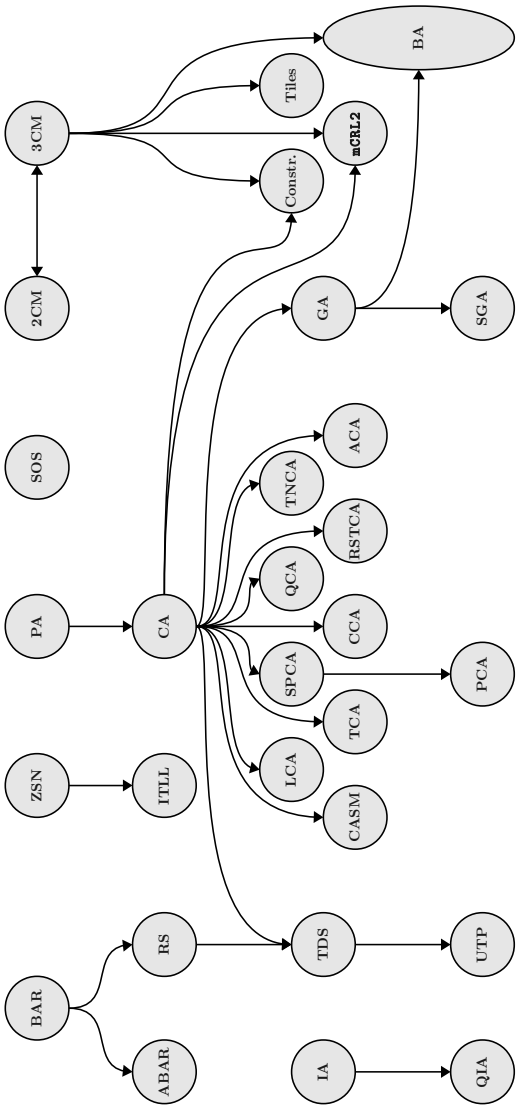
When modeling a Reo connector Conn with a zero-safe net, every epoch of this net represents a single execution step. In Clarke's encoding, zero places ensure that the nodes in Conn cannot fire multiple times during a single epoch; otherwise, this would correspond to nodes of Conn having data flow more than once in a single execution step of Conn, which cannot happen. Other comparisons between Reo and Petri nets appear in [7, 50].

In [27], Clarke also casts Reo and zero-safe nets into *intuitionistic temporal linear logic*. Clarke does this not only for reasoning about coordination models, but also to enable more refined behavioral descriptions of connectors (e.g., to distinguish internal from external choices).

Unifying theories of programming In [52], Sun Meng and Arbab define the semantics of Reo connectors in terms of the *unifying theories of programming* (UTP) [37]. The UTP can provide a formal semantics for various languages, and thus, facilitates the specification of *similar* features in *different* languages in a *similar* style. This enables a straightforward comparison and analysis of different languages in the same framework.

Sun Meng and Arbab model Reo connectors as UTP *designs*. A UTP design comprises a pair of predicates of the form $P \vdash Q$ with P an *assumption* on the input (of a connector) and Q a *commitment* about the output (of a connector). The predicates P and Q induce sets of admissible tuples of *timed data sequences*: structures that resemble tuples of timed data streams (see Section 3.1), but which can have a finite length.

In [55], Sun Meng et al. use the UTP semantics of Reo for model based testing through refinement and test case generation (also based on the work presented in [1]).



2CM : Coloring models with two colors [28, 29, 33]
3CM : Coloring models with three colors [28, 29, 33]
ABAR : Augmented BAR [39, 40]
ACA : Action CA [46]
BA : Behavioral automata [61]
BAR : Büchi automata of records [38, 40]
CA : Constraint automata [10, 17]
CASM : CA with state memory [60]
CCA : Continuous-time CA [18]
Constr. : Propositional constraints [30, 31, 32]
GA : Guarded automata [20, 21]
IA : Intentional automata [33]
ITLL : Intuitionistic temporal linear logic [27]
LCA : Labeled CA [44]
mCRL2 : Process algebra [47, 48, 49]
PA : Port automata [45]
PCA : Probabilistic CA [15]
QCA : Quantitative CA [12, 53]
QIA : Quantitative IA [13]
RS : Record streams [38, 40]
RSTCA : Resource-sensitive timed CA [51]
SGA : Stochastic GA [56, 57]
SOS : Structural operational semantics [58]
SPCA : Simple PCA [15]
TCA : Timed CA [8, 9]
TDS : Timed data streams [4, 5, 14, 62]
Tiles : Tile models [11]
TNCA : Transactional CA [54]
UTP : Unifying theories of programming [55, 52]
ZSN : Zero-safe nets [27]

Figure 7: Known relations between semantic formalisms. An arrow from formalism X to formalism Y means: if one can model the behavior of a connector Conn in X, one can model the behavior of Conn in Y without loss of information.

3.5 Summary

Figure 7 summarizes the relations between the semantic formalisms discussed in this section.

4 Data-Aware Coloring Models

As demonstrated in the previous section, constraint automata and coloring models influenced and formed the basis of many other classes of semantic models: CA inspired at least TCA, PCA, CCA, QCA, RSTCA, TNCA, while CMs inspired tile models, the `mCRL2` models, and the models based on constraints. Formally establishing a correspondence between CA and CMs, therefore, has great value: it paves the way for the application of tools and extensions devised for CA to CMs and vice versa.

In this section, we take a first step towards this goal: we make traditional CMs *data-aware* by extending them with constraints similar to those carried by transitions in CA. We introduce this extension, because one of the transformation operators that we define later in this paper lacks a desirable property otherwise: it would map *many*-to-one instead of *one*-to-one. More precisely, the transformation from CA to CMs would map *different* CA, namely those whose transitions carry different data constraints but equal firing sets, to the *same* CM. Alternatively, to gain this one-to-one property, we could have narrowed our scope to PA, which abstract from data constraints (see Section 3.2.2). We favor an extension of CMs for generality.

We add data-awareness to CMs, independently of the number of colors, by associating each coloring with a data constraint. Such a *constraint coloring* describes an execution step of a connector wherein data items that satisfy the data constraint flow through the nodes marked by the flow color. Below we give the formal definition. With respect to notation, we place a $\sim$ above those symbols that denote constituents of data-aware CMs (but we use the same symbols as for CMs without constraints).

Definition 19 (Constraint coloring)**.** *A constraint coloring* $\widetilde{\mathsf{c}}$ *over* $[N \subseteq \textsc{Node}, DC \subseteq \mathbb{D}\textsc{c}(N)]$ *is a pair* $\widetilde{\mathsf{c}} = \langle \mathsf{c}, dc \rangle$ *with* c *a coloring over* N *and* $dc \in DC$ *a data constraint such that* $dc \in \mathbb{D}\textsc{c}(F)$ *with* $F = \{n \in N \mid \mathsf{c}(n) = \text{——}\}$*. We denote the set of all constraint colorings over* $[N, DC]$ *by* $\mathbb{CC}\textsc{ol}(N, DC)$*.*

Note that our definition does not exclude a constraint coloring $\widetilde{\mathsf{c}} = \langle \mathsf{c}, dc \rangle$ with inconsistent c and dc. For example, c may mark some node n with the no-flow color, while dc entails the flow of some data item through n. We do

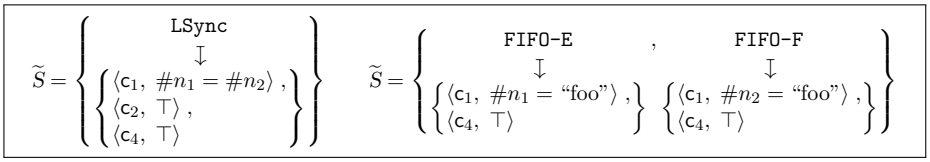


Figure 8: Constraint CTMs of LossySync and FIFO for $\mathbb{D}\text{ATA} = \{\text{“foo”}\}$.

not forbid such constraint colorings, because they do not impair the models in which they appear: they merely describe behavior that never arises.

Next, we incorporate data constraints in the definitions of the other constituents of ordinary CMs (as presented in Section 3.3). This turns out straightforwardly. To summarize the upcoming definitions: (i) a *constraint coloring table* is a set of constraint colorings, (ii) a *constraint* CTM is a map from indexes to constraint coloring tables, (iii) a *constraint next function* is a map from [index, constraint coloring]-pairs to indexes, and (iv) a *constraint* CM (CCM) is a [constraint next function, index]-pair, i.e., a data-aware CM.

Definition 20 (Constraint coloring table)**.** *A constraint coloring table* $\widetilde{T}$ *over* $[N \subseteq \mathbb{N}\text{ODE}\,, DC \subseteq \mathbb{D}\text{C}(N)]$ *is a set* $\widetilde{T} \subseteq \mathbb{CC}\text{OL}(N, DC)$ *of constraint colorings over* $[N, DC]$.

Definition 21 (Constraint CTM)**.** *A constraint* CTM $\widetilde{S}$ *over* $[N \subseteq \mathbb{N}\text{ODE}\,, DC \subseteq \mathbb{D}\text{C}(N)\,, I \subseteq \mathbb{I}\text{NDEX}]$ *is a function* $\widetilde{S} : I \to \wp(\mathbb{CC}\text{OL}(N, DC))$ *that maps an index* i *to a constraint coloring table* $\widetilde{S}(i)$ *over* $[N, DC]$.

Definition 22 (Constraint next function)**.** *Let* $\widetilde{S}$ *be a constraint* CTM *over* $[N, DC, I]$. *A constraint next function* $\widetilde{\eta}$ *over* $\widetilde{S}$ *is a partial function* $\widetilde{\eta} : I \times \mathbb{CC}\text{OL}(N, DC) \rightharpoonup I$ *that maps every [index, constraint coloring]-pair* $\langle i, \widetilde{\mathsf{c}} \rangle$ *such that* $\widetilde{\mathsf{c}} \in \widetilde{S}(i)$ *to an index* $\widetilde{\eta}(i, \widetilde{\mathsf{c}})$.[25]

Definition 23 (CCM)**.** *Let* $\widetilde{S}$ *be a constraint* CTM *over* $[N, DC, I]$. *A* CCM $\widetilde{\mathtt{CM}}$ *over* $\widetilde{S}$ *is a pair* $\widetilde{\mathtt{CM}} = \langle \widetilde{\eta}, i_0 \rangle$ *with* $\widetilde{\eta}$ *a constraint next function over* $\widetilde{S}$ *and* $i_0 \in I$.

To illustrate the previous definitions, Figure 8 shows the constraint CTMs of LossySync and FIFO for $\mathbb{C}\text{OLOR} = \{\text{——}, \text{- - - -}\}$. The colorings c_i with $i \in \{1, 2, 3, 4\}$ refer to the colorings in Figure 5. For instance, constraint coloring $\langle \mathsf{c}_1, \#n_1 = \#n_2 \rangle$ in the constraint CTM of LossySync describes the

[25] Even if c and dc in $\widetilde{\mathsf{c}} = \langle \mathsf{c}, dc \rangle$ are inconsistent. Cf. a transition in a CA labeled with $\neg\top$ also goes to some state.

execution step of LossySync wherein the same data item flows through n_1 and n_2. Constraint coloring $\langle \mathsf{c}_4, \top \rangle$, describes the "execution step" wherein a connector idles. Due to the constraint $\top$, this may always happen. Similarly, LossySync can always behave as described by $\langle \mathsf{c}_2, \top \rangle$, but whereas $\langle \mathsf{c}_4, \top \rangle$ entails no flow at all, $\langle \mathsf{c}_2, \top \rangle$ entails flow through c_1 and no flow through c_2. Here, $\top$ specifies that we do not care about which data item flows through c_1. With respect to FIFO, for simplicity of the example, we assume the universe of data items a singleton as before (see Section 3.3).

Finally, we update the join operators for CMs to incorporate data constraints. For brevity, we give these definitions only for constraint colorings and constraint coloring tables. The join operators for constraint CTMs (symbol: $\widetilde{\odot}$), constraint next functions (symbol: $\widetilde{\otimes}$), and CCMs (symbol: $\widetilde{\bowtie}$) resemble their respective join operators in Section 3.3: essentially, it suffices to replace S_1, S_2, η_1, η_2, $\mathtt{CM}_1$, and $\mathtt{CM}_2$ in Definitions 16–18 with their $\sim$ versions $\widetilde{S}_1$, $\widetilde{S}_2$, $\widetilde{\eta}_1$, $\widetilde{\eta}_2$, $\widetilde{\mathtt{CM}}_1$, and $\widetilde{\mathtt{CM}}_2$. We require only these minor updates, because our extension of CMs with data constraints affects only the definition of colorings directly. For completeness, in Appendix A, we give the definitions of the remaining join operators.

Definition 24 (Join of constraint colorings)**.** *Let* $\widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1 \rangle$ *and* $\widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2 \rangle$ *be constraint colorings over* $[N_1, DC_1]$ *and* $[N_2, DC_2]$ *such that* $\mathsf{c}_1(n) = \mathsf{c}_2(n)$ *for all* $n \in N_1 \cap N_2$*. Their join, denoted by* $\widetilde{\mathsf{c}}_1 \mathbin{\widetilde{\cup}} \widetilde{\mathsf{c}}_2$*, is a constraint coloring over* $[N_1 \cup N_2, DC_1 \wedge DC_2]$ *defined as:*

$$\widetilde{\mathsf{c}}_1 \mathbin{\widetilde{\cup}} \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_1 \cup \mathsf{c}_2, dc_1 \wedge dc_2 \rangle.$$

Definition 25 (Join of constraint coloring tables)**.** *Let* $\widetilde{T}_1$ *and* $\widetilde{T}_2$ *be constraint coloring tables over* $[N_1, DC_1]$ *and* $[N_2, DC_2]$*. Their join, denoted by* $\widetilde{T}_1 \mathbin{\widetilde{\cdot}} \widetilde{T}_2$*, is a constraint coloring table over* $[N_1 \cup N_2, DC_1 \wedge DC_2]$ *defined as:*

$$\widetilde{T}_1 \mathbin{\widetilde{\cdot}} \widetilde{T}_2 = \left\{ \widetilde{\mathsf{c}}_1 \mathbin{\widetilde{\cup}} \widetilde{\mathsf{c}}_2 \;\middle|\; \begin{array}{l} \widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1 \rangle \in \widetilde{T}_1 \text{ and } \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2 \rangle \in \widetilde{T}_2 \\ \text{and } \mathsf{c}_1(n) = \mathsf{c}_2(n) \text{ for all } n \in N_1 \cap N_2 \end{array} \right\}.$$

Note that by taking their conjunction, we combine data constraints in Definition 24 in the same way as the join operator for CA in Definition 7. The lemmas that we formulate and prove in the subsequent sections establish the appropriateness of taking the conjunction of data constraints in the context of CMs.

To illustrate the previous definitions, Figure 9 shows the constraint CTM of LossyFIFO. The colorings c_i with $i \in \{12, 24, 43, 44\}$ refer to the colorings in Figure 6.

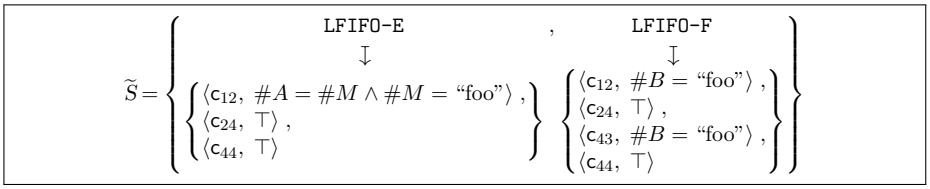


Figure 9: Constraint CTM of LossyFIFO for $\mathbb{D}\text{ATA} = \{\text{“foo”}\}$. Let `LFIFO-E` denote $\langle$`LSync`, `FIFO-E`$\rangle$, and let `LFIFO-F` denote $\langle$`LSync`, `FIFO-F`$\rangle$.

We remark that in [61], Proença uses pairs of colorings and records (as in Definition 3) as transition labels of his behavioral automata (see Section 3.2.3) to account for the transfer of data that takes place through data-flows described by those colorings. This suggests using [coloring, record]-pairs as a data-aware CM. However, our constraint-based extension offers a more concise formalization. For instance, in Proença's model, to give the semantics of LossySync, one must include a [coloring, record]-pair in its coloring table for each data item in $\mathbb{D}$ATA. With our constraint-based extension, in contrast, we capture this with a single constraint coloring as shown in Figure 8.

5 From CCMs to CA

In this section, we present a unary operator, denoted by $\mathbb{L}$, which takes a CCM as its input and outputs an *equivalent* CA; shortly, we elaborate on the meaning of "equivalence" in this context. We call this process of transforming a CCM to a CA the $\mathbb{L}$-*transformation*. By defining the $\mathbb{L}$-transformation for any CCM, it follows that the class of connectors that we can model with a CA *includes* those that we can model with a CCM. It follows that CA are at least as expressive as data-aware CMs.

Suppose we wish to transform a CCM $\widetilde{\texttt{CM}} = \langle \widetilde{\eta}, i_0 \rangle$ over $\widetilde{S}$ over $[N, DC, I]$. The $\mathbb{L}$-operator derives a CA from $\widetilde{\texttt{CM}}$ as follows. First, $\mathbb{L}$ instantiates the set of states of this derived CA with the set of indexes I: this seems reasonable as I denotes the set of indexes that represent the states of the connector that $\widetilde{\texttt{CM}}$ models. Next, $\mathbb{L}$ constructs a transition relation T based on the mappings in $\widetilde{\eta}$: for each $[\langle i, \langle \mathsf{c}, dc \rangle\rangle \mapsto i'] \in \widetilde{\eta}$, the $\mathbb{L}$-operator creates a transition from state i to state i', labeled with dc as its data constraint and with the set of nodes to which c assigns the flow color as its firing set. Finally, i_0 becomes the initial state of the new CA.

Definition 26 ($\mathbb{L}$). *Let* $\widetilde{\mathtt{CM}} = \langle \widetilde{\eta}, i_0 \rangle$ *be a* CCM *over* $\widetilde{S}$ *over* $[N, DC, I]$. *The* $\mathbb{L}$*-transformation of* $\widetilde{\mathtt{CM}}$, *denoted by* $\mathbb{L}(\widetilde{\mathtt{CM}})$, *is defined as:*

$$\mathbb{L}(\widetilde{\mathtt{CM}}) = \langle I, \mathcal{T}, i_0 \rangle$$

$$\textit{with: } \mathcal{T} = \left\{ \langle i, F, dc, \widetilde{\eta}(i, \widetilde{\mathtt{c}}) \rangle \;\middle|\; \begin{array}{l} i \in I \textit{ and } \widetilde{\mathtt{c}} = \langle \mathtt{c}, dc \rangle \in \widetilde{S}(i) \\ \textit{and } F = \{\, n \in N \,|\, \mathtt{c}(n) = \text{——} \,\} \,\} \end{array} \right\}.$$

The proposition below states that the application of $\mathbb{L}$ to a CCM yields a CA.

Proposition 1. *Let* $\widetilde{\mathtt{CM}}$ *be a* CCM *over* $\widetilde{S}$ *over* $[N, DC, I]$. *Then,* $\mathbb{L}(\widetilde{\mathtt{CM}})$ *is a* CA *over* $[N, DC]$.

Proof. See Appendix B. □

In the rest of this section, we prove the equivalence between a CCM $\widetilde{\mathtt{CM}}$ and the CA that results from applying $\mathbb{L}$ to $\widetilde{\mathtt{CM}}$. Additionally, we prove the compositionality of $\mathbb{L}$.

5.1 Correctness of $\mathbb{L}$

In this subsection, we prove the *correctness* of $\mathbb{L}$: we consider $\mathbb{L}$ correct if its application to a CCM yields an *equivalent* CA. We call a CCM and a CA equivalent if there exists a *bisimulation relation* that relates these two models. Informally, a CA $\mathtt{CA}$ is bisimilar to a CCM $\widetilde{\mathtt{CM}}$ if, for each mapping in the constraint next function of $\widetilde{\mathtt{CM}}$, there exists a *corresponding* transition in the $\mathtt{CA}$ and vice versa, i.e., a transition that describes the same behavior in terms of the nodes that fire, the data items that flow, and the change of state.

Definition 27 (Bisimulation). *Let* $\mathtt{CA} = \langle Q, \mathcal{T}, q_0 \rangle$ *be a* CA *over* $[N, DC]$ *and* $\widetilde{\mathtt{CM}} = \langle \widetilde{\eta}, i_0 \rangle$ *a* CCM *over* $\widetilde{S}$ *over* $[N, DC, I]$. $\mathtt{CA}$ *and* $\widetilde{\mathtt{CM}}$ *are bisimilar, denoted as* $\mathtt{CA} \sim \widetilde{\mathtt{CM}}$, *if there exists a relation* $\mathcal{R} \subseteq Q \times I$ *such that* $\langle q_0, i_0 \rangle \in \mathcal{R}$ *and for all* $\langle q, i \rangle \in \mathcal{R}$:

(I) *If* $\langle q, F, dc, q' \rangle \in \mathcal{T}$ *then there exists an* $i' \in I$ *such that:*

- $[\langle i, \widetilde{\mathtt{c}} \rangle \mapsto i'] \in \widetilde{\eta}$ *with* $\widetilde{\mathtt{c}} = \langle \mathtt{c}, dc \rangle$;
- $\langle q', i' \rangle \in \mathcal{R}$;
- $F = \{\, n \in N \,|\, \mathtt{c}(n) = \text{——} \,\}$.

(II) *If* $[\langle i, \widetilde{\mathtt{c}} \rangle \mapsto i'] \in \widetilde{\eta}$ *with* $\widetilde{\mathtt{c}} = \langle \mathtt{c}, dc \rangle$ *then there exists a* $q' \in Q$ *such that:*

- $\langle q, F, dc, q' \rangle \in \mathcal{T}$;
- $\langle q', i' \rangle \in \mathcal{R}$;
- $F = \{\, n \in N \,|\, \mathtt{c}(n) = \text{——} \,\}$.

In that case, $\mathcal{R}$ *is called a bisimulation relation.*

In the previous definition, we compare data constraints syntactically. Alternatively, one can define bisimulation by comparing data constraints logically, i.e., in terms of the records that satisfy a data constraint. The latter yields a weaker notion of bisimulation than the one formalized in Definition 27. Because we can prove the stronger form between a CCM $\widetilde{\texttt{CM}}$ and its $\mathbb{L}$-transformation $\mathbb{L}(\widetilde{\texttt{CM}})$, however, we compare data constraints syntactically. In our proof, which appears in the appendix, we choose the diagonal relation on the set of indexes as a bisimulation relation.

Lemma 1. *Let* $\widetilde{\texttt{CM}}$ *be a* CCM*. Then,* $\mathbb{L}(\widetilde{\texttt{CM}}) \sim \widetilde{\texttt{CM}}$*.*

Proof. See Appendix B. □

5.2 Compositionality of $\mathbb{L}$

We end this section with a compositionality lemma for $\mathbb{L}$. Informally, it states that it does not matter whether we first join CCMs $\widetilde{\texttt{CM}}_1$ and $\widetilde{\texttt{CM}}_2$ and then apply $\mathbb{L}$ to the resulting composite or first apply $\mathbb{L}$ to $\widetilde{\texttt{CM}}_1$ and $\widetilde{\texttt{CM}}_2$ individually and then join the resulting transformations; the resulting CA equal each other. The relevance of this result lies in the potential reduction in the amount of overhead that it allows for when applying the $\mathbb{L}$-operator in practice. This works as follows. There exist tools for Reo that operate on CA and that have built-in functionality for the computation of their join. By the compositionality lemma for $\mathbb{L}$, to use such a tool, we need to transform the common primitives only once, store these in a library, and use this library together with the built-in functionality for join computation to construct the CA of composites (on which the tool subsequently operates). Thus, the overhead of this approach remains constant. In contrast, the overhead of the alternative—first joining CCMs and then transforming the resulting composites using $\mathbb{L}$—grows linearly in the number of composites one wishes to apply the tool on.[26] In Section 7, we illustrate the foregoing with a concrete example; here, we proceed with the lemma.

Lemma 2. *Let* $\widetilde{\texttt{CM}}_1$ *and* $\widetilde{\texttt{CM}}_2$ *be* CCM*s. Then,*
$\mathbb{L}(\widetilde{\texttt{CM}}_1) \bowtie \mathbb{L}(\widetilde{\texttt{CM}}_2) = \mathbb{L}(\widetilde{\texttt{CM}}_1 \mathbin{\widetilde{\bowtie}} \widetilde{\texttt{CM}}_2)$.

Proof. See Appendix B. □

[26] For now, we assume the computation of the join of two CCMs and two CA equally expensive. In Section 7, we argue for the merits of our approach when the cost of joining CCMs differs from the cost of joining CA.

Although we consider only CCMs with two colors, one can apply $\mathbb{L}$ also to CCMs with three colors. In fact, Lemma 1 (correctness) would still hold! Essentially, this means that CCMs with three colors do not have a higher degree of expressiveness than CCMs with two colors. In contrast, Lemma 2 (compositionality) does *not* hold if we consider CCMs with three colors. See Appendix B for details.

6 From CA to CCMs

In this section, we demonstrate a correspondence between CCMs and CA in the direction opposite to the previous section's: from the latter to the former. Our approach, however, resembles our approach in Section 5: we present a unary operator, denoted by $\frac{1}{\mathbb{L}}$, which takes a CA as its input and produces an equivalent, i.e., bisimilar, CCM.[27] We call our process of transforming a CA to a CCM the $\frac{1}{\mathbb{L}}$-*transformation* and define the $\frac{1}{\mathbb{L}}$-operator for any CA. It follows that the class of connectors that we can model with CA includes those that we can model with a CCM. Since the previous section gave us a similar result in the opposite direction, we conclude that CCMs and CA have the same degree of expressiveness.

The $\frac{1}{\mathbb{L}}$ operator works as follows; suppose we wish to transform a CA $\mathtt{CA}$ over $[N, DC]$. The $\frac{1}{\mathbb{L}}$-operator derives a CCM from $\mathtt{CA}$ as follows: for each transition $\langle q, F, dc, q' \rangle$ in the transition relation of $\mathtt{CA}$, $\frac{1}{\mathbb{L}}$ includes a mapping from state q and a constraint coloring $\widetilde{\mathtt{c}} = \langle \mathtt{c}, dc \rangle$ to state q', where $\mathtt{c}$ assigns the flow color to all and only the nodes in F.

Definition 28 ($\mathtt{col}$)**.** *Let $N, F \subseteq \mathbb{N}\text{ODE}$. Then:*

$$\mathtt{col}(N, F) = \left\{ n \mapsto \kappa \;\middle|\; n \in N \text{ and } \kappa = \begin{pmatrix} \text{——} & \text{if } n \in F \\ \text{- - - -} & \text{otherwise} \end{pmatrix} \right\}.$$

Definition 29 ($\frac{1}{\mathbb{L}}$)**.** *Let $\mathtt{CA} = \langle Q, \mathsf{T}, q_0 \rangle$ be a* CA *over $[N, DC]$. The $\frac{1}{\mathbb{L}}$-transformation of $\mathtt{CA}$, denoted by $\frac{1}{\mathbb{L}}(\mathtt{CA})$, is defined as:*

$$\tfrac{1}{\mathbb{L}}(\mathtt{CA}) = \langle \widetilde{\eta}, q_0 \rangle$$

[27] Recall from Section 3.2.1 (and Footnotes 8 and 9) that, without loss of generality, we consider only CA whose transition relations satisfy the following condition:

$$\langle q, F_1, dc_1, q_1' \rangle, \langle q, F_2, dc_2, q_2' \rangle \in \mathsf{T} \text{ and } q_1' \neq q_2' \text{ implies } \langle F_1, dc_1 \rangle \neq \langle F_2, dc_2 \rangle$$

This condition ensures that we can cast transition relations into transition functions.

with: $\widetilde{\eta} = \{\langle q, \langle \mathtt{col}(N,F), dc\rangle\rangle \mapsto q' \,|\langle q, F, dc, q'\rangle \in \mathcal{T}\}$.

The proposition below states that the application of $\frac{1}{\mathbb{L}}$ to a CA yields a CCM.

Proposition 2. *Let* $\mathtt{CA} = \langle Q, \mathcal{T}, q_0\rangle$ *be a* CA *over* $[N, DC]$. *Then,* $\frac{1}{\mathbb{L}}(\mathtt{CA})$ *is a* CCM *over* $\widetilde{S}$ *over* $[N, DC, Q]$ *defined as:*

$$\widetilde{S} = \{\, q \mapsto \widetilde{T} \,|\, q \in Q \text{ and } \widetilde{T} = \{\langle \mathtt{col}(N,F), dc\rangle \,|\langle q, F, dc, q'\rangle \in \mathcal{T}\}\,\}.$$

Proof. See Appendix B. □

6.1 Inverse

Having defined $\frac{1}{\mathbb{L}}$, we proceed by proving that it forms the *inverse* of $\mathbb{L}$ (as already hinted at by its symbol) and vice versa. We do this before stating the correctness and compositionality of $\frac{1}{\mathbb{L}}$, because the proofs of these lemmas become significantly easier once we know that $\frac{1}{\mathbb{L}}$ inverts $\mathbb{L}$. The following two lemmas state the inverse properties in both directions.

Lemma 3. *Let* $\widetilde{\mathtt{CM}}$ *be a* CCM. *Then,* $\frac{1}{\mathbb{L}}(\mathbb{L}(\widetilde{\mathtt{CM}})) = \widetilde{\mathtt{CM}}$.

Proof. See Appendix B. □

Lemma 4. *Let* $\mathtt{CA}$ *be a* CA. *Then,* $\mathbb{L}(\frac{1}{\mathbb{L}}(\mathtt{CA})) = \mathtt{CA}$.

Proof. See Appendix B. □

6.2 Correctness and Compositionality of $\frac{1}{\mathbb{L}}$

As mentioned previously, knowing that $\mathbb{L}(\frac{1}{\mathbb{L}}(\mathtt{CA})) = \mathtt{CA}$ simplifies our correctness and compositionality proofs. We start with the former. Lemma 5, which appears below, states the bisimilarity between $\mathtt{CA}$ and its $\frac{1}{\mathbb{L}}$-transformation $\frac{1}{\mathbb{L}}(\mathtt{CA})$.

Lemma 5. *Let* $\mathtt{CA}$ *be a* CA. *Then,* $\mathtt{CA} \sim \frac{1}{\mathbb{L}}(\mathtt{CA})$.

Proof. See Appendix B. □

Finally, Lemma 6 states the compositionality of $\frac{1}{\mathbb{L}}$: informally, this means that it does not matter whether we first join CA $\mathtt{CA}_1$ and $\mathtt{CA}_2$ and then apply $\frac{1}{\mathbb{L}}$ to the resulting composite or first apply $\frac{1}{\mathbb{L}}$ to $\mathtt{CA}_1$ and $\mathtt{CA}_2$ and then join the resulting transformations; the resulting CCMs equal each other. Our proof, similar to that of the previous lemma, relies on the inverse lemmas.

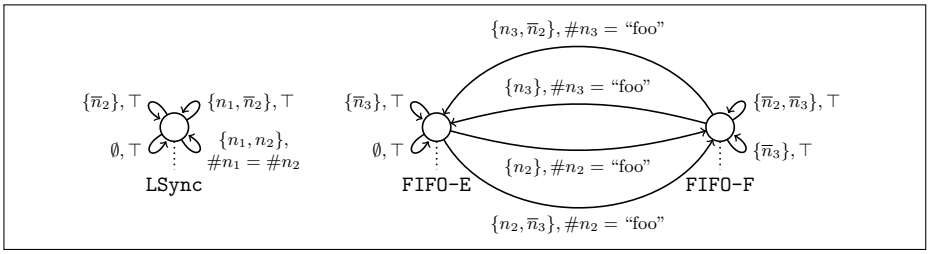


Figure 10: CA obtained by applying $\mathbb{L}$ to the encoded 3CCMs, as 2CCMs, of LossySync and FIFO for $\mathbb{D}\text{ATA} = \{\text{“foo”}\}$.

Lemma 6. *Let* $\mathtt{CA}_1$ *and* $\mathtt{CA}_2$ *be* CA. *Then,* $\frac{1}{\mathbb{L}}(\mathtt{CA}_1) \widetilde{\bowtie} \frac{1}{\mathbb{L}}(\mathtt{CA}_2) = \frac{1}{\mathbb{L}}(\mathtt{CA}_1 \bowtie \mathtt{CA}_2)$.

Proof. See Appendix B. □

7 Application

In this section, we sketch an application of the results presented in Sections 5 and 6: the integration of verification and animation of context-sensitive connectors in Vereofy [16], a model checking tool that operates on CA.[28] Broadly, this application consists of two parts: model checking connectors built from context-sensitive constituents and generating animated counterexamples.

Verification of CCMs Vereofy operates on CA, and therefore, many consider it unable to verify context-sensitive connectors. We mend this deficiency as follows. First, recall from Section 3.3.1 that we can transform CMs with three colors, known for their ability to properly capture context-sensitivity to corresponding CMs with two colors. Because data constraints form an orthogonal concern, this means that CCMs with only two colors can serve as faithful models of context-sensitive connectors. Consequently, the results from Section 5 enable the verification of such connectors with Vereofy: using the $\mathbb{L}$-transformation, we transform context-sensitive CCMs to context-sensitive CA, which we then can analyze with Vereofy.

As an example, Figure 10 shows the CA obtained by applying $\mathbb{L}$ to the encoded 3CCMs, as 2CCMs, of LossySync and FIFO. In this figure, $\overline{n}_1$, $\overline{n}_2$, and $\overline{n}_3$ denote new concrete nodes that the encoding procedure from 3CCMs to 2CCMs introduces (see Section 5): each of the pairs $\langle n_1, \overline{n}_1 \rangle$, $\langle n_2, \overline{n}_2 \rangle$, $\langle n_3, \overline{n}_3 \rangle$

[28] http://www.vereofy.de

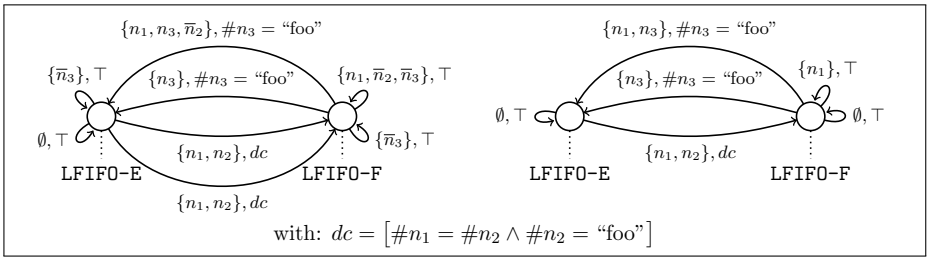


Figure 11: CA of LossyFIFO obtained by joining the CA in Figure 10. Let `LFIFO-E` denote $\langle$`LSync`, `FIFO-E`$\rangle$, and let `LFIFO-F` denote $\langle$`LSync`, `FIFO-F`$\rangle$.

of concrete nodes in the CA represents a single conceptual node that one would draw in a diagram. Note that abstracting away $\overline{n}_1$, $\overline{n}_2$, and $\overline{n}_3$, yields the CA in Figure 3. The left CA in Figure 11 shows the join of the CA in Figure 10; the right CA in Figure 11 shows the same CA but with $\overline{n}_1$, $\overline{n}_2$, and $\overline{n}_3$ abstracted away. Compared to Figure 4, this CA does not contain the transition $\langle$`LFIFO-E`, $\{n_1\}, \top$, `LFIFO-E`$\rangle$, i.e., it models the context-sensitive LossySync rather than its nondeterministic sibling. One can now use Vereofy to analyze this CA. Another example appears in [42].

In this application, the compositionality of $\mathbb{L}$ in Lemma 2 plays an important role (as already outlined in Section 5.2): it facilitates the one-time-application of $\mathbb{L}$ to encoded 3CCMs, as 2CCMs, of Reo's primitives. Subsequently, one can use Vereofy's built-in functionality for joining CA to construct the compound automata that one wishes to analyze. We remark that our compositionality lemmas work also in the opposite direction: if future studies indicate that joining CCMs costs less than joining CA, we can extend Vereofy with a module to automatically (i) transform CA of primitives to CMs with $\frac{1}{\mathbb{L}}$, (ii) join the resulting CCMs, and (iii) transform the resulting composite back to a CA with $\mathbb{L}$. (To truly gain in performance, however, the costs of transforming forth and back should not exceed the benefits of joining CCMs instead of CA.)

Animation of CA Vereofy facilitates the generation and inspection of counterexamples, an important feature that distinguishes it from `mCRL2` (recall from Section 3.4 that, alternatively, we can verify Reo connectors with the analysis tools of `mCRL2`). When used together with the ECT, Vereofy can in *some* cases display counterexamples as connector animations. These animated counterexamples comprise a graphical model of a connector through

which data items visually flow for each execution step in an error trace.

Although such animations improve the ease with which Vereofy users can analyze counterexamples, the opportunity to actually provide these visualizations depends on the availability of a CM of the connector under investigation (in addition to the CA that Vereofy's verification algorithm operates on). Moreover, the standalone version of Vereofy, a command-line tool, does not facilitate the animation of counterexamples at all. The results in Section 6, however, enable animated counterexamples for any CA: in case of unavailability of a CA, Vereofy can simply generate such a model with the $\frac{1}{\mathbb{L}}$-transformation.

8 Conclusion

In this paper, we gave an overview of all the existing semantic formalisms for modeling Reo connectors. Furthermore, we showed that once extended with data constraints, coloring models with two colors and constraint automata have the same degree of expressiveness by defining two operators that transform such data-aware CMs to corresponding CA and vice versa. Moreover, we showed the compositionality of these operators. Though primarily a theoretical contribution, we illustrated how our results can broaden the applicability of Reo's tools.

With respect to future work, we would like to implement the transformation operators and the sketched extension to Vereofy. Another application worth investigating comprises the development of an implementation of Reo based on transforming behavioral models of connectors back and forth to improve performance. Finally, we would like to study correspondences between other classes of semantic models.

Acknowledgements

We thank the reviewers and the members of the ICE 2011 forum nicknamed `gege`, `wind`, `wolf` and `xyz` for their valuable comments.

References

[1] Bernhard Aichernig, Farhad Arbab, Lăcrămioara Aştefănoaei, Frank de Boer, Sun Meng, and Jan Rutten. Fault-Based Test Case Generation

for Component Connectors. In *Proceedings of TASE 2009*, pages 147–154, 2009.

[2] Rajeev Alur and David Dill. A theory of timed automata. *TCS*, 126(2):183–235, 1994.

[3] Farhad Arbab. Coordination of Mobile Components. *ENTCS*, 54(1):1–16, 2001.

[4] Farhad Arbab. Abstract Behavior Types: A Foundation Model for Components and their Composition. In Frank de Boer, Marcello Bonsangue, Susanne Graf, and Willem-Paul de Roever, editors, *Formal Methods for Components and Objects*, volume 2852 of *LNCS*, pages 33–70. Springer, 2003.

[5] Farhad Arbab. Reo: a channel-based coordination model for component composition. *MSCS*, 14(3):329–366, 2004.

[6] Farhad Arbab. Abstract Behavior Types: a foundation model for components and their composition. *Sci. Comput. Program.*, 55(1–3):3–52, 2005.

[7] Farhad Arbab. Coordination for Component Composition. *ENTCS*, 160(1):15–40, 2006.

[8] Farhad Arbab, Christel Baier, Frank de Boer, and Jan Rutten. Models and Temporal Logics for Timed Component Connectors. In *Proceedings of SEFM 2004*, pages 198–207, 2004.

[9] Farhad Arbab, Christel Baier, Frank de Boer, and Jan Rutten. Models and temporal logical specifications for timed component connectors. *SoSyM*, 6(1):59–82, 2007.

[10] Farhad Arbab, Christel Baier, Jan Rutten, and Marjan Sirjani. Modeling Component Connectors in Reo by Constraint Automata (Extended Abstract). *ENTCS*, 97(1):25–46, 2004.

[11] Farhad Arbab, Roberto Bruni, Dave Clarke, Ivan Lanese, and Ugo Montanari. Tiles for Reo. In Andrea Corradini and Ugo Montanari, editors, *Recent Trends in Algebraic Development Techniques*, volume 5486 of *LNCS*, pages 37–55. Springer, 2009.

[12] Farhad Arbab, Tom Chothia, Sun Meng, and Young-Joo Moon. Component Connectors with QoS Guarantees. In Amy Murphy and Jan Vitek, editors, *Coordination Models and Languages*, volume 4467 of *LNCS*, pages 286–304. Springer, 2007.

[13] Farhad Arbab, Tom Chothia, Rob van der Mei, Sun Meng, Young-Joo Moon, and Chrétien Verhoef. From Coordination to Stochastic Models of QoS. In John Field and Vasco Vasconcelos, editors, *Coordination Models and Languages*, volume 5521 of *LNCS*, pages 268–287. Springer, 2009.

[14] Farhad Arbab and Jan Rutten. A Coinductive Calculus of Component Connectors. In Martin Wirsing, Dirk Pattinson, and Rolf Hennicker, editors, *Recent Trends in Algebraic Development Techniques*, volume 2755 of *LNCS*, pages 34–55. Springer, 2003.

[15] Christel Baier. Probabilistic Models for Reo Connector Circuits. *JUCS*, 11(10):1718–1748, 2005.

[16] Christel Baier, Tobias Blechmann, Joachim Klein, Sascha Klüppelholz, and Wolfgang Leister. Design and Verification of Systems with Exogenous Coordination Using Vereofy. In Tiziana Margaria and Bernhard Steffen, editors, *Leveraging Applications of Formal Methods, Verification, and Validation*, volume 6416 of *LNCS*, pages 97–111. Springer, 2010.

[17] Christel Baier, Marjan Sirjani, Farhad Arbab, and Jan Rutten. Modeling component connectors in Reo by constraint automata. *Sci. Comput. Program.*, 61(2):75–113, 2006.

[18] Christel Baier and Verena Wolf. Stochastic Reasoning About Channel-Based Component Connectors. In Paolo Ciancarini and Herbert Wiklicky, editors, *Coordination Models and Languages*, volume 4038 of *LNCS*, pages 1–15. Springer, 2006.

[19] Jan Bergstra and Jan Willem Klop. Process algebra for synchronous communication. *IC*, 60(1–3):109–137, 1984.

[20] Marcello Bonsangue, Dave Clarke, and Alexandra Silva. A model of context-dependent component connectors. *Accepted for publication in Sci. Comput. Program.*

[21] Marcello Bonsangue, Dave Clarke, and Alexandra Silva. Automata for Context-Dependent Connectors. In John Field and Vasco Vasconcelos, editors, *Coordination Models and Languages*, volume 5521 of *LNCS*, pages 184–203. Springer, 2009.

[22] Roberto Bruni, Ivan Lanese, and Ugo Montanari. A basic algebra of stateless connectors. *TCS*, 366(1–2):95–120, 2006.

[23] Roberto Bruni and Ugo Montanari. Zero-Safe Nets: Comparing the Collective and Individual Token Approaches. *IC*, 156(1–2):46–89, 2000.

[24] Tom Chothia and Jetty Kleijn. Q-Automata: Modelling the Resource Usage of Concurrent Components. *ENTCS*, 175(2):153–167, 2007.

[25] Dave Clarke. Reasoning About Connector Reconfiguration II: Basic Reconfiguration Logic. *ENTCS*, 159(1):61–77, 2006.

[26] Dave Clarke. A Basic Logic for Reasoning about Connector Reconfiguration. *Fundam. Inform.*, 82(4):361–390, 2008.

[27] Dave Clarke. Coordination: Reo, Nets, and Logic. In Frank de Boer, Marcello Bonsangue, Susanne Graf, and Willem-Paul de Roever, editors, *Formal Methods for Components and Objects*, volume 5382 of *LNCS*, pages 226–256. Springer, 2008.

[28] Dave Clarke, David Costa, and Farhad Arbab. Connector Colouring I: Synchronisation and Context Dependency. *ENTCS*, 154(1):101–119, 2006.

[29] Dave Clarke, David Costa, and Farhad Arbab. Connector colouring I: Synchronisation and context dependency. *Sci. Comput. Program.*, 66(3):205–225, 2007.

[30] Dave Clarke and José Proença. Coordination via Interaction Constraints I: Local Logic. *EPTCS*, 12(1):17–39, 2009.

[31] Dave Clarke, José Proença, Alexander Lazovik, and Farhad Arbab. Deconstructing Reo. *ENTCS*, 229(2):43–58, 2009.

[32] Dave Clarke, José Proença, Alexander Lazovik, and Farhad Arbab. Channel-based coordination via constraint satisfaction. *Sci. Comput. Program.*, 76(8):681–710, 2011.

[33] David Costa. *Formal Models for Component Connectors*. PhD thesis, Free University Amsterdam, 2010.

[34] Fabio Gadducci and Ugo Monanari. The tile model. In Gordon Plotkin, Colin Stirling, and Mads Tofte, editors, *Proof, language, and interaction: essays in honour of Robin Milner*, Foundations of Computing, pages 133–166. The MIT Press, 2000.

[35] Jan Friso Groote, Jeroen Keiren, Aad Mathijssen, Bas Ploeger, Frank Stappers, Carst Tankink, Yaroslav Usenko, Muck van Weerdenburg, Wieger Wesselink, Tim Willemse, and Jeroen van der Wulp. The mCRL2 toolset. In *Proceedings of WASDeTT 2008*, 2008.

[36] Theo Haerder and Andreas Reuter. Principles of Transaction-Oriented Database Recovery. *ACM CSUR*, 15(4):287–317, 1983.

[37] Tony Hoare and He Jifeng. *Unifying Theories of Programming*. Prentice Hall, 1998.

[38] Mohammad Izadi and Marcello Bonsangue. Recasting Constraint Automata into Büchi Automata. In John Fitzgerald, Anne Haxthausen, and Husnu Yenigun, editors, *Theoretical Aspects of Computing—ICTAC 2008*, volume 5160 of *LNCS*, pages 156–170. Springer, 2008.

[39] Mohammad Izadi, Marcello Bonsangue, and Dave Clarke. Modeling Component Connectors: Synchronisation and Context-Dependency. In *Proceedings of SEFM 2008*, pages 303–312, 2008.

[40] Mohammad Izadi, Marcello Bonsangue, and Dave Clarke. Büchi automata for modeling component connectors. *Software and Systems Modeling*, 10(2):183–200, 2011.

[41] Sung-Shik Jongmans and Farhad Arbab. Correlating Semantic Models of Reo Connectors: Connector Coloring and Constraint Automata. *EPTCS*, 59(1):84–103, 2011.

[42] Sung-Shik Jongmans, Christian Krause, and Farhad Arbab. Encoding Context-Sensitivity in Reo into Non-Context-Sensitive Semantic Models. In Wolfgang de Meuter and Gruia-Catalin Roman, editors, *Coordination Models and Languages*, volume 6721 of *LNCS*, pages 31–48. Springer, 2011.

[43] Ramtin Khosravi, Marjan Sirjani, Nesa Asoudeh, Shaghayegh Sahebi, and Hamed Iravanchi. Modeling and Analysis of Reo Connectors Using Alloy. In Doug Lea and Gianluigi Zavattaro, editors, *Coordination Models and Languages*, volume 5052 of *LNCS*, pages 169–183. Springer, 2008.

[44] Sascha Klüppelholz and Christel Baier. Symbolic Model Checking for Channel-based Component Connectors. *ENTCS*, 175(2):19–37, 2007.

[45] Christian Koehler and Dave Clarke. Decomposing Port Automata. In *Proceedings of SAC 2009*, pages 1369–1373, 2009.

[46] Natallia Kokash, Behnaz Changizi, and Farhad Arbab. A Semantic Model for Service Composition with Coordination Time Delays. In Jin Song Dong and Huibiao Zhu, editors, *Formal Methods and Software Engineering*, volume 6447 of *LNCS*, pages 106–121. Springer, 2010.

[47] Natallia Kokash, Christian Krause, and Erik de Vink. Data-Aware Design and Verification of Service Compositions with Reo and mCRL2. In *Proceedings of SAC 2010*, pages 2406–2413, 2010.

[48] Natallia Kokash, Christian Krause, and Erik de Vink. Time and Data-Aware Analysis of Graphical Service Models in Reo. In *Proceedings of SEFM 2010*, pages 125–134, 2010.

[49] Natallia Kokash, Christian Krause, and Erik de Vink. Verification of Context-Dependent Channel-Based Service Models. In Frank de Boer, Marcello Bonsangue, Stefan Hallerstede, and Michael Leuschel, editors, *Formal Methods for Components and Objects*, volume 6286 of *LNCS*, pages 21–40. Springer, 2010.

[50] Christian Krause. Integrated Structure and Semantics for Reo Connectors and Petri Nets. *EPTCS*, 12(1):57–69, 2009.

[51] Sun Meng and Farhad Arbab. On Resource-Sensitive Timed Component Connectors. In Marcello Bonsangue and Einar Broch Johnsen, editors, *Formal Methods for Open Object-Based Distributed Systems*, volume 4468 of *LNCS*, pages 301–316. Springer, 2007.

[52] Sun Meng and Farhad Arbab. Connectors as Designs. *ENTCS*, 255(1):119–135, 2009.

[53] Sun Meng and Farhad Arbab. QoS-Driven Service Selection and Composition Using Quantitative Constraint Automata. *Fundam. Inform.*, 95(1):103–128, 2009.

[54] Sun Meng and Farhad Arbab. A Model for Web Service Coordination in Long-Running Transactions. In *Proceedings of SOSE 2010*, pages 121–128, 2010.

[55] Sun Meng, Farhad Arbab, Bernhard Aichernig, Lăcrămioara Aştefănoaei, Frank de Boer, and Jan Rutten. Connectors as designs: Modeling, refinement and test case generation. *Accepted for publication in Sci. Comput. Program.*

[56] Young-Joo Moon, Alexandra Silva, Christian Krause, and Farhad Arbab. A compositional model to reason about end-to-end QoS in Stochastic Reo connectors. *Accepted for publication in Sci. Comput. Program.*

[57] Young-Joo Moon, Alexandra Silva, Christian Krause, and Farhad Arbab. A Compositional Semantics for Stochastic Reo Connectors. *EPTCS*, 30(1):93–107, 2010.

[58] Mohammed-Reza Mousavi, Marjan Sirjani, and Farhad Arbab. Formal Semantics and Analysis of Component Connectors in Reo. *ENTCS*, 154(1):83–99, 2006.

[59] Gordon Plotkin. A Structural Approach to Operational Semantics. *J. Log. Algebr. Program.*, 60-61:17–139, 2004.

[60] Bahman Pourvatan, Marjan Sirjani, Hossein Hojjat, and Farhad Arbab. Automated Analysis of Reo Circuits using Symbolic Execution. *ENTCS*, 255(1):137–158, 2009.

[61] José Proença. *Synchronous Coordination of Distributed Components.* PhD thesis, Universiteit Leiden, 2011.

[62] Jan Rutten. Component Connectors. In Prakash Panangaden and Franck van Breugel, editors, *Mathematical Techniques for Analyzing Concurrent and Probabilistic Systems*, volume 23 of *CRM*, pages 73–87. AMS, 2004.

[63] Wolfgang Thomas. Automata on Infinite Objects. In Jan van Leeuwen, editor, *Formal Models and Semantics*, volume B of *HTCS*, pages 133–191. The MIT Press, 1990.

A Appendix: Composition Operators

In this appendix, we give the formal definitions of the join operators whose definition we gave only informally in Section 4. More specifically, we give the definitions of the join operators for constraint CTMs, constraint next functions, and CCMs. The definitions of the join operators for constraint colorings and constraint coloring tables appear in Section 4. As mentioned in that section, we obtain the operators that we define below by replacing S_1, S_2, η_1, η_2, $\mathtt{CM}_1$, and $\mathtt{CM}_2$ in Definitions 16–18 with their $\sim$ versions.

Definition 30 (Join of constraint CTMs)**.** *Let $\widetilde{S}_1$ and $\widetilde{S}_2$ be constraint* CTM*s over $[N_1, DC_1, I_1]$ and $[N_2, DC_2, I_2]$. Their join, denoted by $\widetilde{S}_1 \mathbin{\widetilde{\odot}} \widetilde{S}_2$, is a constraint* CTM *over $[N_1 \cup N_2, DC_1 \wedge DC_2, I_1 \times I_2]$ defined as:*

$$\widetilde{S}_1 \mathbin{\widetilde{\odot}} \widetilde{S}_2 = \{\langle i_1, i_2\rangle \mapsto \widetilde{S}_1(i_1) \mathbin{\widetilde{\cdot}} \widetilde{S}_2(i_2) \mid i_1 \in I_1 \text{ and } i_2 \in I_2\}.$$

Definition 31 (Join of constraint next functions)**.** *Let $\widetilde{\eta}_1$ and $\widetilde{\eta}_2$ be constraint next functions over $\widetilde{S}_1$ and $\widetilde{S}_2$ over $[N_1, DC_1, I_1]$ and $[N_2, DC_2, I_2]$. Their join, denoted by $\widetilde{\eta}_1 \mathbin{\widetilde{\otimes}} \widetilde{\eta}_2$, is a constraint next function over $\widetilde{S}_1 \mathbin{\widetilde{\odot}} \widetilde{S}_2$ defined as:*

$$\widetilde{\eta}_1 \mathbin{\widetilde{\otimes}} \widetilde{\eta}_2 = \left\{ \begin{array}{c} \langle i_1, i_2\rangle, \widetilde{\mathtt{c}}_1 \mathbin{\widetilde{\cup}} \widetilde{\mathtt{c}}_2 \\ \downarrow \\ \langle \widetilde{\eta}_1(i_1, \widetilde{\mathtt{c}}_1), \widetilde{\eta}_2(i_2, \widetilde{\mathtt{c}}_2)\rangle \end{array} \middle| \begin{array}{c} \langle i_1, i_2\rangle \in I_1 \times I_2 \\ \text{and} \\ \widetilde{\mathtt{c}}_1 \mathbin{\widetilde{\cup}} \widetilde{\mathtt{c}}_2 \in (\widetilde{S}_1 \mathbin{\widetilde{\odot}} \widetilde{S}_2)(\langle i_1, i_2\rangle) \end{array} \right\}.$$

Definition 32 (Join of CCMs)**.** *Let $\widetilde{\mathtt{CM}}_1 = \langle \widetilde{\eta}_1, i_0^1\rangle$ and $\widetilde{\mathtt{CM}}_2 = \langle \widetilde{\eta}_2, i_0^2\rangle$ be* CCM*s over $\widetilde{S}_1$ and $\widetilde{S}_2$. Their join, denoted by $\widetilde{\mathtt{CM}}_1 \mathbin{\widetilde{\bowtie}} \widetilde{\mathtt{CM}}_2$, is a* CCM *over $\widetilde{S}_1 \mathbin{\widetilde{\odot}} \widetilde{S}_2$ defined as:*

$$\widetilde{\mathtt{CM}}_1 \mathbin{\widetilde{\bowtie}} \widetilde{\mathtt{CM}}_2 = \langle \widetilde{\eta}_1 \mathbin{\widetilde{\otimes}} \widetilde{\eta}_2, \langle i_0^1, i_0^2\rangle\rangle.$$

B Appendix: Proofs

Proof of Proposition 1 (𝕃 is well-defined). By Definition 26 of 𝕃, we must show that $\mathbb{L}(\widetilde{\texttt{CM}}) = \langle I, \mathcal{T}, i_0 \rangle$ is a CA over $[N, DC]$. To demonstrate this, by Definition 5 of CA, we must show that $\mathcal{T} \subseteq I \times \wp(N) \times DC \times I$. Now, suppose $\widetilde{\texttt{CM}} = \langle \widetilde{\eta}, i_0 \rangle$. Because, by the premise, $\widetilde{\texttt{CM}}$ is a CCM over $\widetilde{S}$ over $[N, DC, \mathcal{I}]$, by Definition 22, the co-domain of $\widetilde{\eta}$ is I. Also, by Definition 26, for all $\langle i, F, dc, \widetilde{\eta}(i, \widetilde{\mathsf{c}}) \rangle \in \mathcal{T}$ with $\widetilde{\mathsf{c}} = \langle \mathsf{c}, dc \rangle$, it holds that $i \in I$, $F \subseteq N$, and $dc \in DC$ (this latter follows from Definition 19). □

Proof of Lemma 1 (correctness of 𝕃). Suppose $\widetilde{\texttt{CM}} = \langle \widetilde{\eta}, i_0 \rangle$ over $[N, \widetilde{S}]$ over $[N, DC, \mathcal{I}]$. Additionally, suppose $\mathbb{L}(\widetilde{\texttt{CM}}) = \langle I, \mathcal{T}, i_0 \rangle$. We show that $\mathcal{R} = \{\langle i, i \rangle \mid i \in I\}$ is a bisimulation relation by demonstrating that it satisfies (I) and (II) in Definition 27. Let $\langle i, i \rangle \in \mathcal{R}$.

(I) Suppose $\langle i, F, dc, i' \rangle \in \mathcal{T}$. Then, by Definition 26 of 𝕃, there exists a $\widetilde{\mathsf{c}} = \langle \mathsf{c}, dc \rangle \in \widetilde{S}(i)$ such that $i' = \widetilde{\eta}(i, \widetilde{\mathsf{c}})$. Hence, $[\langle i, \widetilde{\mathsf{c}} \rangle \mapsto i'] \in \widetilde{\eta}$. Also, by the definition of $\mathcal{R}$, $\langle i', i' \rangle \in \mathcal{R}$. Finally, by Definition 26 of 𝕃, $F = \{\, n \in N \mid \mathsf{c}(n) = \text{——} \,\}$. Therefore, $\mathcal{R}$ satisfies (I).

(II) Suppose $[\langle i, \widetilde{\mathsf{c}} \rangle \mapsto i'] \in \widetilde{\eta}$ with $\widetilde{\mathsf{c}} = \langle \mathsf{c}, dc \rangle$. Then, by Definition 22 of $\widetilde{\eta}$, it holds that $i \in I$ and $\widetilde{\mathsf{c}} \in \widetilde{S}(i)$, hence by Definition 26 of 𝕃, $\langle i, F, dc, i' \rangle \in \mathcal{T}$ with $F = \{\, n \in N \mid \mathsf{c}(n) = \text{——} \,\}$. Also, by the definition of $\mathcal{R}$, $\langle i', i' \rangle \in \mathcal{R}$. Therefore, $\mathcal{R}$ satisfies (II).

Thus, $\mathcal{R}$ satisfies (I) and (II). Moreover, because $i_0 \in I$ by Definition 23, $\langle i_0, i_0 \rangle \in \mathcal{R}$. Hence, $\mathcal{R}$ is a bisimulation relation. Therefore, $\mathbb{L}(\widetilde{\texttt{CM}}) \sim \widetilde{\texttt{CM}}$. □

Proof of Lemma 2 (compositionality of 𝕃). Suppose $\widetilde{\texttt{CM}}_1 = \langle \widetilde{\eta}_1, i_0^1 \rangle$ and $\widetilde{\texttt{CM}}_2 = \langle \widetilde{\eta}_2, i_0^2 \rangle$ over $\widetilde{S}_1$ and $\widetilde{S}_2$. Applying Definition 26 of 𝕃 to the left-hand side (LHS) and Definition 32 of $\widetilde{\bowtie}$ (informally on page 231) to the right-hand side (RHS) yields:

$$\langle I_1, \mathcal{T}_1, i_0^1\rangle \bowtie \langle I_2, \mathcal{T}_2, i_0^2\rangle$$
$$=$$
$$\mathbb{L}\left(\left\langle \left\{ \begin{array}{c} \langle i_1, i_2\rangle, \widetilde{\mathsf{c}}_1 \mathbin{\widetilde{\cup}} \widetilde{\mathsf{c}}_2 \\ \downarrow \\ \langle \widetilde{\eta}_1(i_1, \widetilde{\mathsf{c}}_1), \widetilde{\eta}_2(i_2, \widetilde{\mathsf{c}}_2)\rangle \end{array} \middle| \begin{array}{c} \langle i_1, i_2\rangle \in I_1 \times I_2 \\ \text{and} \\ \widetilde{\mathsf{c}}_1 \mathbin{\widetilde{\cup}} \widetilde{\mathsf{c}}_2 \in (\widetilde{S}_1 \mathbin{\widetilde{\odot}} \widetilde{S}_2)(\langle i_1, i_2\rangle) \end{array} \right\}, \langle i_0^1, i_0^2\rangle \right\rangle\right)$$

$$\text{with: } \mathcal{T}_1 = \left\{ \langle i_1, F_1, dc_1, \widetilde{\eta}_1(i_1, \widetilde{\mathsf{c}}_1)\rangle \middle| \begin{array}{l} i_1 \in I_1 \text{ and } \widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1\rangle \in \widetilde{S}_1(i_1) \\ \text{and } F_1 = \{\, n \in N_1 \mid \mathsf{c}_1(n) = \text{——} \,\} \end{array} \right\}$$

$$\text{and: } \mathcal{T}_2 = \left\{ \langle i_2, F_2, dc_2, \widetilde{\eta}_2(i_2, \widetilde{\mathsf{c}}_2)\rangle \middle| \begin{array}{l} i_2 \in I_2 \text{ and } \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2\rangle \in \widetilde{S}_1(i_2) \\ \text{and } F_2 = \{\, n \in N_2 \mid \mathsf{c}_2(n) = \text{——} \,\} \end{array} \right\}$$

Applying Definition 7 of $\bowtie$ (to LHS), and Definition 26 of $\mathbb{L}$ (to RHS) yields:

$$\langle I_1 \times I_2, \mathcal{T}, \langle i_0^1, i_0^2\rangle\rangle = \langle I_1 \times I_2, \mathcal{T}', \langle i_0^1, i_0^2\rangle\rangle$$

$$\text{with: } \mathcal{T} = \left\{ \langle \langle i_1, i_2\rangle, F_1 \cup F_2, dc_1 \wedge dc_2, \langle i_1', i_2'\rangle\rangle \middle| \begin{array}{l} \langle i_1, F_1, dc_1, i_1'\rangle \in \mathcal{T}_1 \text{ and} \\ \langle i_2, F_2, dc_2, i_2'\rangle \in \mathcal{T}_2 \text{ and} \\ F_1 \cap N_2 = F_2 \cap N_1 \end{array} \right\}$$

$$\text{and: } \mathcal{T}' = \left\{ \langle i, F, dc, (\widetilde{\eta}_1 \mathbin{\widetilde{\otimes}} \widetilde{\eta}_2)(i, \widetilde{\mathsf{c}})\rangle \middle| \begin{array}{l} i \in I_1 \times I_2 \text{ and} \\ \widetilde{\mathsf{c}} = \langle \mathsf{c}, dc\rangle \in (\widetilde{S}_1 \mathbin{\widetilde{\odot}} \widetilde{S}_2)(i) \text{ and} \\ F = \{\, n \in N_1 \cup N_2 \mid \mathsf{c}(n) = \text{——} \,\} \end{array} \right\}$$

What remains to be shown is $\mathcal{T} = \mathcal{T}'$. This follows from Figure 12. □

Interestingly, Lemma 2 (compositionality) does *not* hold if we consider CCMs with three colors: the fourth—counted from top to bottom—equality in Figure 12 on page 249 ("Because, by the definition of F_1 and F_2...") becomes invalid if we consider CCMs with three colors. This means that, although CCMs with two and three colors have the same degree of expressiveness, *they compose differently*: paradoxically, the addition of a third color restricts, as intended, the number of compatible colorings. This restriction allows one to model context-sensitive connectors compositionally with three colors: it serves as a filter for nondeterministic behavior that should not occur due to context-sensitivity.

Proof of Proposition 2 ($\frac{1}{\mathbb{L}}$ is well-defined). By Definition 29 of $\frac{1}{\mathbb{L}}$, we must show that $\frac{1}{\mathbb{L}}(\mathtt{CA}) = \langle \widetilde{\eta}, q_0\rangle$ is a CCM over $\widetilde{S}$. To demonstrate this, by Definition 23 of CCM, we must show that $\widetilde{\eta}$ is a constraint next function over $\widetilde{S}$. First, by Definition 28, all colorings that occur in the domain of $\widetilde{\eta}$ have N as their domain. Next, by Definition 29, all indexes that occur in the domain

$$
\begin{array}{ll}
 & T \\
= & \text{/* By the definition of } T \text{ in Lemma 2 */} \\
 & \{\langle\langle i_1,i_2\rangle, F_1\cup F_2, dc_1\wedge dc_2, \langle i_1',i_2'\rangle\rangle \,|\, \langle i_1,F_1,dc_1,i_1'\rangle \in T_1 \text{ and } \langle i_2,F_2,dc_2,i_2'\rangle \in T_2 \text{ and } F_1\cap N_2 = F_2\cap N_1\} \\
= & \text{/* By the definitions of } T_1 \text{ and } T_2 \text{ in Lemma 2, and by introducing } i=\langle i_1,i_2\rangle \text{ */} \\
 & \left\{ \langle i, F_1\cup F_2, dc_1\wedge dc_2, \langle i_1',i_2'\rangle\rangle \;\middle|\; \begin{array}{l} i_1\in I_1 \text{ and } \widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1\rangle \in \widetilde{S}_1(i_1) \text{ and } i_1' = \widetilde{\eta}_1(i_1,\widetilde{\mathsf{c}}_1) \text{ and } F_1 = \{\, n\in N_1 \,|\, \mathsf{c}_1(n) = \text{——} \,\} \text{ and} \\ i_2\in I_2 \text{ and } \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2\rangle \in \widetilde{S}_2(i_2) \text{ and } i_2' = \widetilde{\eta}_2(i_2,\widetilde{\mathsf{c}}_2) \text{ and } F_2 = \{\, n\in N_2 \,|\, \mathsf{c}_2(n) = \text{——} \,\} \text{ and} \\ F_1\cap N_2 = F_2\cap N_1 \text{ and } i=\langle i_1,i_2\rangle \end{array} \right\} \\
= & \text{/* Because, by the Cartesian product, } [i_1\in I_1 \text{ and } i_2\in I_2] \text{ iff } \langle i_1,i_2\rangle \in I_1\times I_2 \text{ */} \\
 & \left\{ \langle i, F_1\cup F_2, dc_1\wedge dc_2, \langle i_1',i_2'\rangle\rangle \;\middle|\; \begin{array}{l} \widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1\rangle \in \widetilde{S}_1(i_1) \text{ and } i_1' = \widetilde{\eta}_1(i_1,\widetilde{\mathsf{c}}_1) \text{ and } F_1 = \{\, n\in N_1 \,|\, \mathsf{c}_1(n) = \text{——} \,\} \text{ and} \\ \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2\rangle \in \widetilde{S}_2(i_2) \text{ and } i_2' = \widetilde{\eta}_2(i_2,\widetilde{\mathsf{c}}_2) \text{ and } F_2 = \{\, n\in N_2 \,|\, \mathsf{c}_2(n) = \text{——} \,\} \text{ and} \\ F_1\cap N_2 = F_2\cap N_1 \text{ and } i=\langle i_1,i_2\rangle \in I_1\times I_2 \end{array} \right\} \\
= & \text{/* Because, by the definition of } F_1 \text{ and } F_2 \text{ in Lemma 2, } [F_1\cap N_2 = F_2\cap N_1] \text{ iff } [\{\, n\in N_1\cap N_2 \,|\, \mathsf{c}_1(n) = \text{——} \,\} = \{\, n\in N_1\cap N_2 \,|\, \mathsf{c}_2(n) = \text{——} \,\}], \text{ and because, as } \mathsf{c}_1 \text{ and } \mathsf{c}_2 \text{ are 2-colorings, } [\{\, n\in N_1\cap N_2 \,|\, \mathsf{c}_1(n) = \text{——} \,\} = \{\, n\in N_1\cap N_2 \,|\, \mathsf{c}_2(n) = \text{——} \,\}] \text{ iff} \\
 & [\mathsf{c}_1(n) = \mathsf{c}_2(n) \text{ for all } n\in N_1\cap N_2] \text{ */} \\
 & \left\{ \langle i, F_1\cup F_2, dc_1\wedge dc_2, \langle i_1',i_2'\rangle\rangle \;\middle|\; \begin{array}{l} \widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1\rangle \in \widetilde{S}_1(i_1) \text{ and } i_1' = \widetilde{\eta}_1(i_1,\widetilde{\mathsf{c}}_1) \text{ and } F_1 = \{\, n\in N_1 \,|\, \mathsf{c}_1(n) = \text{——} \,\} \text{ and} \\ \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2\rangle \in \widetilde{S}_2(i_2) \text{ and } i_2' = \widetilde{\eta}_2(i_2,\widetilde{\mathsf{c}}_2) \text{ and } F_2 = \{\, n\in N_2 \,|\, \mathsf{c}_2(n) = \text{——} \,\} \text{ and} \\ [\mathsf{c}_1(n) = \mathsf{c}_2(n) \text{ for all } n\in N_1\cap N_2] \text{ and } i=\langle i_1,i_2\rangle \in I_1\times I_2 \end{array} \right\} \\
= & \text{/* Because, by Definition 25 of } \widetilde{\cdot}, \; [\widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1\rangle \in \widetilde{S}_1(i_1) \text{ and } \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2\rangle \in \widetilde{S}_2(i_2) \text{ and } \mathsf{c}_1(n) = \mathsf{c}_2(n) \text{ for all } n\in N_1\cap N_2] \text{ iff} \\
 & [\widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2 \in \widetilde{S}_1(i_1)\,\widetilde{\sim}\,\widetilde{S}_2(i_2)] \text{ */} \\
 & \left\{ \langle i, F_1\cup F_2, dc_1\wedge dc_2, \langle i_1',i_2'\rangle\rangle \;\middle|\; \begin{array}{l} \widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1\rangle \text{ and } i_1' = \widetilde{\eta}_1(i_1,\widetilde{\mathsf{c}}_1) \text{ and } F_1 = \{\, n\in N_1 \,|\, \mathsf{c}_1(n) = \text{——} \,\} \text{ and} \\ \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2\rangle \text{ and } i_2' = \widetilde{\eta}_2(i_2,\widetilde{\mathsf{c}}_2) \text{ and } F_2 = \{\, n\in N_2 \,|\, \mathsf{c}_2(n) = \text{——} \,\} \text{ and} \\ \widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2 \in \widetilde{S}_1(i_1)\,\widetilde{\sim}\,\widetilde{S}_2(i_2) \text{ and } i=\langle i_1,i_2\rangle \in I_1\times I_2 \end{array} \right\} \\
= & \text{/* By Definition 30 of } \widetilde{\odot} \text{ (informally on page 231) */} \\
 & \left\{ \langle i, F_1\cup F_2, dc_1\wedge dc_2, \langle i_1',i_2'\rangle\rangle \;\middle|\; \begin{array}{l} \widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1\rangle \text{ and } i_1' = \widetilde{\eta}_1(i_1,\widetilde{\mathsf{c}}_1) \text{ and } F_1 = \{\, n\in N_1 \,|\, \mathsf{c}_1(n) = \text{——} \,\} \text{ and} \\ \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2\rangle \text{ and } i_2' = \widetilde{\eta}_2(i_2,\widetilde{\mathsf{c}}_2) \text{ and } F_2 = \{\, n\in N_2 \,|\, \mathsf{c}_2(n) = \text{——} \,\} \text{ and} \\ \widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2 \in (\widetilde{S}_1 \,\widetilde{\odot}\, \widetilde{S}_2)(i) \text{ and } i=\langle i_1,i_2\rangle \in I_1\times I_2 \end{array} \right\} \\
= & \text{/* Because, by Definition 31 of } \widetilde{\otimes} \text{ (informally on page 231), } [i=\langle i_1,i_2\rangle \in I_1\times I_2 \text{ and } \widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2 \in (\widetilde{S}_1 \,\widetilde{\odot}\, \widetilde{S}_2)(i) \text{ and } i_1' = \widetilde{\eta}_1(i_1,\widetilde{\mathsf{c}}_1) \text{ and} \\
 & i_2' = \widetilde{\eta}_2(i_2,\widetilde{\mathsf{c}}_2)] \text{ iff } [\langle i_1',i_2'\rangle = (\widetilde{\eta}_1 \,\widetilde{\otimes}\, \widetilde{\eta}_2)(i,\widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2)] \text{ */} \\
 & \left\{ \langle i, F_1\cup F_2, dc_1\wedge dc_2, \langle i_1',i_2'\rangle\rangle \;\middle|\; \begin{array}{l} \widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1\rangle \text{ and } F_1 = \{\, n\in N_1 \,|\, \mathsf{c}_1(n) = \text{——} \,\} \text{ and} \\ \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2\rangle \text{ and } F_2 = \{\, n\in N_2 \,|\, \mathsf{c}_2(n) = \text{——} \,\} \text{ and} \\ \widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2 \in (\widetilde{S}_1 \,\widetilde{\odot}\, \widetilde{S}_2)(i) \text{ and } i\in I_1\times I_2 \text{ and } \langle i_1',i_2'\rangle = (\widetilde{\eta}_1 \,\widetilde{\otimes}\, \widetilde{\eta}_2)(i,\widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2) \end{array} \right\} \\
= & \text{/* By introducing } F=F_1\cup F_2, \text{ and by applying } \langle i_1',i_2'\rangle = (\widetilde{\eta}_1 \,\widetilde{\otimes}\, \widetilde{\eta}_2)(i,\widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2) \text{ */} \\
 & \left\{ \langle i, F, dc_1\wedge dc_2, (\widetilde{\eta}_1 \,\widetilde{\otimes}\, \widetilde{\eta}_2)(i,\widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2)\rangle \;\middle|\; \begin{array}{l} \widetilde{\mathsf{c}}_1 = \langle \mathsf{c}_1, dc_1\rangle \text{ and } \widetilde{\mathsf{c}}_2 = \langle \mathsf{c}_2, dc_2\rangle \text{ and} \\ F = \{\, n\in N_1\cup N_2 \,|\, \mathsf{c}_1(n) = \text{——} \text{ or } \mathsf{c}_2(n) = \text{——} \,\} \text{ and} \\ \widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2 \in (\widetilde{S}_1 \,\widetilde{\odot}\, \widetilde{S}_2)(i) \text{ and } i\in I_1\times I_2 \end{array} \right\} \\
= & \text{/* Because, by Definition 14 of } \cup, \; [\mathsf{c}_1(n) = \text{——} \text{ or } \mathsf{c}_2(n) = \text{——}] \text{ iff } [(\mathsf{c}_1\cup\mathsf{c}_2)(n) = \text{——}], \text{ and by applying } \langle \mathsf{c}, dc\rangle = \widetilde{\mathsf{c}} = \widetilde{\mathsf{c}}_1 \,\widetilde{\cup}\, \widetilde{\mathsf{c}}_2 = \\
 & \langle \mathsf{c}_1\cup\mathsf{c}_2, dc_1\wedge dc_2\rangle \text{ */} \\
 & \{\langle i, F, dc, (\widetilde{\eta}_1 \,\widetilde{\otimes}\, \widetilde{\eta}_2)(i,\widetilde{\mathsf{c}})\rangle \,|\, F = \{\, n\in N_1\cup N_2 \,|\, \mathsf{c}(n) = \text{——} \,\} \text{ and } \widetilde{\mathsf{c}} = \langle \mathsf{c}, dc\rangle \in (\widetilde{S}_1 \,\widetilde{\odot}\, \widetilde{S}_2)(i) \text{ and } i\in I_1\times I_2\} \\
= & \text{/* By the definition of } T' \text{ in Lemma 2 */} \\
 & T'
\end{array}
$$

Figure 12: Proof: $T = T'$.

and co-domain of $\widetilde{\eta}$ are states that appear in elements of the transition relation T; therefore, by Definition 5, all indexes come from Q. Similarly, by Definition 29, all data constraints that occur in the domain of $\widetilde{\eta}$ also appear in elements of T, hence come from DC. Finally, by Definition 22, we must show that $\widetilde{\eta}$ maps every $\langle i, \widetilde{\mathsf{c}}\rangle$ such that $\widetilde{\mathsf{c}} \in \widetilde{S}(i)$ to an index in Q. This follows from Definition 29 and the definition of $\widetilde{S}$ in the premise. □

Proof of Lemma 3 (left-inverse for $\mathbb{L}$). Suppose $\widetilde{\mathtt{CM}} = \langle \widetilde{\eta}, i_0\rangle$ is defined over $\widetilde{S}$ over $[N, DC, I]$. Then, $\frac{1}{\mathbb{L}}(\mathbb{L}(\widetilde{\mathtt{CM}})) = \widetilde{\mathtt{CM}}$ follows from Figure 13. □

$\frac{1}{\mathbb{L}}(\mathbb{L}(\widetilde{\mathtt{CM}}))$
$=$ /* By Definition 26 of $\mathbb{L}$, and because $\widetilde{\mathtt{CM}}$ is defined over $\widetilde{S}$, which is defined over $[N, DC, I]$ */
$\frac{1}{\mathbb{L}}(\langle I, T, i_0\rangle)$ with T as in Definition 26
$=$ /* By Definition 29 of $\frac{1}{\mathbb{L}}$, and because $\mathbb{L}(\widetilde{\mathtt{CM}})$ is a CA over $[N, DC]$ by Proposition 1 */
$\langle\{\langle i, \langle \mathtt{col}(N, F), dc\rangle\rangle \mapsto i' \,|\, \langle i, F, dc, i'\rangle \in T\}, i_0\rangle$ with T as in Definition 26
$=$ /* Because $\langle i, F, dc, i'\rangle \in T$ iff $[i \in I$ and $\widetilde{\mathsf{c}} = \langle \mathsf{c}, dc\rangle \in \widetilde{S}(q)$ and $F = \{\, n \in N \,|\, \mathsf{c}(n) = \text{——} \,\}$ and $i' = \widetilde{\eta}(i, \widetilde{\mathsf{c}})]$ by Definition 26 of $\mathbb{L}$ */
$\langle\{\langle i, \langle \mathtt{col}(N, F), dc\rangle\rangle \mapsto \widetilde{\eta}(i, \widetilde{\mathsf{c}}) \,|\, i \in I$ and $\widetilde{\mathsf{c}} = \langle \mathsf{c}, dc\rangle \in \widetilde{S}(i)$ and $F = \{\, n \in N \,|\, \mathsf{c}(n) = \text{——} \,\}\}, i_0\rangle$
$=$ /* Because $\mathsf{c} = \mathtt{col}(N, F)$ iff $F = \{\, n \in N \,|\, \mathsf{c}(n) = \text{——} \,\}$ by Definitions 9 and 28 */
$\langle\{\langle i, \langle \mathsf{c}, dc\rangle\rangle \mapsto \widetilde{\eta}(i, \widetilde{\mathsf{c}}) \,|\, i \in I$ and $\widetilde{\mathsf{c}} = \langle \mathsf{c}, dc\rangle \in \widetilde{S}(i)\}, i_0\rangle$
$=$ /* By Definitions 22 and 23 */
$\langle \widetilde{\eta}, i_0\rangle = \widetilde{\mathtt{CM}}$

Figure 13: Proof: $\frac{1}{\mathbb{L}}(\mathbb{L}(\widetilde{\mathtt{CM}})) = \widetilde{\mathtt{CM}}$.

Proof of Lemma 4 (right-inverse for $\mathbb{L}$). Suppose $\mathtt{CA} = \langle Q, T, q_0\rangle$ is defined over $[N, DC]$. Then, $\mathbb{L}(\frac{1}{\mathbb{L}}(\mathtt{CA})) = \mathtt{CA}$ follows from Figure 14. □

Proof of Lemma 5 ($\frac{1}{\mathbb{L}}$ is correct). Suppose $\widetilde{\mathtt{CM}} = \frac{1}{\mathbb{L}}(\mathtt{CA})$. Then, $\mathtt{CA} \sim \frac{1}{\mathbb{L}}(\mathtt{CA})$ iff $\mathbb{L}(\frac{1}{\mathbb{L}}(\mathtt{CA})) \sim \frac{1}{\mathbb{L}}(\mathtt{CA})$ iff $\mathbb{L}(\widetilde{\mathtt{CM}}) \sim \widetilde{\mathtt{CM}}$ by Lemma 4. The latter follows from Lemma 1. □

Proof of Lemma 6 (compositionality of $\frac{1}{\mathbb{L}}$). Follows from Figure 15. □

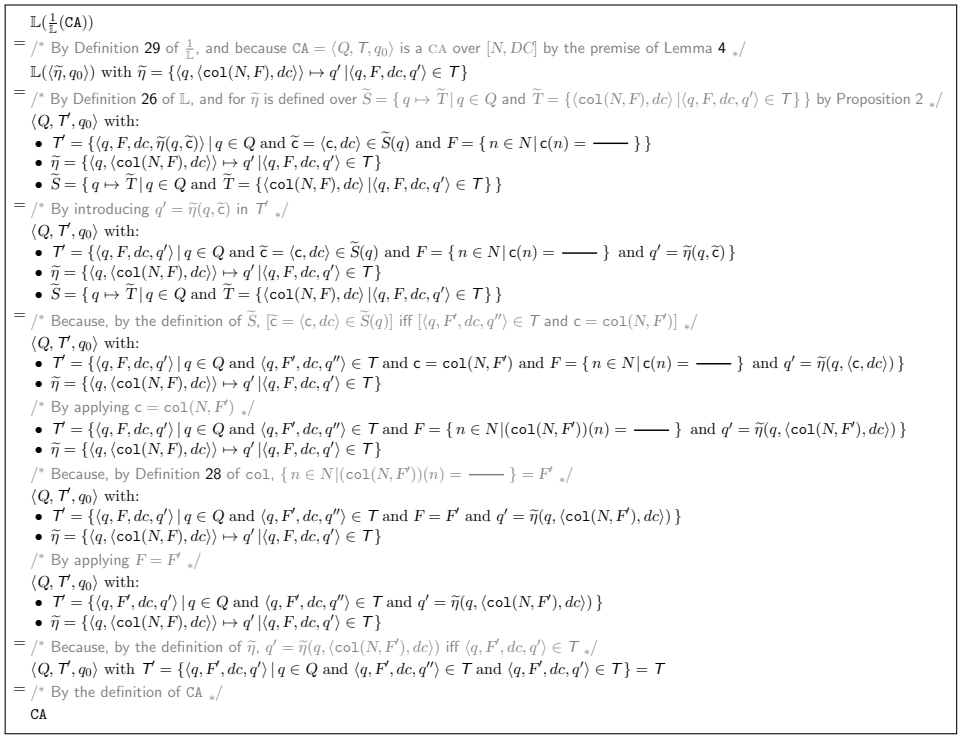


Figure 14: Proof: $\mathbb{L}(\frac{1}{\mathbb{L}}(\mathtt{CA})) = \mathtt{CA}$.

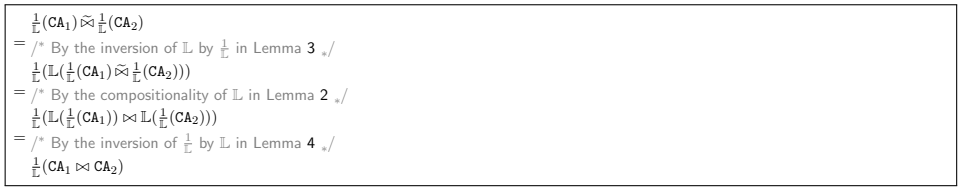


Figure 15: Proof: $\frac{1}{\mathbb{L}}(\mathtt{CA}_1) \widetilde{\bowtie} \frac{1}{\mathbb{L}}(\mathtt{CA}_2) = \frac{1}{\mathbb{L}}(\mathtt{CA}_1 \bowtie \mathtt{CA}_2)$.