MULTIFRACTAL ANALYSES OF GRAYSCALE AND BINARY SOIL THIN SECTION IMAGES

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Received December 21, 2010
Accepted February 20, 2011

Abstract

Multifractal analyses of binary images of soil thin sections (STS) are widely used to characterize pore structure. However, no geometrical model is known to exist for a binary multifractal. Thus, the multifractality of binary images, and the accuracy of multifractal parameters estimated from them, need to be carefully evaluated. We captured 8-bit depth resolution digital grayscale images of three STS images with dimensions of 1024 × 1024 pixels and a pixel length of 1.9 μm. Random grayscale geometrical multifractal fields (GMF) with similar dimensions and known multifractal parameters were constructed using generators extracted from the STS images. The STS and GMF grayscale images were objectively thresholded to give six binary images. The method of moments was used to compute the log-transformed partition function, log(χ(q, δ)) versus log(δ) where δ is box size, for each grayscale image and its binary counterpart.

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Lipschitz-Hölder exponents and/or the generalized dimensions (Dq) computed from the linear portions of the binary log-transformed partition functions were significantly over estimated for q \ll 0 and underestimated for q \gg 0 relative to corresponding Dq values for the grayscale images. Based on these results we contend that binary images are not mathematical multifractals, and that generalized dimensions estimated from them cannot be used to quantify pore space geometry. Instead we encourage further exploration of the use of grayscale images for multifractal characterization of soil structure. This direct approach is theoretically sound and does not require any intermediate thresholding step, which is known to influence the results of multifractal analyses.

Keywords: Multifractal; Grayscale Images; Binary Images; Soil Thin Section.

1. INTRODUCTION

Multifractal analysis (MFA) is increasingly being used to quantitatively characterize the scaling of spatially heterogeneous systems. A variety of digital images from a wide range of origins and scales, including remote sensing imagery, X-ray computed tomography images, magnetic resonance images, atomic force microscope images, and optical images, have all been analyzed using this technique. Most of the images analyzed are in grayscale format. The grayscale values are transformed to mass fractions, and their scaling behavior is then characterized by the Lipschitz-Hölder exponents and/or the generalized dimensions. Zhou et al. have shown that such analyses are quite sensitive to the bit depth resolution used to acquire the images.

In soil science, binary (black and white) soil thin section images or computed tomography images are commonly analyzed by MFA to study the spatial distribution of soil pore structures. However, there are several theoretical and practical limitations regarding the MFA of binary images. Firstly, no geometrical binary multifractal field has been constructed on a sound mathematical basis. Saucier and Muller generated “two-dimensional binary multifractal fields” using a multiplicative process. However, the pore space in the resulting fields was represented by a continuous measure (“porosity”) rather than a binary value, zero or unity. Kravchenko et al. recently pointed out that, because their Lipschitz-Hölder exponents are either 0 or 2 at the pixel scale, binary images are not multifractal in a strict mathematical sense. As a result, empirical MFA’s of binary images must be restricted to box sizes much greater than the pixel scale so as to produce a continuous measure by averaging the binary values.

A practical problem often encountered in the MFA of binary images is that the log-transformed partition function curve, the linearity of which is critical to the multifractal analysis, is usually curvilinear when the mass order q \ll 1. Although multifractal spectra or generalized dimensions can still be calculated from such curves, the results have no physical meaning. A compromise is to choose a limited range of scales over which the log-transformed partition function curve is linear based on the coefficient of determination computed from regression analysis or configuration entropy calculations. However, this often limits the MFA to less than two orders of magnitude, which is detrimental to any interpretation of fractal or multifractal scaling. Another practical problem with the MFA of binary images is that the thresholding method, which is used to obtain a black and white image from the original grayscale data, can have a pronounced effect on the porosity and resulting generalized dimensions.

Despite the above mentioned limitations, MFA of binary soil images using a limited range of box sizes is a widely accepted method for characterizing soil structure. However, because no theoretical multifractal parameters are available for natural images, the veracity of such MFA’s cannot be established.

ABBREVIATIONS
GMF : geometrical multifractal field
MFA : multifractal analysis
MOM : method of moments
STS : soil thin section image

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Consistent linearity was observed in the resulting functions for the grayscale images, indicating, by definition, multifractal behavior. In contrast, the log(\chi(q, \delta)) versus log(\delta) plots for the binary images exhibited a two-region response, with a flat plateau at small scales and linearity at larger scales, indicating they were not true multifractals. Generalized dimensions (Dq) from regression analysis based on the coefficient of determination computed or configuration entropy plots for the multi-fractal spectra or generalized dimensions. Zhou et al. have shown that such analyses are quite sensitive to the bit depth resolution used to acquire the images.

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1. INTRODUCTION

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Despite the above mentioned limitations, MFA of binary soil images using a limited range of box sizes is a widely accepted method for characterizing soil structure. However, because no theoretical multifractal parameters are available for natural images, the veracity of such MFA’s cannot be established.
As a result, the relationship between multifractal parameters and soil structure must be interpreted subjectively.

The objective of this paper is to compare the generalized dimensions obtained from a MFA of grayscale images of soil thin sections with those obtained from the corresponding thresholded binary images. A similar comparison is performed using grayscale images of geometrical multifractal fields (GMF) constructed by the multiplicative cascade method, and then thresholded to produce binary images. Since the GMF’s have known analytical properties, it is possible to quantitatively evaluate the accuracy of the generalized dimensions estimated from the thresholded binary images.

2. METHODS

2.1. Soil Thin Section Images

Three soil thin section images with different pore size distributions were selected for study. Detailed information about soil sampling, soil thin section preparation and image capture can be found in Zhou et al. Briefly, plane polarized soil thin section images were acquired under the same light intensity with a Nikon DS-Fil digital camera equipped on a Nikon Eclipse Lx100Pol polarized microscope. All images were captured at 8 bit depth resolution and stored as grayscale fields in “tiff” format. Individual pixel values ranged from 0 to 255, and were converted to mass fractions by division with the sum of all the pixels values in a field. The dimensions of the images were 2560 × 1920 pixels, with a pixel length of 1.9 µm. To avoid edge effects, only the central part of each image (1024 × 1024 pixels) was used in the analyses.

2.2. Geometrical Multifractal Fields

Random geometrical multifractal grayscale fields were constructed from an initiator of unit length by the multiplicative cascade method. First, the initiator was subdivided into grid of \(b^2 = 16\) cells, where \(b\) is an integer scale factor. (Other \(b\) values were investigated, but \(b = 4\) proved to be most compatible with the soil images). Mass fractions \(\mu_{i,j} = 1 \text{ to } b^2\), where the subscript 1 signifies the first iteration of the multiplicative cascade) were then randomly assigned to the \(b^2\) cells, to produce a generator. Values for the mass fractions were obtained by subdividing the grayscale soil thin section images into a grid of \(b^2 = 16\) cells and summing the mass fractions within each cell. The resulting generator mass fractions, extracted from the three soil images, are given in Table 1.

Each GMF generator was then applied onto itself. At the second iteration level, the grid was further subdivided into \(j = 1 \text{ to } (b^2)^2\) cells. The corresponding mass fractions \(\mu_{i,j}\) were calculated as:

\[
\mu_{2j} = \mu_{j} \times \mu_{i}
\]

and randomly assigned to the \((b^2)^2\) cells in the grid.

Repetition of this multiplicative process to the \(i\)th iteration level produced a random grayscale multifractal field, comprised of \((b^2)^i\) cells. To permit direct comparison with the soil thin section images, the random multiplicative cascade with \(b = 4\) was terminated at \(i = 5\) resulting in GMF’s comprised of 1024 × 1024 cells.

2.3. Visualization and Thresholding

In order to visualize the soil thin section (STS) images and random multifractal fields (RMF), the mass fractions were normalized with respect to the minimum and maximum values present within each field. The resulting normalized mass fractions represent different shades of gray, bounded by black (zero) and white (unity). The three grayscale STS images are shown in Figs. 1a–1c, with the corresponding grayscale RMF’s shown in Figs. 1d–1f.

The mass fractions within the STS images and RMF’s were objectively thresholded using Otsu’s method. The resulting binary images are shown in...
2.4. Empirical Generalized Dimensions

The method of moments (MOM) box counting method was used to conduct MFA’s on the grayscale and thresholded STS images and the thresholded RMF’s. Since detailed explanations of this method can be found in other publications,\textsuperscript{8,13,20} the calculation procedures are only briefly described here.

The images were subdivided into square grids with variable box lengths, $\delta = \beta^m$, where $\beta$ is the box counting scale factor, and $m$ is an integer value between zero and $\log_{10}(b')$. In this study, $\beta = b = 4$, with $0 \leq m \leq 5$. The probability density value of the $k$th box in a superimposed grid, $\rho_k(\delta)$, was calculated as the sum of grayscale (or binary) values within that box divided by the sum of the grayscale (or binary) values for the whole image. The partition function of order $q$, $\chi(q, \delta)$, was computed according to:

$$\chi(q, \delta) = \sum_{k=1}^{N_{\delta}} (\rho_k(\delta))^q,$$

where $q$ is any real number between $\pm\infty$. For a multifractal field, this partition function scales with the box size as:

$$\chi(q, \delta) \propto \delta^{-\tau(q)},$$

where $\tau(q)$ is the mass exponent for $q$. By taking the logarithms of both sides of Eq. (3), the value $\tau(q)$ can be estimated by linear regression analysis for any given value of $q$. The generalized
Multifractal Analyses of Grayscale and Binary Images

Fig. 2  Binary soil thin section images and multifractal fields comprised of 1024 × 1024 pixels: (a) STS1, (b) STS2, (c) STS3, (d) GMF1, (e) GMF2, and (f) GMF3. Frame length of STS images: 1.95 × 1.95 mm. STS = soil thin section image; GMF = geometrical multifractal field.

dimensions, \( D_q \), were then calculated from \( \tau(q) \) using the relationship:

\[
D_q = \frac{\tau(q)}{q-1}
\]

(4)

Integer values of \( q \) between ±10 were selected. For \( q = 1 \), \( D_1 \) was estimated directly from the slope of a linear regression analysis performed on:

\[
\sum_{k=1}^{N_j} \rho_k(\delta) \log(\rho_k(\delta)) \text{ versus } \log(\delta).
\]

2.5. Analytical Generalized Dimensions

Since the grayscale RMF’s are geometrical multifractals, their generalized dimensions are known exactly and there is no need to employ the MOM box counting method to estimate \( D_q \). Instead, analytical generalized dimensions were calculated directly from the distribution of mass fractions in the generator:

\[
D_q = \frac{1}{q-1} \log \left( \sum_{j=1}^{\infty} \rho_j(\delta)^{-q} \right) / \log(\delta); \quad q \neq 1,
\]

(5)

where \( D_q \) is the generalized dimension for moment order \( q \). To be consistent with the empirical MFA, integer values of \( q \) between ±10 were selected. When \( q = 1 \), the \( D_1 \) was calculated according to:

\[
D_1 = -\sum_{j=1}^{\infty} \rho_j \times \log(\rho_j) / \log(\delta).
\]

(6)

3. RESULTS AND DISCUSSION

The three soil thin section images used in this study exhibited very different structural characteristics: well developed aggregates with large pores.
GMF3 0.692 0.314
GMF1 0.743 0.430
GMF2 0.722 0.360
GMF3 0.692 0.314

Table 2 Average Properties of the Soil Thin Section Images and Multifractal Grayscale Fields.

<table>
<thead>
<tr>
<th>Field</th>
<th>Mean Intensity of Grayscale Images</th>
<th>Porosity of Binary Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>STS1</td>
<td>0.579</td>
<td>0.502</td>
</tr>
<tr>
<td>STS2</td>
<td>0.507</td>
<td>0.449</td>
</tr>
<tr>
<td>STS3</td>
<td>0.446</td>
<td>0.399</td>
</tr>
<tr>
<td>GMF1</td>
<td>0.743</td>
<td>0.430</td>
</tr>
<tr>
<td>GMF2</td>
<td>0.722</td>
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</tr>
<tr>
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<td>0.692</td>
<td>0.314</td>
</tr>
</tbody>
</table>

Note: STS = soil thin section image, GMF = geometrical multifractal field. GMF1, GMF2 and GMF3 were constructed using $b = 4$, $i = 5$, and generators corresponding to STS1, STS2, and STS3, respectively.

(Fig. 1a); medium sized aggregates with evenly distributed pores (Fig. 1b); and a relatively homogeneous structure (Fig. 1c). These characteristics are also apparent in the thresholded STS images (Figs. 2a–2c). The mean grayscale intensities of the thin sections were 0.579, 0.507 and 0.446 for STS1, STS2 and STS3, respectively (Table 2). The areal porosities of the corresponding binary images decreased from $\sim 0.5$ to $\sim 0.4$ (Table 2).

The GMF’s are multifractal simulants of the three STS grayscale images. They were constructed by iterating mass fractions obtained from each STS image (Table 1). It was assumed that the thin sections were geometrical multifractals, and that a $4 \times 4$ sub-division would yield their generators. As indicated by the variation of the mass fractions in the generators, GMF1 was the most heterogeneous, GMF2 was intermediate, while GMF3 was the most homogeneous (Table 1). This ranking is consistent with a visual assessment of both the STS and GMF grayscale images (compare Figs. 1a–1c and Figs. 1d–1f).

The GMF’s constructed in this study were not intended to exactly simulate the STS images. Rather they are random realizations of a multiplicative cascade process that produces geometrical replicas of the soil thin sections with exact multifractal scaling properties and known parameters. Binary versions of the thresholded GMF’s are shown in Figs. 2d–2f. While the absolute values of the mean grayscale intensities and areal porosities for the GMF’s differed from those for the STS images, they showed very similar trends with respect to differences observed between the three soils (Table 2).

Log-log plots of the partition function, $\chi(q, \delta)$, versus length scale, $\delta$, for the grayscale STS images are shown in Fig. 3. Linear behavior was observed for all three soils, with coefficients of determination, $R^2$, from the regression analyses always $\geq 0.98$. The pronounced linearity of the log-transformed partition functions in Fig. 3 is powerful evidence that soil structure, when analyzed as a grayscale image, can be considered a statistical multifractal. In contrast, after thresholding, log-log plots of $\chi(q, \delta)$ versus $\delta$ for the binary STS images were distinctly nonlinear for $q \leq 0$ (Figs. 4a–4c). Specifically, two different regions could be observed: a flat plateau coupled with a restricted linear scaling response. Consider STS1 (Fig. 4a) for example: when $\delta < \sim 16$, log($\chi(q, \delta)$) is almost constant regardless of $\delta$, while for $\delta \geq \sim 16$, log($\chi(q, \delta)$) decreases linearly with increasing $\delta$. All three thresholded GMF’s, exhibited similar non-linear behavior to the thresholded STS images (Figs. 4d–4f). (Note: log-transformed partition functions for the grayscale GMF’s were not presented because they are known analytically, and are perfectly linear.)

The phenomenon of two scaling regions in log-log plots of $\chi(q, \delta)$ versus $\delta$ is commonly observed in the MFA of binary images. It can be attributed to the presence of a large number of zero values at small scales, where the change in box size has little effect on the calculation of $\chi(q, \delta)$. As $\delta$ increases beyond a critical value, however, these zero values disappear due to averaging and linear scaling behavior becomes apparent.

Krivchenko et al. have pointed out that binary images are not multifractal because they are composed of ones and zeros, and consequently their Lipschitz-Hölder exponent is either 2 or 0 at the pixel scale. However, more generally, every digital image, grayscale or binary, possesses a limited range of pixel values and consequently limited Lipschitz-Hölder exponents at the pixel scale depending on its bit depth; for example, a maximum of $2^8$ grayscale values for the 8 bit depth images investigated here. Our earlier research found that any reduction in bit depth compared to a true GMF will bias the MFA results, but that coarse graining to a larger scale will eliminate this effect. Binary images can be thought of as being the result of a reduction in bit depth to 1 using a thresholding value. The above partition function analysis has shown that binary images are not multifractal over the whole range of scales. However, linear scaling behavior appears when larger box sizes are selected (Fig. 4). Many previous studies have characterized pore space geometry using multifractal parameters computed over this limited
scaling range. A logical question to ask in this context is how well do the results of MFA's on coarse-grained binary images compare with those from grayscale images, which we have shown to be multifractal over the full range of box sizes?

To answer this question, generalized dimensions \((D_q)\) were computed for both the grayscale and thresholded STS and GMF images based on the MOM box counting method. The nonlinear behavior of \(\log(\chi(q, \delta))\) versus \(\log(\delta)\) in the case of the binary images was most pronounced when \(q = -10\) (Fig. 4). Therefore, the split between the two scaling regions was determined by simultaneously fitting two linear models to the \(q = -10\) log-transformed partition functions using segmented regression (SAS/STAT software, version 9.1.3, SAS Institute Inc. Cary, NC). The two-region model fit the data very well, with \(R^2\) values from the segmented regression analyses \(\geq 0.98\). The intersection of the two linear models, which represents the cut point, \(\delta_c\), was computed from the two sets of slope and intercept estimates. Values for \(\delta_c\) ranged from 11 to 29 pixels for the STS binary images and from 8 to 29 for the thresholded GMF’s. The divisions between the two linear scaling regions are indicated by the dashed vertical lines in Fig. 4. Estimates of \(D_q\) were then obtained from \(\tau(q)\)’s estimated over the restricted scale range \(\delta_c \leq \delta\) using linear regression analysis applied to all of the \(q\) values. In the case of the grayscale images, the \(D_q\) values were computed from the \(\tau(q)\)’s estimated over the entire range of \(\delta\) using linear regression analysis.

The generalized dimensions for the grayscale and binary images are shown in Fig. 5. The uncertainty in the estimation of \(D_q\) was greater for the binary images than for the grayscale images as shown by the 95% confidence intervals. (Note: the \(D_q\) values for the grayscale GMF’s have no error bars since they are known exactly.) This difference can be attributed to the different ranges of scales over which the regression analyses were performed. The grayscale images were analyzed over...
Fig. 4 Log-log plots of the partition function, $\chi(q, \delta)$ versus box size $\delta$ in pixels) for the binary STS images and GMF's: (a) STS1, (b) STS2, (c) STS3, (d) GMF1, (e) GMF2, and (f) GMF3. STS = soil thin section image, GMF = geometrical multifractal field. Note: $q$ increases from $-10$ to $+10$ from top to bottom. The vertical dashed lines indicate the divisions between the two scaling regions determined by segmented linear regression.

The entire range of $\delta$ and included more data points (i.e., less variability in the parameter estimates) than the MFA's on the binary images, which were restricted to a limited range of scales, and involved fewer points (i.e., more variability in the parameter estimates).

Regardless of image type all of the $D_q$ values decreased with increasing $q$, which is a
Fig. 5  Generalized dimensions, $D_q$ versus $q$, for the grayscale and binary images: (a) STS1, (b) STS2, (c) STS3, (d) GMF1, (e) GMF2, and (f) GMF3. STS = soil thin section image, GMF = geometrical multifractal field.
characteristic of multifractality (Fig. 5). The $D_0$ values were all close to the Euclidean dimension of two for both the grayscale and binary images. This is because all of the boxes involved in the calculation of $D_q$ possess a mass fraction. The generalized dimensions for the grayscale and binary images were quite different from each other when $q \neq 0$ (Fig. 5). For the soil thin sections, the binary $D_q$ values were significantly larger than their counterparts for the corresponding grayscale images when $q < 0$, and vice versa when $q > 0$.

Analytical $D_q$’s for the grayscale GMFs were calculated and compared to the empirical $D_q$’s for the corresponding thresholded images (Fig. 5). Similar to the results for the STS images, the generalized dimensions of the binary images were larger than the $D_q$ values for the grayscale images when $q < 0$, and smaller when $q > 0$. The only exception was GMF1 where, although the same trend was apparent, the differences were not statistically significant. The analytical $D_q$ spectra of the GMF2 and GMF3 grayscale images are quite flat and close to the Euclidean dimension of two. However, these fields are still multifractals in a strict mathematical sense. On the other hand, the $D_q$ spectra for the GMF binary images, which showed a pronounced “S” shape and, thus, might be thought of as more “multifractal”, are not at all accurate and can only be termed “apparent spectra”.

Given the above results, and the absence of any geometrical binary multifractal model, it is logical to query the correctness and accuracy of generalized dimensions estimated from binary images, even those calculated over a limited range of scales. In our opinion, the MFA of binary images to characterize and compare different soil structures is highly questionable. Mathematically, binary images cannot be considered multifractals at small scales. Apparent multifractal behavior does appear after coarse graining to a given scale, however, the generalized dimensions estimated over this restricted scaling range are not accurate, and thus any physical-chemical properties or processes that might be inferred from them will likely be erroneous.

The digital images analyzed in this study covered approximated three orders of magnitude and were based on light transmittance as measured by soil microscopy. We anticipate that similar results will be obtained in the case of images derived from the variation in bulk density measured by computer-assisted X-ray tomography. Research on other grayscale data sets, obtained using different experimental methods, is needed to confirm this. Additional studies with images covering a larger range of scales would also be useful.

4. CONCLUSIONS

In this study the method of moments was used to analyze the multifractal properties of grayscale soil thin section images and geometrical multifractal fields (constructed using generators derived from the soil images) and their binary counterparts. For the grayscale images, linear behavior was observed in the log-log plots of the partition function, $\chi(q, \delta)$, versus box size $\delta$, signifying, by definition, multifractality. In contrast, log transformed plots of $\chi(q, \delta)$ versus $\delta$ after thresholding, exhibited a two-region behavior, with a flat plateau at small scales and a limited linear response at larger scales, indicating that the binary images were not true multifractals. Generalized dimensions ($D_q$) calculated from the linear portions of the log($\chi(q, \delta)$) versus log($\delta$) plots for the binary images were significantly different from those obtained over the full range of $\delta$ or the grayscale images. The binary $D_q$’s were generally overestimated for $q < 0$ and underestimated for $q > 0$ relative to the corresponding grayscale values.

From the above analyses, we conclude that binary images cannot be considered multifractals at small scales. Furthermore, their generalized dimensions, calculated over the linear scaling region at larger scales, are inaccurate and can lead to erroneous interpretations of about the scaling properties of soil structure. Additional theoretical and empirical research is needed to fully explore the relationship between multifractal parameters estimated from grayscale and binary images. Method of moment analyses performed on larger images captured using alternative experimental methods such as computer-assisted X-ray tomography would be useful in this context.

ACKNOWLEDGMENTS

This work was supported in part by NKBRSF 2009CB118607.

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