Adaptive Gait Pattern Control of a Quadruped Locomotion Robot

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keyword: Quadruped locomotion robot, Gait pattern control, Non-linear oscillators

Abstract
The authors have proposed a control system of a quadruped locomotion robot by using nonlinear oscillators. It is composed of a leg motion controller and a gait pattern controller. The leg motion controller drives the actuators of the legs by using local feedback control. The gait pattern controller involves nonlinear oscillators with mutual interactions. In this paper, capability of adaptation of the proposed control system against variance of the environment is verified through numerical simulations and hardware experiments: With the input signals from the touch sensors at the tips of the legs, the nonlinear oscillators tune the phase differences among them through mutual entrainments. As a result, a gait pattern corresponding to the states of the system or to the properties of the environment emerges. And the robot changes its gait pattern adaptively to variance of the environment and establishes a stable locomotion while suppressing the energy consumption.

1 Introduction

Locomotion is one of the basic functions of a mobile robot. Using legs is one of the strategies for accomplishing locomotion. It allows the robot to move on rough terrain. Therefore, a considerable amount of research has been done on motion control of legged locomotion robots. This paper deals with the motion control of a quadruped locomotion robot. Usually, motion control of a walking robot has been achieved by using a model-based approach[1][2]. The model-based approach is based on control theory; the design of the trajectories of the legs are implemented through optimization based on the model of the robot. The motion controller which realizes the designed trajectories of the legs is also based on the model of the robot.

In the future, a walking robot will be required which can carry out tasks in the real world, where the geometric and kinematic conditions of the environment are not specially structured. A walking robot is required to realize the real-time adaptability to a changing environment. However, it is difficult for the robot with the model-based control system to carry out various tasks in a changing environment or to adapt to variations of the environment.

The walking motion of an animal seems to offer a solution to the problem; During a walking, a lot of joints and muscles are organized into a collective unit to be controlled as if it had fewer degrees of freedom but to retain the necessary flexibility for a changing environment [3].

Based on the latest achievements of neurobiology and ethology, a new approach to robotics has been developed. Brooks[4],[5] has proposed the subsumption architecture as a principle of design of an autonomous mobile robot which can carry out tasks in the real world. The control system is composed of behavior-generating units. Each unit responds to the changes in the environment and generates a stereotyped action. Responses from all units compete with each other, but only one of them determines the action of the robot. Using the subsumption architecture, Brooks developed a six-legged robot, Genghis to walk over a rough terrain. Although the trajectory of the body was not specified, the robot successfully navigated on a rough terrain.

On the other hand, research has been done on a control system for walking robot which enables to adapt to variances of the environment based on the CPG(Central
Pattern Generator) principle\textsuperscript{[6]}\textendash\textsuperscript{[11]}. M.A.Lewis et al. developed a VLSI CPG Chip and using the chip, they implemented experiments of control of an underactuated running robotic leg: Periodic motion of the hip is driven by an oscillator, and then by controlling phase of oscillator using sensor signal, they established a stable running motion of the leg\textsuperscript{[6]}. K.Akimoto et al. designed a locomotion controller for hexapod robot by using CPG\textsuperscript{[7]}. Oscillators, which are assigned for each leg, drive the periodic motion of each leg. The phase of oscillator is controlled by evaluating energy consumption of motors at joints of the legs. By using this control system, they realized a hexapod robot which can change the gait pattern adaptively to the walking velocity.

This paper deals with the design method of the control system of a quadruped locomotion robot by using nonlinear oscillators: A nonlinear oscillator is assigned to each leg. The nominal trajectory of the leg is determined as a function of phase of its oscillator. We design the local feedback controller for each joint of the legs using the nominal trajectories as the reference. Touch sensors at the tips of the legs are used as triggers on which the dynamic interactions of the legs are based. The mutual entrainment of the oscillators with each other generate a certain combination of phase differences, which leads to the gait pattern. As a result, a gait pattern that can satisfy the requirements of the state of the system or the properties of the terrain emerges and the robot establishes a stable locomotion. The performance of the proposed control system is verified by numerical simulations and hardware experiments.

2 Model

Consider the quadruped locomotion robot shown in Fig. 1, which has four legs and a main body. Each leg is composed of two links which are connected to each other through a one degree of freedom (DOF) rotational joint. Each leg is connected to the main body through a one DOF rotational joint. Legs are enumerated from leg 1 to 4, as shown in Fig. 1. The joints of each leg are numbered as joint 1 and 2 from the main body toward the tip of the leg. We define \( r_i^{(0)} \) and \( \theta_i^{(0)} \) \((i = 1, 2, 3)\) as the components of position vector and Euler angle from inertial space to the coordinate system which is fixed on the main body, respectively. We also define \( \theta_j^{(i)} \) as the joint angle of link \( j \) of leg \( i \).

![Fig. 1 Schematic model of a quadruped robot](image)

The state variable is defined as follows:

\[
q^T = \begin{bmatrix} r_k^{(0)} & \theta_k^{(0)} & \theta_j^{(i)} \end{bmatrix} (i = 1, \ldots, 4, \ j = 1, 2, \ k = 1, 2, 3)
\]

Equations of motion for state variable \( q \) are derived using Lagrangian formulation as follows:

\[
M \ddot{q} + H(q, \dot{q}) = G + \sum (\tau_j^{(i)}) + \Lambda
\]

where \( M \) is the generalized mass matrix and the term \( M \ddot{q} \) expresses the inertia. \( H(q, \dot{q}) \) is the nonlinear term which includes Coriolis forces and centrifugal forces. \( G \) is the gravity term. \( \tau_j^{(i)} \) is the input torque of the actuator at joint \( j \) of leg \( i \). \( \Lambda \) is the reaction force from the ground at the point where the tip of the leg makes contact. We assume that there is no slip between the tips of the legs and the ground.

3 Control system

The architecture of the proposed control system is shown in Fig. 2. The control system is composed of leg motion controllers and a pattern controller. The leg motion controllers drive all the joint actuators of the legs so as to realize the desired motions that are generated by the gait pattern controller. The gait pattern controller involves nonlinear oscillators corresponding to each leg. The gait pattern controller receives the commanded signal of the nominal gait pattern as the reference. It also receives the feedback signals from the touch sensors at the tips of the legs. The gait pattern is determined by the phase differences between the nonlinear oscillators. A modified gait pattern is generated from the nominal gait pattern through the mutual entrainment of the oscillators with the feedback signals of the touch sensors.
The generated gait pattern is given to the leg motion controller as the commanded signal of the locomotion pattern of the legs.

3.1 Design of the gait

3.1.1 Design of the trajectories of the legs

The position of the tip of the leg where the transition from the swinging stage to the supporting stage occurs is called the anterior extreme position (AEP). Similarly, the position where the transition from the supporting stage to the swinging stage occurs is called the posterior extreme position (PEP) [12]. We determine the nominal trajectories which are expressed in the coordinate system which is fixed on the main body. First, we define the nominal PEP \( \hat{r}_{eP}^i \) and the nominal AEP \( \hat{r}_{eA}^i \). The index \( \hat{\ast} \) indicates the nominal value.

The trajectory for the swinging stage is a closed curve given as the nominal trajectory \( \hat{r}_{eF}^i \). This curve involves the points \( \hat{r}_{eA}^i \) and \( \hat{r}_{eP}^i \). On the other hand, the trajectory for the supporting stage is a linear trajectory given as \( \hat{r}_{eS}^i \). This linear trajectory also involves the points \( \hat{r}_{eA}^i \) and \( \hat{r}_{eP}^i \). The position of each leg on these trajectories is given as functions of the phase of the corresponding oscillator. The state of the oscillator for leg \( i \) is expressed as follows:

\[
\hat{z}^i = \exp(j \, \hat{\phi}^i) \tag{3}
\]

where \( \hat{z}^i \) is a complex number representing the state of the oscillator, \( \hat{\phi}^i \) is the phase of the oscillator and \( j \) is the imaginary unit.

The nominal phases at AEP and PEP are determined as follows:

\[
\hat{\phi}^i_A = \hat{\phi}_{eA}^i \quad \text{at AEP}, \quad \hat{\phi}^i = 0 \quad \text{at PEP} \tag{4}
\]

The nominal trajectories for swinging stage \( \hat{r}_{eF}^i \) and for supporting stage \( \hat{r}_{eS}^i \) are given as functions of the phase \( \hat{\phi}^i \) of the oscillator and are alternatively switched at every step of AEP and PEP.

\[
\hat{r}_{e}^i(\hat{\phi}^i) = \begin{cases} 
\hat{r}_{eF}^i(\hat{\phi}^i) & 0 \leq \hat{\phi}^i < \hat{\phi}_{eA}^i \\
\hat{r}_{eS}^i(\hat{\phi}^i) & \hat{\phi}_{eA}^i \leq \hat{\phi}^i < 2\pi 
\end{cases} \tag{5}
\]

3.1.2 Design of the gait pattern

The gait patterns, which are the relationships between motions of the legs, are designed. The gait patterns for a quadruped robot are divided into three groups: One is group of the patterns in which three legs support the main body at any instant during locomotion such as Walk. Another is group of the patterns in which two legs support the main body at any instant during locomotion. The nominal duty ratio \( \hat{\beta}^i \) for leg \( i \) is defined to represent the ratio between the nominal time for the supporting stage and the period of one cycle of the nominal locomotion.

\[
\hat{\beta}^i = 1 - \frac{\hat{\phi}_{eA}^i}{2\pi} \tag{6}
\]
locomotion such as Trot, Pace, Bounce. The other is group of the patterns in which less than one leg support the main body at any instant during locomotion such as Gallop. In this paper, the former two groups are considered.

Each pattern is represented by a matrix of phase differences $\Gamma_{ij}^{(m)}$ as follows:

$$\phi^{(j)} = \phi^{(i)} + \Gamma_{ij}^{(m)}$$  \hspace{1cm} (7)

where, $m = 1, 2$ represent transverse walk pattern and rotary walk pattern, respectively. $m = 3, 4, 5$ represent trot pattern, pace pattern and bounce pattern, respectively.

![Fig. 4: The trot and the transverse walk patterns](image)

**3.2 Locomotion control**

**3.2.1 Leg motion control**

The angle of joint $j$ of leg $i$ is derived from the geometrical relationship between the trajectory $\hat{r}_{e}^{(i)}(\hat{\phi}^{(i)})$ and the joint angle. $\dot{\theta}_{j}^{(i)}$ is written as a function of phase $\hat{\phi}^{(i)}$ as follows:

$$\dot{\theta}_{j}^{(i)} = \hat{\theta}_{j}^{(i)}(\hat{\phi}^{(i)})$$  \hspace{1cm} (8)

where $\hat{\theta}$ indicates the nominal value. The commanded torque at each joint of the leg is obtained by using local PD feedback control as follows:

$$\tau_{j}^{(i)} = K_{p j}(\hat{\theta}_{j}^{(i)} - \dot{\theta}_{j}^{(i)}) + K_{D j}(\dot{\theta}_{j}^{(i)} - \ddot{\theta}_{j}^{(i)})$$  \hspace{1cm} (9)

where $\tau_{j}^{(i)}$ is the actuator torque at joint $j$ of leg $i$.

**3.2.2 Gait pattern control**

We design the phase dynamics of the oscillators corresponding to each leg as follows:

$$\dot{\phi}^{(i)} = \omega + g_{l}^{(i)} + g_{2}^{(2)} \quad (i = 1, \ldots, 4)$$  \hspace{1cm} (10)

$$g_{l}^{(i)} = -K \left( \phi^{(i)} - \phi^{(j)} - \Gamma_{ij}^{(m)} \right)$$  \hspace{1cm} (11)

$$g_{2}^{(i)} = (\hat{\phi}^{(i)} - \phi_{A}^{(i)})\delta$$  \hspace{1cm} (12)

where $K$ is a constant number and $\delta$ is Delta-Function.

Function $g_{2}^{(i)}$ is designed in the following way: Suppose that $\phi_{A}^{(i)}$ is the phase of leg $i$ at the instant when leg $i$ touches the ground. Similarly, $r_{eA}^{(i)}$ is the position of leg $i$ at that instance. When leg $i$ touches the ground, the following procedure is undertaken.

1. Change the phase of the oscillator for leg $i$ from $\phi_{A}^{(i)}$ to $\hat{\phi}_{A}^{(i)}$.
2. Alter the nominal trajectory of the tip of leg $i$ from the swinging trajectory $\hat{r}_{e}^{(i)}$ to the supporting trajectory $r_{eA}^{(i)}$.
3. Replace parameter $r_{eA}^{(i)}$ that is one of the parameters of the nominal trajectory $\hat{r}_{e}^{(i)}$, with $r_{eA}^{(i)}$.

The oscillators form a dynamic system that affect each other through two types of interactions. One is continuous interactions from $g_{l}^{(i)}$ which depends on the nominal gait pattern. The other is the pulse-like interactions $g_{2}^{(i)}$ which is caused by the feedback signals from the touch sensor. Through these interactions, the oscillators generate gait patterns that satisfy the requirements of the environment.

**4 Numerical simulation**

Numerical simulations are implemented to verify the performances of the proposed control system. Table 1 shows the physical parameters of the robot which are used in numerical simulations.

<table>
<thead>
<tr>
<th>Table 1</th>
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<tr>
<td><strong>Main body</strong></td>
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<td>Width</td>
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<tr>
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<td><strong>Legs</strong></td>
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<td>Length of link 1</td>
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<td>Length of link 2</td>
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<td>Mass of link 1</td>
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The nominal time period of the swinging stage is chosen as 0.20 [sec].

First, numerical simulations are carried out to evaluate stability of the locomotion. A steady locomotion is a periodic motion and its stability can be verified by using
Floquet Theorem. Selecting $q = (\theta_1^{(0)}, \theta_2^{(0)}, \dot{\theta}_1^{(0)}, \dot{\theta}_2^{(0)}) \in \mathbb{R}^4$ as the state variables of periodic motion, Poincaré map along the periodic orbit of $q$ at Poincaré section is obtained. If the maximum norm of eigen values of Poincaré map of periodic orbit of $q$ is smaller than one, the periodic motion is stable. Numerical simulations are implemented and then stability of the locomotion is evaluated by checking the maximum norm of eigen values of Poincaré map. One example is shown in Fig. 5. The nominal duty ratio $\hat{\beta}$ is selected as a parameter. We can find that the robot established stable locomotion in a wide parameter range.

Then, in order to clarify capability of adaptation of the proposed control system, we investigated variance of the gait pattern. An order parameter $D^{(m)}$ is defined, which is to measure the similarity between the obtained gait pattern and the nominal gait pattern.

$$D^{(m)} = \frac{1}{4} trace (W^{(m)T} W) \quad (14)$$

$$W_{ij} = \frac{<\zeta_i \zeta_j >}{\sqrt{<\zeta_i > \sqrt{<\zeta_j >}}} \quad <*> = \int * dt$$

$$\zeta_i = \begin{cases} 1 - \frac{1}{\beta} & \text{Swinging stage} \\ -\frac{1}{\beta} & \text{Supporting stage} \end{cases}$$

Figure 6 shows the variances of parameter $D^{(m)}$ against the variance of the nominal duty ratio $\hat{\beta}$. From Fig. 6, we can find that although trot pattern is given as the nominal gait pattern, similarity between the obtained gait pattern and transverse walk pattern increases as duty ratio $\hat{\beta}$ increases. The gait patterns generated in a steady locomotion are examined. Figures 7(a) and 7(b) are examples of the obtained gait pattern. In these figures, solid line indicates supporting stage and blank is swinging stage and the nominal gait pattern is set to trot. The figures show that in the case where duty ratio $\hat{\beta}$ is small, trot is established whereas in the case where duty ratio $\hat{\beta}$ is large, transverse walk is established.

Variance of the gait pattern according to change of the walking speed during locomotion is also checked. The nominal gait pattern is fixed to transverse walk. Figure 8 (a) is the case where duty ratio $\hat{\beta}$ is fixed to 0.75 up to 10 [sec], from 10 [sec] to 15 [sec] duty ratio $\hat{\beta}$ is commanded to change from 0.75 to 0.50 continuously, and after 15 [sec] duty ratio $\hat{\beta}$ is again fixed to 0.75. To the contrary, Fig. 8 (b) is the case where duty ratio $\hat{\beta}$ is changed from 0.50 to 0.75 during the period from 10 [sec] to 15 [sec]. The figures show that as duty ratio $\hat{\beta}$ decreases, the gait pattern changes from transverse walk to trot smoothly, while as duty ratio $\hat{\beta}$ increases, the gait pattern changes from trot to transverse walk smoothly.

Figure 9 shows the state transition diagram with respect to three parameters of the environment, walking velocity, load offset and inclination angle of the ground. In Fig. 9, suffix $*_{0}, *_{H}$ and $*_{L}$ indicate that the value * is zero, low and high, respectively. For example, when the commanded walking velocity $V$ are changed from a small value to large one and other values are given as zero, the obtained gait pattern changes from transverse walk(T.Walk) to Trot. The transition from one gait pattern to another in the figure is observed through numerical simulations when the condition parameters are changed. This figure shows that the proposed control system has capability to emerge various gait patterns according to the conditions of the environment.

Lastly, variance of energy consumption of actuators $E_c$ is investigated, selecting duty ratio $\hat{\beta}$ as a parameter. Energy consumption of actuators $E_c$ is defined as

$$E_c = \frac{< \sum_{i,j} \tau_{ij}^{(d)} \dot{\beta}_{ij}^{(d)} >}{<v>}$$

where, $v$ is walking velocity.

Figure 10 shows the result where "fixed" means that the gait pattern is fixed to a transverse walk. From Fig. 10, we can see that the value of $E_c$ of the proposed control system and their variance with respect to the variance of the duty ratio are smaller than those where the gait pattern is fixed.

From this result, it may be noted that by changing its gait pattern according to variance of the environment, the quadruped locomotion robot with the proposed control system can suppress the energy consumption.
Nominal gait pattern: Transverse walk ($\Gamma^{(1)}$)

Fig. 6. Similarity of the gait pattern

(b) $\beta = 0.75$

Nominal gait pattern: Trot ($\Gamma^{(3)}$)

Fig. 7. Gait pattern diagram

(a) $\beta = 0.50$

Nominal gait pattern: Transverse walk ($\Gamma^{(1)}$)

Fig. 8. Gait pattern diagram

Nominal gait pattern: Trot ($\Gamma^{(3)}$)

Fig. 7. Gait pattern diagram

Walking velocity $V = (V_L, V_H)$

Load offset $L = (L_0, L_H)$

Inclination of the ground $I = (I_0, I_H)$

Fig. 9. State transition diagram

5 Hardware experiment

Performance of the proposed control system is verified by hardware experiments. Figure 11 shows the mechanical system. This quadruped robot has two DOF for each leg. Each joint is driven by Harmonic Drive DC actuator. Maximum actuator torque is 4.6 [Nm] and maximum rotational speed is 3.5 [rad/s]. Load cell is equipped at the tip of each leg and play roles of touch sensor and force sensor. Actuator drivers and amplifiers for force sensors are mounted on the main body.

Figure 12 shows data processing system. The controller is programmed on DSP board (TI 320C44 × 2). Sampling frequency is 1.0 [kHz]. Signals from force sensors are input to 12bit A/D converters and are sent to DSPs. Optical encoder signals from the actuators at the joints are input to 24bit pulse counters, are transformed to angle data and are also sent to DSPs. Torque commands for actuators are calculated on DSP board on real-time using proposed control method, and sent from
16-bit D/A converters to the actuator drivers. Personal computer is connected to DSPs through PCI bus and plays roles of supervisor of DSPs and debugger for control program. Figure 13 shows the photograph of the hardware equipment.

Figures 14(a) and 14(b) are examples of the obtained gait pattern of steady locomotion. In these case, the nominal gait pattern is set to transverse walk. The robot established steady locomotion for each duty ratio $\hat{\beta}$ by changing its gait pattern.

Variance of the gait pattern according to change of the walking speed during locomotion is investigated. The nominal gait pattern is fixed to transverse walk. Figure 15(a) is the case that duty ratio $\hat{\beta}$ is fixed to 0.75 up to 10 [sec], from 10 [sec] to 15 [sec] duty ratio $\hat{\beta}$ is commanded to change from 0.75 to 0.50 continuously, and after 15 [sec] duty ratio $\hat{\beta}$ is again fixed to 0.75. To the contrary, Fig.15(b) is the case that duty ratio $\hat{\beta}$ is changed from 0.50 to 0.75 during the period from 10 [sec] to 15 [sec].

From these results, it is also verified as well as numerical simulations that the robot using the proposed control system has a capability of adaptation to variance of duty ratio $\hat{\beta}$ by changing the gait patterns autonomously.

6 Conclusions

We proposed a control system for a walking robot with a hierarchical architecture which is composed of leg motion controllers and a gait pattern controller. The leg motion controller drives the actuators at the joints of the legs by use of high-gain local feedback based on the commanded signal from the gait pattern controller.
Whereas the gait pattern controller alternates the motion primitives by synchronizing with the signals from the touch sensors at the tips of the legs, and stabilizes the phase differences among the motions of the legs adaptively. The robot with the proposed control system changes its gait pattern according to variance of the environment while suppressing the energy consumption. In this paper, the nominal gait pattern is given as the command. In the future, we are planning to design the control system in which the nominal gait pattern is selected or generated according to the state of the robot. Using such a control system, it is expected that adaptability of the robot to variations of the environment will be highly improved.

Acknowledgments
The authors were funded by grants from the The Japan Society for the Promotion of Science (JSPS) as the Research for the Future program (RFTF) and from the Japan Science and Technology Corporation (JST) as the Core Research for Evolutional Science and Technology program (CREST).

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