

Fraction Models That Promote Understanding For Elementary Students

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FRACTION MODELS THAT PROMOTE UNDERSTANDING FOR ELEMENTARY
STUDENTS

by

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B.S. University of Central Florida, 1998

A thesis submitted in partial fulfillment of the requirements
for the degree of Master of Education
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in the College of Education
at the University of Central Florida
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ABSTRACT

This study examined the use of the set, area, and linear models of fraction representation to enhance elementary students' conceptual understanding of fractions. Students' preferences regarding the set, area, and linear models of fractions during independent work was also investigated. This study took place in a 5th grade class consisting of 21 students in a suburban public elementary school. Students participated in classroom activities which required them to use manipulatives to represent fractions using the set, area, and linear models. Students also had experiences using the models to investigate equivalent fractions, compare fractions, and perform operations. Students maintained journals throughout the study, completed a pre and post assessment, participated in class discussions, and participated in individual interviews concerning their fraction model preference. Analysis of the data revealed an increase in conceptual understanding. The data concerning student preferences were inconsistent, as students' choices during independent work did not always reflect the preferences indicated in the interviews.

This research is dedicated to the students who approached every new experience as an opportunity to learn and to my incredibly supportive husband, daughter, and family.

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CHAPTER ONE: INTRODUCTION

Rationale

I have always found the understanding of fraction concepts and operations a fascinating topic. My initial decision to become an elementary school teacher was in part influenced by the frustration I witnessed from my high school peers as they struggled with their understanding of fractions in an Algebra II course. As a high school student, I tutored students in Algebra and found much of the tutoring focused on fraction concepts and operations. From my naïve perspective, I thought those elementary school teachers must not be doing their jobs. I pursued a career in elementary education with the belief that when my future students reached high school they would have a firm understanding of fraction concepts and operations. I did not realize what a challenging goal I was setting. I have been teaching fifth grade mathematics for seven years. I now have a great deal of empathy for those elementary teachers that I assumed were lacking in their professional obligations. The teaching and understanding of fractions is quite possibly one of the most challenging tasks encountered in an elementary classroom.

As a fifth grade teacher, my students enter my classroom with a wealth of prior knowledge concerning fractions. Some of it is accurate, and some of it is based on misconceptions. During my first few years of teaching, I made the unfortunate mistake of assuming my students understood the meanings of fractions and they simply needed to master operations involving fractions. It did not take long to realize my assessment was often incorrect. Students would frequently offer unreasonable solutions which showed a lack of understanding of the numbers and relationships represented by fractions.

Once I realized my fifth grade students needed instruction that provided opportunities to develop their conceptual understanding of fractions, student achievement improved moderately. There were still several obstacles interfering with my students' achievement concerning fraction concepts and operations. Too often my students had already been introduced to algorithms they could apply with moderate levels of accuracy to solve fraction problems. Unfortunately, because they had no idea why these algorithms worked or a firm conceptual understanding of the meaning of the fractions, they were not able to catch their mistakes if their solutions were unreasonable. Aksu (1997) addressed this obstacle, "A common type of error in teaching fractions is to have students begin computations before they have an adequate background to profit from such operations. Students must understand the meanings of fractions before performing operations with them" (p. 375).

While I was working on my master's degree, I took a course offered at the University of Central Florida. The course, "Seminar in Mathematics Teaching", focused on the understanding of mathematics content. The instructor used a significant amount of instruction time modeling fractions and problems involving fraction operations. She used a variety of manipulatives and three different models for representing fractions. We were required to investigate fraction problems using area, linear, and set models of fraction representation. Prior to this experience, my learning and instruction of fractions relied almost completely on the area model of fraction representation. The National Council of Teachers of Mathematics also noted the value of multiple models of representing fractions in the Principles and Standards for school Mathematics. "Through the study of various meanings and models of fractions- how fractions are related to each other and to the unit whole and how they are represented- students can gain facility in comparing fractions, often by using benchmarks such as $\frac{1}{2}$ or 1" (NCTM, 2000, p.

148). Learning to represent fractions in many different ways was an eye opening experience for me. It was this experience that prompted me to focus my action research on the relationship of multiple models of fraction representation and student understanding.

Purpose

The purpose of this action research was to examine my own teaching practices and my students' understanding of fraction concepts and operations. I wanted to determine how students use different models of fraction representation to increase their understanding of fraction concepts and operations. Two questions guided my research.

1. Will experience with multiple models of fraction representation, specifically area, linear, and set models, increase students' understanding of fraction concepts and operations?
2. Will students have a preferred model (area, set, or linear) for representing fractions when working independently?

Definitions

Fraction concepts- what fractions are, how they are represented, and how they are related to whole numbers (NCTM, 2000)

Fraction operations- addition or subtraction involving fractions

Area model- representing fractions by shading or covering equal parts of a region

Set model- representing fractions with a group of objects as the whole and each individual object as an equal part of the group

Linear model- Representing fractions based on the length of the whole or representing fractions on a number line

Manipulatives- physical objects that students may use to assist them in representing fractions (pattern blocks, Cuisenaire rods, colored chips, fraction circles, etc.)

Significance

“No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratios, and proportionality” (Smith, 2002, p. 3). Over the last several years I have asked my incoming fifth graders to talk about what they are looking forward to in the fifth grade and what they are nervous about as they begin their fifth grade year. Every year since I have started asking the question, fractions have been high on the list of topics in mathematics that cause my students anxiety. Every year I have observed my students try to apply algorithms they have been taught in previous years, but have not mastered. The unreasonable solutions they offer without hesitation demonstrate a lack of complete conceptual understanding. Results from national assessments indicate the difficulties experienced by my students are not uncommon. “Results from large-scale assessments such as the National Assessment of Educational Progress (NAEP) reveal that fourth-grade students have limited understanding of fractions” (Cramer, Post, & delMas, 2002, p. 111).

One reason that fractions cause so much difficulty in elementary school may be the focus on operations and algorithms. “The rules for fraction computation can be relatively simple to teach and learn. Thus, students can become proficient at finding common denominators to add and subtract fractions or to multiply or divide fractions” (Aksu, 1997, p. 375). Several studies

(Carpenter, Corbit, Kepner, Lindquist, & Reyes, 1981) indicate that solving word problems that involve fractions is much more difficult than performing simple computational tasks. These results may indicate students are applying algorithms without any real understanding of fraction concepts. Aksu (1997) also found significant differences between students' abilities to solve fraction problems in different contexts. "Student performance of the four operations declined significantly when the operations were presented in the form of word problems" (Aksu, 1997, p. 379).

The Principles and Standards for School Mathematics (NCTM, 2000) supports the goal of grades 3-5 mathematics teachers focusing on building students' conceptual understanding of fractions. NCTM recommends students exploring a variety of models of fractions that focus on familiar fractions. According to NCTM, computational fluency need not be a primary goal until grades 6-8. Typically, the students in the elementary school where I teach have not had the experience with various concrete models to explore fractions suggested by the National Council of Teachers of Mathematics. The type of activities and instruction my students participated in during this study allowed them to work with various concrete objects and manipulatives which allowed them to develop a richer understanding of fraction concepts.

The purpose of this study was to examine the use of multiple models of representing fractions and the effects of these experiences on students' understanding of fraction concepts and operations. I believe this study was significant because it provided valuable information about how my students develop their understanding of fractions and how to improve the fraction instruction in my classroom.

This chapter included a description of the purpose and significance of this study. Chapter two contains a review of the literature related to the teaching and learning of fractions. The

design and procedures of the study are described in the third chapter. The fourth chapter includes an analysis of the data collected during this study. Finally, the fifth chapter provides a summary of the study, discusses implications of the research, and suggestions are made for further research.

CHAPTER TWO: LITERATURE REVIEW

The purpose of this study was to examine the teaching and learning of fractions in my classroom. I wanted to examine the use of multiple models of fraction representation and the effect their use had on students' conceptual understanding of fractions and their ability to solve problems that involved operations with fractions. There has been a great deal of research concerning the teaching and learning of fractions and the use of manipulatives to enhance conceptual understanding.

Conceptual Understanding

Several studies have reported that students' abilities to solve problems involving the use of fractions vary according to the context of the problems. Aksu's (2001) research indicated student performance varies significantly according to the context of the problem. In his study, 155 sixth grade students were presented test questions in three different formats. Aksu studied student responses to questions about fraction concepts, operations involving fractions, and word problems including fractions. Aksu observed the lowest student performance on the problem solving portion of the test. Interestingly, he did not find any difference in the students' ability to accurately perform the four operations in computation problems. When those same operations were required in word problems, there was a difference in student performance.

Niemi (1996) conducted a study which also indicated students have significantly different levels of performance according to the types of fraction problems presented. In his study, 540 fifth grade students participated in a variety of activities designed to assess the students' understanding of fraction concepts and problem solving abilities. The tasks students were

required to complete in this study involved the graphic and symbolic representation of equivalent and non-equivalent fractions. Students were also required to solve problems including fractions and to explain their solutions verbally or with graphic representations. His results also showed inconsistency in students' performance. Students were more successful with problems that had direct representations than problems that required an understanding of equivalent fractions. Ninety-eight percent of the students in his study circled at least one distracter when identifying equivalent fractions.

Research studies reporting differences in student performance according to the context of the problems may indicate a lack in students' conceptual understanding of fractions. "Conceptual knowledge refers to understanding relationships that are integrated or connected to other mathematical ideas and concepts" (Aksu, 2001, p. 375). Aksu stated one possible reason for this deficit in students' understanding of fraction concepts may be a common error in the usual instructional practices of teachers. He stated, "A common type of error in teaching fractions is to have students begin computations before they have an adequate background to profit from such operations. Students must understand the meanings of fractions before performing operations with them" (Aksu, 2001, p. 375).

There have been many educators and researchers who have attempted to explain the importance of helping students develop a strong conceptual understanding of fractions. Niemi (1996) found students with a solid concept of fractions have less difficulty understanding procedures and principles in mathematics. The National Council of Teachers of Mathematics (NCTM) also advocated the importance of conceptual knowledge. The NCTM Standards (1989) stated the following:

A conceptual approach enables children to acquire clear and stable concepts by constructing meaning in the context of physical situations and allows mathematical abstractions to emerge from empirical experiences. A strong conceptual framework also provides anchoring for skill acquisition. Skills can be acquired in ways that make sense to children and in ways that result in more effective learning. A strong emphasis on mathematical concepts and understandings also supports the development of problem solving (1989, p. 17).

Use of Multiple Models of Fraction Representation

The Rational Number Project (RNP) sponsored by the National Science Foundation has researched students' understanding of fractions for several years. One product of their project was a curriculum that helped students develop fraction concepts by working with physical models of fractions and a variety of fraction representations. Cramer, Post, and delMas (2002) used this curriculum in their study to compare the effects of commercial curricula with the effects of the RNP curriculum. Over 1,600 fourth and fifth graders were involved in this study. Posttest and retention test scores were significantly higher for the students using RNP project materials. The data collected from interviews during this study showed the students that had experiences with multiple representations had better mental images of fractions and were able to approach problems conceptually. They reported, "The results show the importance of providing students with instruction that involves multiple representations- particularly multiple manipulative models- over extended periods to help them develop initial fraction ideas" (2002, p. 138). These types of activities that give students the opportunity to develop conceptual

understanding are critical in the upper elementary grades. Niemi (1996) reported students who do not construct conceptual understanding of fractions by the end of elementary school are unlikely to get additional opportunities through instruction. This statement reinforces the suggestions of Cramer, Post, and delMas (2002) to provide children in grades three through five with learning experiences that offer a variety of modes of representations and multiple concrete models.

Research has also shown a relationship between students' with high levels of conceptual understanding and the ability to form multiple representations. Niemi (1996) found that students who demonstrated high representational knowledge of fractions were able to generate more correct solutions and more graphical and verbal justifications for their solutions than students who had a low representational knowledge of fractions.

Area Model

Teachers use the area model as one of the most common models for developing fraction concepts. Area models of fraction representation focus on the area of the whole unit being divided into equal portions. Armstrong and Larson (1995) researched how these types of experiences related to students' conceptual understanding of fractions and the students' abilities to compare rational numbers. They studied thirty-six students in grades four, six, and eight. Students were interviewed and asked to complete 21 tasks requiring use of the area model of fraction representation. "Students gain insights into part-whole relationships during formal instructional experiences that have been carefully designed and sequenced to include interactions with concrete and pictorial models, associations of language and symbols with models, and comparison and ordering tasks" (p. 3).

The National Council for Teachers of Mathematics Principles and Standards (2000) also stated the importance of using the area model of fraction representation. “By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole, and find equivalent fractions” (p. 149).

Linear Model

Results from the study by Niemi (1996) indicated that students who received instruction using a linear model performed significantly better on assessments that measured conceptual understanding of fractions than other students who received only instruction with the set and area models in a traditional textbook. The results of his research also suggested a need for more discourse in the mathematics classroom concerning fraction concepts. Activity based approaches which do not include discourse concerning the mathematics concepts are not likely to encourage strong conceptual understanding among students.

Results from D’Ambrosio and Mewborn’s (1994) study also had implications for using linear models. They stated the following:

The Fraction Project has found the linear model to be quite effective in children’s constructions of fraction concepts, especially in the use of the unit fraction as an iterative unit that can be used repeatedly to build the whole. The linear model tends to minimize the need for children to understand area relationships as they are striving to make sense of fractional constructs. (p. 155)

Despite the findings from several studies concerning the benefits of exposing students to the linear model of fraction representations, traditional textbooks and classroom practices rarely

incorporate the use of the linear model (D'Ambrosio & Mewborn, 1993). Niemi (1996) cited a possible reason for the under use of number lines when working with linear models of fraction representations. "Successful use of number lines requires at least two types of knowledge not implied by other representations, that is, coordination of multiple units simultaneously and understanding that fractions are numbers representing relations between other numbers" (p. 353).

Set Model

The set model, also referred to as a discrete model, requires students to understand that a group of objects is considered the whole and the individual objects would be subsets or parts of the whole. There is conflicting research concerning the usefulness of this model to promote conceptual understanding of fractions. Research by Payne (Suydam, 1978) suggests work with the set model of fraction representation may interfere with the learning students have already constructed concerning fractions when using area and linear models of fraction representation. Other researchers (Behr, Wachsmuth, & Post, 1988) claim that what could be referred to as interference may actually promote students to reexamine their understanding of fractions and develop a greater conceptual understanding of fractions.

Behr, Wachsmuth, and Post (1988) studied comparison groups of fourth and fifth graders over an eighteen-week period. The students in the experimental groups received instruction that incorporated the use of manipulatives to model continuous representation of fractions. They were interviewed midway through the study. After the midpoint interview, students experienced instruction that incorporated the discrete model of fraction representation. Students were interviewed again at the conclusion of the study. The control group received instruction from a

typical textbook series that had picture representations of fractions, but their instruction did not incorporate the use of the manipulatives. They reported, “The performance of all individual children in the experimental group increased or remained consistent. Since these children received instruction between assessments based on both discrete and continuous models, this information lessens the strength of the research by Payne reported in Suydam (1976) which made the claim instruction based on discrete models interferes with children’s performance on tasks based on continuous models” (p. 8).

Use of Manipulatives

NCTM (2000) Principles and Standards stresses the importance of lessons that are engaging and active in the elementary classroom. “Instruction at this level must be active and intellectually stimulating and must help students make sense of mathematics” (p. 143). Use of manipulatives in the classroom is one way to provide students with active, engaging lessons that promote conceptual understanding.

Research by Butler, Miller, Crehan, Babbitt, and Pierce (2003) supported the use of concrete materials to improve conceptual understanding of fractions. Their study involved fifty middle school students who had diagnosed learning disabilities. The students were divided into two treatment groups. One group followed a lesson sequence that went from concrete manipulative devices, to representational drawings of fractions, to abstract symbols. This group was called the concrete-representational-abstract (CRA) group. The second treatment group was the representational-abstract (RA) group. These students received an instruction sequence that did not involve the concrete models and went straight from representational pictures to abstract

symbols. They stated, “On all achievement measures, students in the CRA group had overall higher mean scores than did students in the RA group” (p. 99). The findings of this study show the benefits of using concrete manipulatives to promote conceptual understanding of fractions.

Conclusion

The teaching and learning of fractions is difficult and complex work. Using multiple models of fraction representations strengthens students’ conceptual understanding of fractions. Students who have a deep understanding of fractions should be able to recognize multiple ways to represent fractions and realize that different representations do not necessarily mean different fractions (Niemi, 1996). A lack of conceptual understanding may be one explanation for discrepancy in students’ performance when performing fraction operations in different contexts as reported in studies by Aksu (2001) and Niemi.

In addition to providing students with experiences with multiple models of fraction representation, quality fraction instruction should also include increased opportunities for student discourse and activities which allow students to manipulate concrete materials. Manipulative materials play an important role in the development of student’s conceptual understanding. Circular and rectangular pieces, colored chips, and Cuisenaire rods should be critical component of instruction (Behr, Lesh, Post, & Silver, 1983).

CHAPTER THREE: METHODOLOGY

Introduction

The purpose of this action research was to examine my own teaching practices and my students' understanding of fraction concepts and operations. I wanted to determine if the use of three different models of fraction representation during instruction would increase students' understanding of fraction concepts and operations. The models of fraction representation used during this study were the area model, set model, and linear model. After the initial instruction, I wanted to determine if students preferred a certain model of fraction representation when they were working independently.

Design of Study

Because the focus of my study was my teaching practices and my students' learning, my design study was primarily action research. Mills (2003) defined action research as “any systematic inquiry conducted by teacher researchers, principals, school counselors, or other stakeholders in the teaching/learning environment to gather information about how their particular schools operate, how they teach, and how well their students learn” (p. 5).

I used a mixed methods approach with this study. There were several methods for collecting qualitative data during this study. Student reflections, teacher field notes, observations, and interviews were all used as sources of data. In an effort to make sure that my data was

triangulated, I also collected some quantitative data. Student responses on pre and post assessments were analyzed.

Limitations

There were a few limiting factors that might have effected the results of my research. Due to the requirements of my school's administrator and the county of which I am an employee, I was limited to one hour and fifteen minutes per day for mathematics instruction. A second limitation to this study was student attendance. Because of the nature of the teacher modeling, class discussion, and group activities involved in the lessons, activities were unable to be repeated for students who were absent from the lesson. The majority of the participants of this study were in attendance for the lessons. There were students who missed portions of the lessons because of their attendance in other mandatory programs within the school. There was one student who missed several weeks of the unit, because he had unexpected major surgery.

Setting

The setting of this study was a fifth grade classroom in a public elementary school in central Florida. The school services students in kindergarten through fifth grade. The school also has a small prekindergarten program for students with special needs and a program for children who have autism. The school is in an upper-middle class community with only about 13% of the students qualifying for free or reduced lunches. There is a great deal of parental involvement at the school and the majority of the parents take an active role in their children's education.

The school where this study took place is fairly large with a student population of about 970 students. There is not a great deal of racial diversity at the school. Approximately 77.6% of the student body is classified as white, non-Hispanic. Other ethnic groups are represented at our school in much smaller numbers. The student population is 9.4% Hispanic, 5.6% multiracial, 4.1% Asian or Pacific Islander, 3.1% African American, and .2% Native American.

The participants of this study were all assigned to my classroom at the beginning of the school year by the school's administrator. Efforts were made to provide a heterogeneous group of students. My class consisted of twenty-two students of varying abilities. Four of the participants qualified for services from the gifted program. Those students missed one day of instruction every week during this study. One student was diagnosed with Asperger's Syndrome, a condition related to autism.

Informed Consent

Prior to beginning this study, approval was granted from the Institutional Review Board (IRB) (see Appendix A). I informed the parents of the students in my class about the content and purpose of my research (see Appendix B). I obtained written consent from the parent or legal guardian of every student participant. Verbal consent was given by each participant after he or she was read the student assent form (see Appendix C). Approval from the school's administrators was also received before the research study began.

Instruments

The assessment instrument that was used as the pre and post assessment was developed by the researcher (see Appendix D). The instrument was reviewed by several professional educators as well as another researcher who has conducted several studies concerning students' understandings of fractions. The instrument was reviewed by these professionals in an effort to ensure the assessment's validity.

Items one and two were used to assess students' conceptual understanding of fractions. The purpose of items three through five was to assess the students' ability to identify fractions using the set, area, and linear models. Item number six on the assessment was adapted from a research study conducted by Niemi and published in his article *Assessing Conceptual Understanding in Mathematics: Representations, Problem Solutions, Justifications, and Explanations* (1996). Item seven evaluated the students' ability to represent equivalent fractions. Items seven through sixteen were used to evaluate students' abilities to add and subtract fractions with like and unlike denominators.

Methods of Data Collection

Several methods of collecting data were used during this study in order to ensure triangulation of the data. I was an active participant observer in the study. Mills (2003) describes teachers as active participant observers, because they are constantly monitoring the effects of their teaching and making the necessary adjustments to instruction. I took extensive daily field notes throughout the study in order to document the progression of the lessons, students' comments during class discussions, and challenges that were observed during independent and

group work. I took notes during the lessons and I also reflected on the activities and discussions at the conclusion of each lesson.

Data were collected from the pre and post assessments that were completed by each participant in the study. The students also completed some portions of the daily activities in individual journals. Data were collected for this study from the participants' journal responses. Students' completion of written assignments and worksheets were also collected and analyzed. I interviewed each participant individually at the conclusion of the study. When reporting the data in this research, pseudonyms were used throughout the text.

Procedures

All of the participants were administered an individual written assessment before the unit of fraction instruction began. Students were allowed to take as much time as they needed to complete the assessment. The purpose of this written assessment was to determine each participant's prior knowledge of fraction concepts and operations. Students participated in eight weeks of fraction instruction that incorporated the use of the set, area, and linear models of fraction representation. The written assessment was readministered to each participant at the conclusion of this study.

Fraction Representation

On the first day of instruction the students were introduced to representing fractions using the area model and the linear model. We started the lesson by representing fractions using the area model. Each student was given a set of pattern blocks that was made from colored

construction paper. The pattern blocks were reproduced from the Activities Integrating Mathematics and Science (AIMS, 1998) Actions With Fractions resource book. Each set contained two yellow hexagons, four red trapezoids (one half the area of a hexagon), six dark blue rhombuses (one third the area of a hexagon), eight light blue trapezoids (one fourth the area of a hexagon), 12 green equilateral triangles (one sixth the area of a hexagon), and 24 purple right triangles (one twelfth the area of a hexagon). I chose to use the AIMS paper pattern blocks instead of the traditional plastic or wood pattern blocks, because of the additional pieces that would allow the students to experiment with fourths and twelfths.

As a class, we discussed the fractional name of each piece if the yellow hexagon was considered the whole. Students used their paper sets to cover the entire area of the yellow hexagon with congruent pieces and we discussed results as a class. Students also explored improper fractions and mixed numbers. Students were asked to use their pieces to represent fractions such as eleven sixths. Then students worked with a neighbor to determine another name for the area they covered. The class discussed why eleven sixths and one and five sixths could both be used to describe the area covered with 11 green triangles. After the class discussion, the students were asked to write an entry in their fraction journal. Students responded to the question, "If the red trapezoid is one whole, what fractional part is a green triangle?" Students were asked to draw and explain their solutions.

After all of the students completed their journal entries, the class began experimenting with the linear model of fraction representation. Each pair of students was given a set of Cuisenaire rods. As a class, we discussed examples using the orange rod as one whole. Each pair of students determined the fractional part of the other rods based on the orange piece as the whole. After the class discussion, each pair of students chose another rod to be the whole and

worked to determine the fractional parts of the other rods. The students then made a second entry in their fraction journal. Students were instructed to use the Cuisenaire rods to model the fraction four fifths. They were asked to draw and explain their answers.

We began the second day of our fraction unit by sharing journal responses from the previous day. Then we began to represent fractions using the set model. The students used two-color counters to represent some of the same fractions that were discussed the day before using the area model and the linear model. After the class discussed several examples of sets displayed on the overhead projector, small groups of students made sets using attribute blocks. Each group created a set of shapes using the attribute blocks. Then they brainstormed as many ways as possible to describe their sets using fractions. For example, three sevenths of the set are triangles, two sevenths of the set are blue, or five sevenths of the shapes are thick. Then students completed an entry in their fraction journal. Each student chose a fraction and represented it using the set model. The students were asked to explain why the set they drew represented the fraction they chose.

The third day of the unit began with a journal entry. At this point in the study, the only manipulatives that had been used for representing the area model of fractions were the pattern blocks. I displayed a fraction circle on the overhead. The circle had five eighths of the area covered with one eighth pieces. I asked the students to draw the figure and explain in their journal what fraction was shown. Then I displayed a two by two square drawn on grid paper on the overhead. I challenged students to try to find a way to shade half of the area so that the remaining unshaded portion was also a square. We discussed their solutions.

The students also completed a set of worksheets that were reproduced from the AIMS (1998) resource guide. The worksheets required students to name the fractions that were

represented by shaded circles, rectangles, and sets. Another worksheet gave students a shape and they were required to shade it according to the fraction that was named. Students completed this activity independently.

Over the course of the study, activities were used periodically to review fraction representation using different models. One day students were given a card with a fraction written on it. In their journals, students drew a representation of the fraction using any model they chose and explained it. In a reflective newsletter students wrote independently to their parents, each student modeled one fraction two different ways and explained why their drawings represented the fraction. Students were given unconventional drawings to represent fractions. They were required to determine the shaded portion of the figure and explain their reasoning. Students were asked to divide a rectangle into fourths in as many different ways as they could. Representations of fractions using number lines were also explored in class. Students were shown examples of number lines and determined the fractional part shown on the number line. Students used number lines to show fractional values and explained their reasoning. Another activity required that students work with a partner to look at pattern block designs. In each of these activities, the design was considered the whole, and students had to determine the fractional parts that were represented by the individual pattern blocks.

Equivalent Fractions

Once the students had many experiences representing fractions using different models and manipulatives, the class began to explore equivalent fractions. We started investigating equivalent fractions using the area model with their individual sets of paper pattern blocks. I

displayed a model of two-thirds using the yellow hexagon and two dark blue rhombuses on the overhead. The class identified the fraction shown as two thirds without difficulty. I wrote the symbol $\frac{2}{3}$ under the figure and displayed another yellow hexagon. I asked the students to use their manipulatives to try to figure out what other congruent figures they could use to cover the same area. I displayed their results on the overhead and we discussed that these fractions were all equivalent, because they covered the same area of the whole. Students investigated many more examples of equivalent fractions using their pattern blocks. Their results were displayed on the overhead and labeled. After the groups' discussion, students were asked to find a fraction that was equivalent to one-fourth. Students were asked to draw and explain their solutions in their journal.

The following day the class investigated equivalent fractions using the linear model and Cuisenaire rods. On the overhead, I identified the dark green rod as one whole. I displayed a light green rod directly underneath it and asked the class to identify the fraction represented. Students easily identified the fraction as one-half and modeled it on their desks with their own Cuisenaire rods. I then asked the class to use their rods to find congruent pieces that would be as long as the light green piece. The students discovered that three white rods were as long as one light green rod. Through class discussion we determined that it would take six white rods to equal the whole, therefore three white rods were three sixths of the whole and one half was equivalent to three sixths. The students worked through several more examples finding equivalent fractions based on the lengths of their Cuisenaire rods.

The class also used the area model with fraction circles to investigate equivalent fractions. Each student worked with a partner to model equivalent fractions using sets of fraction

circles. The students worked cooperatively to model, draw, and label as many equivalent fractions as they could on chart paper.

Students investigated equivalent fractions using the set model. Students were each given a set of two-colored counters. Students were asked to model the fraction two fifths on their desks. I modeled it on the overhead and we discussed why our models represented two fifths. We discussed as a class how we could model a fraction that was equal to two fifths using the two-color counters. We built on to the set using the same proportion of red and white counters. The class discussed that four tenths was equivalent to two fifths. Students verified those results with fraction circles.

Comparing Fractions

The students' work with comparing fractions began with the area model using fraction circles. Students worked with a partner to represent one third and one sixth. We discussed how the fractions they represented compared to one another. Students discussed which fraction covered a larger area of the whole. After the students modeled the comparison using manipulatives, they drew the figures, labeled the fractions, and wrote the inequality. Students completed several more examples comparing fractions with the same numerator. As a class, the students looked for patterns and ways to generalize their findings. I recorded their observations on the overhead. These steps were repeated to compare fractions that had the same denominator and fractions that had different numerators and denominators.

During this study, students also compared fractions using a linear model of representation. Students used the orange Cuisenaire rod as a whole. They represented two fifths

underneath the orange. Then they represented two tenths underneath the two fifths. Students compared which fraction was longer and wrote the inequality represented by their rods. Students completed several similar examples using different rods as the whole.

Addition and Subtraction of Fractions

We began addition and subtraction of fractions using linear model representations. The class started with examples that had the same denominator with solutions that were less than one. We moved on the examples that had unlike denominators, but still had a solution less than one. We followed the same steps for all of our examples. We defined the whole, represented the fractions to be added or subtracted, and then determined the solution by adding or subtracting the units using the Cuisenaire rods. Students always drew their examples and named all of the fractions in their drawings. The students also wrote the mathematical sentences using the fraction symbols. After many experiences working with a partner and adding and subtracting fractions, the students were asked to solve a question independently in their journals. Students were asked to use the orange as one whole and add one half and three fifths. This was the first example where the solution was greater than one. Students were asked to draw and explain their solutions.

During the next day of instruction the students continued adding and subtracting fractions, but this time they were using the area model of fraction representation. Students worked with fraction circles to model addition and subtraction of fractions. Students were always asked to model the problem with their manipulatives, then draw the problem and solution, and label their drawings with the fraction symbols. Once the students had many experiences using the manipulatives, they completed AIMS (1998) worksheets that had only the picture

representations. The students labeled and colored the pictures according to the addition and subtraction problems shown.

The following day the students investigated using the set model to add and subtract fractions with like and unlike denominators. We worked through several problems on the overhead using two-colored counters. The class discussed the need to build on to the sets before they could add or subtract. This was the first activity that required the students to find a common denominator. In all of the other activities the students joined the sets and then renamed the new fraction using their manipulatives.

After students had many experiences adding and subtracting fractions using all three models of fraction representation, they were asked to solve and explain problems in their journals. Students were asked to draw and explain their solutions. All of the manipulatives that were used during instruction were available to the students.

At the conclusion of the unit, the students completed the post assessment for the fraction unit. This was the same assessment that was given at the beginning of the study. Each student also participated in an individual interview. During the interview, the students responded to questions concerning the model of fraction representation they preferred to use when solving fraction problems independently. Students were asked to explain why the model they chose made sense to them.

CHAPTER FOUR: DATA ANALYSIS

Introduction

The teaching and learning of fractions have always been a challenge in my classroom. Based on my readings of the literature related to this topic, I realized I was not alone in my observations. The purpose of this research study was to examine the use of multiple models of fraction representation during fraction instruction. I investigated how the use of area, set, and linear models of fraction representation improved student understanding of fraction concepts and operations. The students' preferences regarding the three models used were also researched.

While investigating the students' understanding of fraction concepts and operations, I discovered additional themes in the research. Other themes identified included difficulty explaining and justifying solutions, improvement in student attitudes regarding fractions and mathematics, an increased amount of discourse during mathematics instruction, and students' application of fraction concepts in other content areas. These themes were identified from students' journals, field notes, observations, and interviews.

Students' Understanding

Several methods of data collection were used in an attempt to assess students' understanding of fraction concepts during this research study. The pre and post assessments provided valuable information concerning the students' development of conceptual understanding. There were sixteen items on the assessment. I used a three- point rubric to assess

each student's response to each individual question. This gave a possible score of 48 for the assessment. Each student was randomly assigned a number for the purpose of analyzing the results of the assessment. The results are displayed on Table 1 below.

Table 1: Pre and Post Assessment Results

Student Number/Pseudonym	Pretest (48)	Posttest (48)	Change
1 Fred	15	21	+6
2 Peter	11	27	+16
3 Mary	29	37	+8
4 Evan	20	37	+17
5 Lisa	8	26	+18
6 Mike	19	44	+25
7 Sam	9	27	+18
8 Tom	16	34	+18
9 Rachel	25	41	+16
10 Jean	9	39	+30
11 Meredith	33	40	+7
12 Brian	29	40	+11
13 Jim	22	31	+8
14 Karen	11	20	+9
15 Janet	17	34	+17
16 Angie	13	25	+12
17 Daniel	15	28	+13
18 Marie	30	38	+8
19 Christina	19	35	+16
20 Derek	17	25	+8
21 Josh	39	41	+2
Mean	19.3	32.9	+13.6

As shown on the table above, all of the twenty-one students who participated in the study showed an increase in conceptual understanding according to this assessment. The increase ranged from 2 to 30 points. The mean improvement was 13.6. Although this assessment showed improvement for all students in conceptual understanding of fractions, some students showed more growth through other methods of data collection.

Item number six on the assessment was not a useful test item for assessing the students' conceptual understanding of fractions. I did not realize until final examination of the data that the examples I used in class to explore number lines only showed the portion of the number line from zero to one. The exclusion of larger portions of the number line during class discussions greatly limited the students' abilities to identify fractions equivalent to one half using the number line examples on item number six.

Although items 11 and 16 were helpful for assessing students' abilities to solve problems involving fractions, they were not useful for assessing students' understanding of fraction concepts. Both of these items were word problems that involved fractions. When I was analyzing the data I noticed that many students were unable to solve these problems correctly. The errors did not appear to be related to a lack of conceptual understanding about fractions. The most common errors concerned understanding which operation to use in order to solve the problem correctly. In fact, some students had drawings and explanations that demonstrated a deep understanding of fraction concepts, but they did not receive the full three points because of their problem solving errors. Two examples of these kinds of errors are shown in Figures 1 and 2.

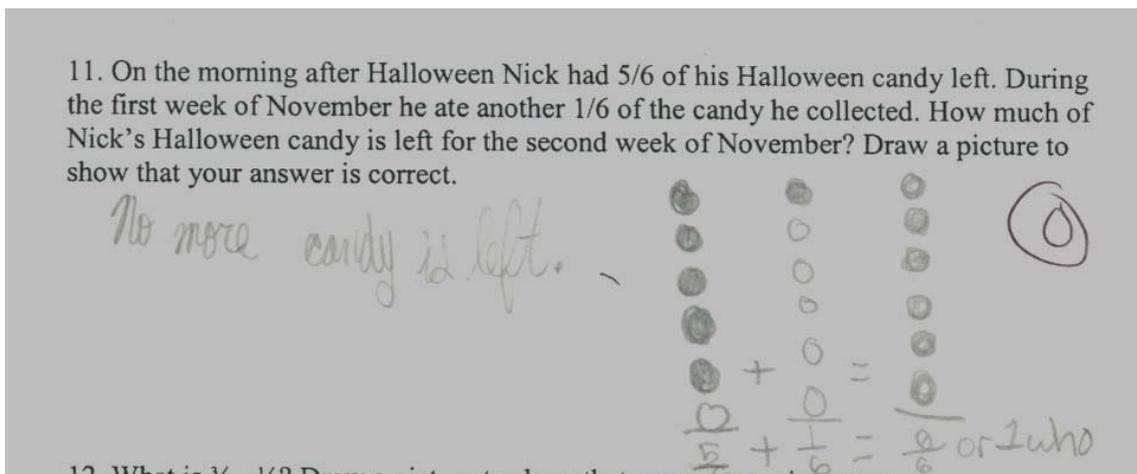


Figure 1: Post Assessment Example

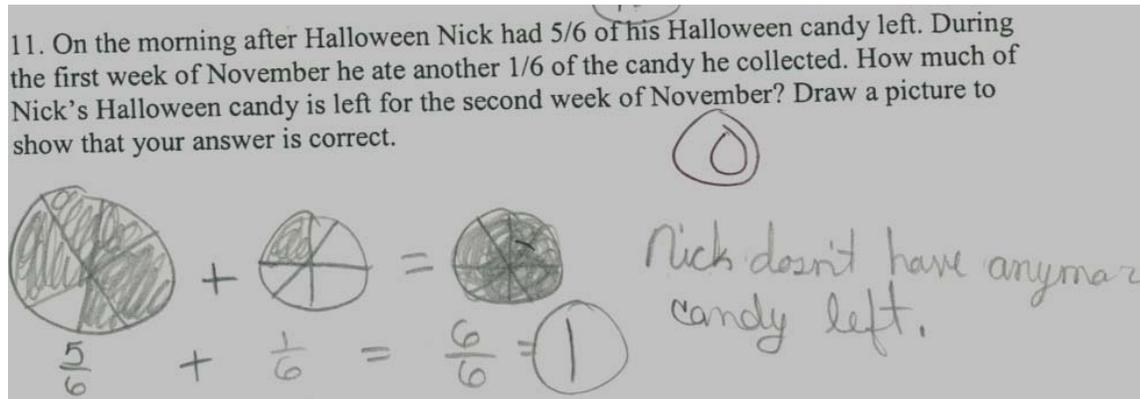


Figure 2: Post Assessment Example

These students did not receive any points for their responses on the post assessment, because they applied the wrong operation when solving the problem. The solutions did show an improvement in their conceptual understanding of fractions. The results concerning students' performance on the items involving problem solving supported the findings of Aksu (1997). He reported, "Students' computational abilities with fractions are better than their ability to solve word problems involving fractions" (p. 375).

Another reason the assessment may not have accurately documented students' conceptual understanding relates to their ability to justify their solutions. Fifteen out of the sixteen items on the pre and post assessment required students to justify their answers with words or pictures. Over the course of the study, this proved to be a difficult task for students. Even when the students were able to solve fraction problems accurately, they had a difficult time explaining their answers. Conversely, some students provided excellent justification for their drawings and solutions, but never provided the fraction symbols for their drawings. The requirement for both the fraction symbols and the written explanations might have contributed to lower scores for

students who displayed deeper understanding of fraction concepts on other methods of data collection.

The ability to justify solutions to fraction problems was an interesting topic to research. Although it proved to be a challenging task for many students which may have contributed to lower post assessment scores, it was also the area that showed the most improvement. Students who were able to justify their answers to fraction problems on the pre assessment showed the least improvement and the students who were unable to justify their responses on the pre assessment showed the greatest improvements.

The results of the pre assessment showed only five students who were able to consistently and accurately justify their solutions. Meredith, Jim, Marie, and Josh were the highest scoring on the pre assessment and they were the students who were most capable of justifying their solutions. They were also among the students that showed the least improvement on the post assessment. It may also be significant to note that these four students have qualified for the gifted program and typically have an advanced understanding of the fifth grade mathematics curriculum. On the pre assessment, Evan was also able to justify solutions, but he did not label his justifications with fraction symbols. On the post assessment his justifications were complete and increased his score seventeen points. Mike, Sam, Tom, and Jean demonstrated the most significant gains on the post assessment. After careful examination of their assessments, it was clear that their improvements could be partially attributed to an increased ability to justify solutions.

Other methods of data collection supported both the difficulties and improvements related to the students' abilities to justify their solutions to fraction problems. Students completed several activities in fraction journals during the course of this study. Many of the students'

entries showed an increase in conceptual understanding and in the ability to justify solutions. After our first lesson which involved using the area relationships of pattern blocks to represent fractions, students completed a similar activity in their fraction journal. During the lesson, the entire class was successful using the manipulatives to represent the fractions. They were able to both identify the fractions represented by others and they were able to represent the fractions independently. However, when they were required to represent and explain the fraction one third, only ten students in the class were able to transfer the understanding they demonstrated in class to a written drawing and explanation. Jean's first journal entry is shown in Figure 3. Her entry was typical of students who were unable to completely represent and justify a representation of one third using pattern blocks.

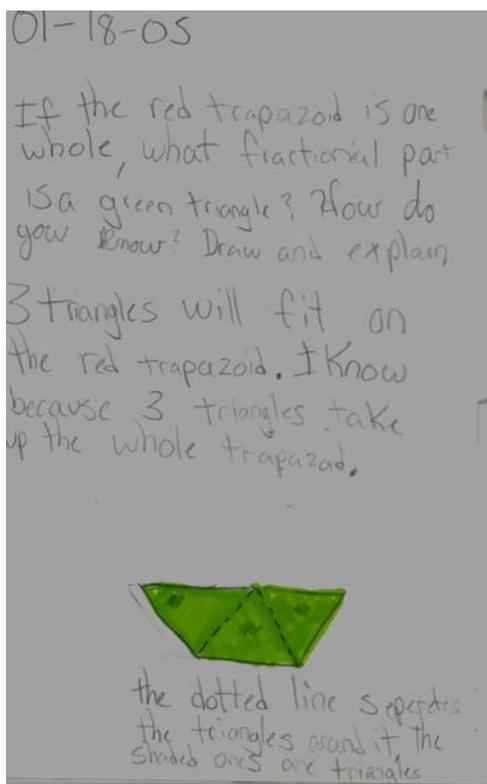


Figure 3: Jean's Early Journal Entry

Despite this student's difficulty representing and explaining a fraction at the beginning of this unit, she greatly improved her understanding of fraction concepts and operations that involve fractions. After a little over a week of instruction that involved multiple models of fraction representation and experience justifying solutions, the students were given a more challenging task to complete in their journals. Students were asked to use Cuisenaire rods to model the addition of one half and one third. Again students were asked to draw and explain their solutions. This journal assignment was the first time the students encountered a solution that was greater than one whole and involved a denominator in the sum that was unlike the denominators used for the addends. Even with the challenging aspect of the problem, 15 students were able to solve and explain the problem. Jean was one of those students. Her entry is shown in Figure 4.

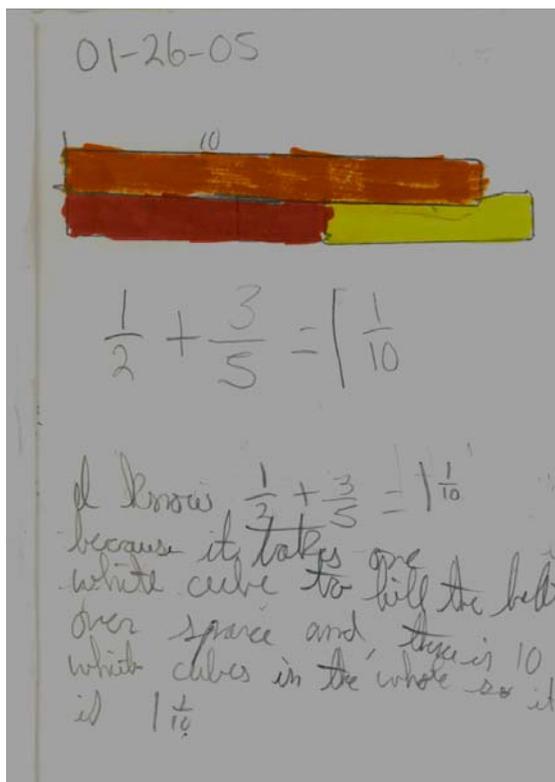


Figure 4: Jean's Later Journal Entry

Jean's improvement in conceptual understanding as shown in her journal was considerable, but it was not an isolated occurrence. Examining the journals of the individual students showed a great deal of growth in conceptual understanding of fractions for many students during this study.

Improved conceptual understanding was also evident when field notes of class discussions were reviewed. One activity during this study required students to look at a two by two grid. Students were asked to shade one half of the figure in a way that the remaining unshaded portion would still be a square. As we discussed possible solutions, an increase in conceptual understanding of fractions was evident for some students. Janet had difficulty justifying her solutions on the pre assessment. However, after participating in activities that involved multiple methods of fraction representation, she was better able to articulate her understanding of fractions. During a class discussion about the accuracy of a possible solution, Janet explained, "I know that is right. If you shade half of every fourth, that has to be one half of the whole thing." The confidence with which she spoke was a clear indication that her conceptual understanding was improving.

One difficulty with conceptual understanding that persisted for many students was the belief that equal pieces must look alike. The students were asked to try to divide a rectangle into fourths as many different ways as they could. Then students drew their solutions on the board and we discussed the accuracy of the solutions. Initially, some students would not accept that the pieces were fourths if they did not look the same. Brian was one student that held firmly to the belief that fourths had to be the same shape. Despite the fact he had one of the highest scores on the post assessment, his conceptual understanding of fractions still focused on equal parts that looked the same. His journal entry for this assignment is pictured in Figure 5.

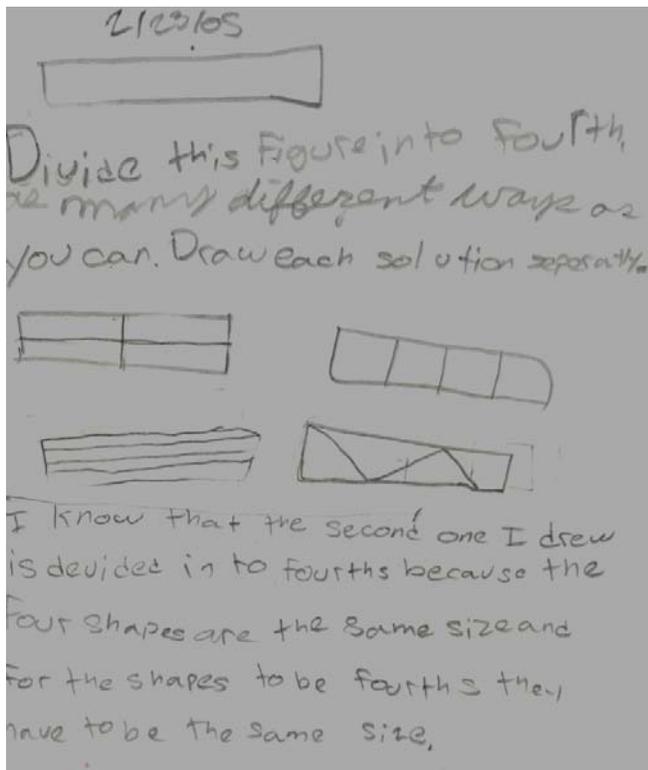


Figure 5: Brian's Explanation of Fourths

Other students thought any way the shape was divided into four pieces would be an example of fourths. As the discussion persisted, the students began to make connections between this activity and others we had done in class. Marie explained one of her drawings by saying, “I divided the rectangle in half, and then divided the halves in half. It was like the square we shaded.” Once students remembered that activity, they were more accepting of some of the students’ solutions that did not involve congruent fourths. The class discussion during this activity revealed some remaining difficulties with conceptual understanding of fractions. D’Ambrosio and Mewborn (1993) reported similar results from their study. “Much of children’s early work with fractions leads them to a strongly held construction that equal means looks alike” (p. 154).

Students' Preferences for Models of Fraction Representation

Data were collected using several different instruments regarding students' preferences for the area, set, or linear model of fraction representations. Student's explanations and use of manipulatives when working independently in their fraction journals were examined, written assignments that allowed students to choose the model they would use to represent fractions, students' explanations when completing the post assessment, and interviews with students were used to triangulate the data collected concerning student preferences.

After several days of activities that provided students with experiences using all three models of fraction representation, each student was given a card with a different fraction written on it. They were asked to draw and explain a representation of the fraction on their card in their fraction journal. Students had access to all of the manipulatives that we used in class. For this particular task, fourteen students chose to use Cuisenaire rods to represent the fraction using the linear model. One student chose to model the fraction using the set model. None of the students participating in this activity represented their fraction using the area model. A few of the fraction cards given to students could not have been represented using the pattern blocks, but most of the fractions would have worked well with any of the three models of fraction representation. There may have been several factors that contributed to the dominance of the linear model in the students' work. Students' desks are arranged in table groups. The students were aware of the model chosen by the other students in their groups. The linear model was the model that we had used in class most recently. The recent use of the model may have influenced their choices.

After a few weeks of working with all three models, students were asked to write a reflective paragraph concerning representations of fractions. Each student was asked to represent

five sixths two different ways. Again, all of the manipulatives used in class were available to the students while they worked independently. The area model was used to represent the fraction nine times, the set model was used eight times, and the linear model was used twelve times. When students were required to use two models to represent fractions the linear model was still the most frequently used, but all three of the models had a similar frequency.

Examination of the written post assessments provided interesting data concerning students' preferences of fraction models when working independently. Table 2 shows the frequency with which each model was shown for the assessment items that required students to justify their solutions. Items which were omitted from this table did not allow students to choose a model of fraction representation. When creating this table, I only counted student responses that strongly indicated one of the three fraction models. Students who chose to justify their solutions using algorithms or students who neglected to justify their solutions were not included on this table.

Table 2: Frequency of Models Used on Post Assessment

Item Number	Content of Problem	Students Using Area Model	Students Using Set Model	Students Using Linear Model
2	Representation	18	3	0
7	Equivalent	16	2	3
8	Addition	15	4	2
9	Addition	13	5	2
10	Addition	18	1	2
11	Subtraction	18	2	1
12	Subtraction	16	4	1
13	Subtraction	17	1	1
14	Subtraction	20	0	0
15	Representation	6	5	2
16	Addition and Subtraction	6	3	3

The area model was the students’ overwhelming preference while working on the post assessment independently. The results for item number fifteen were especially surprising. The problem reads, “A fifth grade class collected data about hair color. Six students had brown hair, four students had blonde hair, two students had red hair, and five students had black hair. What fraction of the class had blonde hair? Explain your answer with words or a picture.” Even though the problem would most logically be solved using the set model, only five students in the class utilized the set model to solve the problem.

The post assessments were analyzed to look for the dominant model used by each individual student. Of the 21 students tested, 15 used predominantly the area model on the post assessment. Eight of the students who preferred the area model used it exclusively. Three students used the area and set models equally on their post assessments. One student used the set model to solve most of the problems and one student used the linear model for most problems. One student worked with all three models of fraction representation on the post assessment and showed no preference.

At the conclusion of the study, each child was interviewed individually in order to obtain data concerning the model of fraction representation they preferred. Each student was asked to explain why they preferred the model they chose. The frequency of the students’ responses are displayed on Table 3.

Table 3: Models Students Preferred According to Their Individual Interviews

Model of Fraction Representation	Frequency
Area	5
Linear	2
Set	13

Based on the individual interviews, the most preferred model of fraction representation was the set model. Students offered a variety of explanations for the model they chose. Many students supported their choice by stating that the model they selected helped them to “see” the fraction better. Interestingly, many students that used a particular model exclusively on the post assessment did not state a preference for that same model in the individual interview. Another interesting observation based on the interviews was the extreme differences in explanations the students offered for their preferences. Students that preferred the area model often explained that this model made more sense to them, because the whole was so clear and they did not have to count separate objects. Students who preferred the set model made the exact opposite claim. They preferred the set model, because it was easier to understand because the whole was spread out and they could count it more easily.

Based on the data collected during this study from the assessments, journal writings, class discussions, and interviews, I was unable to determine a preferred model of fraction representation. The model of fraction representation varied from one activity to the next. Students’ who seemed to have a strong preference in their written work often did not verbalize the same preference in the individual interview. These mixed results indicate experience with all three models is likely to be beneficial to students. Tom was one of the few students that had a consistent preference when working with fraction models. Tom’s written work and interview indicated a strong preference for the set model of fraction representation. He was also a student who showed a great deal of improvement with his conceptual understanding of fractions. Tom improved his score eighteen points on the post assessment. I can not be certain, but I wonder if this student would have had the same level of success if the fraction instruction had not included this model that made so much sense to him.

Improved Student Attitudes

Throughout this unit I observed an improvement in students' attitudes concerning fractions and mathematics. These data were collected by teacher observation, student correspondences, and student interviews.

It was apparent at the beginning of any fraction lesson during this unit that the students' attitudes about mathematics were improving. Daniel often asked me when he entered the classroom if we would "get to do fractions today." His tone of voice indicated work with fractions was a part of the day that he looked forward to and enjoyed. Whenever I gave the instruction for students to take out their fraction journals, students made positive comments about the activities. At the beginning of one lesson a student commented to his neighbor, "All right! It's time to do fractions!" This was not the only positive comment made at the beginning of a lesson by this student. Other students made similar comments throughout the study. I had never observed this attitude about working with fractions prior to the activities in this research study involving the use of manipulatives and multiple ways of thinking about fractions.

Angie made several comments about enjoying learning about fractions during her individual interview. When discussing the model of fraction representation that she most preferred, she stated, "They were all fun to do with fractions, but the set was easier to understand."

Janet affirmed my beliefs about improvements in students' attitudes. One afternoon she sent me an unsolicited email. I was pleased to receive a one line email from her. She stated, "I love this fraction stuff we are doing." I responded that I was pleased she was enjoying the fraction unit. I asked her why she liked it so much. She replied, "The reason why I like fractions

so much is because it is interesting when we learn new things, especially when we work in our fraction journal! When we work in it I like to draw the shapes, explain the fraction, and try to figure out which is greater, or at least I hope we can do fractions more often! I LOVE FRACTIONS!” Her statements were especially encouraging, because she is a student who normally struggles with the mathematics curriculum. The activities we were doing in class using multiple models of fraction representation were helping her understand difficult content in a way that she found enjoyable.

Increased Discourse

An unintended benefit of this research study was an increase in the amount of student discourse during mathematics instruction. The frequent use of manipulatives and problem solving activities naturally required students to communicate more with one another. As the weeks progressed, I found it more and more difficult to record field notes. The main cause for this difficulty was that students were talking more than they were at the onset of this study.

Whenever the students were working with the manipulatives and writing in their fraction journals, the classroom had a more relaxed atmosphere. The students were sharing materials and often discussing their work in small groups. For example, I noted one day when we worked with Cuisenaire rods two students discussing how to represent four fifths. One student asked his neighbor, “Which one was the fifth?” She replied, “It depends which one is the whole. Are you using the yellow as one?” Similar examples of students discussing fraction ideas were a common occurrence during this research.

During the discussion concerning the division of a rectangle into fourths there was a substantial amount of student to student discourse. Students abandoned the usual practice of raising their hand to speak and were talking directly to one another to make their points. The rest of the class was attentive to the speaker and they really set the pace and direction of the discussion with very little teacher intervention. As students attempted to articulate their position about the accuracy of a drawing, they often made comments that referred to another student's remarks. "It's like Mary said before..." Stephan and Whitenack (2003) explained the importance of these kinds of student dialogues. "Conceptually oriented explanations give students opportunities to articulate, and give other students opportunities to understand, reasons for performing certain procedures" (p. 160).

One student even cited the increased discourse as a beneficial part of this unit in the individual interview at the end of the unit. This student preferred the linear model of fraction representation. One reason she gave for her preference focused on the interaction with her classmates. "We worked in groups the most when we used the linear model. That helped me understand more. We shared the rods, so we talked more."

Chapter Five will conclude my study on the use of multiple models of fraction representation. In Chapter Five, I will review the findings and make recommendations for future research in the area of fraction models in the elementary classroom.

CHAPTER FIVE: CONCLUSION

The purpose of this study was to determine if the use of multiple models of fraction representation in a fifth-grade classroom would improve students' understanding of fractions and fraction operations. I also wanted to determine if students had a preferred model of fraction representation when working independently on fraction problems. During this research, data were collected from students' journal entries, field notes, individual interviews, and pre and post assessments. As explained by Mills (2003) action research is conducted with the goals of gaining insight, developing reflective practice, and improving student outcomes. As I analyzed the data collected during this research, I was able to achieve all three of the goals outlined by Mills. I gained considerable insight concerning my students' understanding of fraction concepts and operations, I refined my reflective habits related to improving my instruction, and my students' showed a great deal of growth in their understanding of fractions.

One reason I believe the instructional practices used in this research study were beneficial to students was because of the frequent use of manipulatives to represent fractions. As Niemi (1996) stated, "There is little evidence to suggest that rote acquisition of any type of knowledge is likely to lead to conceptual understanding; instruction should focus instead on developing understanding of concepts through activity and discourse" (p. 360). During this study, students were frequently involved in activities that allowed them to experiment with concrete manipulatives to enhance their understanding of fraction concepts. Although increased discourse in mathematics lessons was not a focus of this study, I observed both an increase in the amount of student to student discourse and positive effects of such discourse on student learning.

All of the students who participated in this study showed improved understanding of fraction concepts according to their pre and post assessments. Some students who showed only a moderate increase on the assessments made greater gains in conceptual understanding that were not shown on the post assessment. As discussed in chapter four, students' problem solving skills and their abilities to justify solutions may have contributed to lower scores on the post assessment. Analysis of students' journals typically indicated an improved conceptual understanding of fractions. Students were able to represent and identify fractions using the set, area, and linear models of fraction representation. Students were able to add and subtract fractions using all three models. The students who participated in the study also showed an improved ability to justify their solutions with words or pictures.

Students were able to apply some of what they learned in our fraction unit to science activities. At the same time the students were studying fractions in mathematics, they were learning about matter in science. One of the science activities required students to record physical properties of an apple. This was the first time the students were ever asked to use a triple beam balance. A volunteer from the class attempted to read the mass of the apple using the triple beam balance. The apple's mass measured 53.8 grams. The student was easily able to report the 53 grams, but he had to think a little about the eight tenths. While he was studying the triple beam balance, I could see him nodding his head slightly and pointing to the marks that divided one gram. He announced with confidence, "The mass is 53 and eight tenths." I asked him how he determined the mass of the apple. He explained, "Well, there are ten lines between the zero and the one and it is balanced on the eighth line." The class had already participated in activities that divided a length into ten equal pieces when we used the Cuisenaire rods. They were easily able to apply this experience to the use of the triple beam balance and the metric

system of measurement. Once the student explained it in this way, all of the students seemed to understand and every cooperative group during that science experiment was able to accurately read the triple beam balance to the nearest tenth of a gram. This was especially noteworthy because the class had such difficulty with the metric system prior to their experience with different models of fraction representation. The application of fraction concepts when working with the metric system was an unexpected theme that occurred near the conclusion of this study. I was not able to triangulate this data, but it is worthy of further investigation.

It was not possible to determine a preferred model of fraction representation for most of the students who participated in this study. The data indicated a wide variety in preferences from student to student and from activity to activity. The students' choices during independent work often did not support the preferences they verbalized during individual interviews. I found this to be a very interesting topic to investigate. The inconsistency of the data concerning students' preferences supported my belief that exposure to multiple models of fraction representation is beneficial to students. Because there was no one particular model that worked in all situations for all students, students benefited from activities allowing experience with the set, area, and linear models of fraction representation.

Recommendations

Recommendations for further research include studying a larger population of students. Findings in one fifth-grade classroom are not able to be generalized to larger settings. Research on a larger scale concerning the relationship of multiple models of fraction representation and students' conceptual understanding of fractions would be worthwhile.

One obstacle students encountered on the post assessment involved the students' ability to understand which operation to use when solving word problems involving fractions. Further research concerning how multiple models of fraction representations can be used to enhance problem-solving ability would be useful.

This study focused on the conceptual understanding of fractions, equivalent fractions, comparing fractions, and the addition and subtraction of fractions. Further studies are needed to assess the relationship of experiences involving multiple models of fraction representation and the students' abilities to solve problems involving multiplication and division of fractions.

Discussion

This study emphasized the use of manipulatives to model fractions using the linear, set, and area models of fraction representation. Students improved their conceptual understanding of fractions, their abilities to complete fraction operations, and their ability to justify their solutions verbally and in writing. Analyzing the data during this study led me to an additional question. How can activities using manipulatives to represent fractions using multiple models be used to enhance students' problem-solving abilities? Even when the context of a problem clearly indicated a particular model of fraction representation, students did not appear to use the context of the problem to help them choose which model of fraction representation to use. Therefore, has the use of multiple models of fraction representation really helped to improve students' abilities to solve problems or just to perform operations?

As a result of this study, I plan to continue to incorporate activities that involve the set, area, and linear models of fraction representation. I plan to provide more opportunities to discuss

solving fraction problems in context and how to use the context of the problem to help determine which model of fraction representation would be most useful. It is my hope that continued use of multiple models of fraction representations will deepen my students' conceptual understanding of fractions and help them to improve their problem-solving skills.

As stated in this research, activities that involve multiple models of fraction representation, use of manipulatives, and student discourse improve students' conceptual understanding of fractions and their ability to perform operations involving fractions. Improving conceptual understanding in mathematics is a clearly stated goal of the National Council of Teachers of Mathematics (1989). "A conceptual approach enables children to acquire clear and stable concepts by constructing meaning in the context of physical situations and allows mathematical abstractions to emerge from empirical experience" (p. 17). I hope that this research will encourage other teachers of elementary mathematics to consciously evaluate their teaching of fractions and to include experiences in their instruction that allow students to use a variety of models of fraction representation.

APPENDIX A: IRB APPROVAL FORM



November 4, 2004

Lynette Hull
Wekiva Elementary School
1450 East Wekiva Trail
Longwood, FL 32779

Dear Ms.Hull:

With reference to your protocol entitled, "Fraction Models that Promote Understanding for Elementary Students," I am enclosing for your records the approved, expedited document of the UCFIRB Form you had submitted to our office.

Please be advised that this approval is given for one year. Should there be any addendums or administrative changes to the already approved protocol, they must also be submitted to the Board. Changes should not be initiated until written IRB approval is received. Adverse events should be reported to the IRB as they occur. Further, should there be a need to extend this protocol, a renewal form must be submitted for approval at least one month prior to the anniversary date of the most recent approval and is the responsibility of the investigator (UCF).

Should you have any questions, please do not hesitate to call me at 407-823-2901.

Please accept our best wishes for the success of your endeavors.

Cordially,

Barbara Ward

Barbara Ward, CIM
IRB Coordinator

Copies: IRB office

Dr. Juli Dixon, Teaching and Learning Principles, Education, Room 123F, 32816-1250

APPENDIX B: INFORMED CONSENT

January 3, 2005

Dear Parent/Guardian:

I am a student in the graduate program at the University of Central Florida. I am conducting research on students' understanding of fractions under the supervision of faculty member, Dr. Juli Dixon. The purpose of this study is to collect and analyze data about the methods of fraction representation that best promote student understanding of fraction concepts. The results of this study may help teachers better understand how to develop better understanding of fraction concepts in the elementary classroom.

With your permission, your child's understanding of fraction concepts and the methods that your child uses to represent fraction problems will be assessed. Students will participate in a pre and post written assessment. Students will be interviewed individually to discuss their understanding of fractions. The interview will be audio taped to ensure the accuracy of the data collected. The audiotape will be accessible only to the research team for verification purposes. Although the children will be asked to write their names on the survey for matching purposes, their identity will be kept confidential to the extent provided by law. I will replace their names with fictitious names. Results will only be reported in the form of individual and group data. Participation or nonparticipation in this study will not affect the children's grades or placement in any programs. All student data will be kept in a secure place by the researcher and upon completion of the study all audiotapes will be destroyed.

You and your child have the right to withdraw consent for your child's participation at any time without consequence. There are no known risks to the participants. No compensation is offered for participation. Group results of this study will be available in May upon request. If you have any questions about this research project, please contact me at (407) 320-3139 or my faculty supervisor, Dr. Juli Dixon, at (407) 823-4140.

Questions or concerns about research participants' rights may be directed to the UCFIRB office, University of Central Florida Office of Research, Orlando Tech Center, 12443 Research Parkway, Suite 207, Orlando, FL 32826. The hours of operation are 8:00 am until 5:00 pm, Monday through Friday except on University of Central Florida official holidays. The phone number is (407) 823-2901.

Sincerely,

Lynette M. Hull

_____ I have read procedure described above.

_____ I voluntarily give my consent for my child, _____,
to participate in Lynette Hull's research study concerning understanding of fraction concepts and models of fraction representation

Parent/Guardian

Date

_____ I would like to receive a copy of the procedure description.

_____ I would not like to receive a copy of the procedure description.

_____ / _____

2nd Parent/Guardian

Date

(or witness if no 2nd Parent/Guardian)

APPENIX C: STUDENT ASSENT

My name is Mrs. Hull and I am a student in the graduate program at the University of Central Florida. I would like to ask you to complete an assessment about your understanding of fractions. The assessment will take place twice over the next ten weeks during class time. I would also like to ask you some questions about fractions at the end of the ten week unit. I would like to audiotape our conversation to make sure that I record all of the information accurately. Only my instructor at the University of Central Florida and I will listen to the audio tapes. After the study is over, the audio tapes will be destroyed. If you choose to participate, you may withdraw at any time. You will not have to answer any questions you do not want to answer. Your name will not be used when the results are written or discussed. There are no risks and you will not be given anything for your participation. If you decide to participate, I hope that I will learn more about how to help you and other students better understand fractions. Are you interested in participating in these activities?

APPENDIX D: PRE AND POST ASSESSMENT

Fraction Pre and Post Assessment

Name _____

Date _____

1. $\frac{3}{4}$ is a fraction. Explain what it means.

2. Draw a picture that shows $\frac{2}{3}$. Explain why your drawing shows $\frac{2}{3}$.

3. What fraction of the pictures are hearts? Explain how you know.



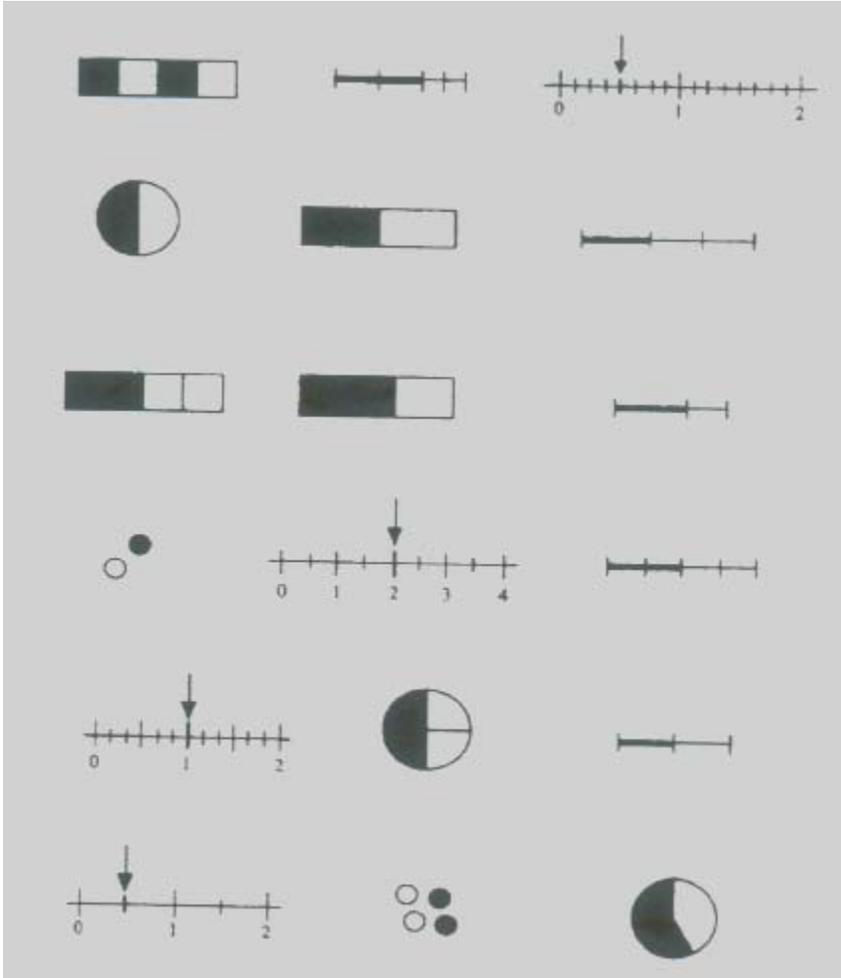
4. What fraction of the rectangle is shaded? Explain how you know.



5. What fraction is shown on the line? Explain how you know.



6. Circle the items below that are the same amount as $\frac{1}{2}$.



7. Write a fraction that is equivalent to $\frac{2}{3}$. Draw a picture to show that your answer is correct.

8. Kelly completed $\frac{1}{6}$ of her homework assignment when she got home from school. She completed $\frac{4}{6}$ of the assignment after swim practice. How much of her assignment has she completed so far? Draw a picture to show that your answer is correct.

9. What is $\frac{1}{3} + \frac{1}{2}$? Draw a picture to show that your answer is correct.
10. What is $1\frac{3}{8} + \frac{2}{8}$? Draw a picture to show that your answer is correct.
11. On the morning after Halloween Nick had $\frac{5}{6}$ of his Halloween candy left. During the first week of November he ate another $\frac{1}{6}$ of the candy he collected. How much of Nick's Halloween candy is left for the second week of November? Draw a picture to show that your answer is correct.
12. What is $\frac{3}{4} - \frac{1}{2}$? Draw a picture to show that your answer is correct.
13. What is $3 - 1\frac{2}{3}$? Draw a picture to show that your answer is correct.
14. Some friends are having a party. They are going to order 3 pizzas. If Adam eats $\frac{1}{4}$ of a pizza, how much pizza is left? Explain your answer with words or a picture.
15. A fifth grade class collected data about hair color. 6 students had brown hair, 4 students had blonde hair, 2 students had red hair, and 5 students had black hair. What fraction of the class has blonde hair? Explain your answer with words or a picture.
16. A worm is trying to crawl across a busy street. He travels $\frac{1}{2}$ of the way across the street before he needs to take a rest. After he rests, he travels another $\frac{1}{3}$ of the distance before he has to stop to let a car pass. What fraction of the street does he still need to cross? Explain your answer with words or a picture.

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