An Experimental Investigation Of Poisson Coordination Games

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Abstract

Poisson games have been proposed to address equilibrium indeterminacy in Coordination games. They model the number of actual players as a Poisson random variable to capture population uncertainty in large games. Two natural questions are (a) whether uncertainty about the number of actual players does have an impact on subjects’ behavior, and if so (b) whether such behavior is consistent with the theoretical prediction of Poisson Coordination games. Investigating these questions is the focus of this paper. We find that uncertainty about the number of actual players may influence subjects’ behavior. Crucially, such behavior is consistent with the theoretical prediction.

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1 Introduction

In many situations in macroeconomics, strategic complementarities arise: individual gains from taking a certain action are non-decreasing in the number of agents who chose the same action. Common Knowledge Coordination games, where “economic fundamentals” (i.e. profitability determinants) and number of agents are assumed to be common knowledge, emphasize that in such environments equilibrium cannot be pinned down uniquely because beliefs are indeterminate. This lack of predictability poses a serious problem for many academics and practitioners when it comes to predicting, for instance, the onset of speculative attacks. Global Coordination games constitute the most popular approach to escape the prediction of equilibrium indeterminacy. They assume instead that agents face idiosyncratic uncertainty about economic fundamentals (see Morris and Shin (1998), Heinemann (2000) and Heinemann and Illing (2002)). Crucially, the prediction about, say, the onset of speculative attacks manifests a threshold level of fundamentals that defines two areas: one in which a successful attack takes place, and another, where a successful attack does not materialize. This has led to a discussion about the relative merits of public information.\footnote{Recent experimental literature finds mixed results on whether subjects’ behavior is consistent with the Global Coordination Game prediction (Heinemann, Nagel, and Ockenfels (2004), Cabrales, Nagel, and Armenter (2007) and Szkup and Trevino (2015)).}

A more recent approach, Poisson Coordination games, is motivated by the fact that the number of potential speculators is by definition very large in macroeconomic environments. Hence, the standard assumption that every player takes every other player’s behavior as given and known when contemplating his/her best response may be violated. In large societies, for instance, it may be prohibitively expensive to collect the necessary information for who all the stakeholders are. Following the suggestion of Myerson (2000), this approach models the number of actual players as a Poisson random variable.\footnote{This modelling choice is driven, in part, by certain convenient properties associated with the Poisson distribution (see Myerson (1998)). As a complementary justification for the latter modelling choice, suppose that the identity of every stakeholder is common knowledge and that binding individual orders for short sales of a currency must arrive with the central bank by a given time. Standard theory suggests that each agent will decide on his/her action by taking the number of orders at the collector’s disposal as given. However, the probability that a phone call to a busy switchboard goes through is...}
prediction, the Poisson Coordination game prediction is that, for any given set of economic fundamentals, no speculative attack will take place as long as the reward from a successful attack, net of the short-selling cost, is sufficiently small (see Section 3 for more details). In the model’s interpretation, creating bigger markets (for instance, by removing trading restrictions and market entry fees and/or improving cooperation between national financial networks) and thereby introducing uncertainty about the number of participants, coupled with a sufficiently high Tobin tax, may be instrumental to ensure financial stability.

In this paper, we conduct an experimental test of a Poisson Coordination game. Specifically, we are concerned with two natural questions: (a) whether uncertainty about the number of actual players does have an impact on subjects’ behavior, and if so (b) whether such behavior is consistent with the prediction of Poisson Coordination games. We thus design a novel experiment to contrast the behavior of subjects in Poisson and Common Knowledge Coordination games (henceforth, for brevity, referred to as Poisson and Common Knowledge games, respectively, unless there is a risk of confusion), and to contrast the behavior of subjects in Poisson games with that predicted by the theory. To the best of our knowledge, we are the first to provide an experimental investigation of Poisson Coordination games. In the context of macroeconomic situations, our setup captures games between players, such as currency or debt speculators, start-up investors and new technology adopters under network externalities. The experimental design is formulated around asking subjects to state their intent to buy a cash amount. Registering to buy the cash amount entails paying a fee, which is less than the cash amount. The fee is non-refundable; that is, once a subject registers to buy the cash amount, the fee is

3Poisson games are not a special case of Global games. As Myerson (1998) discusses both Bayesian games and Poisson games are subsumed by the general class of games with population uncertainty.

4The only other experimental study of Poisson games we know of is that of Ostling, Wang, Chou, and Camerer (2011) who assume Poisson-distributed uncertainty about the number of players participating in the Swedish Lowest Unique Positive Integer (LUPI) game. The behavioral patterns of the field and laboratory data are consistent with the theoretical predictions.

5In the lingo of the speculative attack model of Morris and Shin (1998), registering to buy the cash amount reward is analogous to attacking the currency peg. Alternatively, in the context of investors and technology adopters under network externalities, registering to buy the cash amount is analogous to undertaking the investment opportunity and adopting the new technology, respectively.
subtracted from the subject’s initial endowment. Additionally, registering to buy the cash amount does not imply that the cash amount is awarded. In order to get the cash amount, a threshold number of registrations has to be met. If fewer subjects than the number dictated by the threshold register then the cash amount is not awarded. The experimental sessions are conducted over the Internet. Internet is ideal for Poisson experiments as subjects cannot infer the number of participants, which is typically the case in a laboratory experiment. Crucially, in order to circumvent the difficulties that would arise given the (assumed) unfamiliarity of many subjects with Poisson probabilities, we applied the specific probabilities onto a roulette wheel while noting that the latter is not a standard wheel. In order to maintain consistency with the Poisson experimental sessions, the Common Knowledge sessions were also conducted over the Internet in an analogous setup to the Poisson sessions while accommodating the underlying assumptions of the theory. Once all the relevant information was disclosed, subjects were asked to make a decision whether to buy the cash amount. Our approach resembles how managers and investors commit to their decisions nowadays: after contemplating the pros and cons of various alternatives, managers and investors will often place their (short-selling, purchase or investment) orders online.

Our analysis is comprehensive. We study subjects’ behavior in single shot and in repeated experiments. We find that, in their vast majority, subjects in the Poisson games forego to register to buy the cash amount (i.e. choose the ‘safe’ action). Crucially, such behavior is consistent with the theoretical prediction for the chosen parameters. To compare this behavior with the one in the Common Knowledge games, we focus our attention to the early repeated interactions where experience is limited. We do so because in a changing and volatile world, it is not plausible to expect that agents are faced for a long time with exactly the same strategic environment. We find that subjects’ behavior across the two games is statistically different in each of these early interactions. Robustness checks with smaller and larger sample sizes confirm these results. Therefore, this result suggests that uncertainty regarding the number of actual players may be an important determinant of empirical behavior in volatile macroeconomic environments with strategic complementarities.

The paper adheres to the following plan. We present next the literature review. In Section 3, we review the theoretical predictions of Common Knowledge and Poisson games. In Section 4, the experimental design is presented. In Section 5, we report the results. In Section 6, we conduct a robustness analysis and in Section 7, we discuss
more closely the effect of population uncertainty on subjects’ behavior. Our findings there reinforce that population uncertainty as modelled by Poison games is a good predictor of behavior in the lab. Finally, in Section 8, we offer suggestions for future research.

2 Literature Review

An important issue that arises in environments with strategic complementarities is whether beliefs about equilibrium outcomes can be pinned down uniquely. Most of the received theoretical literature has focused on the interaction of heterogeneity in beliefs or preferences/technologies and uncertainty about economic fundamentals to study uniqueness of equilibrium. The ensuing common view is that in order to escape a prediction of indeterminacy of equilibria a model needs to have a sufficiently large degree of heterogeneity and/or of asymmetric information. In particular, Global games postulate that agents face idiosyncratic uncertainty about economic fundamentals. The predicted equilibrium is in threshold strategies that prescribe the safe action (e.g. do not speculate) if and only if the idiosyncratic signal about the unknown state of the economy is a sufficiently strong indication that profitability is low.

Heinemann, Nagel, and Ockenfels (2004) (henceforth referred to as HNO) study an experiment that resembles the speculative attack model of Morris and Shin (1998) with repeated play. In comparing sessions between Common Knowledge and Global Coordination games, they find that subjects use threshold strategies in both informational protocols. In the Common Knowledge games, the authors find that observed behavior lies between the payoff dominant equilibrium and the Global game solution. In the Global games, they find that observed behavior is closer to the Global game solution. In their setup, the relevant economic fundamental is the profit from short selling the currency, which is drawn anew at the start of each repeated interaction.

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7The context of a subject’s decision differs in our setup compared to the one in HNO. In our setup, a subject has to sacrifice an amount of money from the initial endowment (pay a non-refundable fee) to buy the cash amount. Otherwise, a subject gets to keep the endowed amount. In the study of HNO, subjects are required to decide between the safe and the risky action; however, the risky
Cabrales, Nagel, and Armenter (2007) (henceforth referred to as CNA) study an experiment that resembles the $2 \times 2$ setup of Carlsson and van Damme (1993). Analogous to HNO, CNA also investigate subjects’ behavior in Common Knowledge and Global Coordination games, but distinguish between short-term and long-term play. The authors utilize a discrete state space with five possible states and signals to make the theoretic reasoning simpler. CNA find that in the Global games with long-term play, subjects’ behavior converges towards the Global game solution, which coincides on average with the risk dominant equilibrium. The authors also find that in the Common Knowledge games with short-term play, observed behavior of subjects can be anywhere (weakly) between the payoff dominant equilibrium and the Global game solution. Moreover, CNA establish that subjects’ behavior across the Common Knowledge and Global games with short-term play is similar.

Szkup and Trevino (2015) implement a different informational structure than the two previous studies. Specifically, the authors develop a two-stage model, where each agent, in the first stage, has the possibility to choose, at a cost, the precision of their private signal, and, in the second stage, play the Coordination game, as in Morris and Shin (1998), using the information acquired in the first stage. Szkup and Trevino (2015) prove existence of a unique equilibrium in their model. However, contrary to the theoretical predictions, they find that as subjects choose more precise information they coordinate more often on attacking the currency, and as a result attacks are more successful when agents hold more precise information.

Crucially, the aforementioned literature has not paid particular attention to the implications of the fact that in the above strategic environments, the number of economic agents is often very large. As Myerson (2000) points out, in games with a very large number of players, “it is unrealistic to assume that every player knows all the other players in the game; instead, a more realistic model should admit some uncertainty about the number of players in the game” (p. 7). Following this suggestion, Makris (2008) models the coordination problem as a Poisson game, where it is common knowledge that the number of actual players is a Poisson random variable, and shows that the equilibrium is unique if the transaction cost (fees) relative to the gain from coordination to the risky action is above a well-defined threshold — the unique equilibrium prescribes that all players take the safe action.

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action does not take away any money from their total earnings.
3 Theoretical Predictions

We deploy the canonical Coordination game used in Morris and Shin (1998) (with different notation). Denote by $N > 1$ the number of players, who decide whether to register to buy a cash award (i.e., attack a currency). Denote by $T$ the registration fee (opportunity cost), $Y$ the state of the economy/economic fundamentals, and $Y/2$ the cash award gross of the fee with $Y \in [Y_{\text{min}}, Y_{\text{max}}]$.\(^8\)

The cash amount is awarded if the number of registered players is at least as high as $\alpha(Y)$. Therefore, after letting $\nu$ be the number of other players who register, the payoff of each player is

\[
\begin{align*}
0 & \quad \text{if he does not register,} \\
-T & \quad \text{if he registers and } \nu < \alpha(Y) - 1, \\
Y/2 - T & \quad \text{if he registers and } \nu \geq \alpha(Y) - 1.
\end{align*}
\]

The function $\alpha(.)$ and the registration fee are common knowledge. The minimum number of registrations required for the cash amount to be awarded is set as

\[
\alpha(Y) = C - \frac{Y}{D}
\]

with

\[
C > 0, D > 0 \text{ and}
\]

\[
C - \frac{Y_{\text{max}}}{D} \leq 1.
\]

The last condition states that in the worst economic fundamentals ($Y = Y_{\text{max}}$), the cash amount is awarded even if only one player registers.\(^9\)

Note that for $Y \geq \bar{Y} \equiv \alpha^{-1}(1)$, a single registration is enough for the cash amount to be awarded, while for $Y < \bar{Y}$ more than one registrations will typically be needed.

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8 To map the notation here to that in Morris and Shin (1998) and Heinemann (2000), the interested reader should just use $Y = Y_{\text{max}} - (Y_{\text{max}} - Y_{\text{min}})\theta$, where $\theta \in [0, 1]$ is the state of the economy in these papers. Moreover, $Y_{\text{max}}/2$ is the capital gain from short selling in the worst state of the economy ($\theta = 0$), while $Y_{\text{min}}/2$ is the short-selling reward in the best state of the economy ($\theta = 1$).

9 Here, to fix ideas, a higher cash award corresponds to worse economic fundamentals. This relationship pertains to the example of a speculative attack. For the case of innovation, the converse relationship should be used; that is, a higher cash award would correspond to better economic fundamentals.
We assume that

\[ 2T < \bar{Y} \]

to ensure that it is not weakly dominant to abstain from registering for any cash amount when \( Y < \bar{Y} \). Let \( \bar{Y} \) denote the supremum of all levels of economic fundamentals for which it is not (weakly) profitable to register given that all other \( N - 1 \) players register. That is, \( \bar{Y} \) is the largest of the economic fundamentals \( 2T \) and \( \alpha^{-1}(N) \). The significance of this state of economic fundamentals is that it is dominant to abstain from registering for any state \( Y < \bar{Y} \). This range of fundamentals is non-empty if the best state of economic fundamentals (\( Y = Y_{\text{min}} \)) is smaller than \( \bar{Y} \). This is ensured by our next assumption. In the best state of economic fundamentals, the cash amount awarded is smaller than the fee; that is,

\[ 2T > Y_{\text{min}}. \]

Under common knowledge of economic fundamentals and number of players (i.e. in the Common Knowledge game), zero registrations (the maximin outcome) is the unique equilibrium outcome for \( Y < \bar{Y} \).\(^{10}\) Furthermore, \( N \) registrations (the payoff dominant outcome) is the unique equilibrium for \( Y \geq \bar{Y} \). However, in the “grey area” (i.e. in the remaining area of economic fundamentals) there is multiplicity of equilibria. Depending on self-fulfilling beliefs both the maximin and payoff dominant outcomes (zero and \( N \) registrations, respectively) are equilibria.

In the Poisson game, the cash amount is again common knowledge. However, the number of actual players is a Poisson random variable with the mean \( n \) being common knowledge. Introducing uncertainty about the group size changes the strategic environment in a fundamental way. In particular, it creates conceptual problems in analyzing the resulting game as a Bayesian game. The reason is that introducing population uncertainty implies that players can no longer assign a strategy to other individual players, simply because they are not aware of who they all are.\(^{11}\) This is tackled in Myerson (1998) by laying the foundations for the study of games with

\(^{10}\) If \( Y = 2T \), then contributing when \( Y = \bar{Y} \) is never profitable because even if enough contributions are made so that the cash amount is awarded, the latter just covers the registration fee. Therefore, it is weakly dominant to not contribute when \( Y = \bar{Y} = 2T \).

\(^{11}\) A direct implication of this is that the insights of Global games cannot be used directly to our framework, in contrast to what one might have thought by hastily extrapolating the insights of Global games to a setup where the number of players is treated as yet another uncertain fundamental of the game in hand.
population uncertainty. As Myerson (1998) puts it “... going to a model of population uncertainty requires us to specify a probability distribution over actions for each type of player ... rather than for each individual player...” In essence, each player resolves his decision by formulating ‘beliefs’ over the likely behavior of the typical player in the game and then by calculating which action maximizes his expected utility given the resulting expected action profile. A crucial building block of the analysis of such problem is thus the beliefs of a player who found himself in the game about the number of other players in the game. Being an actual player is a new piece of information that might affect the player’s beliefs about the actual size of the game. On one hand, he might believe that the number of other players is Poisson-distributed with a mean of \( n - 1 \) (since being in the game has lowered the mean of the number of remaining players). On the other hand, the fact that he is an actual player is a clue that the number of players is large — not small. Under the Poisson distribution, the two effects exactly cancel out. According to this “environmental equivalence” property, shown in Myerson (1998), under the Poisson distribution, the mean number of players in the game from the point of view of an outsider (or a potential player) is equal to the mean number of other players in the game from the point of view of a player who has found himself in the game.

Utilizing these foundations, we can show, by following the steps in Makris (2008), that the predictions for economic fundamentals such that \( Y < 2T \) or \( Y \geq \bar{Y} \) coincide with the corresponding predictions of Common Knowledge games. However, for economic fundamentals within the remaining area, the unique equilibrium is the maximin outcome (where no player registers) if and only if

\[
1 - F([\alpha(Y)] - 2 | n) < 2T/Y, \tag{1}
\]

where \( F(\cdot | n) \) is the Poisson cumulative distribution function with parameter \( n \), and the symbolic function \([\cdot]\) rounds-up the fraction to the nearest integer from above. To understand this condition note that an expected utility maximizer will not take a bet if the ratio of the price to the payoff under a win is higher than the probability of winning the bet. Condition (1) is such a condition after identifying the price with the fee, the payoff under a win with the cash amount reward, and the probability of

12The private knowledge of being in the game can therefore be thought of as the counterpart of the idiosyncratic signal about the uncertain economic fundamentals received by a player in a Global game, where the number of players is common knowledge.
winning the bet with the left-hand side of the condition. When the above inequality does not hold then multiplicity of equilibria is predicted instead.

In the experiments, we will choose parameters such that the theoretical prediction prescribes that there is equilibrium indeterminacy in the Common Knowledge games, whereas in the Poisson games, the theoretical prediction prescribes that all players do not register. To justify the latter choice notice that our aim in this paper is to investigate environments where Poisson games predict uniqueness of equilibrium. Lastly, we emphasize one more important feature of our experimental design. Trying to give the worst chance to Poisson games in the lab, we chose parameters to barely satisfy condition (1).

4Experimental Design

Our experimental setup features a coordination problem that is examined under two informational protocols: Poisson games and Common Knowledge games. The experiments were conducted over the Internet. Internet is ideal for Poisson experiments as subjects cannot infer the number of participants, which is typically the case in a laboratory experiment. To maintain consistency with the Poisson treatments, the treatments based on Common Knowledge games were also conducted over the Internet. A disadvantage of running experiments over the Internet is that it becomes very hard to monitor participants’ engagement with the game. In particular, there is no control over what participants are doing. For instance, participants could take a break to call someone, to browse the web, to eat pizza, to have a coffee etc. To safeguard against such distractions and to maintain subjects’ focus to the game, the screens included timers that allowed a limited, but sufficient amount of time to read comfortably the instructions. In addition, the inclusion of timers minimized the possibility of wired or wireless communication. Once the time lapsed, the subjects would concurrently move to the next screen. In the questionnaire that followed the game play stage, none of the subjects reported running out of time while reading the instructions on any of the screens. Next, we provide a detailed description of the experimental design, and subsequently formulate our general hypotheses.
4.1 Treatments

Upon logging in, subjects were endowed with £12 in the single shot experiments, and £15 in the repeated experiments, in lieu of a show-up fee (we justify the reason for the difference in endowments at the very end of this subsection). Subjects were then provided with the instructions. The instructions accommodated the underlying assumptions of the corresponding theories. Right after the delivery of the instructions, subjects were asked to make a decision whether to buy the cash amount. A value added of this approach is that it mimics how managers and investors commit to their decisions nowadays: after contemplating the pros and cons of various alternatives, managers and investors will often place their (short-selling, purchase or investment) orders online. Finally, subjects were asked to complete a short questionnaire consisting of demographic questions. With the conclusion of the experimental session, subjects claimed their earnings from the school office of Social Sciences at the University of Southampton.

First, we describe the Poisson treatments. In the first stage of the experiment, subjects were instructed that there would be a computer draw and that the number drawn would correspond to the number of subjects participating in the second stage of the experiment. Subjects were explicitly told that the number drawn would not be revealed to them. The Poisson process was based on $n = 17$. To circumvent the difficulties that would arise given the (assumed) unfamiliarity of many subjects with Poisson probabilities, we applied the specific probabilities onto a roulette wheel (see Figure 1). We showed the roulette wheel pictorially and noted the following.

You can see that the roulette is not a standard roulette; the number drawn can be any number between 8 and 26, but not all numbers are equally likely to be drawn.

Numbers closer to 17 (the mean) are more likely to be drawn.

The instructions specified that subjects not selected to participate in the second stage of the experiment would be dismissed, but would keep their initial endowment.

In the second stage, subjects had the option to buy a cash amount of £12.50 at a fee of £9 (£10). Subjects were informed that the cash amount of £12.50 would be issued only if a minimum of 16 (15) subjects registered to buy it, and that the fee of £9 (£10) required for the purchase of the cash amount was non-refundable.

\footnote{In each of the Poisson sessions, we sent log in information to 26 subjects. The total number of participants in each Poisson treatment is shown in Table 1.}
Figure 1: **Roulette Wheel in the Poisson Treatments for n = 17**

Notes: We circumvented the difficulties that would arise given the (assumed) unfamiliarity of many subjects with Poisson probabilities by applying the specific probabilities onto a roulette wheel.

and collected immediately. That is, if a subject registered to buy the cash amount of £12.50, the £9 (£10) would be subtracted automatically from the initial endowment regardless of the number of subjects registering. The subjects were then asked to indicate whether they would like to register to buy the cash amount.

In the Common Knowledge treatments, subjects were told the number of participants (i.e. 17), the cash amount (i.e. £12.50), the fee (i.e. £9 or £10) and the threshold number of registrations (i.e. 16 or 15) that needed to be met to earn the cash amount. The subjects were then asked to make a decision, analogous to the Poisson treatments.

In the repeated experiments, subjects in the Poisson and Common Knowledge games were informed that the play would be repeated for 20 periods. Additionally, in the Poisson games, subjects were told that in each period there would be a new draw of the number of active players. At the end of each period, subjects in both game types were provided feedback on the period’s game play. The only difference in

\[15] This parallels the design of HNO in Global games where in every period there is a new draw of the economic fundamentals over which there is uncertainty.
terms of the feedback received was that the feedback in the Poisson games pertained
to the decisions of the active players, whereas in the Common Knowledge games,
the feedback pertained to all participants. Specifically, the feedback consisted of
(i) the (active) player’s decision in the period, (ii) the number of (active) players
in the period, (iii) the number of (active) players who chose to register to buy the
cash amount, and (iv) the number of (active) players who chose not to register to
buy the cash amount. To maintain consistency with the payments in the single
shot experiments, we informed subjects that at the end of the experimental session,
there would be a computer draw where one period (common to all participants in
the session) would be selected for payment. Thus, a participant’s payoff would be
determined based on his/her decision in the drawn period.\footnotemark[16] Otherwise, the design
was analogous to the ones in the corresponding single shot experiments.

The experimental sessions took place in October of 2012 and May of 2013. We
conducted two sessions per treatment. The 231 subjects were recruited from the
undergraduate student population of the University of Southampton. We announced
our experiments via class presentations. In order to participate, students replied by e-
mail. We then indicated to the respondents the date and time of the experiment, and
asked them to confirm their attendance. Those who confirmed were subsequently sent
log in information (username and password) and the url of the website. Most of the
participants majored in business, economics, finance, and mathematics. Participants
were allowed to participate in only one session. In the single shot experiments, average
earnings per participant were £9.72. Specifically, in the Common Knowledge games,
subjects made on average £7.51, whereas in the Poisson games, subjects made on
average £11.51. In the repeated experiments, average earnings per participant were
£13.97; in the Common Knowledge games, subjects made on average £12.62 and
in the Poisson games, subjects made £15.\footnotemark[17] The experimental instructions for all

\footnotetext[16]{The underlying idea of the random lottery incentive scheme is that subjects make a number of
decisions knowing that at the end of the experimental session one of these decisions will
be selected for payment. There is a vast literature testing the validity of this payment scheme. \textit{Laury}\n(2012) finds that subjects do not scale down decisions when they are only being paid for a subset of
these decisions. In addition, \textit{Cubitt, Starmer, and Sugden} (1998) find no evidence that such design
contaminates elicited preferences. \textit{Hey and Lee} (2005) show that subjects separate the various
questions and respond to each question individually and in isolation from the rest; thus, incentives
are retained. A value added of this approach is that it neutralizes the income effect that would
otherwise be experienced as subjects progress through the periods.}

\footnotetext[17]{Recall that in the repeated experiments, one period was selected for payment. Subjects in
the repeated Poisson experiments that were not selected to participate in the second stage of the}
Table 1: Characteristics of the Experimental Sessions

Common Knowledge Games

<table>
<thead>
<tr>
<th># of Subj.</th>
<th># of Ses.</th>
<th>Mean</th>
<th>Threshold</th>
<th>Fee (£)</th>
<th>Amount (£)</th>
<th>Acronym</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>2</td>
<td>-</td>
<td>16</td>
<td>9</td>
<td>12.50</td>
<td>CK169</td>
</tr>
<tr>
<td>34</td>
<td>2</td>
<td>-</td>
<td>15</td>
<td>10</td>
<td>12.50</td>
<td>CK1510</td>
</tr>
<tr>
<td>34</td>
<td>2</td>
<td>-</td>
<td>16</td>
<td>9</td>
<td>12.50</td>
<td>CK169R</td>
</tr>
</tbody>
</table>

Poisson Games

<table>
<thead>
<tr>
<th># of Subj.</th>
<th># of Ses.</th>
<th>Mean</th>
<th>Threshold</th>
<th>Fee (£)</th>
<th>Amount (£)</th>
<th>Acronym</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
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<td>17</td>
<td>16</td>
<td>9</td>
<td>12.50</td>
<td>P169</td>
</tr>
<tr>
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<td>17</td>
<td>15</td>
<td>10</td>
<td>12.50</td>
<td>P1510</td>
</tr>
<tr>
<td>45</td>
<td>2</td>
<td>17</td>
<td>16</td>
<td>9</td>
<td>12.50</td>
<td>P169R</td>
</tr>
</tbody>
</table>

Notes: In the first column, we provide the total number of participants in each treatment. We conducted two sessions per treatment. The number of participants in the Common Knowledge sessions was common knowledge. Notice that the number of participants in each session in the Common Knowledge treatments coincides with the mean $n$ of the Poisson treatments. Moreover, the cash amount is the same in the two game types. The acronyms in the last column consist of the game type (CK for Common Knowledge games or $P$ for Poisson games), the threshold (15 or 16), the fee (9 or 10) and letter “R” for repeated interaction.

At this point and before proceeding to the general hypotheses, we feel compelled to justify our choices with respect to the cash amount (£12.50), (expected) number of players (17), the threshold number of players (15 or 16), the fee (£9 or £10) and the initial endowment (£12 or £15). For design reasons that are related to further experiments we report in Subsection 7.2, we had to conduct a draw for the level of $Y$ in advance of all the experiments run. The details about the generating distribution are in that subsection. The drawn $Y$ was 25. In HNO, $Y$ is identified with the cash experiment in the drawn period were paid their initial endowment and all active players chose to forego registering to buy the cash amount. Thus, all participants in the repeated Poisson experiments earned £15.
amount. However, a cash amount of £25 seems unreasonably high given the short duration of the experiment. Instead, we decided to offer a cash amount of £\frac{Y}{2}$ (i.e. £12.50). Moreover, the number of players had to be large enough to capture the “largeness” of the games while being cost effective. This motivates our choice of the number of players. To ensure comparability across game types, the population mean of the Poisson distribution used in the Poisson games had to also be equal to the number of players in the Common Knowledge games.

The threshold and the fee were chosen next. Presumably, a lower fee and a lower threshold would make subjects more willing to register to buy the cash amount. However, ensuring equilibrium uniqueness in the Poison games implies that we cannot choose low values for both the fee and the threshold number (recall condition (1)). In addition, the threshold number of registrations should not exceed the number of players in the Common Knowledge games (otherwise, subjects would have a dominant strategy to not register). Our chosen parameters struck a balance when faced with a tradeoff between low fees and high threshold numbers at the design stage. To see this, observe the Poisson Cumulative Distribution Table (included in the Appendix) for $n = 17$. Consider first, the lowest fee required for the threshold number of registrations to be equal to the mean number of the Poisson distribution while satisfying condition (1). Looking at the table and applying condition (1) this fee is £7.86. Consider next decreasing the threshold number of registrations by 1 and 2 subjects. The corresponding lowest fee such that condition (1) is satisfied is £9 and £10, respectively. Having a threshold level which is (at least) equal to the mean number of the Poisson distribution could make subjects perceive it as less likely that the cash amount will be awarded. This in turn could make subjects unwilling to register. To make it harder for the theoretical prediction of the Poisson games to be confirmed by subjects’ behavior, we showed preference towards increasing the fee by merely £1.14 and £2.14 in order to decrease the threshold number of registrations by 1 and 2 subjects, respectively. Similarly, lower threshold numbers, such as 11 would imply a fee significantly close to the cash amount. For example, the fee required for a threshold number of registrations of 11 is £12.18. It is therefore highly doubtful that a subject would risk losing the fee of £12.18 to earn the cash amount of £12.50. Instead, we chose to set the fee at £9 and £10 and thereby to set a threshold number of registrations of 16 and 15, respectively to ensure condition (1), while also ensuring that (a) the fee is not very close to the awarded cash amount, and (b) the threshold
number of registrations is less than the mean number of the Poisson distribution.

Finally, considering the duration of the experiments (single shot experiments lasted approximately 20 minutes, whereas repeated experiments lasted approximately 55 minutes) and the minimum wage in UK (≈ £6 per hour), we stipulated that no subject should get a compensation below £2 in the single shot experiments and £6 in the repeated experiments. Therefore, the difference between the highest fee and the endowment should not be less than £2 in the single shot experiments and £6 in the repeated experiments, which led us to provide subjects with an initial endowment of £12 in the single shot experiments, and £15 in the repeated experiments.

4.2 General Hypotheses

We formulate next three hypotheses. The first hypothesis examines the behavioral differences across the two informational conditions. This is important in order to understand whether uncertainty as modelled in a Poisson game influences strategic behavior in macroeconomic environments with strategic complementarities.

**Hypothesis 1** Subjects’ behavior is statistically similar across the Common Knowledge and Poisson games when controlling for the parameter choices of each pairwise comparison.

Finally, the last two hypotheses serve as a direct test of the predictions of the Poisson and Common Knowledge games. Recall that on one hand, the Poisson games for the parameters specified, predict that subjects will forego the opportunity to register to buy the cash amount and will keep their endowment. On the other hand, the Common Knowledge games establish that based on our parameter choices, subjects will either all coordinate on registering to buy the cash amount or all coordinate on foregoing to register to buy the cash amount. The last two hypotheses are formulated as follows.

**Hypothesis 2** Subjects in the Poisson games will choose to forego registering to buy the cash amount in accordance with the prediction of the Poisson games for the parameters specified.

**Hypothesis 3** Subjects in the Common Knowledge games will either all coordinate on registering to buy the cash amount or all coordinate on foregoing to register to
buy the cash amount in accordance with the prediction of the Common Knowledge games for the parameters specified.

5 Results

All hypotheses are formally tested through pairwise $\chi^2$-tests, where the $H_0$ states that behavior across treatments is not statistically different. Each hypothesis is matched with the corresponding result; that is, result $i$ is a report on the test of hypothesis $i$. Note that the decision of a subject in the game is a binary variable. The subjects who chose not to register to buy the cash amount were assigned a value of 1. The subjects who chose to register were assigned a value of 0. We start off with the study of subjects’ behavior in single shot games. We then investigate the behavior of subjects in repeated games.

5.1 Single Shot Experiments

Table 2 reports descriptive statistics on the raw data. Recall that subjects had to decide whether to register to buy a cash amount at a fee or forego this option and keep the endowment of £12. In the table, we display the frequency and percentage of subjects who registered to buy the cash amount, and the frequency and percentage of subjects who chose to keep their endowment. The summary statistics are classified by treatment. With the exception of CK1510, in all other treatments, the subjects who chose not to register outnumbered the ones that chose to register. In the Common Knowledge treatments, the percentages of those who kept the endowment of £12 was 52.9% in CK169 and 47.1% in CK1510. In sharp contrast, the percentages in the Poisson treatments are substantially higher (95.0% in P169 and 95.5% in P1510). Overall, out of 152 subjects, 38 chose to register to buy the cash amount and 114 subjects chose to keep the endowment of £12. The threshold was not met in any of the treatments; consequently, the cash amount was not awarded.

Next, we investigate whether subjects’ decisions varied significantly across the Common Knowledge and Poisson games. The results are displayed in Table 3 and formalized in our first result.
### Table 2: Descriptive Statistics

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Registered Freq.</th>
<th>Registered %</th>
<th>Not Registered Freq.</th>
<th>Not Registered %</th>
<th>Amount Awarded?</th>
</tr>
</thead>
<tbody>
<tr>
<td>CK169</td>
<td>16</td>
<td>47.1</td>
<td>18</td>
<td>52.9</td>
<td>No</td>
</tr>
<tr>
<td>CK1510</td>
<td>18</td>
<td>52.9</td>
<td>16</td>
<td>47.1</td>
<td>No</td>
</tr>
</tbody>
</table>

#### Poisson Games

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Registered Freq.</th>
<th>Registered %</th>
<th>Not Registered Freq.</th>
<th>Not Registered %</th>
<th>Amount Awarded?</th>
</tr>
</thead>
<tbody>
<tr>
<td>P169</td>
<td>2</td>
<td>5.0</td>
<td>38</td>
<td>95.0</td>
<td>No</td>
</tr>
<tr>
<td>P1510</td>
<td>2</td>
<td>4.6</td>
<td>42</td>
<td>95.5</td>
<td>No</td>
</tr>
</tbody>
</table>

**Notes:** The table indicates the number of subjects who registered, and the number of those who did not register to buy the cash amount in each treatment. In addition, we provide the corresponding percentages. The threshold was not met in any of the treatments. The acronyms consist of the game type (CK for Common Knowledge games or P for Poisson games), the threshold (15 or 16) and the fee (9 or 10).

**Result 1** Subjects’ behavior differs significantly between the Common Knowledge and Poisson games when controlling for the parameter choices of each pairwise comparison.

### Table 3: Differences in Subjects’ Behavior Across Game Types

<table>
<thead>
<tr>
<th>Alternative hypothesis:</th>
<th>$\text{decision}_i \neq \text{decision}_j$</th>
<th>$\chi^2$-test p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Knowledge games vs Poisson games</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Notes:** We utilize the $\chi^2$-test to determine whether subjects’ decisions differ across the two game types ($i \neq j$) conditional on the same parameters. The acronyms consist of the type of treatment (CK for Common Knowledge games or P for Poisson games), the threshold (15 or 16) and the fee (9 or 10).

To investigate the consistency of subjects’ behavior with the theoretical predictions, the distribution of each treatment is compared to the predicted distribution of
the corresponding theory. Panel A, in Table 4, indicates the \( p \)-values of the treatments under the \( H_0 \) that the observed distribution and the distribution where all subjects choose to not register are statistically similar. The \( p \)-values in Panel B are based on the assumption that the observed distribution and the distribution where all subjects register to buy the cash amount are statistically similar.

Table 4: Theory and Subjects’ Behavior

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Panel B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Alternative hypothesis:</strong> ( \text{decision}_i \neq 1 )</td>
<td><strong>Alternative hypothesis:</strong> ( \text{decision}_i \neq 0 )</td>
</tr>
<tr>
<td><strong>( p )-values</strong></td>
<td><strong>( p )-values</strong></td>
</tr>
<tr>
<td><em>Common Knowledge games</em></td>
<td><em>Common Knowledge games</em></td>
</tr>
<tr>
<td>CK169</td>
<td>CK169</td>
</tr>
<tr>
<td>CK1510</td>
<td>CK1510</td>
</tr>
<tr>
<td><em>Poisson games</em></td>
<td><em>Poisson games</em></td>
</tr>
<tr>
<td>P169</td>
<td>P169</td>
</tr>
<tr>
<td>P1510</td>
<td>P1510</td>
</tr>
</tbody>
</table>

Notes: The decision of a subject in the game is a binary variable. The subjects who chose not to register to buy the cash amount were assigned a value of 1; otherwise, were assigned a value of 0. We utilize the \( \chi^2 \)-test to determine whether subjects’ decisions in Common Knowledge and Poisson games differ from the theoretical predictions for the parameters specified. Panel A indicates the \( p \)-values in the assumption that the observed distribution and the distribution where all subjects choose to not register are statistically similar. The \( p \)-values in Panel B are based on the assumption that the observed distribution and the distribution where all subjects register to buy the cash amount are statistically similar. The acronyms consist of the type of treatment (\( CK \) for Common Knowledge games or \( P \) for Poisson games), the threshold (15 or 16) and the fee (9 or 10).

The second hypothesis was formulated to test the consistency of subjects’ behavior with the prediction of the Poisson games. The results in Panel A present serious evidence of such consistency for the parameters specified. Given that our sample size is large enough, we also run a probit regression where the dependent variable is a subject’s decision and the four treatments are the covariates with CK169 set as the base. Acknowledging that coefficients in probit models are estimated up to scale and cannot be directly interpreted, we only present marginal effects in Table 5. The standard errors are reported in parentheses. Crucially, the coefficients are statistically significant only in the Poisson games. The marginal effects imply an
increase in probability of 42.1% (P169) and 42.5% (P1510) in not registering to buy the cash amount in the Poisson treatments. We formalize next our second result.

Table 5: MARGINAL EFFECTS

<table>
<thead>
<tr>
<th>Regressor</th>
<th>$dy/dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CK1510</td>
<td>-0.059</td>
</tr>
<tr>
<td></td>
<td>(0.121)</td>
</tr>
<tr>
<td>P169</td>
<td>0.421***</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
</tr>
<tr>
<td>P1510</td>
<td>0.425***</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
</tr>
</tbody>
</table>

Notes: We report marginal effects after a probit regression on decision. CK169 is set as the base against which the estimated parameters are compared. $dy/dx$ for factor levels is the discrete change from the base level. All standard errors are reported in parentheses. The acronyms consist of the game type (CK for Common Knowledge games or P for Poisson games), the threshold (15 or 16) and the fee (9 or 10). *** Significant at the 1% level.

**Result 2** Subjects’ behavior in the Poisson games is consistent with the prediction of the Poisson games for the parameters specified.

The third hypothesis aims to examine whether all subjects coordinate on foregoing registering to buy the cash amount or all coordinate on registering to buy the cash amount in the Common Knowledge treatments. On one hand, the $p$-values of the Common Knowledge treatments in Panel A of Table 4 serve to determine whether subjects coordinate on foregoing registering to buy the cash amount. On the other hand, the $p$-values of the corresponding Common Knowledge treatments in Panel B serve to determine whether subjects coordinate on registering to buy the cash amount. Our third result formalizes our findings.

**Result 3** Subjects’ behavior in the Common Knowledge games differs from the predictions of the Common Knowledge games for the parameters specified.
5.2 Repeated Experiments

In real life, for many applications of Coordination games, there are ample (personal or social) learning opportunities. Therefore, we study here the impact of repeated interactions on subjects’ behavior in the Poisson games, and contrast it to the behavior in the Common Knowledge games. Recall that we ran two sessions in the Common Knowledge game with $N = 17$, and two sessions in the Poisson game with $n = 17$ as the mean of the Poisson distribution (denoted as CK169R and P169R in Table 1). The other two parameters were identical across the two informational protocols (i.e. $a(Y) = 16$ and $T = 9$).

In Figure 2, we display the proportion of subjects who did not register to buy the cash amount over the 20-period span in the treatments with repeated interaction. Based on the earlier results in the single shot Poisson games (around 95% of the subjects chose not to register to buy the cash amount), we hypothesized that a very high percentage in the first period would be a significant deterrent to register in the next period (recall feedback is provided in each period) and so on and so forth. A proportion quite close to 1 is thus expected throughout the 20-period play. As shown in Figure 2, this prediction is confirmed. Through the first five periods in the Poisson games, the proportion of active subjects that chose not to register is over 90%. From the sixth period onwards, all active players chose not to register to buy the cash amount. It is important to reiterate that such behavior is also consistent with the theoretical prediction of the Poisson games. In the Common Knowledge games, convergence to a proportion of 1 is slower. This is in line with earlier results documented in Brandts and Cooper (2006), Brandts, Cooper, and Fatas (2007), Brandts and Cooper (2007), and Cooper, Ioannou, and Qi (2014), where under fixed economic fundamentals and hence a stable strategic environment, some subjects in the Common Knowledge games do not choose the safe action in the early interactions; albeit, all choose the safe action after sufficiently many repeated interactions. Specifically, here, only from the eleventh period onwards the proportion of subjects who chose not to register equals 1. Before convergence, the proportion fluctuates between 0.53 and 0.85. The statistical analysis confirms that subjects’ behavior in the earlier periods (i.e. before the eleventh period) is significantly different across the Poisson and Common Knowledge games in each period. Indicatively, the $p$-values are 0.001, 0.006 and 0.087 in the first, fifth and tenth period, respectively.
Figure 2: Subjects’ Behavior with Repeated Interaction

Notes: The figure displays the proportion of subjects who did not register to buy the cash amount over the 20-period span in the treatments with repeated interaction. The acronyms consist of the game type (CK for Common Knowledge games or P for Poisson games), the threshold (16), the fee (9) and letter “R” for repeated interaction.

6 Robustness Analysis

Contrasting the behavior in the Common Knowledge and Poisson games, we found above that subjects’ behavior across the two game types is statistically different in the early periods. This result suggest that uncertainty regarding the number of actual players may be an important determinant of subjects’ behavior when the strategic environment does not stay the same for long periods. Specifically, in the Common Knowledge games, we found that, in the early periods, subjects split almost evenly between foregoing registering to buy the cash amount and registering to buy the cash amount. However, in the Poisson games, subjects forewent to register to buy the
cash amount, which is in fact consistent with the theoretical prediction of the Poisson games. Both results are important findings that deserve further scrutiny. We thus conduct a number of robustness controls with smaller and larger sample sizes. As mentioned earlier, in a volatile world, it is not plausible to expect that agents are faced for a long time with the exact same strategic environment. Therefore, we focus on single shot experiments, where experience is limited. The characteristics of the robustness sessions are displayed in Table 6, and their corresponding experimental instructions are included in the Appendix.

### Table 6: Characteristics of Robustness Sessions

<table>
<thead>
<tr>
<th>Common Knowledge Games</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td># of Subj.</td>
<td># of Ses.</td>
<td>Mean</td>
<td>Threshold</td>
<td>Fee (£)</td>
<td>Amount (£)</td>
<td>Acronym</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>-</td>
<td>4</td>
<td>10</td>
<td>12.50</td>
<td>CK410</td>
</tr>
<tr>
<td>38</td>
<td>2</td>
<td>-</td>
<td>18</td>
<td>9</td>
<td>12.50</td>
<td>CK189</td>
</tr>
<tr>
<td>38</td>
<td>2</td>
<td>-</td>
<td>17</td>
<td>10</td>
<td>12.50</td>
<td>CK1710</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Poisson Games</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td># of Subj.</td>
<td># of Ses.</td>
<td>Mean</td>
<td>Threshold</td>
<td>Fee (£)</td>
<td>Amount (£)</td>
<td>Acronym</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>12.50</td>
<td>P410</td>
</tr>
<tr>
<td>48</td>
<td>2</td>
<td>19</td>
<td>18</td>
<td>9</td>
<td>12.50</td>
<td>P189</td>
</tr>
<tr>
<td>46</td>
<td>2</td>
<td>19</td>
<td>17</td>
<td>10</td>
<td>12.50</td>
<td>P1710</td>
</tr>
</tbody>
</table>

**Notes:** In the first column, we provide the total number of participants in each treatment. We conducted four sessions in the small sample treatments and two sessions in the large sample treatments in each game. The number of participants in the Common Knowledge sessions was common knowledge. Note that the number of participants in each session in the Common Knowledge treatments coincides with the mean $n$ of the Poisson treatments. This was done to ensure comparability across the two game types. The acronyms consist of the game type (CK for Common Knowledge games or P for Poisson games), the threshold (4 or 17 or 18) and the fee (9 or 10).

First, we investigate subjects' behavior with a smaller sample size. We ran four, Common Knowledge sessions and four, Poisson sessions. In the Common Knowledge games, four subjects participated in each session. The choice of a setup with four subjects was motivated by the extensive literature in the Turnaround games (Brandts and Cooper (2006), Brandts, Cooper, and Fatas (2007)). Consequently, given the choice of $N = 4$, to ensure comparability between the Common Knowledge and
Poisson games, we set the mean of the Poisson distribution to \( n = 4 \). Moreover, in both game types, the threshold was set to \( \alpha(Y) = 4 \). This choice was made for two reasons. First, having a setup where the threshold exceeds the (expected) number of players is problematic because (a) such setup would invite experimenter effects, and (b) it would be dominant for subjects to not register in Common Knowledge games. Second, the only value for the threshold level that does not exceed the mean population and ensures equilibrium uniqueness in the Poisson games is in fact \( \alpha(Y) = 4 \) for the parameters specified (i.e. \( n = 4, \frac{Y}{2} = \£12.50, T \in \{9, 10\} \)). Next, we experimented with a larger sample size. Our choice was to set \( N = 19 \) in the Common Knowledge games and \( n = 19 \) as the mean of the Poisson distribution in the Poisson games. For the larger group size, we decided to run two treatments in an analogous manner to the earlier treatments. The fee was set at either \( T = \£9 \) or \( T = \£10 \), which corresponds to a threshold number of registrations of 18 and 17, respectively. These choices ensured equilibrium uniqueness in the Poisson games in a similar manner to our corresponding parameter choices under the smaller sample sizes.

Table 7: Descriptive Statistics of Smaller & Larger Sample Sizes

<table>
<thead>
<tr>
<th>Common Knowledge Games</th>
<th>Registered</th>
<th>Not Registered</th>
<th>Amount Awarded?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. %</td>
<td>Freq. %</td>
<td></td>
</tr>
<tr>
<td>CK410</td>
<td>7 43.8</td>
<td>9 56.3</td>
<td>No</td>
</tr>
<tr>
<td>CK189</td>
<td>16 42.1</td>
<td>22 57.9</td>
<td>No</td>
</tr>
<tr>
<td>CK1710</td>
<td>18 47.4</td>
<td>20 52.6</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Poisson Games</th>
<th>Registered</th>
<th>Not Registered</th>
<th>Amount Awarded?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. %</td>
<td>Freq. %</td>
<td></td>
</tr>
<tr>
<td>P410</td>
<td>1 6.3</td>
<td>15 93.8</td>
<td>No</td>
</tr>
<tr>
<td>P189</td>
<td>3 6.3</td>
<td>45 93.8</td>
<td>No</td>
</tr>
<tr>
<td>P1710</td>
<td>2 4.4</td>
<td>44 95.7</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes: The table indicates the number of subjects who registered, and the number of those who did not register to buy the cash amount in each treatment. In addition, we provide the corresponding percentages. The threshold was not met in any of the treatments. The acronyms consist of the game type (CK for Common Knowledge games or P for Poisson games), the threshold (4 or 17 or 18) and the fee (9 or 10).
Table 7 reports descriptive statistics on the raw experimental data of smaller and larger sample sizes. Similar to the earlier findings, the threshold was not met in any of the treatments; consequently, the cash amount was not awarded. Furthermore, in the Common Knowledge games, the number of subjects is split between those choosing to register and those choosing not to register. Finally, in the Poisson games, only 6 subjects out of the 110 that participated registered to buy the cash amount. The other 104 subjects forewent registering.

Table 8: Robustness Analysis for Small Samples

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Alternative hypothesis:</th>
<th>decision$_i$ $\neq$ decision$_j$</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Knowledge games vs Poisson games</td>
<td></td>
<td>CK410 &amp; P410</td>
<td>0.019</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Alternative hypothesis:</th>
<th>decision$_i$ $\neq$ 1</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson games</td>
<td></td>
<td>P410</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Notes: The decision of a subject in the game is a binary variable. The subjects who chose not to register to buy the cash amount were assigned a value of 1; otherwise, were assigned a value of 0. For the analysis, we utilize Fisher’s exact test. Panel A calculates the $p$-value under the $H_0$ that behavior across the Common Knowledge and Poisson games ($i \neq j$) is not statistically different conditional on the same parameters. Panel B indicates the $p$-value under the $H_0$ that the observed distribution in the Poisson games and the distribution where all subjects choose to not register are statistically similar. The acronyms consist of the game type (CK for Common Knowledge games or P for Poisson games), the threshold (4) and the fee (10).

In Table 8, we present the robustness analysis for the smaller sample size. For the analysis, we utilize Fisher’s exact test. Panel A calculates the $p$-value to determine whether subjects’ decisions differ across the Common Knowledge and Poisson games conditional on the same parameters. The $H_0$ states that behavior between the two game types is not statistically different. The $p$-value in the pairwise comparison is below the 2% level of statistical significance. Therefore, the $H_0$ is rejected. Furthermore, Panel B displays the $p$-value under the $H_0$ that the observed distribution in
Table 9: **Robustness Analysis for Large Samples**

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Alternative hypothesis:</th>
<th>$decision_i \neq decision_j$</th>
<th>$p$-values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>Common Knowledge games vs Poisson games</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CK189 &amp; P189</td>
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<td></td>
<td></td>
<td>CK1710 &amp; P1710</td>
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<table>
<thead>
<tr>
<th>Panel B</th>
<th>Alternative hypothesis:</th>
<th>$decision_i \neq 1$</th>
<th>$p$-values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>Poisson games</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>P189</td>
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<td></td>
<td>P1710</td>
<td>0.153</td>
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<table>
<thead>
<tr>
<th>Panel C</th>
<th>Dependent variable:</th>
<th>decision</th>
<th>Regressor</th>
<th>$dy/dx$</th>
<th>$p$-values</th>
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<td>P189</td>
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<td></td>
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<td>(0.087)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>P1710</td>
<td></td>
<td>0.378***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.086)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The decision of a subject in the game is a binary variable. The subjects who chose not to register to buy the cash amount were assigned a value of 1; otherwise, were assigned a value of 0. In Panel A, we utilize the $\chi^2$-test to determine whether subjects’ decisions differ across the Common Knowledge and Poisson games ($i \neq j$) conditional on the same parameters. In addition, Panel B indicates the $p$-values in the assumption that the observed distribution in the Poisson games and the distribution where all subjects choose to not register are statistically similar. In Panel C, we report marginal effects after a probit regression on decision. CK189 is set as the base against which the estimated parameters are compared. $dy/dx$ for factor levels is the discrete change from the base level. All standard errors are reported in parentheses. The acronyms consist of the game type (CK for Common Knowledge games or P for Poisson games), the threshold (17 or 18) and the fee (9 or 10). *** Significant at the 1% level.

The Poisson games and the distribution where all subjects choose to not register are statistically similar. The $H_0$ cannot be rejected (the $p$-value is 0.500).

Table 9 presents the robustness analysis for the larger sample size. In particular, Panel A tests whether subjects’ decisions varied significantly across the Common
Knowledge and Poisson games when controlling for the parameter choices. We find that subjects’ behavior differs significantly between the two game types. All the $p$-values in the pairwise comparisons are below the 1% level of statistical significance.

To investigate the consistency of subjects’ behavior in the Poisson games with the respective theoretical prediction, the distribution of each Poisson treatment is compared to the predicted distribution. Panel B indicates the $p$-values of the treatments under the $H_0$ that the observed distribution in the Poisson games and the distribution where all subjects choose to not register are statistically similar. The results present further evidence of such consistency. Finally, in Panel C, we take advantage of the large sample size to present the marginal effects. CK189 is set as the base. The standard errors are reported in parentheses. The coefficients are statistically significant in the Poisson games. More specifically, the marginal effects imply an increase in probability of 35.9% in P189 and 37.8% in P1710 in not registering to buy the cash amount in the Poisson treatments.\footnote{We also ran marginal effects with CK1710 set as the base. With the latter base, the marginal effects imply an increase in probability in the Poisson treatments of 41.1% in P189 and 43.0% in P1710 in not registering to buy the cash amount. Both results are statistically significant at the 1% level.} Overall, the robustness analysis confirms that the first two main results are insensitive to smaller or larger sample sizes in the Common Knowledge and Poisson games.

## 7 Discussion

Population uncertainty seems to be a driving force of behavior when experience is limited. Furthermore, our statistical analysis confirms that subjects forego to register to buy the cash amount in accordance with the prescription of the Poisson games. However, a number of natural questions still remain unanswered. How does subjects’ behavior change as we increase (or decrease) the threshold level? How does subjects’ behavior change in the region where Poisson games predict multiplicity? Is population uncertainty such a big deterrent that subjects will always forego registering? Is it that any noise can, in effect, be the main driver of observed behavior? In other words, will other types of noise also deter subjects from foregoing to register to buy the cash amount? The next two subsections offer answers to these questions.
7.1 Comparative Statics

We conduct next a comparative statics exercise to shed light on the interplay between subjects’ behavior in the single shot games and population uncertainty. Specifically, we vary the threshold in order to observe its impact on the empirical distribution of contributions in a Poisson treatment when the fee is £9, the cash amount is £12.50 and the mean number of the Poisson distribution \( n \) is 17.\(^{19}\) Some general characteristics of the comparative statics sessions are shown in Panel A of Table 10. In Panel B, we report descriptive statistics on the raw data.

In Figure 3, we plot the proportion of subjects who did not register to buy the cash amount over different thresholds. Note that for the threshold levels of 13 – 15, the theoretical prediction is that either all subjects will register to buy the cash amount or none of the subjects will register to buy the cash amount (i.e. condition (1) does not hold, which results in equilibrium indeterminacy). In contrast, for the threshold levels of 16 – 18, condition (1) is satisfied, hence the theoretical prediction is that no subject will register to buy the cash amount. The line is increasing for the thresholds investigated. Consequently, the number of subjects that do not register seems to be increasing in the threshold level. Furthermore, in the Poisson treatments where multiplicity of equilibria is predicted, we still observe a large proportion of subjects not registering, but these proportions are well below the proportion of P169 (i.e. 0.95). We benchmark our statistical analysis on P169 and compare the empirical distribution of that treatment to those of the other treatments. Comparing P169 and P159, we find that the distributions are marginally statistically similar (the \( p \)-value is 0.103). However, when comparing P169 and P149, and P169 and P139, we find that the distributions are statistically different (the \( p \)-values are 0.016 and 0.000, respectively). Thus, the further you move below the threshold the more pronounced the difference in subjects’ behavior becomes. Furthermore, the empirical distribution of P169 is neither statistically different from that of P179 nor from that of P189 (the \( p \)-values are 0.907 and 0.378, respectively). These results are consistent with the theoretical prediction of the Poisson Coordination games.

\(^{19}\)In principle, we could have varied instead the mean of the Poisson distribution, or the fee, or even the cash amount. We showed preference towards changing the threshold simply because it led to the least number of changes in the experimental instructions. For one, changing the mean of the distribution would lead to different roulette wheels and for another, changing the fee or the cash amount would lead to further changes in the final payoffs provided.
Table 10: Characteristics of the Comparative Statics Sessions

<table>
<thead>
<tr>
<th>Panel A</th>
<th># of Subj.</th>
<th># of Ses.</th>
<th>Mean</th>
<th>Threshold</th>
<th>Fee (£)</th>
<th>Amount (£)</th>
<th>Acronym</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>1</td>
<td>17</td>
<td>13</td>
<td>9</td>
<td>12.50</td>
<td>P139</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>17</td>
<td>14</td>
<td>9</td>
<td>12.50</td>
<td>P149</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>17</td>
<td>15</td>
<td>9</td>
<td>12.50</td>
<td>P159</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>2</td>
<td>17</td>
<td>16</td>
<td>9</td>
<td>12.50</td>
<td>P169</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>1</td>
<td>17</td>
<td>17</td>
<td>9</td>
<td>12.50</td>
<td>P179</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>17</td>
<td>18</td>
<td>9</td>
<td>12.50</td>
<td>P189</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Registered</th>
<th>Not Registered</th>
<th>Amount Awarded?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acronym</td>
<td>Freq.</td>
<td>%</td>
<td>Freq.</td>
</tr>
<tr>
<td>P139</td>
<td>9</td>
<td>42.9</td>
<td>12</td>
</tr>
<tr>
<td>P149</td>
<td>4</td>
<td>28.6</td>
<td>10</td>
</tr>
<tr>
<td>P159</td>
<td>3</td>
<td>18.8</td>
<td>13</td>
</tr>
<tr>
<td>P169</td>
<td>2</td>
<td>5.0</td>
<td>38</td>
</tr>
<tr>
<td>P179</td>
<td>1</td>
<td>4.3</td>
<td>22</td>
</tr>
<tr>
<td>P189</td>
<td>0</td>
<td>0.0</td>
<td>15</td>
</tr>
</tbody>
</table>

Notes: In Panel A, we provide some general characteristics of the comparative statics sessions. In the first column, we provide the total number of participants in each treatment. The acronyms in the last column consist of the game type (P for Poisson games), the threshold (13 or 14 or 15 or 16 or 17 or 18) and the fee (9). In Panel B, we report the number of subjects who registered, and the number of those who did not register to buy the cash amount in each treatment. In addition, we provide the corresponding percentages. P169 is reproduced from Tables 1 and 2.

7.2 Alternative Uncertainty

Subjects in the games with population uncertainty forego to register to buy the cash amount. Will other types of noise also deter subjects from foregoing to register to buy the cash amount? Of course the options are several; however, a natural candidate given the received literature on the topic is to investigate a single shot Global Coordination game, where agents face idiosyncratic uncertainty about the economic fundamentals. To ensure comparability across sessions, we design experiments in an
Figure 3: Comparative Statics Over Different Thresholds

Notes: The figure displays the proportion of subjects who did not register to buy the cash amount over different thresholds. Specifically, we vary the threshold in order to observe its impact on the empirical distribution of a Poisson treatment when the fee is £9, the cash amount is £12.50 and the mean number of the Poisson distribution $n$ is 17. For the threshold levels of 13 – 15, the theoretical prediction is that either all subjects will register to buy the cash amount or none of the subjects will register to buy the cash amount, whereas for the threshold levels of 16 – 18, the theoretical prediction is that no subject will register to buy the cash amount.

analogous fashion to those in the Common Knowledge and Poisson games.

In addition, parameters that determine the information structure here are chosen to ensure that the equilibrium prediction in the Global Coordination games is also that subjects should forego registering to buy the cash amount. This has led us to draw the level of $Y$ from the set of integers 5 to 95 according to the uniform distribution. For design reasons, this level of $Y$ was drawn ahead of all experiments conducted. More detailed information on the symmetric Bayesian Nash equilibrium, the experimental design as well as the experimental instructions of the Global Coordination games is provided in the Appendix. Some general characteristics of the Global sessions are shown in Panel A of Table 11. In Panel B, we report descriptive statistics on the raw data.
### Table 11: Characteristics of the Global Sessions

**Panel A**

<table>
<thead>
<tr>
<th># of Subj.</th>
<th># of Ses.</th>
<th>Threshold</th>
<th>Fee (£)</th>
<th>Amount (£)</th>
<th>Acronym</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>2</td>
<td>$22 - \lfloor \frac{Y}{3} \rfloor = 16$</td>
<td>9</td>
<td>$\frac{Y}{2} = 12.50$</td>
<td>G169</td>
</tr>
<tr>
<td>34</td>
<td>2</td>
<td>$21 - \lfloor \frac{Y}{3} \rfloor = 15$</td>
<td>10</td>
<td>$\frac{Y}{2} = 12.50$</td>
<td>G1510</td>
</tr>
</tbody>
</table>

**Panel B**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Registered</th>
<th></th>
<th>Not Registered</th>
<th></th>
<th>Amount Awarded?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq.</td>
<td>%</td>
<td>Freq.</td>
<td>%</td>
<td></td>
</tr>
<tr>
<td>G169</td>
<td>14</td>
<td>41.2</td>
<td>20</td>
<td>58.8</td>
<td>No</td>
</tr>
<tr>
<td>G1510</td>
<td>16</td>
<td>47.1</td>
<td>18</td>
<td>52.9</td>
<td>No</td>
</tr>
</tbody>
</table>

**Notes:** In Panel A, we provide some general characteristics of the Global treatments. In the first column, we provide the total number of participants in each treatment. In the calculation of the threshold in the Global games, the symbolic function $\lceil \cdot \rceil$ rounds up the fraction to the nearest integer from above. The acronyms in the last column consist of the game type (G for Global games), the threshold (15 or 16) and the fee (9 or 10). In Panel B, we report the number of subjects who registered, and the number of those who did not register to buy the cash amount in each treatment. In addition, we provide the corresponding percentages.

We utilize the $\chi^2$-test to determine whether subjects’ decisions in the Global Coordination games differ from the theoretical prediction for the parameters specified. Specifically, the $H_0$ states that subjects’ decisions in the Global sessions are statistically similar to the theoretical predictions for the parameters specified. We find that subjects’ behavior in the Global sessions differs from the prediction of the Global games for the parameters specified (the $p$-values are 0.000 in both G169 and G1510). This suggests that noise per se is not necessarily a driving force of coordination (to the safe action).

Subjects may deviate from the Global game solution towards the risky action in one shot Global Coordination games because they, presumably, believe others will also strive for the efficient (from the point of view of the group) equilibrium. In general, in Coordination games with population uncertainty, even if all other players play the risky action, following the same choice is not profitable if the probability that the actual number of players is below the threshold is large. In Poisson Coordination games, this probability is indeed large, and hence subjects have a very strong incentive to choose the safe action. Therefore, we can conclude that population uncertainty as
modelled by Poisson games is a good predictor of coordination to the safe action in the lab for the chosen parameters.

8 Concluding Remarks

We design novel, online experiments that attempt to capture “large” games between players to study population uncertainty in Coordination games. Our study is the first to investigate experimentally Poisson Coordination games. We find that subjects’ behavior in the Poisson Coordination games is indeed consistent with the theoretical prediction. Our findings also suggest that uncertainty about the number of players may be an important factor of subjects’ behavior in the macroeconomy. These results suggest that we could use the framework of Poisson games to articulate possible policies that promote financial stability. Recalling that Poisson games try to capture the strategic environment in large economies, and that condition (1) states that in such an environment no attack will take place when the cost of attacking is higher than the expected gain, we have immediately the following policy prescription: increasing the size of the markets (by removing trade restrictions and market entry fees and/or improving cooperation between national financial networks) and imposing a Tobin tax on short-selling transactions may reduce the prior probability of an attack by debt or currency speculators.

An important avenue for future research could be the provision of a unified theory of explaining behavior across various environments. Such fruitful attempts have been undertaken by Heinemann, Nagel, and Ockenfels (2009), and Kneeland (2012). The former study estimates various parameters of a Global Coordination game and shows that the estimated model performs well on that front. The latter study utilizes the experimental dataset of HNO to calibrate a model that rests on the limited-depth-of-reasoning solution concept. However, neither study incorporates Poisson treatments. Finally, an engaging future direction could be the investigation of whether our results on Poisson Coordination games carry over to other important contexts, such as Voting games (Bouton and Castanheira (2012)) and Discrete Public Goods games (Makris (2009)).
References


Laury, Susan K. “Pay One or Pay All: Random Selection of One Choice for Payment.”, 2012. Mimeo.


