

Comparison of logistic and neural network models to fit to the egg production curve of White Leghorn hens

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ABSTRACT Neural networks are capable of modeling any complex function and can be used in the poultry and animal production areas. The aim of this study was to investigate the possibility of using neural networks on an egg production data set and fitting models to the egg production curve by applying 2 approaches, one using a nonlinear logistic model and the other using 2 artificial neural network models [multilayer perceptron (MLP) and radial basis function]. Two data sets from 2 generations of a White Leghorn strain that had been selected mainly for egg production were used. In the first data set, the mean weekly egg-laying rate was ascertained over a 54-wk egg production period. This data set was used to adjust and test the logistic model

and to train and test the neural networks. The second data set, covering 52 wk of egg production, was used to validate the models. The mean absolute deviation, mean square error, and R^2 were used to evaluate the fit of the models. The MLP neural network had the best fit in the test and validation phases. The advantage of using neural networks is that they can be fitted to any kind of data set and do not require model assumptions such as those required in the nonlinear methodology. The results confirm that MLP neural networks can be used as an alternative tool to fit to egg production. The benefits of the MLP are the great flexibility and their lack of a priori assumptions when estimating a noisy nonlinear model.

Key words: logistic, neural network, multilayer perceptron, radial basis function, egg production

2011 Poultry Science 90:705–711
doi:10.3382/ps.2010-00723

INTRODUCTION

The use of mathematical models to estimate egg production curves is of great importance for evaluating egg production over the laying cycle. These models may serve, for example, to estimate the financial loss caused by a decline in egg production, as evinced by a deviation from the expected curve (Johnston and Gous, 2007). Many nonlinear models described in the literature have been used to fit to egg production curves (McNally, 1971; Gavora et al., 1982; McMillan et al., 1986; Cason and Britton, 1988; Yang et al., 1989; Cason and Ware, 1990). Every nonlinear model has 3 or 4 parameters and some of them summarize traits relating to egg laying. Moreover, they allow comparison between different prediction curves and forecasts for total egg production.

Artificial neural networks (ANN) are massively parallel interconnections of simple neurons that function

as a collective system (Kohonen, 1988). Their development was inspired by the artificial model for biological neurons described by McCulloch and Pitts (1943), and they form an alternative to the use of regression models for studying the dynamics of biological processes.

The ANN is a methodology that takes into account nonlinearities in the relationship between the input and output information. The advantages of artificial neural networks include knowledge plasticity with regard to changing inputs and outputs, fault tolerance, and interpolation capabilities (Zhang et al., 2007). Artificial neural networks have the ability to learn the patterns of a data set during the training process, thereby providing consistent predictions or generalization capabilities over test sets. Artificial neural networks have a high computation rate provided by their massive parallelism, so that real-time processing of huge data sets becomes feasible with the proper hardware (Bishop, 1995).

Neural networks may be categorized into 2 types: those that learn by updating their connection weights during training, and those in which the weights are invariant over time. The first class of networks passes through 2 phases (i.e., training and testing). During the

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Received February 23, 2010.

Accepted November 12, 2010.

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training phase, each error in membership assignment is fed back and the connection weights are updated. The back-propagated errors are computed with respect to each desired output, which is a membership value denoting the degree of belongingness of the input vector to that class. Hence, the error that is back-propagated for weight updating has inherently more weight in the case of nodes with higher membership values. The contribution of ambiguous or uncertain vectors to the weight connection is automatically reduced. After several cycles, the neural network may converge to a minimum error solution (Pal and Mitra, 1992).

Neural network models have been used in the poultry and animal production areas to predict poultry BW over time (Roush et al., 2006), chicken performance based on food composition (Ajakaiye et al., 2006), and fat and milk production among dairy cattle (Hosseini et al., 2007). Studies are needed to investigate the application of neural networks to egg production.

Multilayer perceptron (**MLP**) and radial basis function (**RBF**) are the 2 architectures of neural networks most often used for nonlinear regression analysis. Multilayer perceptron neural networks have been found to be particularly effective for problems that can make use of supervised training. This type of neural network is extremely popular for both classification and prediction (Haykin, 1999). The input layer receives the data and transfers it to the next layer after applying an activation function, which is usually a sigmoid function. The hidden layer receives this output from the input layer and multiplies these values by a weighting factor called the synaptic weight, which represents the reinforcement of learning through the neural network. These values are summed and go through a sigmoid function before the result is sent to the next layer. This process is repeated until the values reach the output layer.

Radial basis function neural networks are able to perform nonlinear tasks the same as MLP networks can. The RBF networks are constructed with only a single hidden layer of neurons with Gaussian transfer functions and a linear activation function. In comparison with MLP networks, RBF networks have greater robustness in relation to data variation, and much faster training procedures (Derks et al., 1995).

Multilayer perceptron and RBF networks differ in certain respects. Radial basis function neural networks have a single intermediate layer of neurons, whereas MLP networks may have one or more hidden layers. The hidden layer of the RBF network is modeled with Gaussian activation functions, whereas this is done with sigmoid functions in MLP networks. The argument of the transfer function in RBF networks is the Euclidian distance between the input vector and the center of the radial function, whereas MPF networks form the inner product from the input vector and synaptic weight vectors (Haykin, 1999).

The aim of the present study was to investigate the possibility of using 2 types of neural networks (MLP

and RBF) for nonlinear regression analysis on curve fitting for egg production from laying hens, and to compare their performance with a nonlinear logistic model that has been widely used in the literature for this type of curve fitting.

MATERIALS AND METHODS

Data Source

Two data sets from 2 generations of a strain of White Leghorn hens that have been developed and maintained under selection by Embrapa Swine and Poultry (Concórdia, Santa Catarina, Brazil) were used. This strain has been selected to improve egg production, egg weight, feed efficiency, viability, sexual maturity, fertility, hatchability, and egg quality.

The first data set contained the mean weekly egg-laying rate of 1,569 hens over a 54-wk period of egg laying, starting at wk 17 of age and finishing at wk 70 of age. The second data set related to the hens of the next generation and contained the mean weekly egg-laying rate of 636 hens over a 52-wk period of egg laying, starting at wk 19 of age and finishing at wk 70 of age.

Eggs were collected on 5 d/wk. According to Wheat and Lush (1961), this measure presents a correlation of 0.99 with egg production over the 7 d of the week. The weekly egg production rate was measured as a percentage, and the maximum number of eggs that a chicken could lay per week was 5.

Nonlinear Model

The curve that was fitted to the egg production from 17 to 70 wk of age was obtained from the nonlinear logistic function used by Nelder (1961). This was found to be the best out of 10 nonlinear curves for egg production that were tried in relation to the present database: Brody, Von Bertalanffy, Richards, logistic, Wood, compartmental, McNally function, modified logistic form, modified compartmental form, and modified Von Bertalanffy form [Brody (1945), Von Bertalanffy (1957), Richards (1959), Nelder (1961), Wood (1967), McMillan et al. (1970), McNally (1971), Cason and Britton (1988), Yang et al. (1989), and Cason and Ware (1990), respectively]. The nonlinear logistic model is represented by equation 1:

$$y_t = a \left\{ 1 + e^{[b-(ct)]} \right\}^{-1} e^{-xt}, \quad [1]$$

where y is the egg production rate over t weeks of production (range from 1 to 54), a is the asymptotic value that estimates the egg production at the peak, b is a value associated with the growth of the curve, c is a constant, and x is a value associated with the persistence of egg production. The extra term $e^{(-xt)}$ was add-

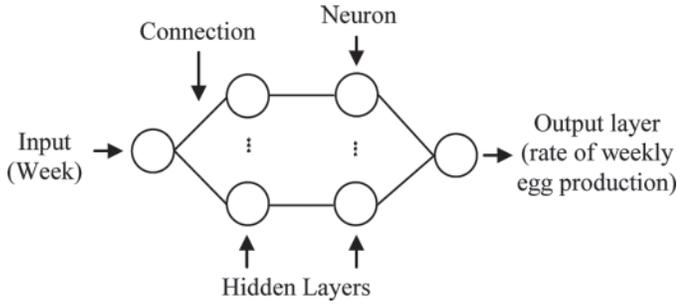


Figure 1. Multilayer perceptron neural network model with 2 hidden layers.

ed to the model to make the curve fit better after the peak (Brody et al., 1923; Cason and Ware, 1990).

MLP

The MLP is a feed-forward ANN model and was used to predict the weekly egg production rate (Figure 1). The algorithm used for training this neural network was the back-propagation algorithm (Bryson and Ho, 1969).

The back-propagation learning algorithm can be divided into 2 phases: propagation and weight update. Each propagation involves forward propagation of a training pattern input through the neural network to generate the output activation of the propagation, along with back propagation of this output activation through the neural network, using the training pattern target to generate the deltas for all the output and hidden neurons. In the weight update phase, the output delta of each synaptic weight is multiplied by the input activation to obtain the gradient of the weight and bring the weight in the opposite direction to the gradient by subtracting a ratio of the gradient of the weight from the weight (Pal and Mitra, 1992).

The MLP neural network used in this study had 2 hidden layers containing 5 neurons in each layer. These were interconnected by connection forces represented by values that are called synaptic weights, which are responsible for storing the acquired knowledge. The

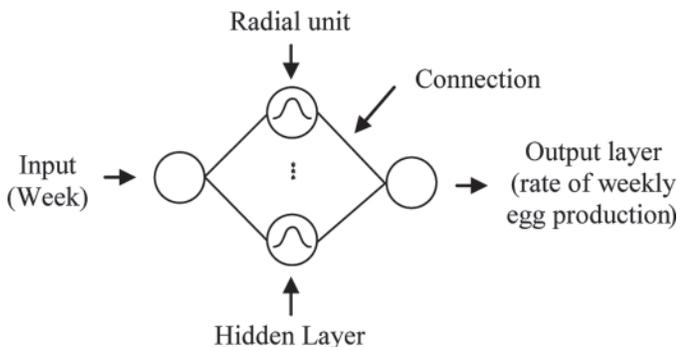


Figure 2. Radial basis function neural network model with 8 neurons in the hidden layer.

Table 1. Observed weekly egg production rate (OEP), and predictions from the logistic and neural network models [multilayer perceptron (MLP) and radial basis function (RBF), respectively]¹

Week	OEP	Logistic	MLP	RBF
1	0.020	0.050	-0.140	0.019
2	0.080	0.130	0.090	0.244
3	0.278	0.270	0.319	0.421
4	0.529	0.480	0.527	0.558
5	0.659	0.680	0.693	0.661
6	0.785	0.800	0.801	0.737
7	0.836	0.860	0.848	0.791
8	0.855	0.880	0.852	0.828
9	0.861	0.880	0.838	0.851
10	0.813	0.880	0.826	0.865
11	0.860	0.880	0.826	0.871
12	0.846	0.870	0.835	0.872
13	0.853	0.870	0.847	0.871
14	0.868	0.860	0.858	0.867
15	0.867	0.860	0.865	0.862
16	0.840	0.850	0.868	0.858
17	0.841	0.850	0.867	0.853
18	0.861	0.840	0.863	0.849
19	0.838	0.840	0.858	0.845
20	0.831	0.830	0.852	0.841
21	0.840	0.820	0.846	0.837
22	0.848	0.820	0.840	0.834
23	0.839	0.810	0.835	0.830
24	0.838	0.810	0.830	0.827
25	0.829	0.800	0.825	0.822
26	0.835	0.800	0.820	0.818
27	0.804	0.790	0.816	0.813
28	0.812	0.790	0.811	0.808
29	0.812	0.780	0.807	0.802
30	0.800	0.780	0.802	0.796
31	0.803	0.770	0.798	0.790
32	0.793	0.770	0.793	0.785
33	0.779	0.760	0.788	0.779
34	0.771	0.760	0.783	0.774
35	0.780	0.750	0.777	0.769
36	0.768	0.750	0.772	0.764
37	0.751	0.740	0.766	0.760
38	0.760	0.740	0.760	0.756
39	0.751	0.730	0.753	0.751
40	0.732	0.730	0.746	0.746
41	0.730	0.720	0.739	0.741
42	0.728	0.720	0.731	0.734
43	0.728	0.720	0.724	0.727
44	0.714	0.710	0.715	0.717
45	0.700	0.710	0.707	0.706
46	0.699	0.700	0.698	0.694
47	0.680	0.700	0.689	0.680
48	0.678	0.690	0.680	0.665
49	0.678	0.690	0.670	0.650
50	0.654	0.680	0.660	0.636
51	0.641	0.680	0.650	0.624
52	0.633	0.670	0.640	0.619
53	0.622	0.670	0.629	0.621
54	0.626	0.670	0.619	0.637

¹The odd weeks were used for training the neural networks and adjusting the logistic model, whereas the even weeks were used for testing the models.

values used in the input layer were normalized in accordance with equation 2:

$$y_i = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}}, \tag{2}$$

where x_i is a value of the input vector (week), x_{\min} is the minimum value, and x_{\max} is the maximum value of

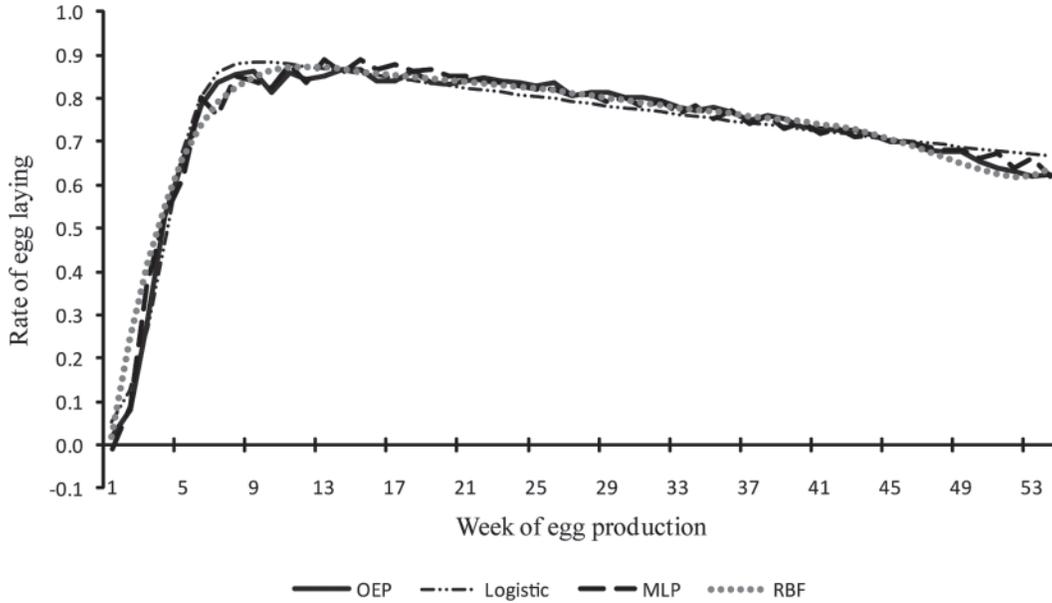


Figure 3. Observed weekly egg production rate (OEP) and the values fitted using the logistic, multilayer perceptron (MLP), and radial basis function (RBF) neural network models in the training and testing phases.

the egg-laying week. The output value of each neuron in the k layer is expressed by $y_k = g(a_k)$, where g is the hyperbolic activation function of a_k and a_k is the synaptic function, which is a linear combination of the input normalized values and the synaptic weights, as shown in equation 3:

$$a_k = \sum_j y_j w_{kj}, \quad [3]$$

where w_{kj} are synaptic weights linking the y_j input values with each k neuron. The transfer or activation function used in the neurons of each hidden layer was the hyperbolic function shown in equation 4:

$$g(a_k) = \frac{e^{a_k} - e^{-a_k}}{e^{a_k} + e^{-a_k}}, \quad [4]$$

where e is the Neperian algorithm.

RBF

The RBF has only one hidden layer, and each neuron contains a radial basis function. An RBF with 8 neurons was used in the present study (Figure 2).

A Gaussian function was used as the RBF in each neuron (Bishop, 1995), and the distance values (deviation) of this function increased or decreased in relation to the midpoint (Haykin, 1999). The Gaussian function was expressed by equation 5:

$$\varphi(v) = \exp\left(-\frac{v^2}{2\sigma^2}\right), \quad [5]$$

where $v = \|x - \mu\|$ is the Euclidian distance between the input vector and the center μ of a Gaussian function, and σ is its width. The Euclidian distance from the input vector to the center μ is the input for the Gaussian function, which gives the activation value of the radial unit. The output of each neuron in the output layer is defined by equation 6:

$$y_j = \sum_{h=1}^H w_{jh} \varphi_h(x_s), \quad [6]$$

where w_{jh} is the synaptic weight between the neuron h of the hidden layer and the neuron j of the output layer, x_s is the training input data set x , and φ_h is the Gaussian function of each neuron in the hidden layer. As in the MLP neural network, the input values were normalized through equation 2, and the output values in the output layer gave the prediction for the egg production rate in each week.

The RBF neural network was trained through the k -means algorithm (Bishop, 1995). This algorithm attempts to select an optimal set of points that are placed at the centroids of the training data patterns.

Training, Testing, and Validation of Models

The first data set was divided into 2 subsets for the training and testing phases of the neural networks and the logistic model. The odd weeks were used for training and the even weeks were used for testing the MLP and RBF models. Thus, for the logistic model, the odd weeks were used for model fitting and the even weeks were used for testing the model.

The second data set was used to validate both the neural network models and the logistic model, to as-

certain whether these models could be used on other data sets that did not participate in the construction of these models. The accuracy of the models was measured at each phase.

The fitting, testing, and validation of the logistic model, and the training, testing, and validation of the MLP and RBF ANN, were done using the Advanced Linear and Nonlinear Models package and the Neural Network package of the Statistica data analysis software system (Statistica 7.0, StatSoft Inc., Tulsa, OK).

Accuracy of the Models

The accuracy of the models was calculated using the R², mean absolute deviation (MAD), and mean square error (MSE). The R² is an indicator of how well the model fits the data. The MAD and MSE are described by equations 7 and 8, respectively:

$$MAD = \frac{\sum_{t=1}^n |y_t - \hat{y}_t|}{n}, \text{ and} \tag{7}$$

$$MSE = \frac{\sum_{t=1}^n |y_t - \hat{y}_t|^2}{n}, \tag{8}$$

where y_t is the egg production record over t weeks, \hat{y}_t is the estimated value for egg production, and n is the number of observations. Smaller values of MAD and MSE indicate better fitting of the models.

RESULTS AND DISCUSSION

Table 1 presents the mean weekly egg production rates observed in the first data set, along with the respective predictions of the models in the training-fitting and testing phases, which resulted in Figure 3. Table 2 presents the means observed for the same trait measured from the second data set and the respective predictions of the models in the validation phase, which resulted in Figure 4. The following logistic model (equation 1) was fitted to the data:

$$y_t = 0.9452\{1 + e^{[3.7957 - (0.9684t)]}\}e^{-0.0064t}.$$

The estimate for the parameter a (equation 1) was 0.9452, and it represents the estimated egg production rate at the peak of egg laying. The observed value over this period (around wk 14 of production) was 0.87. The estimate for parameter x was -0.0064 . The lower were the values for this parameter, the greater would be the persistence of egg laying among the hens after the egg-laying peak. This value was close to that reported by Cason and Ware (1990). The estimates for these 2 pa-

Table 2. Observed weekly egg production rate (OEP), and predictions from the logistic and neural network models [multilayer perceptron (MLP) and radial basis function (RBF), respectively] in the validation phase

Week	OEP	Logistic	MLP	RBF
3	0.001	0.043	-0.022	-0.368
4	0.036	0.107	0.062	0.032
5	0.241	0.242	0.245	0.327
6	0.527	0.459	0.501	0.540
7	0.680	0.688	0.702	0.690
8	0.815	0.843	0.812	0.791
9	0.879	0.917	0.867	0.857
10	0.908	0.944	0.895	0.897
11	0.914	0.950	0.911	0.919
12	0.917	0.948	0.919	0.929
13	0.923	0.943	0.923	0.932
14	0.922	0.937	0.924	0.931
15	0.931	0.930	0.924	0.927
16	0.928	0.924	0.922	0.923
17	0.915	0.917	0.920	0.918
18	0.920	0.910	0.916	0.914
19	0.925	0.904	0.912	0.910
20	0.903	0.897	0.908	0.907
21	0.885	0.891	0.904	0.903
22	0.874	0.884	0.899	0.900
23	0.888	0.878	0.894	0.896
24	0.878	0.871	0.889	0.892
25	0.889	0.865	0.884	0.888
26	0.882	0.859	0.879	0.883
27	0.876	0.853	0.873	0.877
28	0.866	0.846	0.867	0.871
29	0.878	0.840	0.862	0.865
30	0.868	0.834	0.856	0.858
31	0.864	0.828	0.849	0.851
32	0.865	0.822	0.843	0.844
33	0.831	0.816	0.836	0.837
34	0.840	0.810	0.830	0.829
35	0.811	0.804	0.823	0.821
36	0.798	0.798	0.815	0.813
37	0.804	0.793	0.808	0.805
38	0.802	0.787	0.800	0.796
39	0.813	0.781	0.793	0.788
40	0.779	0.775	0.785	0.779
41	0.776	0.770	0.777	0.771
42	0.755	0.764	0.769	0.763
43	0.754	0.759	0.760	0.755
44	0.739	0.753	0.752	0.748
45	0.733	0.748	0.743	0.741
46	0.741	0.742	0.734	0.736
47	0.735	0.737	0.726	0.731
48	0.725	0.731	0.717	0.725
49	0.720	0.726	0.708	0.719
50	0.708	0.721	0.699	0.709
51	0.704	0.716	0.690	0.693
52	0.668	0.710	0.681	0.668
53	0.659	0.705	0.673	0.628
54	0.648	0.700	0.664	0.566

rameters, which have a biological interpretation, made it possible to make comparisons with the egg-laying performance of other generations of the same strain, or of different strains. However, fitting the data using nonlinear models has 2 negative points. First, the speed of fitting is directly proportional to the quantity of data and to the initial values of the parameters used to start the iterative process. Second, in these models, data input occurs in series. With neural networks, the fitting process becomes fast because the synaptic weights are

Table 3. Values for coefficients of accuracy in the fitting (F), testing (T), and validation (V) phases of the logistic model and in the training (Tr), testing (T), and validation (V) phases of the neural networks¹

Item	MAD			MSE			R ²		
	F ^{NL} /Tr ^{ANN}	T	V	F ^{NL} /Tr ^{ANN}	T	V	F ^{NL} /Tr ^{ANN}	T	V
Logistic	0.0092	0.0118	0.0196	0.0005	0.0008	0.0007	0.9870	0.9663	0.9823
MLP	0.0243	0.0076	0.0104	0.0010	0.0001	0.0002	0.9693	0.9960	0.9958
RBF	0.0144	0.0200	0.0190	0.0009	0.0014	0.0030	0.9745	0.9657	0.9532

¹MAD = mean absolute deviation; MSE = mean square error; NL = nonlinear; ANN = artificial neural network; MLP = multilayer perceptron; RBF = radial basis function.

adjusted automatically during the learning process and the data are inserted in parallel along the layers.

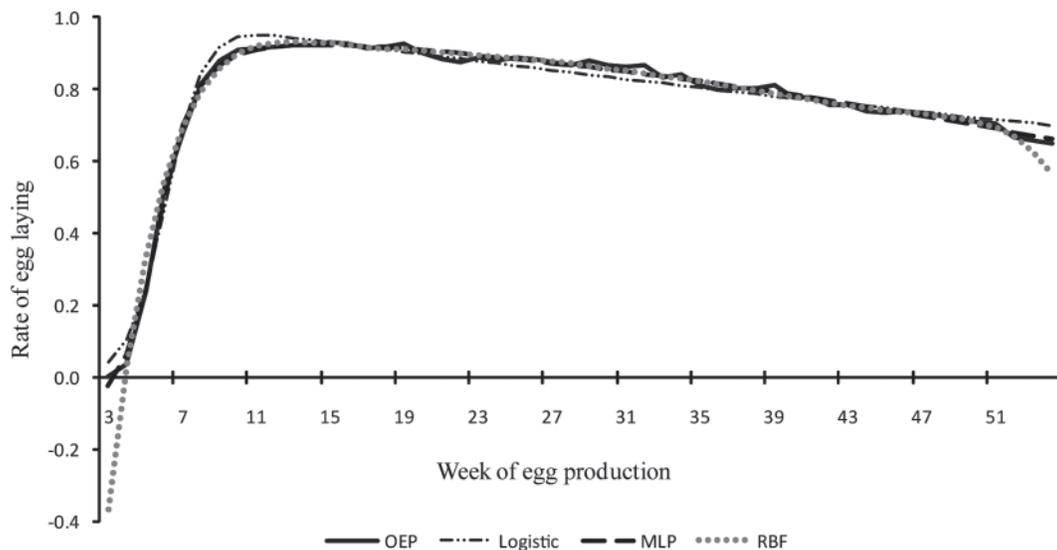
Table 3 shows that the logistic model had lower values for MAD and MSE during the adjustment phase, whereas the MLP network had lower values during the testing and validation phases. The MLP network had higher R² values in the testing and validation phases. Comparing the 2 approaches (nonlinear and neural networks), any model could be used because the values for MAD, MSE, and R² were very close. However, the MLP network had better predictions than those of the RBF network. The latter did not produce a good fit until wk 5 of egg laying and after wk 52 of egg laying, in the model validation phase (Table 2). This may be explained by the greater capacity that MLP has for problem generalization in relation to RBF. Ahmadi and Golian (2008) used a neural network model to predict the weekly egg-laying rate, and they obtained smaller errors and slightly larger R² for neural network training and testing, compared with the nonlinear model used by Grossman et al. (2000).

In general, MLP networks construct overall approximators that have a greater ability to generalize and extrapolate regions without training data. According

to Cybenko (1989), an MLP neural network with one hidden layer can implement any continuous function, whereas according to Cybenko (1988), an MLP neural network with 2 hidden layers can implement any function.

Radial basis function neural networks have some advantages over MLP networks. First, they can model any nonlinear function using a single hidden layer, which removes some design decisions regarding numbers of layers. Second, the simple linear transformation in the output layer can be optimized fully by using traditional linear modeling techniques, which are fast and do not suffer from problems such as the local minimum errors that plague MLP training techniques. The RBF networks can therefore be trained extremely quickly (Hill and Lewicki, 2007).

The advantage of using neural networks is that they can be fitted to any kind of data set and do not require model assumptions of the type required in nonlinear methodologies (Seber and Wild, 2003). One disadvantage of neural network models is that they do not provide parameters that may be useful for comparative and developmental purposes. Roush et al. (2006) reported that it is impossible to compare a neural network with

**Figure 4.** Observed weekly egg production rate (OEP) and the values fitted using the logistic, multilayer perceptron (MLP), and radial basis function (RBF) neural network models in the validation phase.

other models or with data sets from other populations when taking into account the synaptic weights of neural networks.

The present results confirm that MLP can be used as an alternative tool for fitting to egg production. The benefits of the MLP are the great flexibility and their lack of a priori assumptions when estimating a noisy nonlinear model.

Moreover, because the data set used in this study was from an experimental chicken population that had been selected for egg production, few uncontrolled conditions were affecting egg production. Most of these conditions were known, were rigorously controlled, or both; therefore, it may not be appropriate to generalize the finding that the neural network model is superior to traditional nonlinear models. On the other hand, for a commercial data set with a large amount of environmental noise, a neural network would be more appropriate for generalizing the predictions using the input information of the neural network, especially if the MLP was used, given that this type of network was found to more appropriate for generalizing the information than was the RBF.

This paper concentrated on the MLP and RBF approaches, but many neural models exist. Further research should explore the abilities and drawbacks of other ANN.

ACKNOWLEDGMENTS

Financial assistance was provided by Embrapa (Empresa Brasileira de Pesquisa Agropecuária). R. P. Savegnago, B. N. Nunes, and S. L. Caetano were supported by grants from CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior/Programa de Pós-Graduação em Genética e Melhoramento Animal-FCAV/UNESP, Jaboticabal, SP, Brazil).

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