Morphable guidelines for the human head

by

Shelley Y. Gao
B.Sc., University of Victoria, 2010

A Thesis Submitted in Partial Fulfillment of the
Requirements for the Degree of

MASTERS OF SCIENCE

in the Department of Computer Science

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ABSTRACT

Morphable guidelines are a 3D structure that helps users achieve better face warping on 2D portrait images. Faces can be difficult to warp accurately because the rotation of the head affects the shape of the facial features. I bypass the problem by utilizing the popular Loomis ‘ball and plane’ head drawing guideline as a proxy structure. The resulting ‘morphable guidelines’ consist of a simple 3D head model that can be reshaped by the user and aligned to their input image. The vertices of the model go on to act as deformation points for a 2D image deformation algorithm. Thus, the user can seamlessly transform the face proportions in the 2D image by transforming the proportions of the morphable guidelines. This system can be used for both retouching and caricature warping purposes, as it is well-suited for both subtle and extreme modifications. This system is advantageous over previous work in face warping because our morphable guidelines can be used on a wide range of head orientations and do not require the generation of a full 3D model.
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Chapter 1

Introduction

We live in a world where photorealistic imagery is easy to produce by anyone with the right equipment. Thus, the labour-intensive arts of drawing and painting are mainly utilized to produce images that cannot be produced through photography. These are not limited to fictional objects. For example, diagrammatic pictures such as blueprints or signage are generally illustrations because photographs are more difficult to parse at a glance. Illustrative methodologies, or visual styles, are also used in order to assert particular moods or qualities on the image. When applied to human subjects, visual style is often used to exaggerate and heighten features to either reinforce or change our perception of the subject. This type of illustration is commonly known as ‘cartooning’, and can either be done additionally (by exaggerating features) or subtractively (by removing extraneous details). Thus, the predominant paradigm of cartooning is more or less a direct transformation from realism to exaggeration.

This thesis introduces a new ‘morphable guidelines’ system for transforming the proportions of the face from one form to the other. For example, an unedited photograph of a human being can be directly transformed into a properly-proportioned caricature, regardless of previous artistic experience on the part of the user. The output of this system can be combined with non-photorealistic rendering techniques to compound the ‘cartoon’ effect.

The morphable guidelines system is simple and lightweight enough for casual mobile platform use, but is also designed to allow for fine-tuned user control for advanced photo-editing purposes.
1.1 Summary of the problem

Thanks to commercial image editing software, point-based image deformation is a widely available tool that is accessible to users across all skill brackets. Point-based image deformation allows a user to precisely deform an image to their specifications. However, precise modification on the two-dimensional plane does not apply 1:1 in the three-dimensional space. Since photographs are usually pictures of three-dimensional objects, modifications to the two-dimensional picture plane of the photograph must follow the rules of three-dimensional geometry. The failure to properly comply to three-dimensional reality is one of the hallmarks of a bad photo manipulation.

Problems compound when the photo-manipulation subject is the human face – a subject that is a) difficult for a novice artist to render accurately and b) easy for the average viewer to determine as erroneous. Working with faces can be especially problematic when performing photo-manipulation, particularly on faces that do not directly face the camera and are thus asymmetrical on the 2D plane. Given that a relatively small percentage of people are educated in human anatomy or illustration, only a limited number of skilled users can use 2D deformation to manipulate faces and/or simulate caricatures in an effective manner.

Artists have struggled with this issue for centuries, and have subsequently created tools to surmount their difficulties. For example, a primary tool used to simplify difficult subjects is the concept of ‘guidelines’ – rough preliminary lines and shapes which help the artist maintain a sense of three-dimensional space while working on the two-dimensional plane. On the side of human-computer interaction, there are also interfaces that have been designed specifically to allow unskilled artists to design faces. Video games with robust character creation capabilities often include the option to individually manipulate different aspects of the human head. Character creation systems of this type allow the user to change the parameters of particular features using linear sliders as controllers. The user does not have to interact directly with the polygon mesh of the model in order to change it.

1.2 Summary of my solution

In this thesis, I introduce a method for the accurate shape manipulation of a three-dimensional object (namely, the human head) that has been projected in a two-dimensional photograph. I present a simple 3D geometric model as a framework for
placing the key deformation points on the photograph. The 3D model, known as a set of ‘morphable guidelines’, acts as a proxy for the geometry of the target head in the photograph. All changes to the 3D model automatically reflect the perspective and foreshortening changes inherent to the original. Thus, the user can make 3D-space symmetrical changes to the 3D model and have the correct results appear in the 2D plane of the photograph. Morphable guidelines give users of all skill levels the inherent sense of three-dimensional construction that artists normally achieve through intensive training.

The Loomis ball and plane method [16] is the basis of the morphable guidelines, and the controls used to manipulate the parameters of the morphable guidelines resemble those featured in video game character creation systems.
Morphable guidelines are named after a similar system, the morphable model [4]. We share the concept of a generic 3D model having access to a topologically consistent ‘vector space’ of faces, both realistic and stylized. However, morphable guidelines can produce similar results to Blanz and Vetter [4] without multiple input images or the inclusion of a complex model.

1.3 Outline of contributions

The main contributions of my research are as follows:

1. a user-friendly system for easier photo-manipulation of faces, regardless of artistic skill level;

2. a methodology for transferring changes from the 3D space to the 2D space;

3. a simpler and more economical alternative to previous methods of multi-angle face warping.

These three qualities make the morphable guidelines a viable alternative to current face warping techniques, especially where an artistically untrained user is concerned.

1.4 Outline of thesis

The chapters address the following issues:

Chapter 1 contains a statement of the claims which will be proved by this dissertation followed by an overview of the structure of the document itself.
Chapter 2 describes in details the open problem which is to be tackled together with its context, its impact and the overall motivation for the research overall.

Chapter 3 covers the background research on computer-assisted face modification techniques, as well as drafting assistance tools.

Chapter 4 introduces the morphable guidelines.

Chapter 5 describes an example of a system that uses the morphable guidelines in a practical context.

Chapter 6 showcases applications of the system and addresses some limitations that could pose issues to users.

Chapter 7 contains a restatement of the claims and results of the thesis.
Chapter 2

Imitating visual style

‘Style’ in the visual arts is a combination of medium, technique, subject matter and other such visual components that makes a work consistent either with other works by the artist or consistent with other works by other artists. The style of an artwork can be used to identify the artist or a category it belongs to.

In some exceptional cases, an artist’s style can transcend broad categorization and become distinctive in it’s own right. This means that the style itself is not only identifiably unique, but also represents particular aesthetic and emotional qualities that are different from those of their contemporaries.

2.1 Manga as stylistic shorthand: a case study

The difference between a broad style category and a specific style can be demonstrated by observing a particular case: Japanese comic books, or manga. Manga is a gigantic multi-genre industry in Japan, with over fifty magazines each serializing ten to twenty titles of content. The most popular magazine, Shueisha’s Weekly Shônen Jump, requires their creators to produce twenty pages per week, which means each credited artist must manage a squad of assisting artists in order to meet demand. In addition to this extreme flow of professionally published content, there is also the dōjinshi or amateur element. The biannual Comiket in Tokyo is the largest comic convention in the world, and is largely devoted to the sale of self-published comic books. Needless to say, manga is a major cultural force in Japan. It also has a huge influence on other domestic pulp fiction industries such as animation, video games and commercial illustration. Because of this influence, the manga style category is highly recognizable
outside of its actual readership, even on the international level.

Due to the high volume turnout of titles in the manga industry, there is a great
deal of imitation between artists. The long history of the genre has ensured that
common elements used by manga artists, colloquially referred to as the ‘big eyes
small mouth’ style, are a visual canon commonly understood by viewers to exemplify
particular values and traits, especially in terms of youth and beauty. Certain artists
adhere more closely to that visual canon and are often imitated by other artists who
are attempting to fit into the manga style category.

The codification of the style is largely credited to genre pioneer Osamu Tezuka,
whose use of large eyes is again credited to imitation of the early Disney Animation
Studio style a la Mickey Mouse. The continued use of the style markers is due to a
long history of imitation and influence between many different artists.

2.1.1 Akira Toriyama, a standout artist

Most drawn media are dependent on their visual styles to not only to set the tone,
but to act as a form of identification. It can be very easy to evoke and/or parody a
well-known artistic work without actually using intellectual property in the form of
color designs or writing.

As an example within the manga industry, I will focus on the artist Akira Toriyama.
Toriyama is a veteran author of children’s action adventure stories about superheroes
and fantasy. He is internationally famous for his 1984 to 1995 series *Dragon Ball* in
*Weekly Shōnen Jump*. This highly popular manga series was then adapted to several
multi-season anime series by Toei Animation, the latter of which was exported as the
international phenomenon *Dragon Ball Z*.

Toriyama’s illustrations have been used to sell the best-selling video game franchise
*Dragon Quest* since 1986, long before the advent of representational graphics in the
actual video games. The *Dragon Quest* series was first released on the Nintendo
Famicom system, a console-style computer with a 16-colour, highly pixelated graphics
output. However, the games were publicized using Toriyama covers and illustrations.

Enix’s choice of Toriyama as the primary series illustrator most likely had to
do with his preceding reputation as a manga artist in the children’s fantasy genre,
thus making his style a visual shorthand for the atmosphere of family-friendly fantasy
adventure that Enix wanted their games to evoke. In other words, Toriyama’s artistic
style overrides the actual visuals of the video games it represents.
2.2 Reality and stylization

Stylized drawings of faces generally modify the proportions of the facial features to suit a particular end. Much of the time, stylized drawings of faces still maintain most (if not all) of the topological features found in a photograph of the same face.

2.2.1 Categorizing styles

In his 1993 book *Understanding Comics: The Invisible Art*, comics artist and analyst Scott McCloud theorizes on the effectiveness of simplification and stylization in depth [18]. He observes that simple iconography such as the smiley face have more in common with the pictorial elements of written language than they do with visual reality. The more simple a drawing, the more direct and symbolic it becomes. This is the reason why diagrams are usually drawn – the removal of superfluous detail makes diagrams easier for the brain to parse. At a certain point, symbolic pictures become the glyphs we know as language. Lastly, visual art does not simply run the gamut between reality and symbol; visual art can concentrate on qualities such as shape, colour and texture rather than subject matter. This is the realm of abstraction, where the ‘picture plane’ is the primary concern.

With these points in mind, McCloud identifies three points of a triangle that a visual style can be placed on: ‘Reality’, ‘Language’/‘Meaning’ and the ‘Picture Plane’, as depicted in Figure 2.1. An artist’s style can be described by it’s position on the triangle. The more essential and symbolic the style, the further it is to the right corner, whereas the more abstract it is, the closer it is to the top corner. For example, a photo-manipulation caricature would lie closer to the Reality corner than the Language corner, but lie further upwards towards the Picture Plane corner due to the fanciful and abstract aspects of caricature. Manga art is heavily simplified due to rapid-production constraints, so it tends towards the Language corner instead.

McCloud also observes that humans have a tendency to see faces in objects with the barest minimum of human-like characteristics, such as car grills and electrical outlets. This kind of association may be because humans visualize their own faces in the same symbolic, non-realistic way. Humans are unable to assess the actual appearance of their body unless they are currently viewing it in a reflection, so we rely on a fuzzy, general sense of our body parts in order to direct their actions. This could be the reason why of all the main facial features, the nose is the most likely to be omitted; compared to the eyes and mouth, the nose is much less likely to be
manually controlled. McCloud then theorizes that simplified and cartoonish faces are appealing to us because they tap into the way we view ourselves, rather than the way we view other people. In other words, the more iconic the face, the more we empathize and project ourselves onto it. On the other hand, more realistic depictions heighten our consciousness of the face as a separate entity.

2.3 Stylization through direct editing

Some caricature artists use photo-editing tools to warp photographs into caricatures of themselves, as depicted in Figure 2.2. This technique results in realistically-textured caricatures that blend the line between reality and satire.

It should be noted that this technique is often more easily said than done. Though modifying the original photograph may seem much simpler than producing artwork from scratch, there is still a marked difference between what a trained and untrained artist can produce with the same image deformation tools. The challenge is in iden-
2.3.1 Difficulties in accurately manipulating the human face

The average person has a great deal of difficulty recording the visual world as he sees it. Overcoming the natural hurdles of perception and dexterity is a specialized skill set. Accurate drafting of 3D shapes requires superior development of visual memory and perception.

As humans we are accustomed to living in a three dimensional world, full of solid objects that are delineated via volume and depth cues. Thus, a person who wants to convey realism through drawing must have a firm grasp on the way 3D objects interact with space. This can be difficult, depending on the ‘camera angle’ at which the subject is being ‘viewed’/rendered.

Research in visual perception has shown that subjects are much better at drawing when they are ‘tracing’ from a source. For example, drawings done on a glass ‘Da Vinci Window’ (alternatively, a ‘Leonardo’s Window’) are significantly better than
drawings done on a regular surface, but this is simply because a Da Vinci Window allows the subject to directly ‘trace’ the shapes they are drawing through the window, thus facilitating an easy transition from 3D to 2D [22].

2.3.2 Specifics of the human face

The human face is functionally symmetrical across the vertical axis. Thus, the easiest angles to draw a face from are the full-frontal view (two symmetrical halves) and the profile view (one half alone).

However, views between the full frontal view and the profile view require the artist to draw both halves of the face, but in an asymmetrical fashion. Drawing one of these intermediate angles requires an artist to extrapolate the shape of each feature based on their personal understanding of the 3D shape of the head, cobbled together via a combination of memory and guesswork. This is as difficult as it sounds, and most artists spend years developing the sense through training.

The difficulty of maintaining proper proportions is significantly lessened when working off of a subject like a photograph, especially when the canvas is the photograph itself. Photo editing applications allow a user to mould the 2D features of an image to their liking via warping tools. In the hands of a skilled artist, this can work out very well – modern caricature artists such as Rodney Pike (Figure 2.2) often work using a combination of 2D image deformation and digital painting. A novice artist, however, may have difficulty achieving good results without assistance.

2.4 Summary of problem

Visual stylization is highly appealing to the eye and serves many artistic and commercial purposes. However, mastery of visual stylization is inherent to skilled artists, as it often requires a solid background in three-dimensional space and realistic anatomy; colloquially, “learn the rules before you break them.” The challenge is to create a system that allows unskilled users to apply stylistic exaggeration to photographs with a less developed spatial skill set.
Chapter 3

Background

Previous research on the topic of computer-assisted face modification in photographs brings up a myriad of possible methods for an equally diverse set of goals. A wide variety of methods can be used to modify and stylize faces. Some are general and others are specialized. Some are user-driven and others are completely automated. Some of the research is done for practical application purposes; for example, face warping is commonly used in psychology studies, so a survey on ‘morphing techniques’ was published in a 1999 edition of *Behavior Research Methods, Instruments, & Computers* [26]. On the other hand, there are just as many papers that take a solely artistic view of the problem.

At the end of the chapter, I will cover some additional ground on tools that have been developed to help users draw in a more accurate fashion. These systems are not directly related to our topic in terms of subject matter, but are similar in terms of approach and overall goal.

3.1 Generalized methods

Some methods are commonly used for the purpose of modifying and stylizing faces, but are not specifically designed for the purpose. These generalized methods are generally 2D image deformation methods that are components of image editing systems. These tools are applied manually and require a firm grasp on artistic fundamentals to use properly.
The most efficient and lightweight method of applying shape deformation to a 2D image is to use a point-based image deformation algorithm, such as the Puppet Warp tool in Adobe Photoshop CS5+. For a skilled artist, this kind of freehand tool is ideal. Modern caricature artists such as Rodney Pike combine this technique with digital painting to great effect. However, the problems addressed in Chapter 2.2 can limit the effectiveness of freehand tools when the user is not a skilled or experienced artist.

3.2 Specialized Methods

Specialized face warping methods involve one or more structures that have been built specifically to fit faces. Generally this takes the form of either a face-fitting structure and face-tracking package, or a specialized model.

Some seemingly specialized methods are still fairly close to a generalized method – Yang et al.’s Entertaining Video Warping [31] simply detects the position of the face and applies a basic bloat or pucker effect to it. Most of the specialized techniques are largely theoretical and experimental in nature, and only a few of these methods are in any commercially available form.

3.2.1 Active Appearance Model

The Active Shape Model (ASM) [7] and the Active Appearance Model (AAM) [6] (shown in Figure 3.1) are computer vision algorithms that match a statistical model of object shape and appearance to a new image. The statistical model is built by training the model on a large variety of different facial photographs. The statistical model can then be used to detect, track and fit faces in other photographs using the shape and texture data it associates with faces in general.

The AAM are invaluable tools in the field of automatic feature extraction and face detection. As tools they are used to facilitate a large number of automatic face warping systems, as they automatically produce a set of points on the face that can be used as deformation points. However, the large differences in shape between different head rotations means that the same AAM is unlikely to be useful for different head rotations. As a result, most caricature systems use an AAM that has only been trained on front-facing faces. Improvements have been made on the method over time [30], but the commonly available packages are still limited in valid angles.
Additionally, the points on an AAM do not adequately describe the geometry of the face. The AAM describes the face as a flat space defined by an outline of vertices. Thus, the standard AAM does not take into account the three-dimensional, symmetrical nature of facial features because it is a two-dimensional entity in itself.

### 3.2.2 Generic 3D Model

The other main gambit used to morph faces in photographs is the generic 3D model. A model of a generic human head is ‘fitted’ to the face as faithfully as possible as so to create a duplicate of the original head.

The most cited example of this technique in action is Blanz and Vetter’s seminal SIGGRAPH 1999 paper, ‘A Morphable Model For the Synthesis of 3D Faces’ [4]. The original morphable model utilized a 360 degree head-scan repository of 200 young adults (100 male and 100 female) as a set. By interpolating between the faces in the set, the morphable model can mimic a large number of different faces in that particular set (Figure 3.2). The reconstructed face is then combined back into the image. The photograph is then used as a texture for the model.

Blanz and Vetter have since expanded on the morphable model concept in a myriad of papers, adapting the tool for many different applications, such as facial animation [2] and automatic face replacement between two photographs [3].
3.3 Face modification for stylistic purposes

There are two major categories for face stylization. Caricature systems exaggerate features that deviate from the average, while portrait synthesis systems are collage-like systems that match individual facial features to the best match in a library of pre-drawn features [5]. Some portrait synthesis systems [5] include a form of face warping to make the proportions more stylistic before adding the pre-drawn elements.

3.3.1 Caricature Systems

Research on caricature systems usually emphasizes automation of exaggeration. These papers generally describe a method that automatically determines the features that should be exaggerated or minimized, usually by comparing the original face to an average face and then exaggerating the deviations from the average face. The opposite approach also exists, where the original face is warped towards the average rather than away from it. This has a normalizing effect on the face and can make the face appear to be more conventionally proportioned. Leyvand et al. call this a ‘beautification’ technique in their SIGGRAPH 2006 sketch, ‘Digital face beautification’ [14].

Many 2D-based caricature systems use guideline-like systems of varying degrees of complexity. For example, Gooch et al. uses the vertices of a grid to warp front view faces [9], as shown in Figure 3.3. Other systems use basic shapes such as ovals
and triangular shapes instead [12]. The aforementioned Active Appearance Models [6] are heavily used for this purpose because of their self-aligning nature, but they are still unreliable when detecting extreme head alignments. Obaid et al. [20], enact a fairly typical example of this in their 2009 caricature paper, as do Liao and Lai in 2010 [15].

Occasionally the goal isn’t to warp the original photograph, but to create a 3D model with the caricatured characteristics of the original face [32].

![Figure 3.3: A caricature system that uses a grid to place the deformation points.][9]

Lastly, it is important to note that the caricaturization process now goes both ways. Recent work has been done on creating an automated system which can draw parallels between a caricatured face and a real one [11].

### 3.3.2 Portrait synthesis

Portrait synthesis is the practice of assembling a face using a library of prefabricated parts. This method is useful when the desired style to be imitated is heavily stylized and deliberately hard-lined – examples include the pen-and-ink comic book and manga drawing styles meant to be replicated through photocopy. In this case, cobbling the face together using matching pre-drawn parts produces more authentically looking results than direct photograph-warping plus a filter. Early examples do not necessarily use a photograph as a basis; Iwashita et al. [10], use linguistic descriptors instead in their 1999 paper, and a common collaborator Onisawa continues this work in later research on fuzzy systems [21, 1].

Chen et al.’s 2004 paper ‘Example-based composite sketching of human portraits’ uses ‘manga’-inspired parts to assemble a drawing (Figure 3.4). The facial features (eyes, eyebrows, nose and mouth) are all excised from the original image and compared against a library of photographic features with matching manga-esque features. The
library photo with the closest texture match provides the matching manga-esque feature. The final result is a collage of all the different features. Gao et al. iterate on this concept in 2009 [8]. The same method is also used to assemble patterns for traditional Chinese paper-cut art [19], which are akin to reverse stencils and must be composed entirely of connected lines.

3.4 Drawing assistance tools

In addition to the tools that directly affect faces, there is one last category that we must address: tools that assist the user in making better choices when drawing.

Drafting tools do not occupy a particular field of study within computer graphics or human-computer interaction. The research is sporadic and loosely distributed throughout the fields of computational aesthetics and sketch-based modelling.

Guideline systems are sometimes seen in sketch-based modelling systems. Sketch-based modelling systems take in 2D input and produce 3D results, so the shape accuracy of input is of paramount importance. In the 2009 paper Analytic drawing of 3D scaffolds [24], Schmidt et al. describe a box-based guideline system which helps both the user and the system comprehend space parameters. In this case, the scaffold system is based on industrial design drawing conventions that help preserve accuracy of shape and perspective.

Another example is the ShadowDraw system [13], which is an advanced tracing system for a pool of pre-chosen objects. A few images of the chosen object are shown in low opacity under the drawing surface. Redundant images are exchanged for more relevant images as the user draws the indicative lines. For example, if the user wishes to draw a motorcycle from a profile view, the system will automatically weed out the images that do not fit this profile and replace them with profile-view motorcycles.
3.5 Background Summary

Thus far, most work on warping face in photographs emphasizes the computer-controlled, automated side of the process. Most caricature systems are focused on replicating an actual caricaturist’s decisions.

The structural methods used to support these automated face warping techniques each have their own respective weaknesses – face tracking AAM packages are weak when trying to match a wide range of facing angles, and morphable models are overly complex and inefficient for nonprofessional use and minor changes.

Some work has been done on assisting the user’s hand, but most of this work has been done outside of the field of face warping.
Chapter 4

Morphable guidelines

In essence, morphable guidelines make point-based image deformation easier by embedding the deformation points into a rigid structure. The rigid structure is provided by a simple 3D model called a ‘morphable guideline’.

This system relies upon a single input image and provides easy controls on a range of symmetrical warps in the resulting output image. Previous systems such as the classic morphable model [4] are able to create 3D models from several images of faces, but the Blanz and Vetter morphable model system is overly complicated for performing simple facial stylizations and beautifications within a single image.

The morphable guidelines described in this thesis are designed exclusively for faces, but the same concept can be adapted to different subjects with consistent topology.

4.1 An introduction to ‘guidelines’

Living in the 21st century, we often forget that the visual principles that we now regard as obvious were the product of mathematical research. For example, advances in painting realism during the Renaissance period can be partially credited to architects and mathematicians, who codified the rules of vanishing point perspective during the 15th century. Books such as Piero della Francesca’s De Prospectiva Pingendi (Figure 4.1) stringently codified drawing faces affected by perspective in a methodological way. These techniques quickly proliferated through the artistic community and became part of the artistic canon. In other words, mathematics and invention have always been a crucial part of artistic development.

If artists are accustomed to using techniques, methodologies and tools to bolster
their skill sets, non-artists can benefit from new systems as well. Thus, we borrow a fundamental artistic tool and modify it for common use.

In this thesis, I use the word ‘guidelines’ in a literal sense. By this literal definition, ‘guidelines’ are lines that guide the placement of other lines. ‘Guidelines’ are preliminary structural drawings that delineate the composition of a finished piece of artwork, usually consisting of preliminary shapes such as ellipses and boxes. Guidelines help the artist to stay in control of the macro elements of their figure (anatomy, pose) while working on micro elements (outline, rendering). Art instruction using guidelines can be found in a great variety of books throughout a wide variety of audiences and skill levels.

Though usually found in multiples, a ‘guideline’ can be used independently. For example, a single horizontal guideline can be used to denote the horizon in an image. Even then, this kind of horizon guideline is accompanied by several others, in the configuration found in Figure 4.1. This combination of guidelines forms the guideline system that is used to imitate single-point linear perspective. The diagonal guidelines radiate from the vanishing point, which is where all the guidelines converge.

Guidelines can be used to assist the drawing of just about anything with a geometric component and are especially useful for drawing organic entities with movable parts, such as animals and humans. Basic figure drawing instruction trains the macro
instincts via the ten-second pose exercise, which forces the student to draw a simple sketch of a live model within an extremely short timeframe before the model moves to an different pose.

Guideline systems for drawing human beings vary between artists, depending on their means of instruction and personal preferences. Complexity can vary between a few lines delineating the movement of the figure, to complete assemblies of geometrical primitives. There are established guideline systems that concentrate the body, as well as those that focus on the head. The Loomis ball-and-plane guidelines are one of the latter.

## 4.2 Loomis ball-and-plane guidelines

We chose the ball-and-plane guidelines, shown in Figure 4.2, for their popularity, flexibility and classic status among illustrators. American illustrator Andrew Loomis featured the ball-and-plane guidelines in many of his instructional books [16, 17], and his method has been used to teach proper face construction by many art instructors ever since.

The ball and plane method consists of two major parts; a ball representing the top part of the skull and a few lines that delineate planes and represent the jawline and lower part of the skull. The ball portion is bisected by perpendicular arcs, which flesh the circle into a 3D sphere and delineate the area between the brows. The arcs form a crosshair that represents the bare minimum of any guideline, as it succinctly shows
which way the head is facing. The jawline consists of three lines — two representing the sides of the jaw and one representing the jaw.

The ball and plane method is widely used for two reasons. First, the ball and plane method produces a reasonably accurate three-dimensional model of the head, thus making complex angles and orientations simpler to draft. Secondly, the ball and plane method is generalizable to a huge range of different heads and faces. The ball and plane method provides a topologically consistent structure across the entire face vector. By reducing the structure of the skull to a few lines, a trained artist can easily draft the bone structure from which to build the desired face. Loomis shows off this flexibility in his instructional books (Figure 4.2).

4.3 The 3D morphable guidelines

Our morphable guidelines are both more and less complex than guidelines produced by the ball and plane method. Generally the ‘plane’ portion of the analog method incorporates curves, but morphable guidelines are a framework for a system of vertices; thus, all curves and lines are simplified to a series of points. Some points have been added to delineate facial features, such as the eyes and nose.
The base shape of the morphable guidelines is a single stationary circle. It is delineated by two perpendicular ‘cross’ circles that pivot inside, which produces the illusion of a pivoting sphere. The remaining planes are described via a set of vertices connected by line segments. These planes pivot along with the centreline circles. The model thus appears to have a full 360 degrees of rotation around all three axes.

The principal controlling variable of the morphable guidelines is the radius of the circle. All model vertex coordinates are calculated via ratios of the radius. For example, the default measurement between the chin and the base of the nose is calculated as $1/3$ of the radius measure, as with the default measurement between the base of the nose and the brow line. These proportions can be altered by changing the variable ratios that determine the size of each feature. A complete list of the proportion variables and their default values is available in Chapter A.1.
4.4 Proportional controls

Our morphable guidelines are generic across most faces. When the morphable guidelines are first generated by the system, the vertices are set with default proportions determined by a set of default variable ratios, such as the 1/3 rule for the nose and chin placement in Section 4.3. The variables are then adjusted by the user to produce the desired proportions. I call this process ‘proportion warping’.

The key to the morphable guidelines control system is in the constraints. All proportion warping is constrained to be symmetrical in the 3D space – all changes to the vertex positions are reflected across the Y axis. The user can always ensure that the changes they made to the face are technically correct in the 3D space, even though they are viewing the results on the 2D plane only.
Chapter 5

A morphable guideline-based face warping system

Figure 5.1: System flowchart. Images are marked in blue, system attributes are red, and user actions are green.

The system flowchart is presented in Figure 5.1. Our system takes a single portrait-style image as input and overlays a morphable guideline on top of it. The user adjusts the morphable guidelines twice: once to fit it to the face, and once to specify the new face. The system extracts a set of x/y coordinates from each of these configurations, thus providing a set of ‘source’ points and a set of ‘destination’ points. The input image and the source and destination points are then fed into a 2D image deformation algorithm. The image deformation algorithm produces the desired image by warping the corresponding pixels of the source points to the positions of the destination points.
5.1 Input image

The system has some criteria for an ‘ideal’ input image. Most of these criteria are determined either by difficulty of guideline fitting or by interference with our chosen method of image deformation.

We have designed the system to work with photographic portraits of humans. In this case, we define a ‘portrait’ as an image that a) has a human face is the focal point, and b) takes up a significant area of the image. It is possible to use a portrait of a cartoon character as the original image, as long as the proportions of the cartoonist’s style are topologically consistent with the morphable guidelines. It is also possible to create multiple morphable guidelines to morph several faces in a group photo, though the results are less predictable and the incidence of unwanted artifacts may rise.

The face should be reasonably unobstructed by hair, limbs or props, although some obstructions can be accommodated as long as absolute accuracy is not required. Higher resolutions (over 500px x 500px) are ideal, as 2D image deformation is more likely to cause artifacts in lower resolution images.

We strive to emulate as many of the qualities of the ball and plane method as possible, and our system is able to handle a wide range of head facing angles. However, a head with more than 90 degrees of rotation on the X or Y axis is not strictly ideal, as most of the vertices of the model would be occluded by the back of the head.

A small degree of perspective can be accommodated, but extreme up or down head facing angles can make the initial fitting stage difficult.

5.2 Fitting stage

The ‘fitting’ stage is the portion of our system that requires the most user effort. The user must resize, rotate and proportion warp the morphable guidelines to fit the input image and produce the ‘source’ coordinates of the deformation points. The correct placement of deformation points is crucial to achieving good results from this system. Ideally, this stage would be automated, but the face detection systems currently available do not handle the full range of rotation that our system can handle. In many cases, a user can create a better fit than an automatic system.

Our assumption is that morphable guidelines can be fitted easily onto any face that fulfills the input image criteria described in Section 5.1. Depending on the composition of the image, the ease of initial fitting can vary. The fewer axes of
rotation present in the head, the better; more complex rotations generally take longer for the user to fit.

Some cartoon styles are also fairly receptive to fitting, such as the style employed by Nickelodeon’s *The Last Airbender: The Legend of Korra*. As a result, we can consider the proportions of this style fairly easy to mimic using image deformation.

### 5.3 Adjustment stage

After the morphable guidelines have been properly fitted, the user can once again proportion warp the morphable guideline to the desired parameters. This produces a set of ‘destination’ coordinates of the deformation vertices.

The actions taken in the ‘adjustment stage’ are completely determined by the goals and personal creativity of the user. For example, a user who wishes to make minor ‘retouching’ changes to the input image (such as heightening the cheekbones) could tweak the associated vertices slightly, leaving most of the vertices intact. On the other hand, a user aiming to create a caricature could enlarge or shrink large portions of the face to outlandish proportions.

![Figure 5.2: 3/4 view caricature warp. Original image ©Steve Evans, 2008](image)

Figure 5.2 is an example of a portrait taken from a camera angle that produces a 3/4 view of the face. In Adobe Photoshop, freehand warping this image using the Puppet Warp tool would require tedious adjustment of individual deformation points on both sides of the face. With our system, the user enlarges the cheekbones, lowers the tip of the nose and exaggerates the tapering of the face by exerting those changes directly on the morphable guidelines.

Once the desired adjustments are made, the source and destination coordinates are fed into the image deformation algorithm along with the input image.
5.4 Image deformation algorithm

We use the moving least squares (MLS) image deformation method [23] as our image deformation algorithm. Moving least squares image deformation is a common method of 2D image deformation that operates over a triangular mesh. An implementation of this type of image deformation is accessible in Adobe Photoshop CS5+ as the 'Puppet Warp' tool.

MLS image deformation produces instant results when paired with our system. We use the simpler affine similarity deformation described in [23], instead of the rigid deformation. Affine similarity deformation facilitates smoother facial deformation and is slightly more efficient in runtime.

5.5 Software implementation

Figure 5.3: Software interface of implementation
We implemented the system in C++ using the Xcode version of the OpenFrameworks visual toolkit, release 0073 [25]. The MLS image deformation algorithm was achieved through Chen Xing’s Image Warping/Deformation library [29].

Our implementation of the system includes 13 different controls for different facial features, as shown in Figure 5.3. Each of these controls affects one or more equations which control a pair of vertices in the mode. The ratios in these equations are increased or decreased when the user uses the mouse to click and drag along the X and Y axis. This method of feature control is inspired by the character customization systems in video games, where one-dimensional sliders are often used to determine the parameters of a given facial feature. However, the two-dimensional control method is used here for more fine-grained control.

All code used in this implementation has been licensed under nonrestrictive open source – hence, the implementation can be freely distributed with proper attribution.
Chapter 6

Evaluation, Analysis and Comparisons

6.1 Examples of Application

Our implemented system produces attractive results across a wide range of applications. It can produce changes that are subtle enough for retouching (Figure 6.1) while also being powerful enough to make large stylistic changes such as manga illustration style (Figure 6.2) or caricature (Figure 6.3).

The primary application of the morphable guidelines system is photo-editing. As a standalone application, the morphable guidelines system could be included in an editing pipeline, the retouching capabilities in particular. The system is lightweight enough to be implemented on some contemporary mobile hardware, which means that the application could reside on the same hardware as the camera.

With a little modification, morphable guidelines could easily be implemented in existing photo editing software with 3D capabilities. Our methodology is also easy to integrate with existing caricature and beautification literature, possibly as a less automated but more versatile alternative to the popular AAM method [6]. It would be easy to create a set of ‘template’ stylized morphable guidelines to use as destination points, thus allowing users to morph faces into pre-established styles such as manga. This is particularly effective when combined with a ‘toon’ shading filter that imitates the line-work and real-media colouring style of a manga illustration. Figure 6.2 shows an example that uses a toon-shaded input image [28].
Figure 6.1: A ‘beautification’-style face warp in a fashion associated with South Korean popular beauty standards. The jawline has been modified to form a ‘v-line’ shape. Original image ©Frank Kovalche, 2011

Morphable guidelines could also be used as a teaching aid for novice artists. By tying facial anatomy to skeletal guidelines rather than a face outline or a 3D model, we can train novice artists to understand the intrinsic relationship between the underlying structure and the surface appearance of the face.

Lastly, there is no particular reason why this methodology should be restricted to faces only. A similar system could be applied to practically anything with a consistent structure, though the inexact nature of 2D image deformation is best suited to organic subjects.

6.2 Comparison with previous work

The morphable guidelines system is novel largely because it is a solution to an oft-ignored user problem. Morphable guidelines simply make face warping easier to implement properly, and do so in a lightweight way. However, morphable guidelines are named after a similar predecessor, so we shall compare the two.

Blanz and Vetter’s SIGGRAPH 1999 paper, ‘A Morphable Model For the Synthesis of 3D Faces’ [4] is arguably the most prominent paper in face warping. The original morphable model utilized a 360 degree head-scan repository of 200 young
adults (100 male and 100 female) as a set. By interpolating between the faces in the set, the morphable model can mimic a large number of different faces in that particular set. The reconstructed face is then combined back into the image.

The crucial factor in the difficulties of a morphable model is that it produces a three-dimensional model as output. This three-dimensional model must be recomposed into the image. This can become difficult if the shape of the face was affected. If the jawline was minimized, as in the case of Figure 6.2, some extra pains would have to be made to erase the extruding original jawline seamlessly. This kind of procedure would not necessarily be easy for a novice artist. The same problem arises with objects that obscure the face, such as hair or glasses. On the other hand, morphable guidelines affect the 2D image directly and thus bypass these issues entirely.

In conclusion, the characteristics of the morphable model make it useful in a professional production context, but it does not serve the same purpose as morphable guidelines: the facilitation of easier face warping.

Other than Blanz and Vetter’s work, there is not much previous work that is really similar to this system. Caricature systems tend to be designed specifically to showcase an algorithm, rather than to solve the particular artistic problem of creating a caricature. For that reason, caricature papers often limit their input scope.
Figure 6.3: A caricature-style face warp. The indicators of the smile (wide mouth and squinted eyes) have been exaggerated, along with the jaw and cheeks. Original image ©Alvaro Frank, 2012

to front-facing photographs only. This is simply a matter of focus, as caricature papers tend to be about proving the effectiveness of an automated system, rather than the production of a versatile tool.

6.3 Limitations

The limitations of our system are mostly tied to the occasional difficulty of the fitting stage. Optimally, a morphable guidelines system should be able to detect the facing angle automatically, and perhaps the proportions as well. By combining our system with AAM research on face detection, we may be able to fit the guidelines automatically. This could possibly be achieved to a limited extent by commercially available face tracking packages, such as the one in the Kinect SDK.

A major issue with 2D deformation can occur when certain features overlap in model space. For example, consider Figure 6.4a. In a photograph of a face at a 3/4 angle, all of the facial features will generally fall within the outline of the face as long as the proportions are realistic. If the user wishes to elongate the nose past realistic proportions, the nose may break the outline, thus affecting the topology of the image. An example of the error can be seen in Figure 6.4b. As 2D image
Figure 6.4: When the nose should break the outline of the face, the image deforms in a manner that is topologically correct in 2D but incorrect in 3D. 2D deformation fails to produce a satisfactory result in this case.

deformation maintains 2D topology, it is impossible to make the overlap occur under these constraints. This error could be addressed in future versions through more rigorous preprocessing and the addition of ‘cutout’ portions of the image.

Another minor issue worth mentioning is the situation with occluded vertices, which can be superfluous or even detrimental to the final result. These superfluous vertices still deform the image even though they should not exist. We can mitigate this problem by allowing users to remove vertices that they deem superfluous.
Chapter 7

Conclusions

Above all, morphable guidelines are a tool that makes 2D face warping easier to achieve. Though it is completely possible to achieve our results freehand, it is a tedious process involving a large number of separate deformation points. Our system allows the user to concentrate solely on what they want to change about the face, instead of wrestling with the monotonous task of maintaining perspective and symmetry. It is a tool that facilitates creativity by maximizing quality while minimizing labour.

Morphable guidelines may be simple in concept, but the idea could be expanded with further research. For example, the addition of a robust face tracking package may make it possible to warp faces in real-time video. It may even be possible to create a system that allows a user to create custom-built morphable guidelines using their own proxy models. Morphable guidelines are loosely defined entities, and could be quite simple to modularize for easy adaptation.

In conclusion, the morphable guidelines system provides a simple and effective way to warp faces in 2D images, and possibly other objects as well.
Appendix A

Appendix

A.1 Default morphable guideline parameters

Figure A.1: Components used to construct a set of morphable guidelines. These morphable guidelines were initialized with default values.
Morphable guidelines are constructed using a set of ratios relative to the base circle radius $r$. When a set of morphable guidelines is initialized, it is constructed using these variables. The larger the value, the larger the deviation from the centre of the base circle.

Features that lie on the centreline are calculated via the following function:

$$f(varH) = \frac{clineBD+(varH-faceH)(clineUD-clineBD)}{-faceH}$$

Table A.1: Default scaffold variables and multipliers of radius $r$

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<th>Variable set</th>
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<th>Height (y)</th>
<th>Depth (z)</th>
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<td>centreline (bottom)</td>
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<td>-</td>
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<td>1.333</td>
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