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Pulsation modes of Mira stars and questioning of linear modelling: indications from HIPPARCOS and the LMC*

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Abstract. Thorough discussion of the previous theoretical works on the pulsation of Long Period Variables leads us to the conclusion that the mode periods predicted by linear models must significantly differ from the reality, and that, if one nevertheless relies upon such a modelling, it is at least necessary to change the mixing length. The hypothesis that the so–derived mode periods be reasonably reliable is supported by confrontation between a model grid based on these grounds and the luminosities of LPVs in the Large Magellanic Cloud and in LMC clusters, as well as the luminosities and effective temperatures of Miras in the solar neighbourhood. A wide majority of the Miras appear probably pulsating on the first overtone, and the sample Semi–Regulars on the second. However, a significant proportion of Miras seem to be fundamental pulsators. Individual masses are derived. A few stars are probably undergoing hot bottom burning, while two seem to have a peculiar dust envelope.

Key words: stars: AGB – stars: oscillations – stars: fundamental parameters – Magellanic Clouds

1. Introduction

The problem of the identity of the predominant pulsation mode of LPV stars, especially the Miras, has been debated for decades and is still unresolved. One way to conclude is to analyze the light curves of many stars on the long term and to identify several probable modes, whose periods are then compared to a grid of theoretical models. This became possible only recently and was performed for a few stars by Barthès & Tuchman (1994) and Barthès & Mattei (1997a, b, c), using linear pulsation models. Pulsation on the first overtone, with a few other excited modes, appeared the most likely. On the other hand, various attempts at determining the predominant pulsation mode have been done by comparing the main pulsation period and observationally estimated luminosities or radii of many stars to the predictions of linear pulsation models (Willson 1980, Robertson & Feast 1981, Wood & Zarro 1982, Ostlie & Cox 1986, Wood & Sebo 1996, Van Leeuwen et al. 1997, Van Belle et al. 1997). Results were contradictory, as well as those derived from photospheric velocities (Hill & Willson 1979, Willson 1982) and accelerations (Tuchman 1991).

Apart from important observational uncertainties on the luminosities and radii or effective temperatures, even for nearby Miras, numerous sources of inaccuracy pertaining to the theory burden on all published results: for example, nonlinear effects, coupling with the stellar wind at the outer boundary, turbulent pressure and coupling between convection and pulsation are liable to significantly affect the periods and growth rates of the modes at given mass, luminosity, temperature and metallicity. Up to now, it has been impossible to build fully reliable dynamical models, so that we are still compelled to use linear models, assuming perfect reflection at the outer boundary. As a consequence, the debate is still far from being closed.

The primary purpose of this paper is to examine to which extent and under which conditions one may still rely (as usually done) upon linear models so as to fit the observed distributions of the luminosity and temperature with respect to the main period, and to derive the identity of the predominant pulsation mode, together with mass estimates. The main theoretical uncertainties are first reviewed in Sect. 2. This leads to the conclusion that their overall effect is important, but might nevertheless be moderate enough to permit the use of grids of linear models. If this is actually the case, it appears indispensable to allow the mixing–length parameter to depart from the value usually adopted for other types of stars, and possibly to vary along the AGB: in this empirical way, it is accounted (at least partially) for the period and temperature shifts resulting from the simplified input physics of the models. We present, in Sect. 3, the basics of such a model grid, with a mixing length chosen in agreement with the results of lightcurves analysis. In Sects. 4 & 5, the grid is confronted with the period–luminosity distribution of Miras and Semi-Regulars found in the Large Magellanic Cloud and in LMC clusters, as well as to the individual luminosities and effective temperatures of 22 Miras in the solar neighbourhood. The results are discussed in Sect. 6. The grid fits both the LMC and galactic data with masses consistent with evolutionary calculations, the first overtone being the most probable predominant mode for a wide majority of Miras. However, a significant part seem to be fundamental pulsators. Semi–Regulars in the sample

* Based on data from the HIPPARCOS astrometry satellite

probably pulsate on the second overtone. Masses are derived for the nearby Miras.

Indication for hot bottom burning (high mass) is found for at least three stars. Discrepancy between luminosity and temperature suggests that two stars have peculiarly close, dense dust shells, possibly related to the thermal pulse cycle.

The overall agreement between the models and the observations suggests that nonlinear effects might be not as dramatic as suggested by some recent hydrodynamical calculations.

2. The theoretical issue

2.1. Outer boundary

Though usually adopted, perfect reflexion on an isothermal atmosphere does not constitute an appropriate assumption for extended envelopes undergoing a strong stellar wind. Part of the pulsation energy is transmitted throu- gh the outer layers by shock waves, leading to a damping of the pulsation. According to Pijpers (1993), coupling of the pulsation with the strong stellar wind results in the apparition of new modes, including many with periods longer than or intermediate between the classical linear modes. Most of the modes are strongly damped. The effect of the stellar wind onto the modes corresponding to the classical solutions is a significant damping. Their linear periods and period ratios, in these calculations, are larger than in classical models, mainly because the (partial) reflection of the waves occurs at the sonic point (which depends on the frequency) rather than the effective radius or any other fixed boundary. The effects strongly depend on the mass–loss rate: as the latter increases (thus as the star evolves along the AGB and as the pulsation amplitude increases), the sonic point moves inward, which must reduce the linear periods and period ratios, whereas the damping increases. On the other hand, the pulsation–induced extension of of the envelope probably leads to lower effective temperatures, and this effect should increase along the AGB.

2.2. Convection/turbulence

Convection is another major source of uncertainty. It is usually treaten by means of the mixing length theory. In the presence of the pulsation, three cases may be distinguished, according to the ratio of the pulsation period P to the mean eddy lifetime $\tau = \Lambda/\bar{v} \sim t_{ff}(t_{ff}/t_K)^{-1/3}$ (where Λ and $\bar{v}$ are respectively the mixing length and the mean convective velocity, and $t_{ff} \sim G^{-1/2}M^{-1/2}R^{3/2}$ and $t_K \sim GM^2L^{-1}R^{-1}$ are the free–fall and Kelvin times respectively) [Cox & Giuli 1968]: if $P \ll \tau$, convection may be considered frozen–in, as was assumed by Ostlie & Cox (1986). If, on the contrary, $P \gg \tau$, convection instantly adjusts itself to the variation of the local conditions induced by the pulsation, as assumed by Tuchman et al. (1978, 1979), Perl & Tuchman (1990), Barthès & Tuchman (1994) and Barthès & Mattei (1997a, b, c). In fact, the mean eddy lifetime is of the order of the pulsation period (typically ≈ 0.3 times the period of the linear fundamental mode [Ostlie & Cox (1986)]), which means that the convective velocity or flux is not only coupled to the pulsation, but also significantly phase–lagged, and its variation amplitude is lower than in the case of instantaneous adaption to pulsation (Cox & Giuli 1968). Nonlocal effects, due to the irruption of material from nearby eddies into the considered zone, have similar smoothing consequences and should be taken into account for nonlinear calculations (see below).

Up to now, only a few attempts have been made at dealing with time–dependent convection in Miras by using various simplified formulae, such as in Keeley (1970a, b, c), Wood (1974), Fox & Wood (1982), Balmforth et al. (1990), Cox & Ostlie (1993) or Wood & Sebo (1996). Consistently with these models, one can straightforwardly, but qualitatively, guess the effects of phase–lagged convection: in the ionization regions (covering most of the convection zone), convection allows the outward transfer of thermal energy which would otherwise be dammed up by the increasing opacity during the compression phase. In other terms, the κ-mechanism gets somewhat counteracted. Thus, reduced variations of the convective flux over the pulsation cycle, with respect to the instantaneously adapting case, result in a more nonadiabatic pulsation inside the ionization regions (representing most of the convection zone). Increased nonadiabaticity leads to a shorter linear fundamental mode period and smaller period ratios of the fundamental, first and second overtone periods, while the linear growth–rates increase (hence a larger pulsation amplitude). The fundamental mode is the most sensitive.

As the luminosity (as well as the pulsation period) increases along the AGB, while the mass and temperature decrease, the phase–lag may be expected to decrease, since $P/\tau \propto QM^{-1}L^{1/2}T^{-6}$ significantly increases (Q being the "pulsation constant" corresponding to the predominant mode). Hence, there is a reduced shift of the linear periods and period ratios.

According to Ostlie & Cox (1986) and Balmforth et al. (1990), taking the turbulent pressure into account makes it necessary to adjust the mixing length parameter λ (ratio of mixing length to the pressure scale–height) so as to keep the star on the AGB core mass—luminosity relation without changing the fundamental parameters. In models omitting time–dependent convection, Ostlie & Cox (1986) found an increase of the linear fundamental period and period rates generated by turbulent pressure, whatever the sign of the λ change. However, according to calculations including time–dependent convection (Balmforth et al. 1990, Cox & Ostlie 1993), turbulent pressure makes the fundamental period and the period ratios significantly decrease ; it has a destabilizing influence, and the growth–rate of the first overtone strongly increases with respect to the fundamental.

Turbulent viscosity, though moderate, tends to damp the pulsation (and to make the star slightly expand), which may be taken into consideration, together with turbulence energy.

2.3. Nonlinear effects

Lastly, nonlinear effects must be taken into account. They are of two types: mechanical and thermal. The nonlinear convective effects include some important phenomena that linear models can by no means approximate, such as the periodic onset/offset of convection at the outer edge of the convective zone. Nonlinear effects are expected to reach their full efficiency after about a thermal (Kelvin) timescale, which may mean from a few years up to a few centuries. The time–averaged convective flux differs from the static value, which necessarily results in some change of structure and mode periods. Growth– or switching–rates must significantly change, too. Due to computing time requirements, the long–term nonlinear behaviour of Miras had not been really explored until recently. Shorter–term behaviour (over a few ten years) has been more extensively studied. These nonlinear effects may be summarized as follows:

According to Wood (1974), Tuchman et al. (1979), Perl & Tuchman (1990), and Tuchman (1991), over a few ten years, nonlinear pulsation usually makes the star globally shrink ($\overline{R}_m^{\rm dyn} < R_m^{\rm stat}$, $\overline{T}_{\rm eff}^{\rm dyn} > T_{\rm eff}^{\rm stat}$) by about 10 % (though the star may expand in some cases). The period of the fundamental mode and the period ratios slightly decrease (or increase, if the star expands), then stabilize. Among the fundamental and first overtone modes, the predominant one is the latter if its growth–rate *per day* significantly exceeds the one of the fundamental (which may not be the case of the growth–rate *per period*). When the fundamental mode dominates, the pulsation is usually found unstable.

According to long–term hydrodynamical calculations of models having Kelvin timescales of 18 through 87 years and $t_K/P_0 = 9$ through 219 (Yaari & Tuchman 1996), after a first stage of pure overtone pulsation (lasting about $4t_K$), the fundamental mode starts growing, which leads to a mode switching. Then, the period of the star decreases, *as well as* the mean effective temperature (the star expands by a few %). For models with $t_K \lesssim 34$ yrs and $t_K/P_0 \lesssim 35$, the period change is very strong (up to 35 % less with respect to the linear fundamental mode), while the temperature decreases by 10 %. The pulsation gets finally stabilized on a new period lying between the original fundamental and first overtone modes. The total process has taken about 10 times the Kelvin timescale. The joint decrease of period and temperature corresponds to a change of the inner structure of the star (entropy redistribution), due to thermal nonlinear effects. As noticed by Barthès & Mattei (1997a), this unusual phenomenon might be related to the existence of multiple resonances between the linear modes, and the final period is very close to a combination of the resonant linear modes, i.e. the pulsation may still have significant projections onto them.

By the way, a similar, drastic period decrease had already been found by Keeley (1970b,c) in a model using very similar input parameters as the main model presented by Yaari & Tuchman. However the total time required for reaching the new state was only about a Kelvin timescale and the new mean radius and temperature were respectively smaller and higher than their static values, as in the shorter–term calculations. In contrast, Wood (1974) did not observe any such phenomenon over a similar time–span in a model using very similar parameters: the nonlinear periods of the fundamental and first overtone remained close to the linear ones, and the pulsation was a mixture of both modes, the latter being predominant.

The main difference between these models lies in the time–dependent and non–local treatment of convection. Yaari & Tuchman assumed instantaneously adapting convection and represented the spatial coupling of convective zones *lying within a mixing length of* (or at least next to) each other by means of a weighted average of the *superadiabatic gradients*, following the recipe of Spiegel (1963), which uses a smooth weighting function, nonlinear with respect to the radial variable. On the other hand, Wood (1974) modeled the coupling of zones *next to each other* by using a weighted average of the convective *velocities* — with a fully linear weighting function — and time–dependence was modeled by means of a linear phase–lag formula applied to the velocities, too. Lastly, the modeling code of Keeley (1970a, b, c) used similar phase–lag and spatial averaging formulae applied to the convective *flux*.

On the other hand, Wood (1995), using the same code as in 1974, performed hydrodynamical calculations of a model ($t_K \approx 26$ years, $t_K/P_0 \approx 45$) pulsating on the fundamental mode, whose convergence was forced by using a huge artificial viscosity during the first 70 years. He obtained a significant *expansion* of the star, which is consistent with Yaari & Tuchman (1996), but the period *increased*. The mode period and the radius remained close to the values given by the static model. This result, and the similar ones obtained from first overtone models, are consistent both with the previous works which Tuchman was involved in and with Cox & Ostlie (1993), who used a similar scheme as Wood, whithout the huge artificial viscosity but using a smoother (quadratic) time–dependence formula and also including turbulent pressure and viscosity.

Obviously, differences in the treatment of convection/ turbulence have important, contradictory effects onto the structure and pulsation of the star through the highly nonlinear behaviour of convection. As noticed by Wood (1974), Keeley's choice leads to convergence difficulties, perhaps due to the strong decoupling from the local and instantaneous variables. The nonlocal scheme chosen by Yaari & Tuchman might be the most realistic and liable to yield stable nonlinear pulsation. On the other hand, their simplistic treatment of time–dependence (despite the advantage of working on superadiabatic gradients) probably amplifies nonlinear instabilities in the long term. One must also remember that the sensitivity of the nonlinear pulsation to the numerical scheme is probably enhanced by the multiple resonances noticed by Barthès & Mattei (1997a).

Lastly, it is clear from Sect. 2.2 that models including turbulent pressure together with time–dependent convection have a much more excited linear first–overtone. This is likely to significantly change the nonlinear behaviour of the models: in those calculated by Yaari & Tuchman, the initial switch to the fundamental mode could be more difficult ; similarly, Keeley's model might start pulsating on the first overtone, rather than the fundamental mode.

Concluding, it is by no means evident that the surprising results of Yaari & Tuchman (1996) actually represent the behaviour of real stars. It is still a likely hypothesis that the actual pulsation remains close to a linear mode or a superposition of linear modes (though with significant period shifts).

2.4. Summary

Compared to the most classical models, the openness of the outer boundary and the turbulent viscosity tend to increase the period of the fundamental mode and the period ratios of the modes, while reducing the growth rates. Because of the extension of the envelope, effective temperature lower than the static value may be expected. Period shifts due to the outer boundary tend to decrease as the star evolves along the AGB.

On the other hand, phase–lagged time–dependent convection and turbulent pressure, taken altogether, tend to reduce the fundamental period, the period ratios, and to increase the effective temperature. The linearly predominating mode (the first overtone in most cases) gets more excited with respect to the others. These effects decrease as the star evolves along the AGB.

Nonlinear effects, if we assume that they remain moderate, lead to converse variations of the fundamental periods and period ratios on the one hand, and effective temperatures on the other hand. One might expect this shift (whatever its sign) to get larger as the star evolves along the AGB toward larger amplitudes ; the concomitant reduction of t_K and t_K/P may enhance or reduce this effect, according to its sign.

Evaluating the overall result would require comprehensive dynamical codes which are not yet available. However, one can make this important remark : those joint, converse shifts of fundamental period and period ratios on the one hand and temperatures on the other, are similar (at least qualitatively) to the effect of changing the ratio λ of the mixing length to the pressure scale–height at fixed M, L and Z.

On the other hand, it seems very likely that the overall difference between the results of classical and more sophisticated models evolves along the AGB in the LPV domain. However, the respective evolutions of the various period and temperature shifts often contradict each other, so that the overall change along the AGB might be relatively small. Anyway, this cannot be decided at the very outset.

These considerations lead us to the conclusion that, if one leaves aside the possiblity of drastic changes involved by long–term thermal effects pointed out by Yaari & Tuchman (1996), and thus attempts at determining the predominant pulsation mode of Miras by comparing observational estimates of the fundamental parameters of some sample of stars to linear models, one may assume at the very outset *neither* a fixed mixing length parameter, *nor* the same value as for non–AGB or non–LPV stars, as Wood & Sebo (1996) and others did. One *must* adopt some ”effective” value (which, of course, no more represents the convection efficiency alone), so as to empirically minimize the errors resulting from the simplified input physics.

This is confirmed by the small λ derived from the confrontation between a linear model grid and the results of a Fourier and wavelet analysis of the lightcurves of four Miras (Barthès & Tuchman 1994, Barthès & Mattei 1997a, b, c).

In the next sections, we reexamine the issue by fitting to observational data some theoretical period—luminosity (or temperature)—mass—metallicity relations based on a fixed λ law. The degree of compatibility of this model grid with the observation is expected to tell us whether long–term nonlinear effects actually remain moderate.

3. Modelling

As in Barthès & Tuchman (1994) and Barthès & Mattei (1997a), our grid of linear pulsation models is based on the code of Tuchman et al. (1978). We use the OPAL opacity tables, complemented by molecular opacities of Alexander & Ferguson (1992, 1994). Convection is treated according to the mixing length formalism of Cox & Giuli (1968), i.e. without turbulent pressure, viscosity nor energy. Convective velocity is assumed to adjust instantaneously. The outer boundary, where perfect reflection is assumed, is set at optical depth $\tau = 0.01$. The core–mass—luminosity relation of Paczyński (1970) is enforced so as to make the star belong to the AGB. Using this code, we have made model grids for two metallicities ($Z = 0.02$ and $Z = 0.001$; $X = 0.7$), three values of the mixing length parameter ($\lambda = 1$, 1.5 and 2), masses ranging from 0.6 to 5 $M_\odot$ and luminosities from 1000 to 50000 $M_\odot$.

In the preceding section, we have shown that the real behaviour of the pulsating star differs from this classical scheme by an amount that may vary along the AGB in a way that cannot be guessed at the very outset, but might be, at least partially, accounted for by varying the mixing length parameter. Low values of λ ($\lesssim 1.3$) were found necessary in studies using this code (Barthès & Tuchman 1994, Barthès & Mattei 1997a, b) for explaining the power spectra and scalograms of o Cet (period 332 days), R Leo (315 d) and S CMi (334 d), while fitting their observed luminosities and temperatures. In calculations by Wood & Sebo (1996), a high value ($\lambda = 2.5$) was required for fitting the base of the AGB, which lies below the short–period boundary of the LPV domain, and where the effective temperature is much better defined than on the rest of the branch. With our code and opacities, the same calibration is obtained with the value $\lambda = 1.63$ which was found appropriate for the Sun and for RGB stars (Bressan et al. 1993, Claret & Giménez 1992, Schaller et al. 1992). It thus seems reasonable to assume for our model grid, despite all other uncertainties, that the adopted mixing length for an LPV star must be lower than 1.6 and decreases along the AGB. That is, λ must decrease as the period of any given mode increases (preferably the fundamental or first overtone, since the higher modes are more sensitive to the outer boundary conditions). Considering the numerous, recent observational evidences for a predominating first overtone (Barthès & Tuchman 1994, Haniff et al. 1995, Feast 1996, Barthès et al. 1997a, b, c, Van Leeuwen et al. 1997), we assumed a linear $\log \lambda$—$\log P_1$ relation, with $\lambda = 1.2$ at 330 days and $\lambda = 1.3$ at 80 days (the beginning of the Mira strip). As will be shown in the next section, this provides an excellent fit to the Mira strip in the period–luminosity diagram of the Large Magellanic Cloud.

Table 1. Periods, mean apparent magnitudes, parallaxes (or statistical distances), and effective temperatures at phase φ of nearby Miras (see text for the meaning of the temperature source–codes).

Star	P [d]	$\overline{m}_{\rm bol}(\pm 0.15)$	$\varpi \pm \sigma_\varpi$ [mas]	$T_{\rm eff}$ [K]	φ	$m_{\rm bol}(\varphi)$	Source
R Aql	284	2.34	4.73 ± 1.19	2539 ± 113	0.31	2.25	K
R Aqr	387	2.26	5.07 ± 3.15	2568 ± 136	0.34	2.00	K
R Car	309	1.74	7.84 ± 0.83				
S Car	149	4.65	2.47 ± 0.63				
R Cas	430	1.40	9.37 ± 1.10	2300 ± 150	$\overline{\rm max, min}$		BB
				2625 ± 150	0.07		BB
				2580 ± 190	0.04	1.1	RI
				2954 ± 174	0.81	1.88	K
T Cas	445		0.59 ± 1.07	2795 ± 128	0.30	1.97	K
			$d = 570 \pm 31$ pc				
R Cen	546	2.38	1.56 ± 0.84				
T Cep	388	1.50	4.76 ± 0.75	2359 ± 302	0.72	$\overline{m}_{\rm bol}$	RI
o Cet	332	0.70	7.79 ± 1.07	2371 ± 101	0.85	0.65	K
				2381 ± 150	$\overline{\rm max, min}$		BB
				2436 ± 174	$\overline{\rm max, min}$	$\overline{m}_{\rm bol}$	RI
R Cnc	361		$d = 366 \pm 132$ pc		0.14	2.12	
χ Cyg	408	1.39	9.43 ± 1.36	2242 ± 140	0.32	$\overline{m}_{\rm bol}$	RI
R Hor	408	2.22	3.25 ± 1.08				
R Hya	389	0.66	1.62 ± 2.43	2661 ± 213	0.28	$\overline{m}_{\rm bol}$	RI
			$d = 195 \pm 44$ pc				
R Leo	310	0.69	8.81 ± 1.00	2443 ± 121	0.21	0.50	K
				2315 ± 128	0.27	$\overline{m}_{\rm bol}$	RI
R Lep	427	3.45	3.99 ± 0.85				
U Ori	368	2.54	1.52 ± 1.65				
			$d = 378 \pm 112$ pc				
X Oph	329		$d = 322 \pm 145$ pc	3041 ± 160	0.75	1.96	K
				2857 ± 160	0.75	1.96	K
S Peg	319		3.08 ± 2.05	2107 ± 295	0.28	4.24	K
			$d = 449 \pm 136$ pc				
RR Sco	281	2.88	2.84 ± 1.30				
R Ser	356		3.58 ± 1.51	2804 ± 144	0.32	2.83	K
R Tri	267	4.04	2.51 ± 1.69				
			$d = 420 \pm 95$ pc				
S Vir	375		$d = 417 \pm 134$ pc	2787 ± 150	$\overline{\rm max, min}$		BB
				2821 ± 050	$\overline{\rm max, min}$	3.47	H'

A good fit would anyway be obtained by simply assuming a constant value $\lambda = 1.2$.

Under these assumptions, interpolation in the original model grid yields a new one in which the mixing length is no longer a free parameter. For metallicities other than $Z = 0.02$ and 0.001, a linear $\log P$—$\log Z$ relation will be assumed at given (L, M).

4. Period–luminosity distribution

The period–dependence of the mixing length parameter results in slightly smaller slopes for the period—luminosity relations at given mass and metallicity. Fig. 1 shows the theoretical lines obtained for the fundamental, first and second overtone modes. The masses are 0.6, 0.8, 1 and 1.5 $M_\odot$ for the first overtone and, for sake of clarity, 0.8, 1 and 1.5 $M_\odot$ for the other modes. The assumed metallicity is $Z = 0.008$.

We show in the same figure the P—$M_{\rm bol}$ fit line corresponding to the LPVs of the Large Magellanic Cloud with periods shorter than 420 days (the reason for this cut–off is that, as the convection zone comes deeper, stars with longer periods are likely to depart significantly from the standard core–mass—luminosity relation, as shown by the sudden change of slope of the P—L relation). This relation is:

$$M_{\rm bol} = -3 \log P + 2.81$$

(Feast et al. 1989), assuming an LMC distance modulus of 18.54. It is nearly indistinguishable, in the relevant period range, from the one derived by Hughes & Wood (1990) from a much larger sample, including carbon stars. The dashed lines on both sides correspond to ± 0.2 mag., the standard residual deviation about the fit line [the larger residual σ of Hughes & Wood (1990) includes variability effects, since the magnitudes were taken at

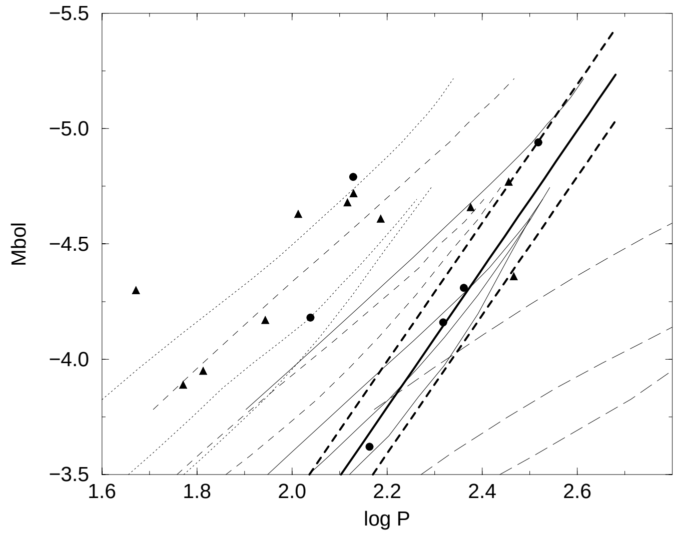

Fig. 1. Periods of the fundamental (long–dashed lines), first (solid), second (dashed) and third (dotted) overtone modes of model AGB stars of masses 0.6, 0.8, 1 and 1.5 $M_\odot$ (for the 1st overtone) or 0.8, 1 and 1.5 $M_\odot$ (other modes), for LMC metallicity. The thick, solid and dashed lines represent the mean ($M_{\rm bol}$—P) relation ($\pm\sigma$) for Miras in the LMC. Circles are LPVs near NGC 1850 and triangles are LPVs near NGC 2058/65.

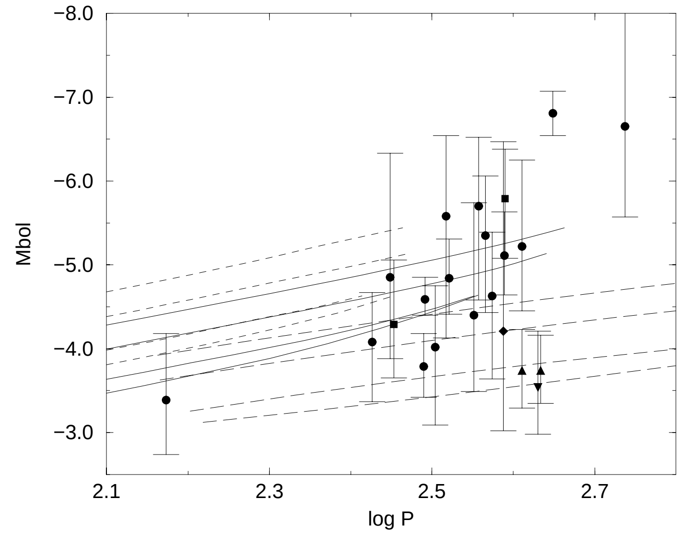

Fig. 2. Periods of the fundamental (long–dashed lines), first (solid), and second (dashed) overtone modes of model AGB stars of masses 0.8, 1, 1.5 and 2 $M_\odot$ for near–solar metallicity. Squares are the He–flashing Miras R Aql and R Hya. The diamond is the symbiotic Mira R Aqr. Triangles (up) are R Cas and χ Cyg. The triangle (down) is the Carbon star R Lep. Other Miras are represented by circles.

arbitrary phases]. Individual LPVs observed at arbitrary phases in the LMC clusters NGC1850, NGC2058 and NGC2065 are plotted, too (Sebo & Wood 1995, Wood & Sebo 1996).

As one can see, the first overtone lines fit well the LMC Mira strip and a first set of 7 cluster stars, for masses ranging from 0.6 to about 1.5 $M_\odot$: fortunately, this is the most likely mass range, according to evolutionary calculations (Butcher 1977, Wood 1990, Bertelli et al. 1992). A second, larger–luminosity (or shorter–period) set of 9 cluster stars lies within a second strip, parallel to the classical Mira strip and as broad as it: it probably corresponds to some semiregulars, as confirmed by the small amplitude of these stars (Sebo & Wood 1996). This strip stops around 130 days according to the individual data. Between magnitudes -3.5 and -4.8, it fits the second overtone for masses ranging from 0.9 to 1.7 $M_\odot$. Third overtone pulsation, seems possible too (but then, one must account for the accoustic cut–off period).

A single star, with very short period, lies above this second LPV strip: probably a third–overtone pulsating star.

Theoretical relations assuming near–solar metallicity ($Z = 0.02$) are shown in Fig. 2. A metallicity decrease by a factor 20 would result in a luminosity increase by about 0.43 magnitude.

In the same figure are plotted the estimated mean absolute bolometric magnitudes of 22 Miras whose distance is now better known thanks to HIPPARCOS (ESA 1997). In the case of R Leo, as in Van Leeuwen et al. (1997), the satellite–measured parallax was averaged with the ground–based, more precise estimate published by Gatewood (1992). For a few stars having either very imprecise or no usable parallaxes, we have adopted a distance derived from their kinematics and parallaxes (if any) by means of statistical calibration of the absolute magnitudes in the K band or (for T Cas and R Tri) at 1.04 μm, using the LM method of Luri et al. (1996), as explained in Alvarez et al. (1997).

Most apparent bolometric magnitudes were taken from Van Leeuwen et al. (1997). For R Cnc and S Vir, we adopted the values given by Feast (1996). In the case of T Cas, X Oph, S Peg and R Ser, magnitudes around phase 0.25–0.30 or 0.75, i.e. near mean light, were taken from Van Belle et al. (1997). In all cases we have assumed a standard error of ± 0.15 mag. In fact, the values given by Van Leeuwen et al. (1997), which were derived from a blackbody fit to numerous recent (or recently reevaluated) JHKL photometric data, may be systematically underestimated by about 0.1 mag because of H_2O opacity, but also overestimated by 0.1 to 0.2 mag because they do not account for dust emission in the far infrared (Haniff et al. 1995). Van Belle et al. probably did not avoid the latter error by using a bolometric correction *vs.* spectral type relation applied to K magnitudes. All these data are given in Table 1. The derived mean absolute magnitudes may be found in Table 2.

Three stars are denoted by special symbols in Fig. 2: the symbiotic star R Aqr (diamond), which fits the fundamental mode but is also compatible the first overtone, is *a priori* not representative of the Mira family. Neither are R Aql and R Hya (squares) which, according to their slow period drift, are very probably on the stage immediately following a helium shell flash, making them depart toward higher luminosity from the AGB core–mass—luminosity relation adopted in our modelling code (Wood & Zarro 1981). If we nevertheless rely on the model grid, R Aqr is found most probably on the fundamental mode, but also compatible with the first overtone, whereas R Aql and R Hya are definitely first overtone pulsators.

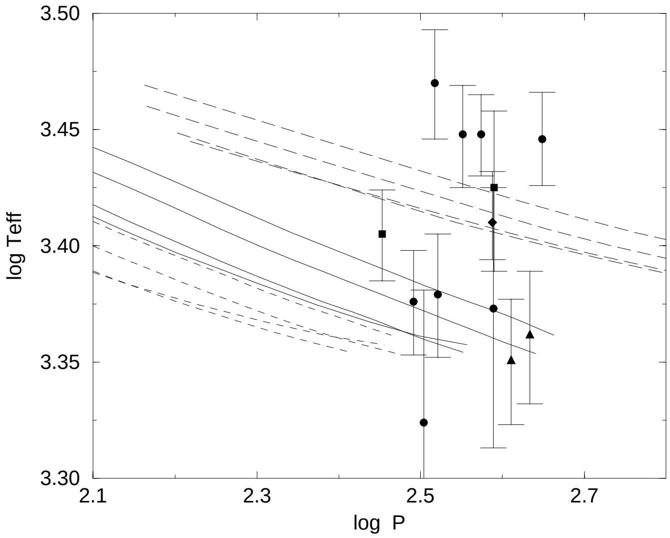

Fig. 3. Effective temperatures of model AGB stars of masses 0.8, 1, 1.5 and 2 $M_{\odot}$ and near–solar metallicity, pulsating on the fundamental, first or second overtone modes. Individual stars are also shown. Same symbols as in Fig. 2.

Among the 19 remaining stars, owing to the 1σ error bars, 14 are compatible with fundamental mode pulsation with masses lower than 2 $M_{\odot}$. T Cep and R Leo are most probably first overtone pulsators with masses $\lesssim 2M_{\odot}$. The 3 remaining stars (T Cas, R Cen and U Ori) are first overtone pulsators with large masses ($\gtrsim 2.5M_{\odot}$) and/or low metallicity.

On the other hand, if we rely on the mid–point values of the magnitudes, 9 out of the 19 Miras are most probably pulsating on the first overtone ($M \approx 0.6$ through 1.9 $M_{\odot}$ for five out of them). One is ambiguous. The 9 remaining stars match the fundamental mode with masses lower than $2.2M_{\odot}$: five fit the first overtone, too, within the error bars, but R Cas and χ Cyg (triangles up), as well as the carbon star R Lep (triangle down) and R Car are definitely too faint to be first overtone pulsators with this metallicity.

At this stage, pulsation on the first overtone may seem a little bit more likely than the fundamental mode for a majority of the sample stars, but no definitive conclusion can be drawn.

5. Effective temperatures

In order to enforce additional constraints onto the theory, observational estimates of the effective temperatures of 13 nearby stars, most of them having a usable parallax too, are listed in Table 1.

In most cases, these temperatures were derived from angular diameters measured in the near–infrared K or H' band ("K" or "H' " source code in Table 1). The reason for this choice, rather than shorter wavelengths, is that, according to photospheric models, the intensity profiles in these bands are close to uniform disk and the correction to apply in order to obtain the effective radius ($r(\tau_{\rm ross} = 1)$) is small : thus the so–derived values of the effective temperature are the least model–dependent. For S Vir, the adopted temperature is the average of the values derived by Feast (1996) from H' diameters near maximum and minimum light ; for the others, it was taken from Van Belle et al. (1997) at a phase corresponding, more or less, to mean light and mean temperature (this holds for o Ceti too, because the nearest light maximum was peculiarly faint, and so the V and bolometric magnitudes at the considered date were close to the usual mean). The scaling factors applied to the K diameters by these authors are the values (0.98 or 1.11) derived from photospheric models taken at the maximum or minimum of the pulsational cycle, according to whether the star is closest to maximum or minimum light. For X Oph we give two values, obtained from the same diameter with the two different scaling factors, because the star is exactly at phase 0.75.

In the case of R Cas, we have adopted a mean temperature derived from JHKL blackbody fits at maximum and minimum light (Bessell et al 1989), and applied the following correction, derived from photospheric models of Bessell et al. (1989) : $T_{\rm eff} \simeq T_{\rm BB} + 2.25(T_{\rm BB} - 2300)$. The error bars are about $\pm 150K$. As a check of the consistency of temperatures yielded by various sources and methods, Table 1 also mentions the effective temperature of R Cas near maximum visual brightness, derived from blackbody fit (accounting for the abovementioned correction) and from angular diameters evaluated by Haniff et al. (1995) at wavelengths 0.833 and 0.902 μm ("RI" code), corrected (as in that paper) according to Model E of Bessell et al. (1995) : they perfectly agree. However, the temperature derived by Van belle et al (1997) from K diameters at about the same phase is significantly higher. This discrepancy will be discussed later.

We have also added the mean corrected blackbody temperatures for o Ceti and S Vir : they perfectly agree with the values derived from K and H' diameters. All temperatures derived from blackbody fits are denoted by the "BB" code in Table 1.

For R Hya, χ Cyg, and T Cep, we used the angular diameters evaluated by Haniff et al. (1995) at 0.833 and 0.902 μm ("RI" code in Table 1) near mean light, corrected according to Model E of Bessell et al. (1995), and we combined them with the accurate mean bolometric magnitudes of Van Leeuwen et al. (1997). Temperatures from the same origin have also been added for o Ceti and R Leo: they perfectly agree with the others.

The agreement of the temperatures derived from the different sources and methods (except for R Cas) makes us confident in the values listed in Table 2 which are, for each star, the average of all estimates near mean light mentioned in Table 1.

Fig. 3 shows, for the same metallicity and masses as in the preceding section, the theoretical $\log T_{\rm eff}$—$\log P$ relations pertaining to each pulsation mode, compared to the observed values. It must be remembered that a metallicity decrease by a factor 20 would make $\log T_{\rm eff}$ increase by about 0.036. The individual temperatures listed in Table 2 are shown, too, using the same symbols as in Fig. 1.

Six or seven stars have definitely too low a temperature for fundamental mode pulsation. Surprisingly, they include χ Cyg and R Cas, which were found definitely too faint for first overtone in Fig. 2.

Six stars have temperatures compatible with fundamental pulsation and incompatible or hardly compatible with first overtone at this metallicity. For three out of them (including R Aqr), fundamental mode had also been derived from the mid–point magnitude. The temperatures of T Cas and R Hya strongly suggest fundamental mode pulsation (with $M \approx 4.5$ and 2.1 $M_\odot$ respectively), but this is incompatible with the magnitude. The other possibility is first overtone pulsation with a relatively high mass : $M \geq 3M_\odot$ for R Hya and $M \geq 5M_\odot$ for T Cas. In the case of X Oph, the temperature is incompatible with the first overtone derived from the magnitude ; fundamental mode pulsation is permitted by the mid–point value of the magnitude and corresponds to a mass $\approx$ 4–5 $M_\odot$.

To these informations, we may add the anomalous early spectral type of R Cen (period 546 d): it is the one expected for a star with about half its period, probably indicating a temperature significantly higher than the average value extrapolated along the Mira strip for its long period (Keenan et al. 1974, Barthès et al. 1992). Owing to its high luminosity, this star probably is a high–mass ($> 3M_\odot$, $\approx 5M_\odot$), or low–metallicity, first overtone pulsator. Actually, in our model grid, the effective temperature at 1 $M_\odot$ and period 270 days is the same as the one at 5 $M_\odot$ and 546 days for solar metallicity.

6. Discussion

Concerning the three LMC clusters, it is worth noting that the maximum period reached by the second LPV (probably SR) strip — about 130 days — coincides with the second overtone period at which, for a star of about 1.5–1.7 $M_\odot$ (the maximum mass of the strip, according to our model grid), the linear growth–rate of this mode starts being exceeded by the first overtone. Moreover, whatever the mass, the linear growth–rates switch at a period which nearly coincides with a 3:2 resonance between the first and second overtones, which is likely to favour the mode switch. To this, we must add the fact that the models fit the Mira strip of the LMC and clusters for first overtone pulsation with masses perfectly consistent with the evolutionary results. Consistently, the linear growth rate of the first overtone is the largest. These consistent evidences support the hypothesis that it is possible to use linear models in the way we have chosen (especially the mixing–length calibration) and that most LMC Miras are pulsating on the first overtone, while the SR of our sample would be on the second overtone.

This conclusion differs from the one of Wood & Sebo (1996), who stated that Miras pulsate on the fundamental mode and semi–regulars on the first or second overtone. This has two explanations:

— first, these authors have adopted the same mixing–length parameter as for non–LPV stars, which leads to more compact envelopes, thus shorter fundamental and first–overtone periods : we have shown that, owing to the uncertainties pertaining to the modelling scheme, this is a fully arbitrary choice ;

— second, the LMC period–luminosity relation that they have adopted is, among the three published by Feast et al. (1989), the one mixing Oxygen stars with a large proportion of Carbon stars, which emit a much larger proportion of their luminosity in the far–infrared : as their bolometric magnitudes were derived by fitting a blackbody to near–infrared fluxes, they were significantly overestimated ; this, combined with the longer period of Carbon Miras, must significantly affect the slope. The P–L relation that we have adopted in this paper was derived by Feast et al. (1989) for Oxygen stars alone, and is nearly indistinguishable from the one of Hughes & Wood (1990), which includes a significant proportion of Carbon stars.

The observational data (L, $T_{\rm eff}$) concerning the 22 nearby Miras and the mass estimates derived from each of them with the help of the pulsation models are listed in Table 2. Metallicity $Z = 0.02$ was assumed. The error bars adopted for the masses represent the mass range involved by the observational error bars *and* by the models for the considered mode (in particular, the minimum and maximum allowed masses are assumed to be 0.5 and 5 $M_\odot$ respectively).

In addition to our remarks concerning the LMC and its clusters, five facts support the reliability of our modelling :

— First, for eleven out of the thirteen nearby Miras for which both the absolute bolometric magnitudes and the effective temperatures are available, consistent pulsation modes and theoretical masses are derived from either of them (masses often agree within $0.4M_\odot$).

— Second, the mixing length was not chosen arbitrarily, but was derived from the three probable radial modes identified in the lightcurves of o Cet and R Leo. Moreover, the theoretical luminosities and effective temperatures derived on that occasion are in excellent agreement with the observed values (Barthès & Mattei 1997a, b).

— Third, the model grid perfectly predicts the anomalous spectral type of R Cen, which would result from its high mass ($\approx 5M_\odot$).

— Fourth, at about this mass ($4.5M_\odot$), the linear model grid predicts the existence of three resonances liable to generate the peculiar shape of the lightcurve of R Cen (double maximum) : $P_0/P_1 \simeq 5$, $P_1/P_2 \simeq 5/3$ and $P_1/P_3 \simeq 2$.

— Fifth, if one varies the mixing length so that a larger number of stars match one of the two possible modes in the period–luminosity plane, then a lesser number match this mode in the period–temperature plane, since both the luminosity and the temperature increase. This confirms that our mixing–length calibration is a much better compromise than the standard (solar and RGB) value within the frame of the linear approximation.

Two stars, χ Cyg (S–type) and R Cas (M–type), have been found definitely pulsating on the fundamental mode according to their magnitudes, and definitely on the first overtone according to their temperatures. Despite many efforts, no change of chemical composition (X, Z) was found able to reconcile the observation and the theory within the error bars (not even within 2σ), even if the mixing length was changed.

Van Leeuwen et al. (1997) had found R Cas and χ Cyg simply pulsating on the fundamental mode. The disagreement may have two complementary explanations. First, these authors made, for the first overtone, a rough analysis using effective radii together with the so–called "pulsation constant", taken

Table 2. Periods, mean absolute bolometric magnitudes, mean effective temperatures, probable predominant modes and theoretical masses derived from the magnitudes and from the temperatures ($Z = 0.02$ assumed)

Star	P [d]	$\overline{M}_{\rm bol}$	$\overline{M}_{\rm bol}(+\sigma)$	$\overline{M}_{\rm bol}(-\sigma)$	$\overline{T}_{\rm eff}$ [K]	Mode	$\mathcal{M}\,(M_{\rm bol})$	$\mathcal{M}\,(T_{\rm eff})$	Remarks
R Aql	284	-4.29	-3.65	-5.06	2539 ± 113	1O	$0.8^{+1.5}_{-0.3}$	$2.8^{+1.0}_{-1.0}$	He–flashing
R Aqr	387	-4.21	-3.02	-6.47	2568 ± 136	F	1.5^{+3}_{-1}	$1.2^{+1.3}_{-0.7}$	Symbiotic
R Car	309	-3.79	-3.42	-4.18		F	$1.1^{+0.5}_{-0.3}$		
S Car	149	-3.39	-2.74	-4.18		F	$1.2^{+1.0}_{-0.7}$		
						1O	≤ 1.6		
R Cas	430	-3.74	-3.35	-4.16	2300 ± 150	F	$0.9^{+0.5}_{-0.3}$		
						1O		$1.8^{+1.4}_{-0.7}$	
T Cas	445	-6.81	-6.54	-7.07	2795 ± 128	1O	≥ 5	≥ 5	
R Cen	546	-6.65	-5.57	-8.48		1O	$4.5^{+1.5}_{-1.5}$		Double max.
T Cep	388	-5.11	-4.64	-5.63	2359 ± 302	1O	$1.7^{+1.0}_{-0.7}$	$2^{+3.0}_{-1.4}$	
o Cet	332	-4.84	-4.41	-5.31	2396 ± 145	1O	$1.5^{+0.9}_{-1.0}$	$1.9^{+1.5}_{-1.4}$	
R Cnc	361	-5.70	-4.58	-6.52		1O	$3.5^{+2.0}_{-2.3}$		
χ Cyg	408	-3.74	-3.29	-4.23	2242 ± 140	F	$0.9^{+0.6}_{-0.3}$		S–type
						1O		$1.2^{+1.1}_{-0.1}$	
R Hor	408	-5.22	-4.45	-6.25		1O	$1.8^{+2.0}_{-0.8}$		
R Hya	389	-5.79	-5.08	-6.38	2661 ± 213	1O	$3.5^{+1.5}_{-1.8}$	≥ 3	He–flashing
R Leo	310	-4.59	-4.35	-4.85	2379 ± 124	1O	$1.3^{+0.4}_{-0.5}$	$1.6^{+1.0}_{-1.0}$	
R Lep	427	-3.54	-2.98	-4.21		F	$0.8^{+0.7}_{-0.3}$		C–type
U Ori	368	-5.35	-4.43	-6.06		1O	$2.5^{+2.0}_{-1.0}$		
X Oph	329	-5.58	-4.13	-6.54	2949 ± 160	F	$5_{-2.0}$	$4_{-3.5}$	
S Peg	319	-4.02	-3.09	-4.75	2107 ± 295	1O	≤ 1.4	≤ 1.8	
RR Sco	281	-4.85	-3.88	-6.33		1O	$1.8^{+3.5}_{-1.3}$		
R Ser	356	-4.40	-3.49	-5.74	2804 ± 144	F	$1.8^{+2.2}_{-1.0}$	$3.2^{+0.8}_{-0.9}$	
R Tri	267	-4.08	-3.37	-4.67		F	$1.6^{+1.0}_{-0.8}$		
						1O	< 1.6		
S Vir	375	-4.63	-3.64	-5.39	2804 ± 112	F	$2.2^{+2.3}_{-1.3}$	$3.1^{+1.2}_{-1.3}$	

equal to 0.04, whereas, in fact, it significantly increases with the period : using the same model grid as for the fundamental mode, it appears that, at $P_1 \simeq 420$ d, $Q_1 \simeq 0.055$, which makes the radius smaller by 24 %. As a consequence, the correct first overtone radius for $M \leq 1 M_\odot$ falls within the error bars of χ Cyg, generating a mode ambiguity which Van Leeuwen et al. could not see. Second, as radius mixes luminosity and temperature, it may little vary when both are decreasing ; it is thus less discriminant than temperature.

At first sight, the "exotic" long–term nonlinear behaviour observed by Yaari & Tuchman (1996) in their hydrodynamic models — switch onto to the fundamental mode, then drastic decrease of its period, together with its temperature — might be the explanation of the anomaly. However, the models predict that the new pulsation period lies *between* the linear fundamental and first overtone (it must be 10 to 35 % shorter than the fundamental), which is not the case of χ Cyg and R Cas. Moreover, the observed periods are approximately 4 times longer than the linear first overtone at this luminosity: this is hardly reached by extrapolating the results of Yaari & Tuchman, and only at *high* luminosities, while the two stars are among the faintest of our sample. Lastly, it remains to explain why the other stars (especially the more luminous ones) do not exhibit a similar behaviour.

On the other hand, we have investigated the possibility that the discrepant behaviour of these two stars be due to deviation from the standard AGB core mass–luminosity relation because of the thermal pulse phenomenon (Wood & Zarro 1981, Lattanzio 1986, Boothroyd & Sackmann 1988a). That is, we have computed linear pulsation models having the same luminosities and periods as R Cas and χ Cyg, but core masses up to 0.1 $M_\odot$ higher or lower than the value given by Paczyński's relation (this corresponds to huge deviations from the standard luminosity at a given core mass). This led to very small shifts of mass (a few %) and temperature (1 %). Thermal pulses are thus unable to explain the discrepancy between luminosity and temperature in the two stars. But this is rather good news for our whole work (in particular, our conclusions concerning R Aql and R Hya are not influenced by their recent helium flash) and for others, since the AGB M_c–L relation is far from being well fixed (Boothroyd & Sackmann 1988b, Vassiliadis & Wood 1993).

The most likely explanation of the peculiarity of R Cas and χ Cyg is the presence of a dense dust shell close to the star (at about 10 R_*), making the color temperature lower and increasing the angular diameter at relatively short wavelengths (scattering). This is supported by the very significant silicate emission excess of R Cas at 8 μm (Vardya et al. 1986, Little–Marenin & Little 1990). Since dust opacity evolves as $\lambda^{-1.n}$, angular diameters measured in K or H' should be the only reliable (hence a temperature about 14 % higher, according to the angular diameters of R Cas near maximum light). Then, R Cas and χ Cyg would be fundamental pulsators with masses $\approx 1.1 M_{\odot}$. It has been suspected for long time that thermal pulses generate important mass–loss variations, leading to important spatial and temporal variations of the dust density around the star (Willems & de Jong 1988, Oloffson et al. 1990, Zijlstra et al. 1992) : this might be the origin of the peculiarity of these two stars.

Concerning the carbon star R Lep, whose magnitude indicates fundamental mode pulsation, and remembering that only the higher–mass AGB stars can become carbon–rich, the theoretical mass ($0.8 \pm 0.7 M_{\odot}$) seems unlikely — though not impossible, owing to 1σ or 2σ error bars. This star may simply be a fundamental pulsator with an overestimated bolometric magnitude, because of strong obscuration by carbon dust in the visible and near–infrared bands.

T Cas was found having a high mass ($\geq 5 M_{\odot}$). It is thus probably undergoing hot bottom burning (Blöcker & Schönberner 1991, Boothroyd & Sackmann 1992, Vassiliadis & Wood 1993). This may be the case of R Cen and X Oph, too.

The symbiotic star R Aqr probably is pulsating on the fundamental mode: a moderate metallicity decrease could allow the star to only marginally fit the first overtone at about 3 $M_{\odot}$ but the star would nevertheless remain both at the higher luminosity and lower temperature error bars.

The other 13 stars may be considered as "ordinary Miras". Eight out of them are found most probably pulsating on the first overtone. Three are most probably fundamental pulsators. Two may pulsate on either of these two modes with similar probabilities.

7. Conclusion

Thorough discussion of the previous attempts at modelling the pulsation of Long Period Variable stars led us to the following conclusion : if, owing to the uncertainties pertaining to hydrodynamical models, the possibility of drastic structural changes generated by thermal nonlinear effects (Yaari & Tuchman 1996) is left aside, then it is possible and thus necessary to minimize the significant errors on the mode periods and effective temperatures involved by the classical linear schemes, by allowing the mixing length parameter to differ from the one adopted for non–LPV stars and, possibly, to change as the pulsating star evolves along the AGB. That is, the mixing length parameter becomes an effective parameter embracing various phenomena in addition to convection. On this ground, we have built a linear model grid assuming a mixing–length parameter lower than the value required for the Sun or for RGB stars, and slightly decreasing along the AGB (in the LPV domain) according to a fixed law, based on the values that were derived from the several probable radial modes identified in the lightcurves of three oxygen–rich Miras (Barthès & Tuchman 1994, Barthès & Mattei 1997a, b).

The periods of the radial modes predicted by the model grid were confronted to the bolometric magnitudes and periods of the numerous Miras in the Large Magellanic Cloud and of LPVs in three LMC clusters. We find that the two sequences of LMC clusters stars, in the $M_{\rm bol}$—$\log P$ plane, correspond to two different predominant pulsation modes. The lower–luminosity/longer–period sequence, coinciding with the Mira strip of the LMC, is the one of the first overtone. This is consistent with the linear growth rates. The probable masses range from 0.6 to 1.5 for stars with periods lying between 100 and 400 days, which is consistent with the values commonly admitted on evolutionary grounds. Assuming a constant mixing length parameter does not basically change this conclusion.

The other, larger–luminosity/shorter–period (and lower–amplitude) sequence, corresponding to some Semi–Regulars, is composed of stars pulsating on the second overtone. The observed switch from the second to the first overtone is correctly predicted by the model grid. For every assumed mass, it occurs at the period where the linear growth rate of the first overtone starts exceeding the second ; this coincides with a 3:2 resonance between the two modes.

Thanks to the new parallaxes and proper motions made available by HIPPARCOS, and to recent, relatively accurate determinations of the mean effective temperatures, we have been able to check the validity of the abovementioned conclusions on Miras of the solar neighbourhood. Pulsation on the first overtone is found the most probable for 60 % of the sample Miras (and about 70 % of the most "standard" ones) ; consistent masses are derived. The other 40 % (30 %)seem to be fundamental pulsators. It is worth noting that our modelling straightforwardly explains both the unusual spectral type and variability of R Cen (high mass and multiple resonances).

The high masses derived in the case of T Cas, R Cen and X Oph make them likely candidates for hot bottom burning.

Discrepancy between the predominant modes derived from the magnitude and from the temperature led us to set apart two very peculiar stars : R Cas and χ Cyg. Their color temperatures and angular diameters at wavelengths shorter than 1 μm seem to be strongly affected by dust shells. They might be at a special phase of the thermal pulse cycle when, because of mass–loss variations, the dust density close to the star is peculiarly high.

The overall agreement between our models and the observations (including the probable modes detected in the lightcurves of a few stars) gives support to our initial hypothesis that nonlinear effects do not yield any dramatic change of structure such as found by Yaari & Tuchman (1996), whose calculations might suffer from their simplistic treatment of time–dependent convection/turbulence. It thus seems that linear modelling, if implemented with care, can still provide fair approximations to the pulsation (pseudo)periodicities for given fundamental parameters. This is rather good news, considering the amount of works (including evolutionary calculations) that require P–M–L relations.

Knowing the period of the dominant mode and the bolometric magnitude and/or the effective temperature, one thus may derive a theoretical mass for a given metallicity (which can be roughly estimated, e.g., from the kinematics). More precise results may be obtained if one or two other modes are known from the Fourier and time–frequency analysis of the light curves, even if the luminosity or temperature is ignored (Barthès & Tuchman 1994 and Barthès & Mattei 1997a, b, c). We must however stress that precision does not not exclude some inaccuracy, i.e. possible systematic shifts resulting from the fact that open outer boundary, phase–lagged convection, nonlinear effects, etc. are not really equivalent to changing the mixing–length parameter. We also stress that, as far as we know, our mixing–length calibration is *not* suitable for calculating evolutionary tracks, at least outside the LPV domain, which is only a limited part of the Asymptotic Giants Branch.

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