OFDM Interference Mitigation Algorithms for Doubly-Selective Channels

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Abstract—Frequency Division Multiplexing (OFDM) system may suffer from significant Inter-carrier Interference (ICI) and Inter-block Interference (IBI) under a time- and frequency-selective (or doubly-selective) channel. We propose a novel hybrid architecture where a time-domain FIR filter mitigates IBI by shortening the channel to the cyclic prefix length. This is followed by ICI cancellation in the frequency-domain using few FIR taps per subchannel combined with successive cancellation. We also propose an iterative joint data and channel estimation scheme to improve the channel estimation performance.

I. INTRODUCTION

(OFDM) is the modulation scheme of choice for emerging broadband wireless communication standards. These include IEEE 802.16e, 802.11a, 802.15.3 and other emerging cellular wireless systems like 3GPP evolution and 4G.

In OFDM, a cyclic prefix (CP) of length greater than or equal to the channel memory in each information block is inserted to eliminate IBI at the expense of a rate loss. In this case, subcarrier orthogonality is maintained if the channel is assumed to be time-invariant within an OFDM block, which simplifies equalization at the receiver to a single-tap frequency-domain equalizer (FEQ) per subcarrier. However, future wireless systems are expected to operate at higher carrier frequencies, higher levels of mobility, and higher capacities, resulting in doubly-selective fading, i.e., frequency-selective (due to long multipath delay) and time-selective (due to mobility) [1] [2] [3]. This double selectivity results in two forms of interference: 1) IBI when a short CP is used to reduce CP rate overhead [4]; 2) ICI due to mobility or short CP which destroys subcarrier orthogonality [5].

To compensate for the interference caused by the short CP, several channel-shortening time-domain equalizer (TEQ) algorithms have been proposed in the literature. One simple and effective design was proposed by Melsa [6], which shortens the channel impulse response to the CP length. In the frequency-domain, several FEQ algorithms have also been proposed to deal with ICI [1] [7] [8].

One of the main contributions of this paper is to combine both time- and frequency-domain equalization into a hybrid two-stage structure for robust and reliable reception in OFDM system over a doubly-selective fading environment. We use an FIR TEQ to shorten the channel and minimize the effect of IBI and ICI due to short CP. To further mitigate the residual ICI due to time-variation of the channel, we propose a low complexity FEQ using few FIR taps per subchannel. Another contribution of this paper is to combine this FEQ with iterative detection to further combat ICI at high Doppler. Finally, we propose an iterative joint channel estimation and data detection method to further improve performance.

This paper is organized as follows. In Section II, the system model is described and properties of ICI and IBI are investigated. An interference cancellation (IC) algorithm is presented in Section III. In Section IV, a channel estimation method is proposed. Simulation results are given in V, and finally conclusions are drawn in Section VI.

II. SYSTEM MODEL

The system block diagram is shown in Fig 1. At each symbol index \(i\), an OFDM symbol \(X(i) = [X_0^{(i)}, ..., X_{N-1}^{(i)}]^T\) is converted into time domain samples \(x(i) = [x_0^{(i)}, ..., x_{N-1}^{(i)}]^T\) using the \(N\)-point Inverse Fast Fourier Transform (IFFT).

\[
x(i) = F^H X(i)
\]

where \(F^H\) is the \(N\)-point IFFT matrix, \((\cdot)^T\) denotes the transpose, and \((\cdot)^H\) denotes the complex conjugate transpose. If the CP length \(L_g\) is shorter than the maximum channel delay spread \(L_c\), i.e., \(L_g < L_c\), the \(i\)-th received block \(y(i) = [y_0^{(i)}, ..., y_{N-1}^{(i)}]^T\) after removal of CP is [4]

\[
y(i) = H^{(i)} x(i) + y_3^{(i)} + y_4^{(i)} + v(i)
\]

where \(y_3^{(i)} = B^{(i)} x(i-1)\) is the IBI term and \(y_4^{(i)} = -A^{(i)} x(i)\) is the ICI term, both of which are due to short CP. Both \(A^{(i)}\) and \(B^{(i)}\) are sparse matrices with structure shown
in Fig. 2, and \( \mathbf{v}^{(i)} \) is the time-domain noise vector. If the CP length is greater than channel memory, i.e., \( L_g \geq L_c - 1 \), then \( \mathbf{A}^{(i)} \) and \( \mathbf{B}^{(i)} \) become all-zero matrices and the terms \( r_3^{(i)} \) and \( r_4^{(i)} \) vanish.

\( \mathbf{H}^{(i)} \) is an \( N \times N \) matrix with elements given by \( h^{(i)}(k, n) = h^{(i)}(k, (k-n)_N) \) where \( h^{(i)}(k, n) \) is the channel impulse response (CIR) during the \( i \)-th block interval at lag \( n \) and instant \( k \), and \( (\cdot)_N \) denotes the modulo-\( N \) operation. For doubly-selective channels, \( h^{(i)}(k, n) \) is time-variant during an OFDM block and the first term in (2) can be written as the sum of two components as follows

\[
\begin{align*}
\mathbf{y}^{(i)} &= \mathbf{H}^{(i)}\mathbf{x}^{(i)} + \mathbf{y}_3^{(i)} + \mathbf{y}_4^{(i)} + \mathbf{v}^{(i)} \\
&= \mathbf{y}_1^{(i)} + \mathbf{y}_2^{(i)} + \mathbf{y}_3^{(i)} + \mathbf{y}_4^{(i)} + \mathbf{v}^{(i)}
\end{align*}
\tag{3}
\]

where

\[
\begin{align*}
\mathbf{y}_1^{(i)} &= \mathbf{H}_{\text{ave}}^{(i)}\mathbf{x}^{(i)} \\
\mathbf{y}_2^{(i)} &= \mathbf{H}_{\text{var}}^{(i)}\mathbf{x}^{(i)}
\end{align*}
\tag{4}\tag{5}
\]

and \( \mathbf{H}_{\text{ave}} \) is an \( N \times N \) circulant matrix with elements given by \( H_{\text{ave}}^{(i)}(k, n) = \frac{1}{N} \sum_{k=0}^{N-1} h^{(i)}(k, (k-n)_N) \) where \( h^{(i)}(k, n) \) is the time average of \( h^{(i)}(k, n) \) at lag \( n \) for the \( i \)-th block, and \( \mathbf{H}_{\text{var}}^{(i)} = \mathbf{H}^{(i)} - \mathbf{H}_{\text{ave}}^{(i)} \) is the matrix denoting the channel fluctuation from the average. From the structure of \( \mathbf{H}_{\text{ave}} \) and \( \mathbf{H}_{\text{var}} \), it is obvious that \( \mathbf{y}_1^{(i)} \) is the time-domain desired term without ICI and IBI, and \( \mathbf{y}_2^{(i)} \) is the time-domain ICI component due to channel variations.

By taking the \( N \)-point FFT of the received vector, we demodulate the signal and obtain the output

\[
\mathbf{Y}^{(i)} = \mathbf{F}\mathbf{y}^{(i)} = \mathbf{Y}_1^{(i)} + \mathbf{Y}_2^{(i)} + \mathbf{Y}_3^{(i)} + \mathbf{Y}_4^{(i)} + \mathbf{v}_f^{(i)}
\tag{6}
\]

where

\[
\begin{align*}
\mathbf{Y}_1^{(i)} &= \mathbf{F}\mathbf{y}_1^{(i)} = \mathbf{F}\mathbf{H}_{\text{ave}}^{(i)}\mathbf{X}^{(i)} = \mathbf{G}_1^{(i)}\mathbf{X}^{(i)} \\
\mathbf{Y}_2^{(i)} &= \mathbf{F}\mathbf{y}_2^{(i)} = \mathbf{F}\mathbf{H}_{\text{var}}^{(i)}\mathbf{X}^{(i)} = \mathbf{G}_2^{(i)}\mathbf{X}^{(i)} \\
\mathbf{Y}_3^{(i)} &= \mathbf{F}\mathbf{y}_3^{(i)} = \mathbf{F}\mathbf{B}^{(i)}\mathbf{H}^H\mathbf{X}^{(i-1)} = \mathbf{G}_3^{(i)}\mathbf{X}^{(i-1)} \\
\mathbf{Y}_4^{(i)} &= \mathbf{F}\mathbf{y}_4^{(i)} = -\mathbf{F}\mathbf{A}^{(i)}\mathbf{H}^H\mathbf{X}^{(i)} = \mathbf{G}_4^{(i)}\mathbf{X}^{(i)}
\end{align*}
\tag{7}\tag{8}\tag{9}\tag{10}
\]

and \( \mathbf{v}_f^{(i)} = \mathbf{F}\mathbf{v}^{(i)} \).

### III. INTERFERENCE CANCELLATION SCHEME

In this section, we first propose ICI mitigation algorithms in the frequency domain. We assume that the CP length is greater than the channel memory, hence the IBI term \( \mathbf{y}_3^{(i)} \) and ICI term \( \mathbf{y}_4^{(i)} \) in (2) vanish.

#### A. FIR-MMSE FEQ

For simplicity, we drop the block index superscript \( i \) and take the FFT of both sides of (3) resulting in

\[
\mathbf{Y} = \mathbf{F}\mathbf{y} = \mathbf{F}\mathbf{H}^H\mathbf{X} + \mathbf{v}_f = \mathbf{G}\mathbf{X} + \mathbf{v}_f
\tag{11}
\]

If the channel is time-invariant, \( \mathbf{H} \) is circulant and \( \mathbf{G} \) becomes a diagonal matrix. Hence \( \mathbf{G}^{-1} \) is also a diagonal matrix and a one-tap FEQ is optimal. However, for a doubly-selective channel, \( \mathbf{H} \) is not circulant and \( \mathbf{G} \) is not diagonal. The conventional linear Minimum Mean Square Error (MMSE) block estimator is given by

\[
\hat{\mathbf{X}} = (\mathbf{G}^H\mathbf{G} + \sigma^2\mathbf{I}_N)^{-1}\mathbf{G}^H\mathbf{Y}
\tag{12}
\]

where \( \sigma^2 \) is the noise variance and \( \mathbf{I}_N \) is the \( N \times N \) identity matrix. Equation (12) requires an \( N \)-by-\( N \) matrix inversion which requires \( O(N^3) \) complex multiplications, making it impractical for large \( N \) (as in the DVB-H standard). Hence a reduced-complexity FEQ algorithm is needed.

To explain our proposed low-complexity FEQ method, we study the structure of the frequency-domain channel matrix \( \mathbf{G} \). Due to the time-varying nature of the channel, \( \mathbf{G} \) is not diagonal, but can be approximated as a banded matrix as illustrated in Fig. 3. This matrix has non-zero elements only at \( 2D+1 \) main diagonals and two \( D \times D \) triangular matrices in the top-right and bottom-left corners. This banded structure indicates that ICI energy is concentrated in adjacent subchannels, which will be used to simplify our FEQ algorithm.

For the \( m \)-th subchannel, the proposed FEQ algorithm uses an FIR filter applying MMSE criterion (FIR-MMSE FEQ) which is based only on the following \( (2D+1) \times (4D+1) \) submatrix of \( \mathbf{G} \), as is shown in Fig. 3:
The $Q$-tap ($Q = 2D + 1$) FIR-MMSE FEQ is given by

$$w_m = g_m^H (G_m G_m^H + \sigma^2 I_Q)^{-1}$$

where $g_m = G_m(:, 2D + 1)$ is the middle column of $G_m$.

Hence, the detector output for the $m$-th subchannel is

$$\hat{X}_m = w_m Y_m$$

where $Y_m = [Y_{(m-D)N}, \ldots, Y_{(m+D)N}]^T$.

The resulting MMSE for the $m$-th subchannel is $MSE_m = 1 - w_m g_m$. Instead of detecting all subchannels of an OFDM block simultaneously using an $N \times N$ matrix inversion, our proposed FIR-MMSE detects each subchannel individually, requiring $N$ matrix inversions each of size $Q \times Q$ (where $Q \ll N$), which significantly reduces the complexity.

As an example, even for a small FFT size of $N = 64$ and assuming cubic complexity for matrix inversion, a full MMSE FEQ requires $O(N^3)$ = 262144 operations, while a 3-tap FIR-MMSE FEQ requires only $O(NQ^3)$ = 1728 operations. Larger complexity reductions are achieved for larger $N/Q$.

Choosing the number of FIR taps $Q$ represents a tradeoff between complexity and performance.

**B. Ordered Successive Interference Cancellation**

As we will show in Fig 4, an error floor is observed from the FIR-MMSE FEQ performance curves at high SNR. Iterative symbol detection is a common method used to improve performance [1] [2] [3]. In this paper, we combine our FIR-MMSE with Successive Interference Cancellation (FIR-MMSE-SIC), which also utilizes the available time diversity while suppressing the residual interference.

We first detect $i$-th subchannel with the minimum MMSE (based on the first FIR-MMSE FEQ stage), and make a hard decision on subchannel $X_i$. Then we subtract its interference from the received signal as follows

$$Y_{\text{new}} = Y_{\text{old}} - G(:, i) \hat{X}_i$$

where $\hat{X}_i$ is the $i$-th detected subchannel. As long as this hard decision data is correct, the new vector $Y_{\text{new}}$ suffers from less interference. The column vector $G(:, i)$ is then replaced by an all-zero vector. The updated channel matrix $G(:, i)$ is then used to find the new minimum MMSE, and the same procedure is repeated until all subchannels are detected. The detailed detection procedures are given as follows.

1) For each subchannel $k$, calculate equalizer coefficient vector: $w_k = g_k (G_k G_k^H + \sigma^2 I_Q)^{-1}$.

2) Choose $i = \arg \min_k \text{MMSE}_{k} = \arg \min_k (1 - w_k g_k)$.

3) Hard decision: $\hat{X}_i = w_i Y_i$.

4) Subtract its interference from $Y : Y = Y - G(:, i) \hat{X}_i$.

5) Remove $i$-th column from $G : \hat{G}(:, i) = [0, \ldots, 0]^T$.

6) $j = j + 1$.

7) If $j < N$ go to step 1; else, end of loop.

The ordering influences the complexity and overall performance of the algorithm. We may run the ordering procedure only once during initialization to reduce complexity. These different options will be compared in subsection V.

**C. A Novel Hybrid IC Architecture**

IBI and ICI arise for frequency-selective channels when the CP length is smaller than the delay spread ($\gamma_3$ and $\gamma_4$ terms in (2) respectively). To deal with this problem, we implement FIR equalization in the time-domain before the FFT operation at the receiver to shorten the CIR length to the CP length.

This results in a hybrid time-frequency IC architecture. In this paper, we implement the TEQ algorithm proposed by Melsa [6] which minimizes the energy outside a length $\nu + 1$ window (called the ‘wall’) of the effective channel $c = h * w$, while constraining the energy in the desired window of $c$ to one.

Melsa’s algorithm was originally applied to time-invariant channels where it mitigates IBI and ICI caused by insufficient CP. We will combine it with our FEQ algorithms resulting in a two-stage equalization structure:

- In the time domain, using an FIR TEQ according to Melsa’s algorithm to shorten the channel and minimize the effect of IBI and ICI due to insufficient CP.
- In the frequency domain, we use an FIR-MMSE FEQ with $Q$ taps per subchannel to eliminate the residual ICI due to channel variation. At high Doppler, we can combine it with SIC to further improve the performance.

**IV. CHANNEL ESTIMATION**

Channel estimation is a challenging problem in OFDM systems with severe Doppler. In this paper, we use the channel estimation algorithm in [9], where equispaced groups of pilot tones are inserted on the FFT grid. We first parameterize the channel matrix $H$ by using a small number, $M$, of its rows. The channel estimation problem then amounts to estimating $ML$ parameters which are grouped in an $ML \times 1$ vector

$$\hat{\mathbf{h}} := [\mathbf{h}^{T}_{(m+1)}, \ldots, \mathbf{h}^{T}_{m(M)}]^T$$

where $\mathbf{h}^{(i)}_{m}$ is the $m(i)$-th row of the channel matrix $H$. Assuming that there are $P$ pilots with $P \geq ML$, which are placed at subchannels $p(1), \ldots, p(P)$, a $P \times ML$ system of linear equations is formed [9]

$$\begin{bmatrix}
  y(p(1)) \\
  \vdots \\
  y(p(P))
\end{bmatrix} = \begin{bmatrix}
  \hat{\mathbf{b}}^{p(1),p(1)} \\
  \vdots \\
  \hat{\mathbf{b}}^{p(P),p(P)}
\end{bmatrix} \hat{\mathbf{h}} + \begin{bmatrix}
  v(p(1)) \\
  \vdots \\
  v(p(P))
\end{bmatrix}$$

or $Y(p) = \hat{\mathbf{B}}(p) \hat{\mathbf{h}} + \mathbf{v}(p)$ and $\hat{\mathbf{b}}^{p(1),p(1)}$ is related to the pilot symbols and defined in [9]. Calculating $\hat{\mathbf{h}}$ as the least-squares solution of this system of linear equations, we have

$$\hat{\mathbf{h}} = \mathbf{B}^+ (p) Y(p)$$.
The Doppler frequency is normalized to 10% of subchannel spacing unless otherwise stated.

We first test the performance of the FIR-MMSE FEQ algorithm with and without SIC presented in subsections III-A and III-B, respectively, for a 5-tap time-varying channel with sufficient CP \((L_g = L_c = 5)\). Fig. 4 shows BER curves for different numbers of FEQ taps per subchannel. As expected, the performance improves as the number of taps increases with substantial gains achieved (over single tap case). We observe an error floor for the FIR-MMSE FEQ case which is eliminated when it is combined with SIC. We also note that a 3-(or 5-)tap FIR MMSE-SIC’s performance is close to that of full N-tap case at much lower complexity.

Instead of ordering MMSE’s of each subchannel during each iteration, we could do the ordering only once during initialization. We also tested SIC performance without any ordering, i.e., detect the subchannels sequentially. Fig. 5 shows that MMSE-ordered SIC significantly outperforms the unordered case. Also, one-time MMSE ordering during initialization performs almost as good as ordering during each iteration for a wide range of SNR, but has a much lower complexity, hence it is attractive especially when \(N\) is large.

In the rest of this paper, we use MMSE ordering during each iteration.

Fig. 6 illustrates the effects of channel estimation which is performed using the iterative data and channel estimation method described in Section IV with \(M = 2\) (only the top and bottom rows of \(\mathbf{H}\) are estimated and the rest of the rows are obtained using linear interpolation). Initially, we use 16 pilots equispaced onto the FFT grid (i.e. subchannels \([6:9, 22:25, 38:41, 54:57]\) are pilot tones). Then, all detected subcarriers are used as pilot tones for channel estimation. From Fig. 6, we observe a significant reduction in the error floor even with one iteration. This performance improvement is even larger for a mild Doppler of 5% as shown in Fig. 7.

Next, we present performance results of joint hybrid time/frequency equalization for doubly-selective channels at a normalized Doppler frequency of 5%. Fig. 8 compares the BER performance using Melsa’s TEQ, combined with a 3-tap MMSE FEQ with and without SIC. Melsa’s TEQ uses 20 taps to shorten the channel from 12 taps to 5 taps. We observe that 3-tap MMSE-SIC FEQ outperforms TEQ and 3-tap MMSE FEQ joint detection, but it still has an error floor. To eliminate this error floor, we combine TEQ with MMSE-SIC FEQ.

Finally, we tackle the more challenging problem of doubly-selective channel estimation. Initially, we use 32 pilots to obtain a rough estimate of the 12-tap channel. Then, we compute Melsa’s TEQ coefficients based on the average channel (i.e. middle row of the channel matrix) and use this TEQ to shorten the channel to 5 taps at all times (i.e. all rows of the channel matrix). A more accurate estimation of the shortened 5-tap channel follows. Finally, a 3-tap MMSE-SIC-TEQ is used for detection for the shortened channel. Fig. 9 shows that this scheme performs reasonably well at a low Doppler of 1%. However, we found that for higher Doppler, the performance is not satisfactory since the TEQ, which is computed based
on the average (middle) row of the channel matrix, is not able to effectively shorten all rows, resulting in significant residual interference.

VI. CONCLUSIONS

In this paper, we propose a low-complexity FIR-MMSE FEQ with only few taps per subchannel which achieves good performance at high Doppler when combined with an SIC scheme. We also design a hybrid time/frequency domain equalization architecture for OFDM on doubly-selective channels. The ICI and IBI due to short CP is suppressed by the TEQ before the FFT operation. Then, the FIR-MMSE-SIC FEQ is used to mitigate the residual ICI due to mobility. Finally, an iterative channel and data detection scheme is shown to further improve performance.

REFERENCES


