New Upper Bounds on the Star Discrepancy of (t, m, s)-Nets and (t, s)-Sequences

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A popular way of measuring the quality of distribution of a point set of N points in the s-dimensional unit cube is to consider its star discrepancy D_N^*. The star discrepancy of (t, m, s)-nets and (t, s)-sequences, which are among the most widely known classes of s-dimensional quasi-Monte Carlo point sets, was studied extensively by Niederreiter. In the case of (t, m, s)-nets, for example, he showed that for the star discrepancy of any (t, m, s)-net in base b with $m > 0$ we have

$$ND_N^* \leq B(s,b)b^t(\log N)^{s-1} + O\left(b^t(\log N)^{s-2}\right),$$

where $N = b^m$, the constant in the O-notation is independent of N, and where $B(s, b)$ is a term depending only on s and b. A similar result exists for the star discrepancy of (t, s)-sequences. Whereas it is believed that the order of magnitude in N in Niederreiter's bounds cannot be improved, it has been shown by several authors that the constants in the leading term (in the case of (t, m, s)-nets, $B(s, b)$) can be replaced by smaller constants for special choices of t, s, and b.

Here, we present improved upper bounds on the star discrepancy of (t, m, s)-nets and (t, s)-sequences that are not limited to special choices of t, m, s, and b but hold for arbitrary values of these parameters. In our investigations, we are mainly interested in the constants occurring in the leading terms. The new upper bounds on the star discrepancy of nets and sequences are of a similar form as those mentioned above, which makes it easy to compare them to earlier results. We also study the asymptotic behavior of the constants in the leading terms for increasing dimension s and present some numerical results.

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