Cadence Control of Stationary Cycling Induced by Switched Functional Electrical Stimulation Control

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Abstract—Cycling induced by functional electrical stimulation (FES) is an effective means for exercise and rehabilitation of individuals suffering from neurological disorders such as stroke, spinal cord injury, and cerebral palsy. To achieve FES-cycling, potential fields are alternately applied across muscle groups in the lower extremities. Alternating stimulation of various muscle groups according to the crank position makes the FES-cycling system a switched system with autonomous state-dependent switching. This paper examines FES-cycling from a switched systems analysis perspective. Specifically, a switched sliding mode controller is developed to yield approximate tracking of a desired cadence despite an uncertain, nonlinear cycle-rider model. Cadence tracking is proven via a common Lyapunov-like function and experimental results are provided to demonstrate the performance of the switched controller.

I. INTRODUCTION

Neurological disorders such as stroke, spinal cord injury, Parkinson’s disease, and cerebral palsy affect a person’s ability to control the lower extremities, thereby inhibiting functional activities such as standing, walking, and cycling. Functional electrical stimulation (FES) has been used as a means to artificially activate paralyzed or weakened muscles and produce or assist a functional movement (e.g., walking [1], grasping [2], and cycling [3]). FES-cycling is an especially attractive method for exercise and rehabilitation since cycling can be performed for long periods of time with a relatively low risk of injury [4], and numerous physiological and psychological benefits of FES-cycling have been reported [5]. Despite these benefits, the power output at the cycle crank and the metabolic efficiency in FES-cycling is much less than in volitional cycling, even for able-bodied subjects [6]. It has been suggested that the problems of power output and efficiency in FES-cycling are caused by poor control of the involved muscles and non-physiological recruitment of the muscle fibers [7]. While the latter cause is a known problem with electrical stimulation in general [8], the former cause follows directly from the control methods used for FES-cycling.

Past FES-cycling control methods have utilized linear controllers [9]–[11], fuzzy logic and sliding mode controllers [12]–[14], or controllers based on experimental data or numerical optimizations [15]–[18]. All of these previous studies lack detailed stability analyses or require exact model knowledge. In addition, all of the aforementioned studies used some combination of cycling cadence, its time derivative, and cycling power as direct feedback. More recently, some researchers have dealt directly with the uncertain, nonlinear nature of the cycle-rider system and included detailed stability analyses [19], [20].

Most FES-cycling studies have utilized stimulation patterns which stimulate various muscle groups in an alternating pattern according to the crank position. However, aside from [20], no prior studies have analyzed the closed-loop control system performance in the context of switched systems theory. In general, FES-cycling requires switching between subsystems controlled by different combinations of muscle groups, creating a switched control system governed by state-dependent switching [21]. Specifically, since the switching happens automatically and is not controlled manually by an actor, the switching is autonomous.

In this paper, the FES-cycling system is considered to be an uncertain, nonlinear model with an added, uncertain, bounded disturbance representative of unmodeled effects such as changes in load. The stimulation pattern is the same as in [20], and only stimulation of the quadriceps femoris muscle groups is considered. Stimulating only the quadriceps is sufficient for FES-cycling; however, this partitions the crank cycle into controlled and uncontrolled regions. The error dynamics in the controlled region are proven to be exponentially stable through a Lyapunov-like stability analysis with a sliding mode controller. In the uncontrolled regions, the error dynamics are proven to be upper bounded by an exponentially increasing function, so that if the time that the system dwells in the uncontrolled regions is finite then the potential growth of the error is bounded. The overall error system is shown to be ultimately bounded in the sense that the cadence error converges to a residual as the number of crank cycles approaches infinity. Performance of the cadence controller is demonstrated through preliminary experimental results with one able-bodied subject.

II. MODEL

A. Model of Stationary Cycle and Rider

The FES-cycling system is modeled as a stationary cycle and a two-legged rider with rigid-body segments and feet.
fixed to the pedals. The rider’s ankles are rigidly held at 90 degrees to prevent unfavorable motion of the ankle joints and to keep the rider’s legs in the sagittal plane while cycling for safety and stability [22]. The ankles could be controlled by stimulating the triceps surae and the tibialis anterior, but stimulating the lower legs would only grant 6% more work capacity than stimulating the upper legs alone [23] and would require a more complex system model. The rider’s hip and knee joints are modeled as revolute joints with axes of rotation perpendicular to the sagittal plane. The effects of more complex knee joint models on the linkage kinematics are negligible [24]. Passive viscoelastic effects of the tissues within and surrounding the rider’s joints are considered and are modeled as in [25] and [26]. Viscous damping in the crank joint bearings is also considered. The crankset is modeled as two rigid crank arms offset by 180 degrees with revolute pedals and a revolute crank axis perpendicular to the sagittal plane. Modeling the FES-cycling system in this manner yields a single degree-of-freedom system [27] that can be expressed as [20]

\[
M \ddot{q} + V \dot{q} + G + \tau_d - \tau_b - P = \sum_{s \in S} B_s^k \Omega^s u^s, \tag{1}
\]

where \( q \in Q \subseteq R \) denotes the crank angle, measured clockwise from the horizontal to the right crank arm by convention; \( M \in R \) denotes the effects of the combined inertia of the rider’s limbs and the crankset; \( V \in R \) denotes Coriolis and centripetal effects; \( G \in R \) denotes the effect of gravity on the system; \( \tau_d \in R \) is an uncertain, time-varying, bounded disturbance; \( \tau_b \in R \) represents viscous damping in the crank bearings, where the unknown constant \( \varepsilon < \) is the damping coefficient; \( P \in R \) represents the viscoelastic effects of the rider’s joints; \( B_s \in R \) is the torque transfer ratio which transforms torque about the knee to torque about the crank; and \( \Omega \in R \) is an uncertain, nonlinear relationship between quadriceps stimulation voltage \( u \in R \) and active torque about the knee. The superscript \( s \in S \triangleq \{ R, L \} \) indicates the rider’s right (R) or left (L) leg and is subsequently omitted unless it is required to add clarity. Fig. 1 depicts the cycle-rider system described by (1), wherein \( q \in R \) is the measurable constant trunk angle, and \( q_h, q_k \in R \) are the hip and knee angles, respectively, which are geometric functions of the measurable, constant horizontal and vertical distances between the hip and crank joint axes, \( l_s, l_y \in R_{>0} \), and the measurable, constant thigh, shank, and crank segment lengths, \( l_t, l_l, l_c \in R_{>0} \), respectively. The following properties of the model in (1), derived from [25], [26], and [28], are used to facilitate the subsequent analysis.

**Property 1.** \( c_m \leq M \leq c_M \), where \( c_m, c_M \in R_{>0} \) are known constants. **Property 2.** \( |V| \leq c_V |\dot{q}| \), where \( c_V \in R_{>0} \) is a known constant. **Property 3.** \( |G| \leq c_G \), where \( c_G \in R_{>0} \) is a known constant. **Property 4.** \( |\tau_d| \leq c_d \), where \( c_d \in R_{>0} \) is a known constant. **Property 5.** \( |B_s^k| \leq c_B \forall s \in S \), where \( c_B \in R_{>0} \) is a known constant. **Property 6.** \( |P| \leq c_{P1} + c_{P2} |\dot{q}| \), where \( c_{P1}, c_{P2} \in R_{>0} \) are known constants. **Property 7.** \( c_{G1} \leq Q_s \leq c_{G2} \forall s \in S \), where \( c_{G1}, c_{G2} \in R_{>0} \) are known constants. **Property 8.** \( \frac{1}{2} M - V = 0 \).

**B. Switched System Model**

The stimulation pattern is defined as in [20], where the sets \( Q_s \subset Q \) are the regions where the right and left quadriceps muscle groups are stimulated and are defined as

\[
Q_s \triangleq \{ q \in Q \mid -B_s^k (q) > \varepsilon \}, \tag{2}
\]

where \( \varepsilon \in R_{>0} \) is a scalable constant, and the set \( Q \triangleq Q_R \cup_{s \in S} Q_s \) is the controlled region. Specifically, increasing \( \varepsilon \) decreases the size of the controlled regions\(^1\), provided \( \varepsilon < \min (-B_s^k) \). The uncontrolled region, i.e., the portion of the crank cycle in which no stimulation is applied, is defined as \( Q_u \triangleq \{ q \in Q \mid -B_s^k (q) \leq \varepsilon \} = Q \setminus Q_c \). Note that \( Q R \cap Q L = \emptyset \) since stimulation is only applied to one leg at a time. The stimulation pattern is designed to avoid stimulation of the muscles near the dead points of the crank cycle, denoted \( q^* \in Q_u \), where the torque transfer ratios are zero and high intensity stimulation yields no effective torque about the crank \( (B_s^k (q^*) = 0) \).

In accordance with the stimulation pattern defined by (2), the switched control input \( u^s \) is designed as

\[
u^s \triangleq \begin{cases} v & \text{if } q \in Q_c \\ 0 & \text{if } q \in Q_u \end{cases}, \tag{3}
\]

where \( v \in R \) is the subsequently designed control input. Substituting (3) into (1) yields the following switched system model with autonomous state-dependent switching:

\[
M \ddot{q} + V \dot{q} + G + \tau_d - \tau_b - P = \begin{cases} B_s^k \Omega^s v & \text{if } q \in Q_c \\ 0 & \text{if } q \in Q_u \end{cases}. \tag{4}
\]

**Assuming that the crank starts in the controlled region, the sequence of switching states is denoted \( \{ q_{00}^0, q_{01}^1 \} \),**

\( n \in \{ 0, 1, 2, ... \} \), where \( q_{00}^0 \) is the initial crank angle and

\( ^1\)To optimize metabolic efficiency by minimizing the muscle stress-time integral, some evidence in the FES-cycling literature (e.g., [29]) suggests that \( \varepsilon \) should be made as large as possible while stimulating with the maximum tolerable intensity.
the subsequent switching states are the limit points of $Q_u$. The corresponding sequence of switching times is denoted $\{t_{on}^n, t_{off}^n\}$. Note that the switching states depend directly on $B_k$ and $\varepsilon$ and are therefore known a priori, but the switching times depend on the uncertain system dynamics and are therefore unknown a priori. The stimulation pattern on $Q$ is the subsequent switching states are the limit points of $Q_u$. Lightly shaded regions are the controlled regions, and darkly shaded regions are the uncontrolled regions.

### III. Control Development

#### A. Open-Loop Error System

The control objective is to track a desired cadence by stimulation of the quadriceps femoris muscle groups during FES-cycling. This objective is quantified by the tracking error $e \in \mathbb{R}$, which is defined as

$$e \triangleq \dot{q}_d - \dot{q},$$

where $\dot{q}_d \in \mathbb{R}$ is the desired cadence, designed to be continuously differentiable with a bounded first time derivative, i.e., $\dot{q}_d \in L_\infty$. The open-loop error system can be obtained by taking the time derivative of (5), multiplying the result by $M$, and substituting (4) and (5), yielding

$$M\dot{e} = \chi - V e - \begin{cases} B_k \Omega^r v & \text{if } q \in Q_c, \\ 0 & \text{if } q \in Q_u, \end{cases}$$

where $\chi \in \mathbb{R}$ is an auxiliary term defined as

$$\chi \triangleq M\ddot{q}_d + V \dot{q}_d + G + \tau_d - \tau_0 - P.$$  

The expression in (7) can be bounded using Properties 1-6 and (5) as

$$|\chi| \leq c_1 + c_2 |e|,$$

where $c_1, c_2 \in \mathbb{R}_{>0}$ are known constants.

#### B. Closed-Loop Error System

The control input is designed as

$$v \triangleq -k_1 e - k_2 \text{sgn}(e),$$

where $\text{sgn}(\cdot)$ is the standard signum function and the constants $k_1, k_2 \in \mathbb{R}_{>0}$ are control gains. Substituting (9) into (6) yields the following switched closed-loop error system for $q \in Q_c$:

$$M\dot{e} = \chi - V e + B_k^r \Omega^r [k_1 e + k_2 \text{sgn}(e)].$$

### IV. Stability Analysis

Let $V_L : \mathbb{R} \to \mathbb{R}$ be a positive definite, continuously differentiable, common Lyapunov-like function defined as

$$V_L \triangleq \frac{1}{2} M e^2.$$  

$V_L$ is radially unbounded and satisfies the following inequalities:

$$\left(\frac{c_m}{2}\right) e^2 \leq V_L \leq \left(\frac{c_M}{2}\right) e^2.$$  

**Theorem 1.** The closed-loop error system in (10) is exponentially stable for $q \in Q_c$ in the sense that

$$|e(t)| \leq \sqrt{\frac{c_M}{c_m}} |e(t_{on})| \exp \left[ -\frac{\lambda_c}{2} (t - t_{on}) \right]$$

$$\forall t \in (t_{on}, t_{off}) \text{ and } \forall n, \text{ where } \lambda_c \in \mathbb{R}_{>0}$$

is defined as

$$\lambda_c \triangleq \frac{2}{c_M} (\varepsilon \Omega_{Q1} k_1 - c_2),$$

provided the following sufficient gain conditions are satisfied:

$$k_1 > \frac{c_2}{\varepsilon \Omega_{Q1}}, \quad k_2 \geq \frac{c_1}{\varepsilon \Omega_{Q1}}.$$  

**Proof:** Let $e(t)$ for $t \in (t_{on}, t_{off})$ be a Filippov solution to the differential inclusion $\dot{e} \in K [h] (e)$, where $K [\cdot]$ is defined as in [30] and where $h : \mathbb{R} \to \mathbb{R}$ is defined by (10) as

$$h \triangleq M^{-1} \{\chi - V e + B_k^r \Omega^r [k_1 e + k_2 \text{sgn}(e)]\}.$$  

The time derivative of (11) exists almost everywhere (a.e.), i.e., for almost all $t \in (t_{on}, t_{off})$, and $\dot{V}_L \triangleq \dot{V}_L$ is the general time derivative of (11) along the Filippov trajectories of $e \in h(e)$ and is defined as [31]

$$\dot{V}_L \triangleq \bigcap_{\xi \in \partial V_L(e)} \xi^T K \begin{bmatrix} h(e) \\ 1 \end{bmatrix},$$

where $\partial V_L$ is the generalized gradient of $V_L$. Since $V_L$ is continuously differentiable in $e$, $\partial V_L = \{\nabla V_L\}$; thus,

$$\dot{V}_L \subseteq \left[ \begin{bmatrix} M e \\ 1 \end{bmatrix} \right]^T K \begin{bmatrix} h(e) \\ 1 \end{bmatrix}.$$  

Using the calculus of $K [\cdot]$ from [31] and substituting (16) into the result yields

$$\dot{V}_L \subseteq \left\{ \chi - V e + B_k^r \Omega^r [k_1 e + k_2 K \text{sgn}(e)]\right\} e + \frac{1}{2} M e^2,$$

where $c_1, c_2 \in \mathbb{R}_{>0}$ are known constants.
where \( K[\text{sgn}] (e) = \text{SGN} (e) \) and
\[
\text{SGN} (e) \equiv \begin{cases} 
1 & \text{if } e > 0 \\
[−1, 1] & \text{if } e = 0 \\
−1 & \text{if } e < 0
\end{cases}.
\] (18)

Using Property 8 allows (17) to be rewritten as
\[
\dot{V}_L \leq \chi e + B_k^n \Omega^a [k_1 e + k_2 \text{SGN} (e)] e.
\] (19)

Since \( \dot{V}_L (e) \in \dot{V}_L (e) \), (19) can be upper bounded using Young’s inequality, the fact that \( B_k < −\varepsilon \) for \( q \in Q_e \), Property 7, (8), and (18) as
\[
\dot{V}_L \leq −(\varepsilon \varepsilon_1 k_2 − c_1) |e| − (\varepsilon \varepsilon_1 k_3 − c_2) e^2,
\] (20)

where \( \text{SGN} (e) e \) was replaced with \(|e|\) since \( \text{SGN} (e) \) is only set-valued for \( e = 0 \). Provided the conditions on the control gains in (15) are satisfied, (12) can be used to upper bound (20) as
\[
\dot{V}_L \leq −\lambda_c V_L,
\] (21)

where \( \lambda_c \) was defined in (14). Following the procedure described in [20] allows the solution of (21) to be determined as
\[
V_L (t) \leq V_L (t_n^{\text{off}}) \exp [−\lambda_c (t − t_n^{\text{off}})]
\] (22)

for all \( t \in (t_n^{\text{off}}, t_n^{\text{on}}) \) and for all \( n \). Rewriting (22) using (12) and performing some algebraic manipulation yields (13).

**Theorem 2.** The closed-loop error system in (10) can be bounded for \( q \in Q_u \) as follows:

\[
|e (t)| \leq \left\{ \frac{1}{c_m} \left[ c_M e \left( t_n^{\text{off}} \right)^2 + 1 \right] \exp \left[ \lambda_n (t − t_n^{\text{off}}) \right] \right. \\
- \frac{1}{c_m} \left. \right\} ^{\frac{1}{2}}
\] (23)

for all \( t \in [t_n^{\text{off}}, t_n^{\text{on}}] \) and for all \( n \).

**Proof:** In the uncontrolled region, the time derivative of (11) can be expressed using (10) and Property 8 as
\[
\dot{V}_L = \chi e,
\]

which can be upper bounded using (8) and (12) as
\[
\dot{V}_L \leq \left( \frac{2c_2}{c_m} \right) V_L + \left( \frac{c_1 \sqrt{2} c_m}{c_m} \right) V_L^\frac{1}{2}.
\] (24)

The right-hand side of (24) can be upper bounded in a piecewise manner as
\[
\dot{V}_L \leq \begin{cases} 
\lambda_n V_L & \text{if } V_L \leq 1 \\
\lambda_u V_L & \text{if } V_L > 1
\end{cases},
\] (25)

where
\[
\lambda_n \equiv 2 \cdot \max \left\{ \frac{2c_2}{c_m}, \frac{c_1 \sqrt{2} c_m}{c_m} \right\}.
\]

Since both \( V_L \) and \( \lambda_n \) are positive, (25) can always be upper bounded as
\[
\dot{V}_L \leq \lambda_u \left( V_L + \frac{1}{2} \right).
\] (26)

The solution to (26) over the interval \( t \in [t_n^{\text{off}}, t_n^{\text{on}}] \) yields the following upper bound on \( V_L \) in the uncontrolled region:
\[
V_L (t) \leq V_L \left( t_n^{\text{off}} \right) \exp [\lambda_n (t − t_n^{\text{off}})] + \frac{1}{2} \{ \exp [\lambda_n (t − t_n^{\text{off}})] − 1 \}
\] (27)

for all \( t \in [t_n^{\text{off}}, t_n^{\text{on}}] \) and for all \( n \). Rewriting (27) using (12) and performing some algebraic manipulation yields (23).

**Remark 1.** The exponential bound in (23) indicates that in the uncontrolled regions, the error growth is bounded by an exponential function of the off-time. Since the error decays at an exponential rate in the controlled regions, as described by (13), then relationships between the exponential time constants \( \lambda_n \) and \( \lambda_u \) and the time spent in each region \( \Delta t_n^{\text{on}} \equiv t_n^{\text{off}} − t_n^{\text{on}} \) and \( \Delta t_n^{\text{off}} \equiv t_n^{\text{on}} + 1 − t_n^{\text{off}} \) can be developed for stability of the overall system. A challenge is that the durations \( \Delta t_n^{\text{on}} \) and \( \Delta t_n^{\text{off}} \) depend on the switching times, which are unknown a priori. However, based on the design of the desired cadence \( \tilde{q}_d \), selection of the control gain \( k_1 \), and choice of \( \varepsilon \), it can be demonstrated that there exist known constant bounds \( \Delta t_n^{\text{min}} \in \mathbb{R}_{>0} \) and \( \Delta t_n^{\text{max}} \in \mathbb{R}_{>0} \) such that \( \Delta t_n^{\text{on}} \leq \Delta t_n^{\text{min}} \) and \( \Delta t_n^{\text{off}} \geq \Delta t_n^{\text{max}} \).

**Theorem 3.** The closed-loop error system in (10) is ultimately bounded in the sense that \( |e (t)| \) converges to a ball with constant radius \( d \in \mathbb{R}_{>0} \) as the number of crank cycles approaches infinity (i.e., as \( n \to \infty \)), where \( d \) is defined as
\[
d \equiv \left[ \frac{\exp \left( \lambda_n \Delta t_n^{\text{off}} \right) − 1}{c_m \left[ 1 − \exp \left( \lambda_n \Delta t_n^{\text{off}} \lambda_c \Delta t_n^{\text{on}} \right) \right]} \right]^\frac{1}{2}.
\] (28)

provided the following gain condition is satisfied:
\[
\lambda_c > \lambda_n \frac{\Delta t_n^{\text{off}}}{\Delta t_n^{\text{on}}},
\] (29)

**Proof:** Using (22) and (27) sequentially and assuming the worst case scenario for each cycle where \( \Delta t_n^{\text{on}} = \Delta t_n^{\text{on}} \) and \( \Delta t_n^{\text{off}} = \Delta t_n^{\text{off}} \), the following upper bound for \( V_L \) at the \( N \text{th} \) on-time can be developed:
\[
V_L (t_n^{\text{on}}) \leq V_L \left( t_0^{\text{on}} \right) \left[ \exp \left( \lambda_n \Delta t_n^{\text{off}} \lambda_c \Delta t_n^{\text{on}} \right) \right]^N + \frac{1}{2} \left[ \exp \left( \lambda_n \Delta t_n^{\text{off}} \lambda_c \Delta t_n^{\text{on}} \right) − 1 \right]
\times \sum_{n=0}^{N-1} \left[ \exp \left( \lambda_n \Delta t_n^{\text{off}} \lambda_c \Delta t_n^{\text{on}} \right) \right]^n.
\] (30)

Taking the limit of (30) as \( N \to \infty \), provided (29) is satisfied, gives the following ultimate bound on \( V_L \) at the
on-times: 
\[ \lim_{N \to \infty} V_L \left( t_{on}^N \right) \equiv V_L \left( t_{on}^\infty \right) \leq \overline{d}, \]
where \( \overline{d} \in \mathbb{R}_{>0} \) is a constant defined as 
\[ \overline{d} \triangleq \frac{1}{2} \left[ \exp \left( \lambda_u \Delta t_{off} \right) - 1 \right] / \left( 1 - \exp \left( \lambda_u \Delta t_{off} - \lambda_c \Delta t_{min} \right) \right). \] (31)

Similarly, the ultimate bound on \( V_L \) at the off-times can be found as 
\[ \lim_{N \to \infty} V_L \left( t_{off}^N \right) \equiv V_L \left( t_{off}^\infty \right) \leq d, \]
where \( d \in \mathbb{R}_{\geq 0} \) is a constant defined as 
\[ d \equiv \lambda \cdot \exp \left( -\lambda_c \Delta t_{min} \right). \] (32)

The monotonicity of the bounds in (22) and (27) can be used to demonstrate that \( V_L \left( t \right) \leq \overline{d} \forall t \geq t_{on}^N \) when \( V_L \left( t_{on}^N \right) \leq \overline{d} \), or, equivalently, \( \forall t \geq t_{off}^N \) when \( V_L \left( t_{off}^N \right) \leq d \). In other words, if the controller is switched on when \( V_L \left( t_{on}^N \right) \leq \overline{d} \), \( V_L \) will remain within that ball for all subsequent time. Using (12), it can then be demonstrated that as \( N \to \infty \), \( |e(t)| \) converges to a ball with constant radius, i.e.,
\[ \lim_{N \to \infty} |e(t)| \leq d, \]
where \( d \) was defined in (28).

V. EXPERIMENTAL RESULTS

An FES-cycling experiment was performed to demonstrate the performance of the switched sliding mode controller in (3), quantified by the tracking error signal \( e \). The experiment was conducted with an able-bodied male subject (age 24 years, height 186 cm, weight 78 kg), as the response of able-bodied subjects to FES is similar to that of paraplegic subjects [32]–[35]. The experiment was approved the University of Florida Institutional Review Board, and the subject gave written informed consent. During the experiment, the subject was not given any indication of the tracking performance and was specifically instructed not to contribute any voluntary cycling effort.

The experiment was performed using a fixed-gear stationary recumbent cycle that was modified to include an optical encoder to measure the crank angle and high-top walking boots rigidly attached to custom pedals. The boots immobilized the subject’s ankles and maintained sagittal alignment of the subject’s joints. The stationary cycle’s magnetically braked flywheel was set to the lowest level of resistance, and the seat was positioned according to the subject’s preference, provided that the seat was neither too close to (allowing for hyperextension of the knees) nor too far from (causing excessive hip flexion) the crank. Measurements of the subject’s upper and lower leg lengths as well as the seat position were made, as described in [20], and used to calculate \( B_k^0 \).

Electrodes were then placed on the anterior distal-medial and proximal-lateral portions of the subject’s left and right thighs. Electrical stimulation was delivered to the subject’s quadriceps femoris muscle groups via bipolar self-adhesive 3” x 5” PALS® Platinum oval electrodes\(^2\) connected to a current-controlled stimulator (RehaStim, Hasomed, GmbH, Germany). The stimulator delivered biphasic pulses with a constant pulse frequency of 40 Hz and a constant pulse amplitude of 100 mA, while the stimulation pulselwidth was varied according to (9).

The desired cadence was designed to start at zero revolutions per minute (RPM) and exponentially approach 60 RPM as \( \dot{\theta}_0 \triangleq 2\pi \left( 1 - \exp \left[ 0.7 \left( t_{on}^0 - t \right) \right] \right) \). The control gains were selected as \( k_1 = 150 \), \( k_2 = 5 \). The stimulation region was determined by defining \( \varepsilon \triangleq 0.4 \) for the subject. In preliminary testing, it was found that a pulselwidth of 30 \( \mu \)s produced a visible contraction of the quadriceps without causing a measurable change in the crank angle, so a constant offset of 30 \( \mu \)s was added to the control input.

The results of the experiment are depicted in Fig. 3, which includes the cadence tracking error \( e \) and the switched control input \( u^e \). The high frequency switching effect of the sliding mode term in the control input was present but limited in the sense that the error typically changed sign during uncontrolled regions (where the control input was deactivated) and the steady state error was strictly positive. The control input was biased in the sense that the magnitude of stimulation of the left quadriceps was generally greater than that of the right quadriceps. This bias may have been due to asymmetry in muscle mass between the subject’s left and right quadriceps muscle groups related to a past knee surgery.

VI. CONCLUSION

A switched sliding mode controller for an uncertain, nonlinear FES-cycling system is developed which guarantees approximate tracking of a desired cadence in the sense that

\(^2\)Surface electrodes for the study were provided compliments of Axelgaard Manufacturing Co.
the norm of the cadence error $|e(t)|$ converges to a residual value as the number of crank cycles approaches infinity. A quadriceps-only FES-cycling stimulation pattern [20] is utilized which results in autonomous state-dependent switching with known switching states but uncertain switching times. Stability is guaranteed through analysis of a common Lyapunov-like function, provided the sufficient condition in (29) is satisfied. Experimental results are provided which corroborate Theorem 3 under typical stationary cycling conditions. Future work will seek to generalize the control development to include additional muscle groups (e.g., gluteal and hamstrings muscle groups) and extend the experimental results to include more subjects, including subjects with neurological disorders.

REFERENCES


