Non-additive Robust Ordinal Regression: a multiple criteria decision model based on the Choquet integral*

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Abstract

Within the multicriteria aggregation-disaggregation framework, ordinal regression aims at inducing the parameters of a decision model, for example those of a utility function, which have to represent some holistic preference comparisons of a Decision Maker (DM). Usually, among the many utility functions representing the DM’s preference information, only one utility function is selected. Since such a choice is arbitrary to some extent, recently robust ordinal regression has been proposed with the purpose of taking into account all the sets of parameters compatible with the DM’s preference information. Until now, robust ordinal regression has been implemented to additive utility functions under the assumption of criteria independence. In this paper we propose a non-additive robust ordinal regression on a set of alternatives $A$, whose utility is evaluated in terms of the Choquet integral which permits to represent the interaction among criteria, modeled by the fuzzy measures, parameterizing our approach.

In our methodology, besides holistic pairwise preference comparisons of alternatives from a subset of reference alternatives $A' \subseteq A$, the DM is also requested to express the intensity of preference on pairs of alternatives from $A'$, and to supply pairwise comparisons on the importance of criteria, and the sign and intensity of interaction among pairs of criteria.

The output of our approach defines a set of fuzzy measures (capacities) such that the corresponding Choquet integral is compatible with the DM’s preference information.

Moreover, using linear programming, our decision model establishes two preference relations: for any $a, b \in A$, the necessary weak preference relation, if for all compatible fuzzy measures the utility of $a$ is not smaller than the utility of $b$, and the possible weak preference relation, if for at least one compatible fuzzy measure the utility of $a$ is not smaller than the utility of $b$.

Keywords: Multiple criteria decision analysis, interacting criteria, Choquet integral, non-additive robust ordinal regression, aggregation-disaggregation approach.

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1 Introduction

In a multiple criteria decision analysis (see [9] for a recent state of art), an alternative \( a \), belonging to a finite set of \( m \) alternatives \( A = \{a, b, \ldots, j, \ldots, m\} \), is evaluated on the basis of a family of \( n \) criteria \( G = \{g_1, g_2, \ldots, g_i, \ldots, g_n\} \), with \( g_i: A \to \mathbb{R} \). For example, in a decision recruitment problem for engaging a new young employee, the alternatives are the candidates and the criteria are some characteristics useful to give a comprehensive evaluation of the candidates, such as educational degree, professional experience, age and job interview. From here on, we will use the terms criterion \( g_i \) or criterion \( i \) interchangeably \((i = 1, 2, \ldots, n)\). For the sake of simplicity, but without loss of generality, we suppose that the evaluations with respect to the considered criteria are increasing with respect to preference, i.e. the “more the better”, defining a marginal weak preference relation as follows:

\[
a \text{ is at least as good as } b \text{ with respect to criterion } i \iff g_i(a) \geq g_i(b).
\]

The purpose of Multi-Attribute Utility Theory (MAUT) ([8], [25]) is to represent the preferences of the Decision Maker (DM) on a set of alternatives \( A \) by an overall value function \( U: \mathbb{R}^n \to \mathbb{R} \) with \( U \mapsto U(g_1(\cdot), \cdots, g_n(\cdot)) \) such that:

- \( a \) is indifferent to \( b \iff U(g(a)) = U(g(b)) \),
- \( a \) is preferred to \( b \iff U(g(a)) > U(g(b)) \),

where for simplicity of notation, we used \( U(g(a)) \), instead of \( U(g_1(a), \cdots, g_n(a)) \).

The principal aggregation model of value function is the multiple attribute additive utility [25], such that:

\[
U(g(a)) = u_1(g_1(a)) + u_2(g_2(a)) + \cdots + u_n(g_n(a)) \quad \text{with } \ a \in A,
\]

where \( u_i \) are non-decreasing marginal value functions for \( i = 1, 2, \cdots, n \).

Even if multiple attribute additive utility is the most well known aggregation model, some critics have been advanced to it, because it does not permit to represent interaction between the criteria under consideration. For example, in evaluating a car the criteria could be maximum speed, acceleration and price. In this case, very often there is negative interaction (redundancy) between maximum speed and acceleration: in fact, a car with a high maximum speed very often has also a good acceleration and therefore, even if these two criteria can be very important for a DM liking sports cars, their comprehensive importance is smaller than the importance of the two criteria considered separately. In the same decision problem, very often there is positive interaction (synergy) between maximum speed and price in evaluating cars: in fact, a car with a high maximum speed, has often also a high price, and therefore a car with a high maximum speed and a moderate price is very well appreciated. So, the comprehensive importance of these two criteria is greater than the importance of the two criteria considered separately. Within Multiple Criteria Decision Analysis (MCDA), the interaction of criteria has been considered through non-additive integrals such as Choquet integral [5] and Sugeno integral [39] (see [17] for a comprehensive survey on the use of non-additive integrals in MCDA.) The Choquet integral, introduced within the field of multiple criteria decision analysis in [31], is a generalization of the common aggregation operator of the weighted sum. In fact, the weighted sum is the discrete form of the Lebesgue integral with respect to additive measures, while the Choquet integral is the discrete form of the generalization of the Lebesgue integral with respect to fuzzy measures [39] (non-additive capacities). Instead, the Sugeno integral, being based
on ordinal properties of evaluations given by the considered criteria of the alternative at hand, can be labelled as the ordinal counterpart of Choquet integral.

Another interesting decision model permitting to represent interactions between criteria is the Dominance - based Rough Set Approach (DRSA) [19], which represents DM’s preferences by means of decision rules, i.e. easily understandable “if..., then...” statements, such as “if the maximum speed is at least 200 Km/h and the price is not larger than 50,000 $, then the car is good”.

In general, we shall call the decision models which, differently from MAUT, permit to represent the interaction between criteria non-additive decision models.

Each decision model requires specification of some parameters. For example, using multiple attribute utility theory the parameters are related to the formulation of functions \( u_i(g_i(a)) \), \( i = 1, 2, \ldots, n \), while using non-additive integrals the parameters are related to fuzzy measures, which permit to model the importance not only of each criterion \( g_i \in G \), but also of any subset of criteria \( R \subseteq G \). Within MCDA, many methods have been proposed to determine the parameters characterizing the considered decision model in a direct way, i.e. asking them directly to the DM, or in an indirect way, i.e. inducing the values of such parameters from some holistic preference comparisons of alternatives supplied by the DM. In general, this is a difficult task for several reasons, for example because it is acknowledged that the DM’s preference information is often incomplete, since the DM is not fully aware of the multicriteria approach adopted or the DM’s preference structure is not well defined in his/her mind (see [26] and [40]).

Recently, MCDA methods based on indirect preference information and on the disaggregation paradigm [24] are considered more interesting, because they require an easy cognitive interpretation for the DM in order to express some preference information. The DM provides some holistic preferences on a set of reference alternatives \( A' \), and from this information the parameters of the decision model are induced using a methodology called ordinal regression. Then, a consistent aggregation model is taken into consideration to evaluate the alternatives from the set \( A \) (aggregation approach.) Typically ordinal regression has been applied to MAUT models, such that in these cases we speak of additive ordinal regression. For example, the well known multicriteria method UTA (see [23]) is mainly based on the additive ordinal regression. The principles of ordinal regression have been applied also to some non-additive decision models. In this case, we shall speak of non-additive ordinal regression and in this context we remember some UTA like-methods within the Choquet integral framework (see [1, 28]) and the DRSA methodology (see [19]).

Usually, among the many sets of parameters of a decision model translating the DM’s preference information, only a specific set is considered. For example, only one among the many utility functions representing the DM’s holistic preference information is selected, like in the paper of Marichal and Roubens (see [28]) where in case of modelling preference with Choquet integral, the authors choose one among many fuzzy measures compatible with the DM’s preferences.

Since such choice is arbitrary to some extent, recently robust ordinal regression (for a recent survey see [22]) has been proposed with the aim of taking into account all the sets of parameters compatible with the DM’s preference information. The first robust ordinal regression method can be considered a recent generalization of UTA, called UTA\(^{GMS} \) [21]. The UTA\(^{GMS} \) is a multiple criteria method, that instead of considering only one additive utility function compatible with the preference information provided by the DM such as in UTA, takes into consideration the whole set of additive utility functions compatible with the preference information provided by the DM. In particular, the UTA\(^{GMS} \) method requires a set of pairwise comparisons on a set of reference alternatives \( A' \subseteq A \) as DM’s preference information. Then, the model, via linear programming, defines two relations on the set \( A \) : the necessary weak preference relation, which holds for any two alternatives \( a, b \in A \) if all compatible utility functions give to \( a \) a value not smaller than the value given to \( b \), and the possible weak preference relation, which holds for this pair if at least one compatible utility function gives to \( a \) a value not smaller than the value given to \( b \).
Recently an extension of UTA\textsuperscript{GMS} has been proposed: the GRIP method \cite{10}, that builds a set of additive value functions, taking into account not only a preorder on a set of alternatives, but also the intensities of preference among alternatives. Such preference information is included in other well-known MCDA methods such as MACBETH \cite{4} and AHP (\cite{35}, \cite{36}).

Both UTA\textsuperscript{GMS} and GRIP are based on the additive robust ordinal regression.

In this paper, we propose a non-additive robust ordinal regression applying the basic ideas of robust ordinal regression to a utility function expressed as Choquet integral of the evaluations of a given alternative with respect to the considered set of criteria in order to explicitly represent positive and negative interactions between criteria. More precisely, in our paper, we suggest an aggregation-disaggregation methodology taking inspiration from the UTA\textsuperscript{GMS} and the GRIP method, but including also some preference information on the sign and intensity of interaction among couples of criteria.

In fact, our methodology processes the following preference information supplied by the DM:

a) pairwise preference comparison on the alternatives from a reference set \( A' \subseteq A \);

b) the intensity of preference of a pair of alternatives, say \( a \) over \( b \), in comparison to the intensity of preference of another pair of alternatives, say \( c \) over \( d \), with \( a, b, c, d \in A' \);

c) pairwise comparison of importance of criteria;

d) pairwise comparison of the differences between importance of criteria;

e) negative and positive interaction expressing redundancy or synergy \cite{12} between couples of criteria;

f) pairwise comparison of interaction intensity among couples of criteria;

g) pairwise comparison of the differences of interaction intensity among couples of criteria.

Although preference information about interaction and importance of criteria are very useful to build a decision model, they are considered often as a difficult technical data to elicitate from the DM (see \cite{14} for a review of the existing methods.) To overcome these difficulties, the preference information of the proposed methodology, can be provided by the DM in a similar way to the approaches of AHP \cite{35} and MACBETH \cite{4}, i.e. using a qualitative ordinal scale such as “null”, “small”, “medium”, “large” and “extreme”.

Finally, our decision model collects all the preference information in a system of linear inequalities. By linear programming, for every pair of alternatives \( a, b \in A \), we evaluate all the sets of fuzzy measures, denoted by \( \mu \), compatible with the DM's preference information on the reference actions \( A' \), building a necessary and possible weak preference relations, respectively, defined as follows:

- “\( a \) is at least as good as \( b \) for all compatible sets of fuzzy measures \( \mu \);”
- “\( a \) is at least as good as \( b \) for at least one compatible set of the fuzzy measures \( \mu \).”

The paper is organized as follows. In Section 2, we present the basic concepts relative to interaction between criteria and to the Choquet integral. In Section 3, the non-additive robust ordinal regression is presented and illustrated by an example in Section 4. Some conclusions and future directions of research are included in Section 5.
2 Importance and interaction of criteria

Let $2^G$ be the power set of $G$ (i.e. the set of all the subsets of the set of criteria $G$); a fuzzy measure (capacity) on $G$ is defined as a set function $\mu : 2^G \rightarrow [0, 1]$ which satisfies the following properties:

1a) $\mu(\emptyset) = 0$ and $\mu(G) = 1$ (boundary conditions),
2a) $\forall T \subseteq R \subseteq G, \mu(T) \leq \mu(R)$ (monotonicity condition).

In the framework of multicriteria decision problems, the value $\mu(R)$ given by the fuzzy measure $\mu$ on the set of criteria $R$ is related to the importance weight given by the DM to the set of criteria $R$ that can be evaluated by the Shapley value, defined later in this section.

A fuzzy measure is said to be additive if $\mu(T \cup R) = \mu(T) + \mu(R)$, for any $T, R \subseteq G$ such that $T \cap R = \emptyset$. An additive fuzzy measure is determined uniquely by $\mu(\{i\}), \mu(\{2\}) \ldots, \mu(\{n\})$. In fact, in this case, $\forall T \subseteq G, \mu(T) = \sum_{i \in T} \mu(\{i\})$. In the other cases, we have to define a value $\mu(T)$ for every subset $T$ of $G$, obtaining $2^{|G|} - 2$ coefficients values. Therefore, we have to calculate the values of $2^{|G|} - 2$ coefficients, since we know that $\mu(\emptyset) = 0$ and $\mu(G) = 1$.

The Möbius representation of the fuzzy measure $\mu$ (cf. [33]) is defined by the function $a : 2^G \rightarrow \mathbb{R}$ (cf. [37]) such that:

$$\mu(R) = \sum_{T \subseteq R} a(T).$$

Let us observe that if $R = \{i\}$ is a singleton, then $\mu(\{i\}) = a(\{i\})$.

If $R = \{i, j\}$ is a couple (non-ordered pair) of criteria, then $\mu(\{i, j\}) = a(\{i\}) + a(\{j\}) + a(\{i, j\})$.

In general, the Möbius representation $a(R)$ is obtained by $\mu(R)$ in the following way:

$$a(R) = \sum_{T \subseteq R} (-1)^{|R-T|} \mu(T).$$

In terms of Möbius representation (see [6]), properties 1a) and 2a) are, respectively, formulated as:

1b) $a(\emptyset) = 0, \sum_{T \subseteq G} a(T) = 1,$

2b) $\forall i \in R$ and $\forall R \subseteq G, \sum_{T \subseteq R} a(T) \geq 0.$

Let us observe that in MCDA, the importance of any criterion $g_i \in G$ should be evaluated considering all its global effects in the decision problem at hand; these effects can be “decomposed” from both theoretical and operational points of view in effects of $g_i$ as single, and in combination with all other criteria. Therefore, a criterion $i \in G$ is important with respect to a fuzzy measure $\mu$ not only when it is considered alone, i.e. for the value $\mu(\{i\})$ in itself, but also when it interacts with other criteria from $G$, i.e. for every value $\mu(T \cup \{i\})$, $T \subseteq G \setminus \{i\}$. There are some indices introduced in game theory [38] that have been used in the context of multicriteria decision making (see e.g. [13], [30], [34]) in order to measure importance and interaction among criteria. The most important are the following.
The importance index or Shapley value of criterion \(i \in G\) with respect to fuzzy measure \(\mu\) is defined by:

\[
\varphi(\{i\}) = \sum_{T \subseteq G \setminus \{i\}} \frac{(|G| - |T| - 1)!|T|!}{|G|!} [\mu(T \cup \{i\}) - \mu(T)],
\]

or equivalently in terms of the Möbius representation:

\[
\varphi(\{i\}) = \sum_{T \subseteq G \setminus \{i\}} \frac{a(T \cup \{i\})}{|T| + 1}.
\]

The interaction index between criteria \(i, j \in G\) with respect to the value \(\mu(T)\) is measured by the Murofushi-Soneda interaction index introduced in [30], that is defined by:

\[
\varphi(\{i, j\}) = \sum_{T \subseteq G \setminus \{i, j\}} \frac{(|G| - |T| - 2)!|T|!}{(|G| - 1)!} [\mu(T \cup \{i, j\}) - \mu(T \cup \{i\}) - \mu(T \cup \{j\}) + \mu(T)]
\]

or equivalently by:

\[
\varphi(\{i, j\}) = \sum_{T \subseteq G \setminus \{i, j\}} \frac{a(T \cup \{i, j\})}{|T| + 1}
\]

in terms of the Möbius representation.

Given \(x \in A\) and \(\mu\) being a fuzzy measure on \(G\), then the Choquet integral [5] is defined by:

\[
C_\mu(x) = \sum_{i=1}^{n} \left[ (g_{(i)}(x)) - (g_{(i-1)}(x)) \right] \mu(A_i),
\]

where \(\cdot\) stands for a permutation of the indices of criteria such that:

\[
g_{(1)}(x) \leq g_{(2)}(x) \leq g_{(3)}(x) \leq ... \leq g_{(n)}(x).
\]

with \(A_i = \{(i), ..., (n)\}, i = 1, ..., n\), and \(g_{(0)} = 0\).

The Choquet integral can be redefined in terms of the Möbius representation [11], without reordering the criteria, as:

\[
C_\mu(x) = \sum_{T \subseteq G} a(T) \min_{i \in T} g_i(x).
\]

One of the main drawbacks of the Choquet integral is the necessity to elicitate and give an adequate interpretation of \(2^{|G|} - 2\) parameters. In order to reduce the number of parameters to be computed and to eliminate a too strict description of the interactions among criteria, which is not realistic in many applications, the concept of fuzzy \(k\)-additive measure has been considered [13].

A fuzzy measure is called \(k\)-additive if \(a(T) = 0\) with \(T \subseteq G\), when \(|T| > k\). We observe that a 1-additive measure is the common additive fuzzy measure. In many real decision problems, it suffices to consider 2-additive measures. In this case, positive and negative interactions between couples of criteria are modeled without considering the interaction among triples, quadruplets and generally \(n\)-tuples, (with \(n > 2\)) of criteria. From the point of view of MCDA, the use of 2-additive measures is justified by
observing that the information on the importance of the single criteria and the interactions between couples of criteria are noteworthy. Moreover, it could be not easy or not straightforward for the DM to provide information on the interactions among three or more criteria during the decision procedure. From a computational point of view, the interest in the 2-additive measures lies in the fact that any decision model needs to evaluate a number \( n \cdot \binom{n}{2} \) of parameters (in terms of Möbius representation, a value \( a(\{i\}) \) for every criterion \( i \) and a value \( a(\{i, j\}) \) for every couple of distinct criteria \( \{i, j\} \).) With respect to a 2-additive fuzzy measure, the inverse transformation to obtain the fuzzy measure \( \mu(R) \) from the Möbius representation is defined as:

\[
\mu(R) = \sum_{i \in R} a(\{i\}) + \sum_{\{i,j\} \subseteq R} a(\{i,j\}), \forall R \subseteq G.
\]

With regard to 2-additive measures, properties \( 1b) \) and \( 2b) \) have, respectively, the following formulations:

1c) \( a(\emptyset) = 0, \sum_{i \in G} a(\{i\}) + \sum_{\{i,j\} \subseteq G} a(\{i,j\}) = 1, \)

2c) \( a(\{i\}) \geq 0, \forall i \in G, a(\{i\}) + \sum_{j \in T} a(\{i,j\}) \geq 0, \forall i \in G \) and \( \forall T \subseteq G \setminus \{i\} \).

In this case, the representation of the Choquet integral of \( x \in A \) is given by:

\[
C_\mu(x) = \sum_{\{i\} \subseteq G} a(\{i\}) g_i(x) + \sum_{\{i,j\} \subseteq G} a(\{i,j\}) \min\{g_i(x), g_j(x)\}.
\]

The importance index is given by:

\[
\varphi(\{i\}) = a(\{i\}) + \sum_{j \in G \setminus \{i\}} \frac{a(\{i,j\})}{2}, \quad i \in G,
\]

while the interaction index for a couple of criteria \( i, j \in G \) is given by:

\[
\varphi(\{i,j\}) = a(\{i,j\}).
\]

3 Description of the methodology

3.1 Some basic definitions

Let us suppose that the preference of the DM is given by a partial preorder \( \succcurlyeq \) on \( A' \subseteq A \).

The preference relation \( \succcurlyeq \) can be decomposed into its symmetric part \( \sim \) and into its asymmetric part \( \succ \), whose semantics are, respectively:

\( a \sim b \iff a \text{ is indifferent to } b, \)

\( a \succ b \iff a \text{ is preferred to } b, \text{ with } a, b \in A'. \)

Before illustrating our method, we need to introduce some basic definitions.
Definition 1 A set of fuzzy measures $\mu$ is called compatible if the Choquet integral, calculated with respect to it, restores the DM’s ranking on $A'$, i.e.

$$a \succeq b \Leftrightarrow C_\mu(a) \geq C_\mu(b) \quad \forall a, b \in A'.$$

We adopt the Choquet integral in terms of Möbius measures (cf. expression 4 in Section 2), since, as it is explained in Section 2, with regard to 2-additive measures the computational issues are reduced to the evaluation of only $n + \binom{n}{2}$ parameters.

Then two binary relations, the necessary weak preference relation $\succeq^N$ and the possible weak preference relation $\succeq^P$ on $A$ are, respectively, defined in the following way:

Definition 2 Necessary weak preference relation:

$$x \succeq^N y \Leftrightarrow C_\mu(x) \geq C_\mu(y)$$

for all compatible sets of fuzzy measures $\mu$, with $x, y \in A$.

Definition 3 Possible weak preference relation:

$$x \succeq^P y \Leftrightarrow C_\mu(x) \geq C_\mu(y)$$

for at least one compatible fuzzy measure $\mu$, with $x, y \in A$.

Let us observe that the binary relations $\succeq^N$ and $\succeq^P$ are meaningful only if there exists at least one compatible set of fuzzy measures.

The procedure proposed is composed of three successive steps:

(I) elicitation of preference information;

(II) evaluation of all the compatible fuzzy measures to establish the preference relations $a \succeq^P b$ and $a \succeq^N b$ on $A$;

(III) exploitation of the results obtained to detect eventual DM’s inconsistencies or to revise the preference model obtained.

In the following subsections, we analyze in detail each stage.

3.2 Elicitation of preference information

In our approach, the DM is asked to provide the following preference information:

(a) a partial preorder $\succeq$ on $A'$, whose semantics is: for $a, b \in A'$

$$a \succeq b \Leftrightarrow a \text{ is at least as good as } b;$$

(b) a partial preorder $\succeq^*$ on $A' \times A'$, whose semantics is: for $a, b, c, d \in A'$

$$(a, b) \succeq^* (c, d) \Leftrightarrow a \text{ is preferred to } b \text{ at least as much as } c \text{ is preferred to } d;$$
(c) a partial preorder $\succeq$ on $G$, for $i, j \in G$, whose semantics is:

\[ i \succeq j \iff \text{criterion } i \text{ is at least as important as criterion } j; \]

(d) a partial preorder $\succeq^*$ on $G \times G$, whose semantics is: for $i, j, l, k \in G$

\[(i, j) \succeq^* (l, k) \iff \text{the difference of importance between criteria } i \text{ and } j \text{ is at least as much as difference of importance between criteria } l \text{ and } k; \]

(e) sign (positive or negative) of interaction of couples of criteria;

(f) a partial preorder $\succeq_{\text{Int}}$ on $G \times G$, whose semantics is: for $i, j, l, k \in G$

\[(i, j) \succeq_{\text{Int}} (l, k) \iff \text{interaction intensity between criteria } i \text{ and } j \text{ is at least as strong as interaction intensity between criteria } l \text{ and } k. \]

(g) a partial preorder $\succeq^*_{\text{Int}}$ on $G \times G$, whose semantics is: for $i, j, l, k, r, s, t, w \in G$

\[ [(i, j), (l, k)] \succeq^*_{\text{Int}} [(r, s), (t, w)] \iff \text{difference of interaction intensity between criteria } i \text{ and } j \text{ and interaction intensity between criteria } l \text{ and } k \text{ is at least as strong as difference of interaction intensity between criteria } r \text{ and } s \text{ and interaction intensity between criteria } t \text{ and } w. \]

In this phase, the DM compares the interaction intensity of pairs of criteria both redundant or synergic.

In the rest of the paper, we will consider the asymmetric ($\succ$) and symmetric ($\sim$) parts of the weak preference relation $\succsim$, but for simplicity only the asymmetric part of the relations defined above in points (b),(c),(d), and (g), will be adopted with the respective symbols: $\succ^*, \succ, \succ^*, \succ_{\text{Int}}$ and $\succ^*_{\text{Int}}$. For example, the relation $\succ$ on $G$ has the semantics for $i, j \in G$: 

\[ i \succ j \iff \text{criterion } i \text{ is more important than criterion } j. \]

Observe that the symmetric parts of relations $\succ^*, \succ, \succ^*, \succ_{\text{Int}}$ and $\succ^*_{\text{Int}}$ defined in points (b),(c),(d), and (g) can be treated similarly to the symmetric part of $\succsim$, i.e., $\sim$.

The preference information of the points (b), (d), (f) and (g), can be provided by the DM using a semantic scale in a similar way to the approaches of MACBETH [4] and AHP [35]. More precisely, given an ordinal scale such as, for example, “null”, “small”, “medium”, “large”, “extreme”, the DM can give information of the type “the preference of alternative $a$ over alternative $b$ is large” or “the difference of importance between criteria $g_i$ and $g_j$ is medium” or “the synergy between criteria $g_i$ and $g_j$ is small.”

With respect to the information of type (b) (intensity of preference), we have already explained in the paper that this information can be given with the same modalities of MACBETH or AHP. However, observe that codification of intensities of preference of MACBETH or AHP is only a specific way of representing a general quaternary relation relative to intensity of preferences. Observe that when you use a certain fixed number of levels of preference such as “null”, “small”, “medium”, “large” and “extreme”, there is always the possibility that the DM says that the intensity of preference between alternatives $x$ and $y$ is intermediate between two degrees, for example between “medium” and “large”. In this case, the
use of the general quaternary relation permits to represent easily this type of information: the difference
between the utility values assigned to \( x \) and \( y \) have to be larger than the difference of values assigned to
all alternatives \( w \) and \( z \) for which the degree of preference is not larger than “medium” and smaller than
the difference of utility values assigned to all alternatives \( u \) and \( t \) for which the degree of preference is not
smaller than “large”.

With respect to type (d) (difference of importance between criteria), let us remember that this is the
information requested by AHP. In fact, AHP has a scale of intensity of importance of nine degrees, more
precisely “equal importance”, “weak importance”, “moderate importance”, “moderate plus importance”, “strong
importance”, “strong plus importance”, “very strong importance”, “very very strong importance” and “extreme
importance”. The fact that AHP is one of the most applied multiple criteria methods is larger than the redundancy between Mathematics and Physics (information of type (f)) or the redundancy between Mathematics and Physics is moderately larger than the redundancy between Chemistry and Physics (information of type (g)). In any case, information of type (b), (d), (f) and (g) are possible and
not necessary for the proposed method which can be implemented also without any of this information.

Finally observe that the methodology we are proposing does not make the assumption that the DM has
the idea of what could be the value of the interaction index. In fact, the Decision Analyst (DA) doesn’t
show him any analytical aggregation function. The DA indicates to the DM only the partial pre-orderings
on the set of all the alternatives resulting from the possible and the necessary preferences output of the
method. Then, the DM would, eventually, decide to add or remove (in case of inconsistencies or revisions)
some preference information.

The preference information of points (a) to (d) are formally formulated as follows:

\[ a \preceq b \iff C_\mu(a) \geq C_\mu(b) \]  
\[ (a, b) \succ^* (c, d) \iff C_\mu(a) - C_\mu(b) > C_\mu(c) - C_\mu(d) \]  
\[ i \triangleright j \iff \varphi(\{i\}) > \varphi(\{j\}) \]  
\[ (i, j) \uparrow \varphi((l, k)) \iff |\varphi(\{i, j\})| - |\varphi(\{l, k\})| > |\varphi(\{r, s\})| - |\varphi(\{t, w\})| \]

where \( \varphi(\{i\}) \) represents the importance or Shapley index in terms of Möbius measures, defined in 5
of Section 2. In the last step, the DM will give some preference information on the sign and intensity
of interaction between couples of criteria with the statements from (e) to (g), that are translated in the
following way:

\[ \varphi(\{i, j\}) \leq 0 \text{ or } \varphi(\{i, j\}) \geq 0, \text{ with } i, j \in G, \]  
\[ (i, j) \uparrow \varphi((l, k)) \iff |\varphi(\{i, j\})| > |\varphi(\{l, k\})| \]  
\[ |\varphi((i, j), (l, k)) \uparrow \varphi((r, s), (t, w)) \iff |\varphi(\{i, j, l, k\})| - |\varphi(\{r, s, t, w\})| > |\varphi(\{i, j, l, k\})| - |\varphi(\{r, s, t, w\})| \]  

with \( i, j, l, k, r, s, t, w \in G \).
where $\varphi(\{i, j\})$ are the interaction indices in the Möbius representation 6 of Section 2.

Let us observe that the interaction indices are considered in modulus, since otherwise, in case of redundancy between criteria, the inequalities comparing the intensity of interaction should be reversed.

Moreover, let us remind that the DM gives only ordinal information on criteria during the MCDA procedure.

3.3 Definition of the set of all compatible fuzzy measures

Finally, the set of all compatible fuzzy measures is described by the following system of linear constraints, summing up the DM’s preference information together with the monotonicity and boundary conditions of fuzzy measures:

\[
\begin{align*}
& a \sim b \Leftrightarrow C_\mu(a) = C_\mu(b), \text{ with } a, b \in A', \\
& a \succ b \Leftrightarrow C_\mu(a) > C_\mu(b), \text{ with } a, b \in A', \\
& (a, b) \succ^*(c, d) \Leftrightarrow C_\mu(a) - C_\mu(b) > C_\mu(c) - C_\mu(d), \text{ with } a, b, c, d \in A', \\
& i \succ j \Leftrightarrow \varphi(\{i\}) > \varphi(\{j\}), \text{ with } i, j \in G, \\
& (i, j) \succ^*(l, k) \Leftrightarrow \varphi(\{i\}) - \varphi(\{j\}) > \varphi(\{l\}) - \varphi(\{k\}), \\
& \varphi(\{i, j\}) \leq 0 \text{ or } \varphi(\{i\}, j) > 0, \text{ with } i, j \in G, \\
& (i, j) \succ_{\text{int}}(l, k) \Leftrightarrow |\varphi(\{i, j\})| > |\varphi(\{l, k\})|, \text{ with } i, j, l, k \in G, \\
& [(i, j), (l, k)] \succ_{\text{int}} [r, s], (t, w) \Leftrightarrow |\varphi(\{i, j\})| - |\varphi(\{l, k\})| > |\varphi(\{r, s\})| - |\varphi(\{t, w\})|, \\
& \text{ with } i, j, l, k, r, s, t, w \in G, \\
& a(\emptyset) = 0, \sum_{i \in G} a(\{i\}) + \sum_{\{i, j\} \subseteq G} a(\{i, j\}) = 1, \\
& a(\{i\}) \geq 0, \forall i \in G, a(\{i\}) + \sum_{j \notin T} a(\{i, j\}) \geq 0, \forall i \in G \text{ and } \forall T \subseteq G \setminus \{i\}.
\end{align*}
\]

Let $p$ be the number of linear constraints of $E^A$, depending on the DM’s initial preferences, while the monotonicity and boundary conditions of Möbius measures are $2n + 2$.

Since linear programming is not able to handle strict inequalities in $E^A$, we put the constraints in the form of weak inequalities, by adding a small arbitrary positive value $\varepsilon$ (see [28] for a result on this topic) as follows:

\[
\begin{align*}
& a \succeq b \Leftrightarrow C_\mu(a) \geq C_\mu(b) + \varepsilon, \text{ with } a, b \in A', \\
& (a, b) \succ^*(c, d) \Leftrightarrow C_\mu(a) - C_\mu(b) \geq C_\mu(c) - C_\mu(d) + \varepsilon, \text{ with } a, b, c, d \in A', \\
& i \succ j \Leftrightarrow \varphi(\{i\}) \geq \varphi(\{j\}) + \varepsilon, \text{ with } i, j \in G, \\
& (i, j) \succ^*(l, k) \Leftrightarrow \varphi(\{i\}) - \varphi(\{j\}) \geq \varphi(\{l\}) - \varphi(\{k\}) + \varepsilon, \\
& \varphi(\{i, j\}) \leq -\varepsilon \text{ or } \varphi(\{i\}, j) \geq \varepsilon, \text{ with } i, j \in G, \\
& (i, j) \succ_{\text{int}}(l, k) \Leftrightarrow |\varphi(\{i, j\})| \geq |\varphi(\{l, k\})| + \varepsilon, \text{ with } i, j, l, k \in G, \\
& [(i, j), (l, k)] \succ_{\text{int}} [r, s], (t, w) \Leftrightarrow |\varphi(\{i, j\})| - |\varphi(\{l, k\})| \geq |\varphi(\{r, s\})| - |\varphi(\{t, w\})| + \varepsilon, \\
& \text{ with } i, j, l, k, r, s, t, w \in G, \\
& a(\emptyset) = 0, \sum_{i \in G} a(\{i\}) + \sum_{\{i, j\} \subseteq G} a(\{i, j\}) = 1, \\
& a(\{i\}) \geq 0, \forall i \in G, a(\{i\}) + \sum_{j \notin T} a(\{i, j\}) \geq 0, \forall i \in G \text{ and } \forall T \subseteq G \setminus \{i\}.
\end{align*}
\]
3.4 Exploitation of preference model obtained

Finally, to establish the necessary preference relation $\succeq^N$ and the possible preference relation $\succeq^P$ on $A$, we solve respectively the following linear programs $\forall x, y \in A$:

\[
\begin{align*}
\max \quad & \varepsilon \\
\text{s.t.} \quad & E^A_\varepsilon \text{ plus the constraint } C_\mu(y) \geq C_\mu(x) + \varepsilon
\end{align*}
\] (7)

and

\[
\begin{align*}
\max \quad & \varepsilon \\
\text{s.t.} \quad & E^A_\varepsilon \text{ plus the constraint } C_\mu(x) \geq C_\mu(y).
\end{align*}
\] (8)

If the problem (7) finds a solution with $\varepsilon \leq 0$, then $C_\mu(x) \geq C_\mu(y)$ for all compatible fuzzy measures $\mu$, that implies $x \succeq^N y$ with $x, y \in A$.

On the contrary, if a positive $\varepsilon$ solves the linear program indicated in (8), then there exists at least one compatible fuzzy measures $\mu$ such that $C_\mu(x) \geq C_\mu(y)$, that implies $x \succeq^P y$ with $x, y \in A$.

Let us remark that, if $x \succeq^N y$, then $x \succeq^P y$ with $x, y \in A$; moreover, $\succeq^N$ is a partial preorder (i.e. reflexive and transitive), while $\succeq^P$ is strongly complete and negatively transitive (for results on this topic, see [21]).

3.5 Dealing with inconsistencies

Let us observe that the set of admissible solutions of the system $E^A_\varepsilon$ could be also an empty set if the solution $\varepsilon$ of problem (7) is negative; one of the most likely explanations could be that the DM gives some inconsistent preference information, i.e. no set of fuzzy measures $\mu$ exists which is compatible with the DM’s preference information. In this case, the DA can interactively help the DM, removing some pairwise comparisons on reference alternatives or some preferential information on importance of criteria and interaction between criteria in terms of sign and intensity.

In case of such inconsistencies, a procedure, described in general terms in [29] and applied in robust additive ordinal regression in UTA$^{GMS}$, can be implemented to obtain the smallest number of linear constraints, apart from the boundary and monotonicity conditions of the Möbius measures, that can be removed from the system of constraints $E^A_\varepsilon$, such that for every $a, b \in A'$ the set of admissible solutions $\mu$ of $E^A_\varepsilon$ is not empty.

Then, the following mixed $\{0, 1\}$ linear program is performed, introducing as many binary variables $\delta_r$ as the number of constraints $p$ (the index $r$ of $\delta_r$ identifies the constraint under consideration):

\[
\min \sum_{r=1}^{p} \delta_r \quad \text{s.t.}
\] (9)

12
\[
\begin{align*}
 a \succ b & \iff C_\mu(a) + K\delta_r \geq C_\mu(b) + \epsilon, \text{ with } a, b \in A', \\
 (a, b) & \succ^* (c, d) \iff C_\mu(a) - C_\mu(b) + K\delta_r \geq C_\mu(c) - C_\mu(d) + \epsilon, \text{ with } a, b, c, d \in A', \\
 i \succ j & \iff \phi(\{i\}) + K\delta_r \geq \phi(\{j\}) + \epsilon, \text{ with } i, j \in G, \\
 (i, j) & \succ^* (l, k) \iff \phi(\{i\}) - \phi(\{j\}) + K\delta_r \geq \phi(\{l\}) - \phi(\{k\}) + \epsilon, \\
 \phi(\{i, j\}) + K\delta_r & \leq -\epsilon \text{ or } \phi(\{i, j\}) + K\delta_r \geq \epsilon, \text{ with } i, j \in G, \\
 (i, j) & \succ \text{Int} (l, k) \iff |\phi(\{i, j\})| + K\delta_r \geq |\phi(\{l, k\})| + \epsilon, \text{ with } i, j, l, k \in G, \\
 [\{i, j\}, \{l, k\}] & \succ^*_\text{Int} [\{r, s\}, \{t, w\}] \iff |\phi(\{i, j\})| - |\phi(\{l, k\})| + K\delta_r \geq |\phi(\{r, s\})| - |\phi(\{t, w\})| + \epsilon, \\
 a(\emptyset) & = 0, \sum_{i \in G} a(\{i\}) + \sum_{\{i, j\} \subseteq G} a(\{i, j\}) = 1, \\
 a(\{i\}) & \geq 0, \forall i \in G, a(\{i\}) + \sum_{j \in T} a(\{i, j\}) \geq 0, \forall i \in G \text{ and } \forall T \subseteq G \setminus \{i\}
\end{align*}
\]

where \(K\) is a positive large value. The computational issue of program (9) depends on the number of constraints \(p\) which is determined by the preference information elicited by the DM.

The optimal solution of program (9) evaluates one of the feasible subsets of constraints of smallest cardinality, in particular, the constraints for which \(\delta_r = 1\) are satisfied for every set of \(\mu\) and consequently can be removed. Other consistent subsets of constraints can be obtained by solving the program (9) with the additional constraint \(\sum_{r \in T} \delta_r \leq |T| - 1\), where \(T\) is the subset of indices of constraints of the optimal solution for which \(\delta_r = 1\) in the optimization problem (9). Such last constraint avoids that by iteration the optimization algorithm (9) can find the same solution. Finally, such subsets of inconsistent constraints are showed to the DM, that may choose which one can be removed. The flow-chart of Fig. 1 synthesizes the decision model proposed.

Figure 1: The flow-chart of the method
Table 1: Evaluation matrix

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Ed</th>
<th>Ex</th>
<th>Ag</th>
<th>In</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odile</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Slobodan</td>
<td>3</td>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Charles</td>
<td>10</td>
<td>9</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Irene</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>Katherin</td>
<td>8</td>
<td>0</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Felix</td>
<td>5</td>
<td>9</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Germaine</td>
<td>8</td>
<td>10</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Benedicte</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>Arthur</td>
<td>0</td>
<td>10</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

4 An example

In this section, we illustrate our decision procedure with an example, inspired from Pomerol and Barba-Romero [32]. Let us consider a recruitment problem, where the executive manager of a company looks for engaging a new young employee.

The manager takes into account the following four criteria:

(1) Educational degree (abbreviated: Ed);
(2) Professional experience (abbreviated: Ex);
(3) Age (abbreviated: Ag);
(4) Job interview (abbreviated: In).

In the example,

\[ A = \{\text{Odile, Slobodan, Charles, Irene, Katherin, Felix, Germaine, Benedicte, Arthur}\} \]

and

\[ G = \{\text{Ed, Ex, Ag, In}\}. \]

The candidates, evaluated by the executive manager and their scores for every criterion on a [0, 10] scale are presented in Table 1. We suppose that the criteria have to be maximized.

Now, suppose that the executive manager (the DM) on the basis of her preference structure is able to give only the following partial information on the reference actions \( A' = \{\text{Odile, Slobodan, Benedicte, Charles, Katherin}\} \):

- the candidate Charles (Ch) is better than the candidate Slobodan (Sl);
- the candidate Germaine (Gm) is better than the candidate Slobodan;
- the candidate Odile (Od) is preferred to Charles more than Benedicte (Bd) is preferred to Slobodan;
- the candidate Benedicte is preferred to Katherine (Kt) more than Charles is preferred to Slobodan.
The executive manager gives the partial preference information on the importance and interaction of criteria:

- Educational degree is more important than professional experience;
- Professional experience is more important than age;
- Age is more important than job interview;
- Educational degree is preferred to professional experience more than age is preferred to job interview.

Finally, the DM provides the following information on interaction among couples of criteria:

- there is a positive interaction between criteria educational degree and professional experience;
- there is a positive interaction between criteria professional experience and age;
- there is negative interaction between criteria professional experience and job interview;
- interaction between educational degree and professional experience is greater than interaction between professional experience and age.

We can observe from this preference information that the DM is more oriented towards a candidate with good marks in educational degree and professional experience.

To find all the Möbius measures compatible with the DM’s necessary preferences, we implement the procedure explained in Section 3.3.

As a result, the following linear program is performed \( \forall x, y \in A \):\(^1\)

\[
\max \ v \ s.t.
\begin{align*}
C(y) & \geq C(x) + \varepsilon \\
C(Ch) & \geq C(Sl) + \varepsilon \iff Ch \succ Sl \\
C(Gm) & \geq C(Sl) + \varepsilon \iff Gm \succ Sl \\
C(Od) - C(Ch) & \geq C(Bd) - C(Sl) + \varepsilon \iff (Od,Ch) \succ^* (Bd,Sl) \\
C(Bd) - C(Kt) & \geq C(Ch) - C(Sl) + \varepsilon \iff (Bd,Kt) \succ^* (Ch,Sl) \\
\phi(Ed) & \geq \phi(Ex) + \varepsilon \iff Ed \succ Ex \\
\phi(Ex) & \geq \phi(Ag) + \varepsilon \iff Ex \succ Ag \\
\phi(Ag) & \geq \phi(In) + \varepsilon \iff Ag \succ In \\
\phi(Ed) - \phi(Ex) & \geq \phi(Ag) - \phi(In) + \varepsilon \iff (Ed,Ex) \succ^* (Ag,In) \\
\phi(Ed,Ex) & \geq \varepsilon \\
\phi(Ex,Ag) & \geq \varepsilon \\
\phi(Ex,In) & \leq -\varepsilon \\
\phi(Ed,Ex) & \geq \phi(Ex,Ag) + \varepsilon \iff (Ed,Ex) \succ^* (Ex,Ag) \\
a(\emptyset) & = 0, \sum_{i \in G} a(\{i\}) + \sum_{\{i,j\} \subseteq G} a(\{i,j\}) = 1, \\
a(\{i\}) & \geq 0, \forall i \in G, a(\{i\}) + \sum_{j \in T} a(\{i,j\}) \geq 0, \forall i \in G и \forall T \subseteq G \setminus \{i\}.
\end{align*}
\]

\(^1\)For all computations a MATLAB code has been used.
The obtained results, relative to the example, are showed in Figure 2, where the bold arrows are relative the DM’s preference information given at the beginning, while the dashed arrows represent the necessary preference relation \( \trianglerighteq^N \) among the candidates.

As it was pointed out in Section 3.5, during the multicriteria process some DM’s inconsistent preference information can arise.

For example, let us suppose that the DM adds the constraint \( K_t \succ G_m \) to the system (10), obtaining an infeasible system.

After performing the 0–1 MILP optimization explained in Section 3.5, we found three alternatives sets of inconsistent constraints:

1) Constraint: \( K_t \succ G_m \) (trivial solution);
2) Constraints: \( G_m \succ S_l \) and \( A_g \succ I_n \);
3) Constraints: \( C_h \succ S_l, E_x \succ A_g, (E_d,E_x) \succ (E_x,A_g) \) and \( \varphi(E_x,I_n) \leq 0 \).

Therefore, DA makes a list of the different sets of conflicting constraints obtained, asking the DM which one she considers the least adherent to her preference structure.

Finally, after the DA’s removal of inconsistent constraints, the multicriteria method is implemented again from the beginning, as shown in the flow-chart of fig. 1.

5 Conclusions

In this paper, we have presented a multicriteria aggregation-disaggregation approach taking inspiration from the UTA\(^{\text{GMS}}\) and GRIP, some recent generalizations of UTA. Both UTA\(^{\text{GMS}}\) and GRIP apply the additive robust ordinal regression, evaluating a set of additive utility functions, not only linear, compatible
with the DM’s preference information on a reference set of actions, defining two preference relations: the necessary weak preference relation, which holds for any two alternatives \(a, b\) from set \(A\) if for all compatible utility functions \(a\) is at least as good as \(b\), and the possible weak preference relation, which holds for this pair if for at least one compatible utility function \(a\) is at least as good as \(b\).

In this paper, we have extended the basic ideas of UTA\textsuperscript{GMS} and GRIP, including DM’s preference information on positive and negative interactions between criteria adopting a non-additive multiple criteria decision model based on the Choquet integral. In this direction, we shall speak of non-additive robust ordinal regression.

The main features of our approach are the following:

- the DM’s information includes statements on the intensity of preference among pairs of alternatives and criteria, expressed by some ordinal judgements, like in MACBETH and AHP;
- the DM can also provide some information on the eventual interaction of couples of criteria, expressing their redundancy or synergy;
- the method also includes the DM’s pairwise comparisons on the interaction intensities, differently from MACBETH and AHP where such preference information is not considered.

The principal advantages of the described methodology are:

- the DM could express a partial or incomplete preference on some criteria and some reference actions;
- the DM gives only ordinal or at least qualitative judgements on the criteria and reference alternatives, always comparing them pairwise, since, as it well-known in literature, for the DM it is very difficult to assess directly the parameters of the adopted preference model;
- once the DM’s preference information is inferred, from the computational point of view the weights and interaction between criteria, expressed mathematically in terms of Möbius measures, are simply calculated by a linear program for every pair of alternatives;
- an optimization procedure of type “either-or” is implemented to detect possible DM’s inconsistencies; on the basis of its results, the DA interactively helps the DM to select which inconsistency is to be removed among the different sets of contradictory preference information.

Let us observe also that an important advantage of our methodology is the possibility to take into consideration DM’s intensity of preference with respect to single criteria or with respect to holistic comparisons. Moreover, our methodology takes into account information about difference of importance between criteria (e.g., information of the above type c), such as “the difference of importance between criteria \(g_i\) and \(g_j\) is greater than the difference of importance between criteria \(g_h\) and \(g_k\)” because, for instance, \(g_i\) is much more important than \(g_j\) and \(g_h\) is only slight more important than \(g_k\). This type of information has been already considered in the literature using the Choquet integral as decision model of multiple criteria aggregation in [7]. However, the MACBETH-like methodology proposed in [7] is different from our approach with respect to many points:

1. we consider a robust ordinal regression approach taking into consideration the whole set of fuzzy measures compatible with the DM’s preference information, while the authors in [7] evaluate only one set of fuzzy measures compatible;
2. in fact, it is not appropriate to say that the authors in [7] obtain a set of fuzzy measures, because no constraint ensuring the monotonicity property (being a basic property of a fuzzy measure) is considered, such that, in general, their “fuzzy measures” do not satisfy monotonicity;

3. in the approach proposed in [7] the DM is asked to compare all pairs of alternatives with respect to all pairs of criteria and to compare all pairs of criteria with respect to their importance, whereas in our paper the preference information can also be incomplete; in general, it is supposed that our methodology works with partial information in order to avoid overloading the DM with request of too many comparisons. In fact our methodology asks to the DM only the preference information he/she wants to supply;

4. our approach permits the DM to specify the sign and intensity of the interaction among a couple of criteria, whereas such information is not included in the approach of [7];

5. the problem of DM’s eventual inconsistent preference information is not indeed investigated in [7], while our approach incorporates a methodology to systematically detect inconsistencies, helping the DM to remove them. Such methodology has been already proposed in [29] and recently implemented in UTA\textsuperscript{GMS} [21].

Future research will be directed to the implementation of the proposed methodology to sorting problems (see [2]). We also aim at extending the non additive robust ordinal regression to some recent generalizations of the Choquet integral (see [3]): i.e. the bipolar Choquet integral (see [15], [16] and [20]) and level dependent Choquet integral (see [18]).

References


[34] Roubens, M., 1996. Interaction between criteria through the use of fuzzy measures, Report 96.007, Institut de Mathématique Université de Liège, Liège 1996.


