

Hall Effect*

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Summary--The Hall effect is one of the rich sources of information about the conduction properties of semiconductors. The mobility and carrier concentration can be obtained from the Hall constant in conjunction with the resistivity; this cannot be done with the resistivity alone. The mobility is pertinent to the understanding of transistors since such things as high-frequency cut-off and the intrinsic current gain of the transistor are related to this property of germanium. The Hall effect and associated thermomagnetic and galvanomagnetic (Ettingshausen, Nernst, Righi-Leduc, and Ohmic) effects are discussed. The elimination of the effect of associated phenomena from the Hall measurement can be achieved in several ways. Some of the methods which are used today in the study of germanium are discussed, and typical apparatus is described.

INTRODUCTION

THE TRANSISTOR makes use of the special conduction properties of semiconductors to gain the advantages it has over the vacuum tube. To understand the advantages and limitations of the transistor one must understand these special conduction properties. For example, the high-frequency cut-off of the type A transistor can be predicted from the injected carrier's transit time which is related to the mobility. The intrinsic current gain in the type A transistor is related to the ratio of the mobility of electrons to the mobility of holes. In dealing with current flow in semiconductors, one must take account of the fact that other carriers than electrons may be present. The nature of these carriers (whether they are electrons or holes), the number of carriers per unit volume, and the ease with which they respond to an applied electric field (mobility) are all important quantities to know. The Hall effect provides a direct measurement of the carrier type and concentration and, in conjunction with the resistivity, yields the mobility. The density of carriers in semiconductors is determined in part by the density of foreign atoms in the material (for example, aluminum in germanium). Thus the Hall measurement is used to determine the density of impurity atoms. This measure of impurity density in high-purity samples is several orders of magnitude more sensitive than the best chemical procedures. Because of this, the measurement of the Hall constant is one of the basic procedures in experimental studies of semiconductors.

The Hall effect occurs when a substance carrying a current is subjected to a magnetic field perpendicular to the direction of the current. If the current is flowing in the x-direction and the magnetic field is applied in the z-direction, a potential gradient will appear across the sample in the y-direction. This transverse potential gradient is found to be proportional to the product of the current density in the sample and the applied mag-

netic field; the constant of proportionality is called the "Hall constant." Mathematically this can be expressed as follows:

$$\text{Grad } V_H = - RiH. \quad (1)$$

Grad V_H —(the transverse potential gradient) = $- E_H$,

where E_H is the Hall field.

i —the current density.

H —the applied magnetic field.

R —the Hall constant.

If the sample is a rectangular solid of width a and thickness b and if the distribution of current is assumed uniform, (1) can be rewritten in terms of the total current I , the Hall voltage V_H , and the dimensions of the sample.

$$\text{Grad } V_H = \frac{V_H}{a} = \frac{- RiH}{ab}$$

$$VII = \frac{- RiH}{b}. \quad (2)$$

The Hall effect can be explained on the basis of the particle nature of conduction. The current consists of streams of charged particles drifting under the influence of the electric field. When not under the influence of the magnetic field, the current flows longitudinally in the sample. On application of the magnetic field, the current carriers experience a force $e/c(xH)$ and are swept to the edges of the sample. The charge continues to build up on the edges of the sample until the field due to the nonuniform charge distribution exerts a force equal to the deflecting force of the magnetic field.

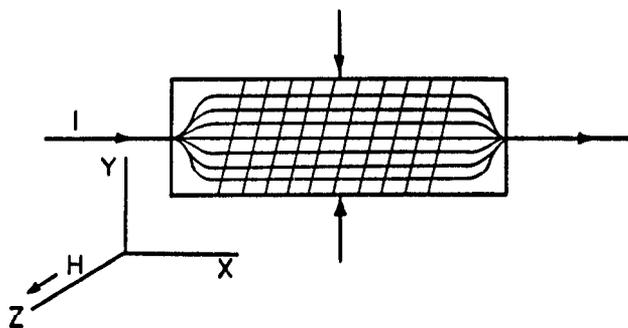


Fig. 1—Rotation of equipotentials by Hall effect. The Hall effect rotates the equipotentials so that they are no longer normal to the current flow.

Fig. 1 shows that the equipotentials are no longer normal to the current flow, but have been rotated through an angle θ , called the "Hall angle." Examination of the vector diagram of the fields in Fig. 2 (opposite page) shows that the Hall angle θ is determined by the following relation:

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$$\tan \theta = \frac{E_H}{E_x} \cong \theta \text{ for small angles.} \quad (3)$$

$$R = \frac{3\pi}{8} \frac{1}{nec} \quad (7)$$

Now $E_H = RiH$ and $E_x = \frac{i}{\sigma}$, thus- $= \frac{RiH\sigma}{i} = RH\sigma$,

where σ is the conductivity.

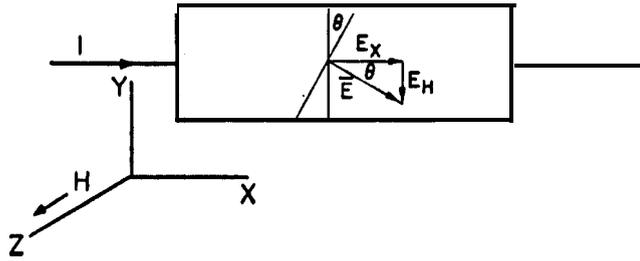


Fig. 2-Vector diagram for the Hall effect. The Hall angle θ is the angle of rotation of equipotentials. E_H is the Hall field if the carriers are electrons.

It is possible to calculate an expression for the Hall constant on the basis of the particle theory. In the derivation we assume that only electrons are present; but if only holes are present, the procedure is analogous. From the discussion above, the condition for the steady state is that the deflecting force of the magnetic field on a current carrier just equals the force exerted by the transverse electric field due to the charge build-up at the edges of the sample. This condition can be met mathematically by setting the y-component of the electric-field force equal in magnitude but opposite in sign to the force experienced by a charge moving in the magnetic field.

$$eE_H = e \frac{V_y H_z}{c} \quad (4)$$

$$E_H = \frac{i_y H_z}{nec}, \text{ the current density } i_x = ncv_x \quad (5)$$

where n the carrier concentration; i_x and H_z can be replaced by i and H since $i_y = i_x = H_x = H_y = 0$.

$$E_H = \frac{iH}{nec} = RiH \quad (6)$$

$$R = \frac{1}{nec}$$

The Hall constant derived by this method is valid only insofar as the particle picture of conduction is valid; that is, it applies to simple metals and impure semiconductors. To obtain an expression of more general applicability the mechanism of conduction must be examined more closely on the basis of a Boltzmann distribution of velocity of carriers. The result of such an investigation gives a relation which is generally used in semiconductor work when only one type of carrier is involved.

¹ W. Shockley, "Electrons and Holes in Semiconductors," D. Van Nostrand Co., Inc., New York, N. Y., p. 277; 1950.

From the measurement of the Hall constant, the carrier concentration can be determined. In an impurity-type semiconductor the carrier concentration is determined by the density of the dominant impurity. For example, in germanium at room temperature the intrinsic carrier concentration is approximately 5×10^{13} carriers per cubic centimeter, and impurity densities ten times as great are commonly found: thus it introduces a small error to attribute the entire carrier concentration to impurity atoms. From (6), the carrier concentration is proportional to $1/R$. This gives a quantitative measure of the impurity density. The value of this procedure can be seen from a calculation of the per cent impurity concentration for an impurity density of 4×10^{16} . There are approximately 4×10^{22} atoms of germanium per cubic centimeter; thus concentrations of one part in 10^7 are commonly met. To use chemical analysis to determine such impurity concentrations is a hopeless task. The Hall voltage, on the other hand, becomes larger for lower impurity concentration, and therefore more easily measured.

The carrier concentration does not give the complete picture of the conduction properties. The conductivity σ is related to the carrier concentration, the charge of the electron and the ease with which the carriers move in an electric field (mobility). The latter is defined as the steady-state average velocity of the conducting particle (cm/sec) in unit electric field (1 volt/cm). In germanium the mobility will vary from 1,000 to 3,600 cm/sec per volt/cm. In a semiconductor with both negative and positive (holes) carriers the conductivity must be a function of the concentration and mobility of both holes and electrons.

$$\sigma = n|e|\mu_n + p|e|\mu_p \quad (8)$$

n -electron concentration (no/cc).

p -hole concentration (no/cc).

μ_n -electron mobility (up to 3,600 $\text{cm}^2/\text{volt-sec}$ in germanium).

μ_p -hole mobility (up to 1,700 $\text{cm}^2/\text{volt-sec}$ in germanium).

In the range where the concentrations of holes and electrons are about the same this expression for conductivity must be used. At the operating temperature of a semiconducting device, the concentration of one type carrier is much greater than the concentration of the other so that the conductivity will be dependent on only one term or the other in (8).

For p-type sample : $\sigma = p e \mu_p$.

For n-type sample: $\sigma = n |e| \mu_n$.

Carrier concentration can be determined from the conductivity if the mobility is known or a value is assumed. The Hall effect gives a method for determining the mobility and the carrier concentration for both n-type and p-type semiconductors without assumptions.

For an n-type sample the Hall constant is negative since e is negative.

$$R = \frac{-3\pi}{8} \frac{1}{n|e|c}. \quad (9)$$

For a P-type sample the Hall constant is positive because the charge of a hole is positive.

$$R = \frac{3\pi}{8} \frac{1}{p|e|c}. \quad (10)$$

The Hall-angle formula differs for holes and electrons since mobility is different for holes and electrons.

$$\text{n-type: } e = \frac{H\mu_n}{c} \quad (11)$$

$$\text{p-type: } e = \frac{H\mu_p}{c} \quad (12)$$

Solving (9) and (10) for the carrier concentrations and (11) and (12) for the mobilities, the following expressions are obtained:

$$n = \frac{3\pi}{8} \frac{-1}{R\sigma c} \quad p = \frac{3\pi}{8} \frac{1}{R\sigma c}$$

$$\mu_n = \frac{\theta c}{H} = |R|\sigma c \quad \mu_p = \frac{\theta c}{H} = R\sigma c.$$

In measurements of the Hall voltage, certain associated effects give rise to potentials which must be corrected in order to avoid error in the measured value. The largest of these effects is the potential which appears because of the experimental difficulty in aligning the measuring probes on the same equipotential plane.

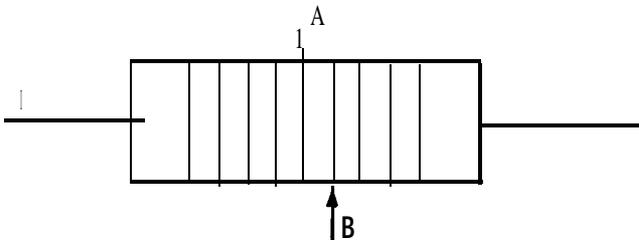


Fig. 3—Source of IR drop error. It is difficult to align the probes A and B so that no voltage will be measured in the absence of a magnetic field.

If Fig. 3, A and B are probes for measuring the Hall potential. With no field applied, the equipotentials are ideally planes perpendicular to the lines of current flow. If probes A and B are not exactly on the same equipotential, a potential will be measured between them, giving a constant error to the Hall voltage measured. This voltage can easily be of the order of magnitude of the Hall voltage itself. This effect is sometimes referred to as the " IR drop." The IR drop is dependent only on the current and the conductivity of the sample, not being affected by a reversal of magnetic field.

In addition to the IR drop, there are three thermomagnetic or galvanomagnetic effects, the Ettingshausen

effect, the Nernst effect, and the Righi-Leduc effect. These effects will give rise to a temperature gradient or a potential gradient when either an electric current or a thermal current is subjected to a magnetic field perpendicular to the direction of current flow. A temperature gradient as well as a potential gradient will cause an error in the Hall-effect measurement. In Fig. 4, junction A is at temperature T_2 and junction B is at temperature T_1 , which is less than T_2 . Since the probe material is in general not the same as the sample material, the probes and the sample form a thermocouple and produce a voltage dependent in sign and magnitude on the materials of the probes and the sample.

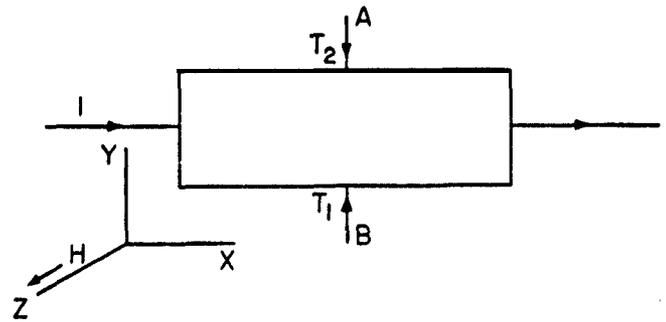


Fig. 4—Ettingshausen effect. The Ettingshausen effect causes the edge at probe A to be at temperature T_2 greater than T_1 , the temperature of the other edge.

In the Ettingshausen effect a permanently maintained temperature gradient will appear if an electric current is subjected to a magnetic field perpendicular to its direction of flow. The temperature gradient is found to be proportional to the product of the current density and the magnetic field.²

$$\frac{T}{a} = PIN \quad i = \frac{I}{ab} \quad (13)$$

$$T = \frac{PIH}{b}$$

ΔT —the difference of temperature between the edges of the sample.

i —current density.

I —total current.

H —magnetic field perpendicular to the direction of current flow.

P —Ettingshausen coefficient.

a —width of sample.

b —thickness of sample.

As in the case of the Hall effect, uniform current distribution and a rectangular solid sample have been assumed.

The Nernst effect and the Righi-Leduc effect are similar to the Hall effect and the Ettingshausen effect except that they are produced by a thermal current and a perpendicular magnetic field rather than an elec-

² P. W. Bridgman, "The Thermodynamics of Electrical Phenomena in Metals," Macmillan Co., New York, N. Y., pp. 135-138; 1934.

tric current and a perpendicular magnetic field.⁸ In the Nernst effect, a potential gradient appears in the y -direction if a thermal current flows in the x -direction and a magnetic field is applied in the z -direction.*

$$E_N = Q \frac{wH}{K}. \quad (14)$$

E_N —Nernst transverse potential gradient.

w —thermal current density.

K —thermal conductivity of the sample.

Q —Nernst coefficient.

The Righi-Leduc effect produces a temperature gradient in the y -direction when a thermal current flows in the x -direction, and a magnetic field is applied in the z -direction.*

$$\frac{\Delta T}{a} = S \frac{wH}{K}. \quad (15)$$

ΔT —difference in the temperature between the edges of the sample.

S —Righi-Leduc coefficient

a, w, H, K —as previously defined.

Associated with a thermal current, there exists an electron current. When two ends of a sample are at different temperatures, the high-velocity electrons at the hot end will diffuse toward the cold end; but for each high-velocity electron that diffuses away, a low-velocity electron must take its place so that a uniform charge distribution will be maintained.

A satisfactory description of the mechanism of the Ettingshausen effect on the basis of the classical theory of conduction by particles has not been given. However, following a method given by Bridgman,³ some justification of the Ettingshausen effect can be made on a thermodynamic basis. Where an electric current flows longitudinally in a material and is subjected to a magnetic field perpendicular to the current flow, a transverse temperature gradient builds up because of a "thermomotive force." This temperature gradient is maintained as long as the current and field are maintained. When the temperature gradient builds up, a transverse thermal current must flow from the cold edge to the hot edge. The energy required for the thermomotive force to drive the thermal current against the temperature gradient must be supplied by the electric current in the sample. Because of the Nernst effect, the transverse thermal current produces a longitudinal potential; the electrical current flowing against this potential provides the energy to build up and maintain the Ettingshausen temperature gradient.

* In this discussion the assumption has been made that the thermal current and the electric current are independent of each other. In reality a part of the total thermal current arises from the electric field present and similarly a part of the total electric current arises from the temperature gradient present. This interdependence is complicated and it is beyond the scope of this article to discuss this effect. The electric current as used here is current as would be measured by a meter and the thermal current is that determined by the temperature gradient and the thermal conductivity.

The Righi-Leduc effect produces a transverse temperature gradient when a longitudinal thermal current flows in a magnetic field. As in the Ettingshausen effect a transverse thermal current will flow from the cold edge to the hot edge while the temperature gradient is being built-up. The thermomotive force set up by this transverse thermal current (Righi-Leduc effect) is in opposition to the longitudinal thermal current, and is therefore able to extract energy from the thermal current and maintain itself.

As has been explained above, the IR drop, the Ettingshausen effect, the Nernst effect, and the Righi-Leduc effect all result in potentials at the probes measuring the Hall potential. Consequently, to measure Hall constant, these sources of error must be eliminated or minimized. The most common method of making Hall measurements utilizes dc magnetic fields and dc currents. The experimental setup is illustrated in Fig. 4. The probes A and B for measuring the Hall voltage are placed in contact with the sample as shown. The current leads are soldered to the ends of the sample. A dc magnetic field is applied in the z -direction and a current flows in the x -direction. If we consider the probes to be perfectly aligned on an equipotential surface at no magnetic field and we also assume that there is no longitudinal thermal current flowing, then the potential measured at A and B will be the sum of the Hall potential and the thermocouple potential due to the Ettingshausen temperature gradient.

$$V_1 = V_H + V_E$$

where V_E , the thermocouple potential due to the Ettingshausen effect, and V_H , the Hall potential, are both dependent on the current and the magnetic field. The only apparent way to separate the two in a dc measurement is to make the probes out of the same material as the sample, and this is usually difficult if not impossible in the case of semiconductors. In making dc Hall measurements in germanium, V_E is usually considered to be of the order of 5 per cent, and is neglected.

From the following considerations it is apparent that a longitudinal thermal current will flow. When a longitudinal electric current flows, there will be a Peltier effect at the junctions where the current leads are soldered to the sample. One junction will become heated and the other cooled, and a temperature difference in the x -direction will be maintained so long as the current flows in the junctions. Such a temperature gradient can also occur due to nonuniform temperature of the sample or its surroundings. In general, the combination of these effects can add up to an important longitudinal temperature difference, and consequently, a longitudinal thermal current will probably exist. Because of the presence of such a thermal current, the Nernst effect and the Righi-Leduc effect should also be considered. The potential E measured at the probes A and B will be

$$E = V_H + V_E + V_N + V_{RL}.$$

V_N = potential due to Nernst effect.

V_{RL} = thermocouple potential due to the Righi-Leduc temperature gradient.

If the sample current is reversed and the potential at the probes, A and B , is remeasured, then the potential will be

$$E_1 = -V_H - V_E + V_N + V_{RL}$$

Since the Hall and the Ettingshausen effects are directly related to the current and the Nernst and the Righi-Leduc are independent of current, only the former change sign when the current is reversed. Subtract E_1 from E ;

$$\begin{aligned} E - E_1 &= 2(V_H + V_E) \\ V_H + V_E &= \frac{E - E_1}{2} \end{aligned} \quad (16)$$

To take account of all first-order associated effects, the IR drop must also be considered. If the probes, A and B , were not properly aligned on equipotentials at no magnetic field (as is always the case when the experiment is performed), then the voltage measured at the probes A and B would be

$$E = V_H + V_E + V_N + V_{RL} + V_{IR}$$

V_{IR} is the potential due to IR drop.

V_{IR} is a function of the current but not the field.

V_H and V_E are dependent on both I and H .

V_N and V_{RL} are dependent on H but not on I .

If a series of four measurements is taken reversing the magnetic field and the sample current in all possible combinations,* then a method is available for eliminating all the undesired associated effects except the Ettingshausen.

$$\begin{aligned} (+H, +I) E_1 &= V_H + V_E + V_N + V_{RL} + V_{IR}, \\ (+H, -I) E_2 &= -V_H - V_E + V_N + V_{RL} - V_{IR}, \\ (-H, -I) E_3 &= V_H + V_E - V_N - V_{RL} - V_{IR}, \\ (-H, +I) E_4 &= -V_H - V_E - V_N - V_{RL} + V_{IR}, \end{aligned}$$

which yield

$$V_H + V_E = \frac{E_1 - E_2 + E_3 - E_4}{4} \quad (17)$$

This method has been outlined in detail because it is a standard method of making Hall measurements in the presence of the associated thermomagnetic and galvanomagnetic effects. The method is not completely satisfactory since it doesn't separate the Ettingshausen effect from the Hall effect and since a series of four measurements requires sufficient time for possible changes to occur in the experimental conditions, especially if measurements are being made as a function of temperature.

* The measurements must be made rapidly so that the part of the temperature gradient due to the Peltier effect does not have time to reverse after the current is reversed.

These objections are overcome by making the measurements with an ac field and an ac sample current. Let the field and the sample current be the same frequency. The Hall potential from (2) is

$$-V_H = \frac{RIH}{b}$$

Substitute the instantaneous value of I and II in (2);

$$\begin{aligned} V_H &= \frac{R}{b} I' \sin \omega t II' \sin \omega t \\ V_H &= \frac{RIH'I'}{2b} - \frac{RIH'I'}{2b} \cos 2\omega t. \end{aligned}$$

Similarly, for the Ettingshausen temperatures gradient

$$\begin{aligned} \Delta T &= \frac{PIN}{b} \text{ in equilibrium} \\ \Delta T &= \frac{PI'H'}{2b} - \frac{PI'H'}{2b} (\cos 2\omega t)(e^{-\omega/\omega_0}). \end{aligned} \quad (18)$$

The exponential is due to the time required to build the temperature up to equilibrium; ω_0 is a constant of the material, inversely proportional to the heat capacity and dependent on certain other properties. The other effects will be ac, and can be represented as follows:

$$\begin{aligned} V_{IR} &= A \sin \omega t \\ V_N &= B \sin \omega t \\ V_{RL} &= C \sin \omega t. \end{aligned}$$

A , B , and C are the appropriate constants. The signal measured at the Hall probes will be the sum of several components. If only the dc component is measured, then the potential will be the sum of the Hall voltage and the voltage due to the Ettingshausen temperature gradient, with the result that the Nernst and Righi-Leduc and the IR drop have been eliminated as a source of error and only one measurement need be made.

The Ettingshausen effect has associated a certain time delay which is required for the temperature gradient to become established. This is accounted for by the exponential in (18). For example, in bismuth the time required for the temperature gradient to reach its maximum value has been measured by one observer to be about one minute. (This corresponds to $\omega_0 = 0.1$.) This makes it apparent that an ac Ettingshausen effect would not be detectable if the period were sufficiently short compared to the response time of the effect. This is the basis of a measurement method which will separate the Hall effect and the Ettingshausen effect. If the field and the current are of different frequencies, then all the resulting signals on the Hall probes will be ac

$$I = I' \sin \omega_1 t \quad H = H' \sin \omega_2 t$$

$$V_H = \frac{RIH}{b} = \frac{RI'H'}{2b} [\cos (\omega_1 - \omega_2)t - \cos (\omega_1 + \omega_2)t]$$

$$V_E = D \frac{PI'H'}{2b} \left[\left(\exp \left[\frac{-|\omega_1 - \omega_2|}{\omega_0} \right] \right) \cos(\omega_1 - \omega_2)t \right. \\ \left. - \left(\exp \left[\frac{-|\omega_1 + \omega_2|}{\omega_0} \right] \right) \cos(\omega_1 + \omega_2)t \right].$$

As in the previous case, b is sample thickness and ω_0 is a constant of the material. D is a constant to account for the thermoelectric effect, and depends on the probe material and the sample material.

$$V_{IR} = A \sin \omega_1 t.$$

$$V_H = B \sin \omega_2 t$$

$$V_{RL} = C \sin \omega_3 t.$$

If the difference frequency is high compared to ω_0 , selecting either the sum or the difference frequency with a suitable filter, the measurement will give the Hall voltage directly. This method gives the Hall voltage free from all associated first-order effects, and requires but a single measurement to do so. The Hall constant is

$$R = \frac{2bV_H'}{I'H'}.$$

The primes indicate peak values and b is the sample thickness.

There are other possible methods for making Hall measurements: these utilize various combinations of ac and dc for the field and the sample current. The methods described above are typical and an analysis of any of the other methods would be similar.

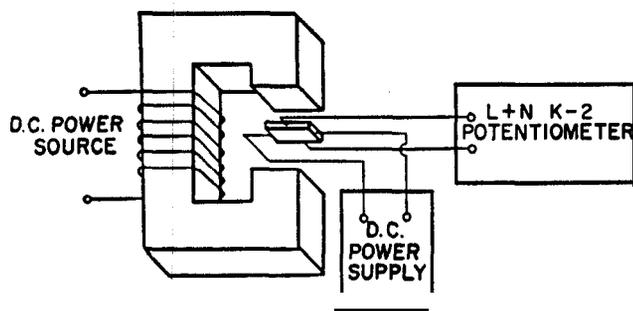


Fig. 5—Typical dc Hall effect apparatus. This is the most common and the simplest method of measuring Hall effect.

Fig. 5 illustrates schematically the apparatus for making dc Hall measurements. The dc electromagnet produces a field up to 5,000 gauss in the air gap. Usually a field of 1,000 gauss provides a Hall voltage large enough to be readily detected. The dc power supply, usually a battery, allows a sample current of 1 or 2 ma. The current should be small to produce as little I^2R heating of the sample as possible and to keep the Peltier heating at the junctions small, but it must be large enough to produce a detectable Hall voltage. A Hall voltage of about 100 μV is desirable if it is to be measured with a potentiometer. If the results of a Hall measurement are as follows,

$$V_H = 100 \mu\text{V}$$

$$H = 1,000 \text{ gauss}$$

$$I = 1 \text{ ma}$$

$$b = 1 \text{ mm},$$

then

$$R = \frac{-V_H b}{HZ} \times 10^8 = \frac{-10^{-4} \text{ volts } 10^{-1} \text{ cm}}{10^3 \text{ gauss } 10^{-3} \text{ amp}} (10^8)$$

$$R = -1,000 \frac{\text{cm}^3}{\text{coulomb}},$$

The factor 10^8 converts voltage and current from electrostatic units to practical units. Equation (7) determines the carrier concentration;

$$n = \frac{3\pi}{8R_{ce}} = \frac{1.17}{10^3 \times 1.6 \times 10^{-19}} = 7 \times 10^{15} \text{ carriers/cc.}$$

In (7) the electronic charge is in electrostatic units: in the expression used above, the charge is in the practical system of units (coulombs). The carriers are electrons since the Hall constant is negative.

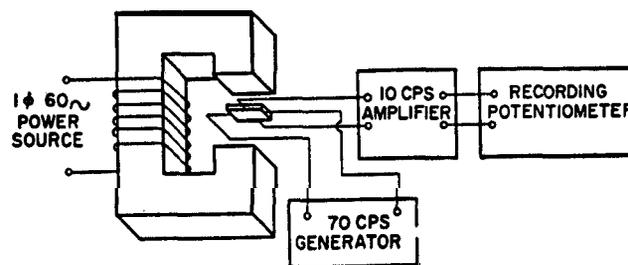


Fig. 6—Typical ac Hall effect apparatus.

Fig. 6 shows an apparatus for making ac Hall measurements. The electromagnet is supplied by a single-phase, 60-cycle power source, and produces a magnetic field in the gap up to about 5,000 gauss. The 70-cycle generator produces a current of 1 or 2 ma in the sample. The Hall-voltage component (the difference frequency) is amplified by a selective amplifier and the output fed into a measuring device, such as a recording potentiometer. The system is required to be stable so that accurate calibration can be obtained. This method is especially adapted to making measurements as a function of temperature where a continuous record of the variation of Hall voltage is desired.

The measurement of the Hall effect may be made in a variety of ways. The ac Hall apparatus is probably the best type for rapid determinations. The measurement is extremely useful in the determination of impurity concentration as such a deduction requires no assumptions as to other properties of the material.

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