Physical-layer Algebraic Network Coding and Superposition Coding for the Multi-Source Cooperation Aided Uplink

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Abstract—In this paper, we consider coding schemes designed for energy efficient multi-source cooperation. More explicitly, we propose both a powerful superposition coding scheme and a physical-layer algebraic network coding scheme. The concept of generalised network coding is introduced and the relation between superposition coding and network coding is revealed. Our simulation results demonstrate that both of the proposed schemes are capable of performing close to the outage probability bound at the lower end of the target transmit power range. Moreover, compared to the superposition coding scheme considered, the proposed algebraic network coding arrangement imposes a lower complexity at the cost of a slight performance degradation, while maintaining the same throughput and delay.

I. INTRODUCTION

Cooperative diversity [1] relying on a distributed (virtual) Multiple Input Multiple Output (MIMO) system is capable of eliminating the correlated fading induced spatial-diversity gain erosion of co-located MIMO elements. Hence this novel technique is capable of improving the achievable performance, while supporting a high throughput as well as providing an improved cell-edge coverage [2]. It has the potential of beneficially combining the traditional infrastructure based wireless networks and the ad-hoc wireless network philosophy [3]. Recently, the Cooperative Multiple Access (CMA) channel has attracted substantial research interests, where multiple sources forming a cluster of cooperating nodes communicate with the destination, which is also known as Multi-Source Cooperation (MSC) [4]–[6]. Inspired by the multilayer turbo Space Time Coding (STC) concept introduced in [7], we proposed in [8] an error-resilient yet high-throughput non-orthogonal interleaved random STC scheme, which was specially contrived for MSC.

In contrast to the uncoded system of [8], in this contribution, we aim for improving the energy efficiency of our proposed MSC framework with the aid of two specifically designed coding schemes, namely SuperPosition Coding (SPC) and a Physical-layer Algebraic Network Coding (PANC) scheme. In contrast to classic time-multiplexing, in the SPC scheme the multiple sources’ information is code-multiplexed in order to generate the superimposed and appropriately rotated composite signal, which results in a high throughput and low crest-factor. Thus we will introduce an outer channel-coded SPC-aided MSC arrangement, which will be used as the benchmark of the proposed PANC scheme.

Network Coding (NC) may be viewed as a technique of conveying a linear combination of multiple information streams, rather than using conventional routing or relaying for delivering these information flows individually with the aid of classic resource allocation, such as time-multiplexing or code-multiplexing. The philosophy of network coding was proposed by Yeung [9] for the sake of enhancing the wired channel’s capacity. The potential diversity gain facilitated by network coding used in wireless networks was then illustrated for example in [10]. Apart from the original network-layer applications, it has recently been recognized that the physical-layer of wireless networks also benefits from NC. The concept of joint channel coding and network coding was proposed in the context of the classic two-way relay channel [11] and the multiple access relay channel [12], where the concept of distributed channel codes was generalised and the redundancy inherent in the network code was exploited in order to support channel decoding. The employment of network coding was proposed for a two-user cooperation-aided scenario in [13], [14], where a promising performance was observed. However, its extension to MSC is not straightforward, since the unique recovery of the information flow from an aggregate of N module 2 superimposed information flows created as $s_1 \oplus s_2, \ldots, s_{N-1} \oplus s_N$ is generally impossible [15]. We therefore generalise the concept of network coding and propose the so-called PANC scheme$^1$.

In a nutshell, the novel contribution of this paper is that we propose both a SPC scheme and a PANC scheme, which are capable of performing close to the best possible outage probability bound in the context of MSC. Our numerical results show that compared to SPC, the novel PANC arrangement exhibits a reduced complexity (a) at the cost of a slight performance degradation ($P_{bl}$), while maintaining the same throughput ($\gamma$) and delay ($\tau$).

The rest of the paper is organized as follows. In Section II, we describe our MSC model and propose the SPC and PANC schemes considered. Furthermore, the iterative receiver structure and the soft PANC decoding algorithm advocated are also discussed. In Section III, the outage probability bound of MSC is analysed and the numerical results characterizing both schemes are provided. Finally, we conclude our discourse in Section IV.

Notation: Throughout the paper, lower (upper) case boldface letters will represent row vectors (matrices). The identity matrix of size $N$ is denoted as $I_N = \text{diag}[1, \ldots, 1]_N$. The superscripts $(\cdot)^T$ denotes transposition and $(\cdot)$ denotes an estimate of the variable $\alpha$. The superscript $(\cdot)^{(1)}$ and $(\cdot)^{(2)}$ denotes Phase-I cooperation and Phase-II cooperation, while $N_I$ and $N_C$ represent the information bit duration and codeword length, respectively.

II. SYSTEM DESCRIPTION

A. Cooperation Model

Consider a cluster of single-antenna sources cooperatively communicating with a destination employing a single receive antenna, which jointly result in a Virtual Multiple Input Single Output (VMISO) system, as seen in Fig 1. In this VMISO cluster, we assume having a total of $N$ Cooperating Sources (CS), $K$ Active Sources (AS) and $N_C$ Cooperating Sources (CS), $K$ Active Sources (AS) and $N_C$.

Acknowledgments: The work reported in this paper has formed part of the Core 4 Research Programme of the Virtual Center of Excellence in Mobile and Personal Communications, Mobile VCE, www.mobilevce.com, whose funding support, including that of EPSRC, is gratefully acknowledged. Fully detailed technical reports on this research are available to Industrial Members of Mobile VCE.

1Network coding was first proposed in the network layer as an alternative to the conventional routing [9]. However, its concept can also be employed in the physical-layer [10]–[15] despite the seemingly contradiction.

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(N − K) Relaying Sources (RS). Our MSC scheme entails two phases. In Phase-I cooperation, the source information emanating from all K ASs is broadcast to all N CSs in a Time Division Duplex (TDD) manner under the assumption of having perfect synchronization. By contrast, Phase-II cooperation is defined as the joint transmission of a combined signal generated by the concerted action of all the N CSs, which will be elaborated on in more detail in the next subsection. Therefore, each CS simultaneously transmits multiple ASs’ information, resulting in a high throughput. This implies that each AS is served simultaneously by multiple CSs and hence the entire set of ASs benefits from a high diversity gain.

B. Cooperative Code Design

In this paper, we focus our attention on developing coding schemes for MSC in Phase-II cooperation, when the so-called decode-forward relaying technique is employed at each of the N CSs.

1) Superposition Coding: Following Phase-I cooperation, the nth of the N CSs retrieves all the K ASs’ information \( s_{k,n}^{(1)} \), \( k \in [1,K] \) and the transmitted codeword is constructed as follows. Firstly, the nth CS forms K parallel codewords

\[
e_{n,k} = \pi_k \left[ f(s_{k,n}^{(1)}) \right], \quad k \in [1,K],
\]

where \( \pi_k \) is referred to as the AS-specific interleaver and \( f(\cdot) \) represents the outer channel coding function, which is assumed to be the same for all ASs. These AS-specific outer codewords are then punctured according to

\[
\hat{e}_{n,k}(i) = e_{n,k}[N(i − 1) + n], \quad i \in [1,N_c/N].
\]

This is followed by a Parallel-to-Serial (P/S) conversion in order to create a single codeword

\[
e_{n} = [\hat{e}_{n,1}(1), \ldots, \hat{e}_{n,K}(1), \ldots, \hat{e}_{n,1}(N_c/N), \ldots, \hat{e}_{n,K}(N_c/N)].
\]

Finally, the composite codeword transmitted from the nth CS is BPSK modulated and linearly superimposed:

\[
x_{n}^{(2)}(i) = \sum_{l=1}^{L_n} \theta_{n,l} e^j \phi_{n,l} x_{n,l}^{(2)}(i),
\]

\[
x_{n}^{(2)}(i) = 2\hat{e}_{n}[L_n(i − 1) + \ell] − 1,
\]

where \( \ell \in [1,N_c/KNL_n] \) and \( L_n \) is referred to as the number of layers contributed by the nth CS’s superposition, while \( \phi_{n,l} \) and \( \theta_{n,l} \) denotes the layer-specific amplitude and phase rotation respectively. In this treatise, we assume \( L_n = L, \rho_{n,l} = \rho, \theta_{n,l} = \theta \), \( \forall n \in [1,N] \).

Remarks: The rationale of allocating a different amplitude \( \rho \) and hence power to each of the L layers is philosophically similar to that of the multilevel coding concept of [16], where we create a number of different protection levels and detect them by gleaning extrinsic information from the previously decoded levels using multistage decoding. Imposing the associated phase rotation \( \theta_{n,l} \) has two benefits, namely that of reducing the Peak-to-Average Power Ratio (PAPR) of the transmitted signal \( x_n^{(2)} \) and making \( x_n^{(2)} \) having \( L \) layers more distinguishable for the detector. In this paper, equal amplitude allocation and uniform phase rotation are employed.

2) Physical-layer Algebraic Network Coding: We generalise the concept of NC as a coding function \( f(\cdot) \), which jointly encodes all the incoming multiple information flows. With the aid of this generalisation, the original NC operation \( \oplus \) of K linearly coded information flows \( s_iG_i, i \in [1,K] \) becomes equivalent to encoding the vectors \( s = [s_1, s_2, \ldots, s_K] \) using a nested Generator Matrix (GM), which can be written as:

\[
e = s_1G_1 \oplus s_2G_2, \ldots \oplus s_KG_K
\]

\[
= [s_1, s_2, \ldots, s_K][G_1, G_2, \ldots, G_K]^T.
\]

We now proceed to describe the construction of codewords for our MSC taking this novel PANC principle into account. After retrieving all the K ASs’ information denoted by \( s = [s_1^{(1)}, \ldots, s_K^{(1)}] \) and having a length of \( KN_i \), the nth CS generates a total of \( \kappa \) number of versions of the differently interleaved information flow and the resultant codeword \( e_n \) of length \( N_c = \kappa KN_i \) is given by:

\[
e_n = \pi_1(s)G_1 \oplus \pi_2(s)G_2, \ldots \oplus \pi_\kappa(s)G_K
\]

\[
= [\pi_1(s), \pi_2(s), \ldots, \pi_\kappa(s)]G
\]

\[
= [G_1, G_2, \ldots, G_K]e_n^T.
\]

where we have \( \pi_1 = \pi, \pi_\kappa = \pi(\pi_{\kappa−1}) \) and \( \pi \) represents a randomly generated interleaver in this paper. Although in principle an arbitrary GM may be applicable, we adopt a simple unity-rate ACcumulate Code (ACC) [17], having a GM and Parity Check Matrix (PCM) represented as

\[
G_{acc} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}
\]

\[
H_{acc} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}
\]

Apart from the non-systematic PANC GM of Eq. 11, we may also generate a systematic PANC by designing the GM as

\[
G = \begin{bmatrix} I_{\kappa'} & 0 \\ 0 & G_{acc} \end{bmatrix}
\]

where \( \kappa' \) number of differently interleaved versions of the original information flows are created, corresponding to \( [\pi_1(s), \pi_2(s), \ldots, \pi_\kappa(s)] \). Finally, the nth CS transmits a BPSK modulated punctured codeword according to

\[
x_n^{(2)}(i) = 2e_n[N(i − 1) + n] − 1 \quad i \in [1,N_c/N].
\]

Remarks: The concept of NC and SPC may have some intrinsic links. In fact, the authors of [14] considered the NC concept as a SPC scheme defined over the Galois Field \( 2 \) (i.e. the operation \( + \) in SPC is replaced by \( \oplus \) in terms of NC), while the authors of [15] considered the SPC concept as a NC scheme defined over the complex field. Therefore, the PANC proposed above may be considered as a conventional NC scheme exhibiting a channel coding gain, which is a benefit of the mutual dependencies introduced by the linear module 2 addition of multiple streams.
C. Receiver Detection and Decoding

1) Receiver Structure: The destination receives $N$ CS's transmitted signals $x_n^{(2)}$, $n \in [1, N]$, which experienced independently faded channels $h_n$, yielding the received signal:

$$y_{MPNC} = \sum_{n=1}^{N} h_n x_n^{(2)} + n,$$  \hspace{1cm} (13)

$$y_{SPC} = \sum_{n=1}^{N} h_n \sum_{l=1}^{L} p_l e^{j \theta_l} x_n^{(2)} + n,$$  \hspace{1cm} (14)

$$= \sum_{n=1}^{N} \sum_{l=1}^{L} h_n x_{n,l}^{(2)} + n,$$  \hspace{1cm} (15)

where $h_{n,l} = h_n p_l e^{j \theta_l}$ denotes the $l$th layer of the $n$th CS's signal's equivalent channel gain and $n \sim CN(0, N_0)$ is the additive circular complex Gaussian process having a variance of $\sigma^2 = N_0/2$ per dimension.

The receiver uses iterative data detection (DET) and channel decoding (DEC) as seen in Fig 2. Both the SPC and PANMC aided MSC may use the same data detection algorithm. However, the soft channel decoder design of SPC aided MSC depends on the choice of the specific outer channel coding function $f(\cdot)$ employed in Eq. 1. Hence here we discuss the soft decoding of PANMC only.

2) Data Detection: Without loss of generality, we consider the detection of the system employing PANMC, but again, the SPC detector may be identical. A host of DET schemes may be invoked, including the powerful but complex Maximum Likelihood (ML) detection scheme, sphere decoding [18], etc. Here we opt for employing a low-complexity soft interference cancellation scheme [7]. Aiming for the detection of $x_n^{(2)}$, Eq. 13 may be written as:

$$y = h_n x_n^{(2)} + \xi,$$  \hspace{1cm} (16)

where $\xi$ denotes the residual interference plus noise. By approximating $\xi$ as a joint Gaussian random vector, which can be justified by the central limit theorem, we can model the extrinsic Logarithmic Likelihood Ratios (LLR) as:

$$\mathcal{L}_{det}(x_n^{(2)}) = \log_2 \frac{p(y|x_n^{(2)} = +1)}{p(y|x_n^{(2)} = -1)}$$

$$= \log_2 \frac{\exp\left(\frac{(y - \hat{\xi} - h_n x_n^{(2)})^2}{2V_{\xi}}\right)}{\exp\left(\frac{(y - \hat{\xi} + h_n x_n^{(2)})^2}{2V_{\xi}}\right)}$$

$$= 2h_n (y - \hat{\xi}) / V_{\xi},$$  \hspace{1cm} (17)

where the estimated value of $\xi$ and its variance may be expressed as:

$$\hat{\xi} = \sum_{n=1}^{N} h_n \hat{x}_n^{(2)} - h_n x_n^{(2)}$$  \hspace{1cm} (18)

$$V_{\xi} = \sum_{n=1}^{N} v_n |h_n|^2 + \sigma^2 - v_n |h_n|^2,$$  \hspace{1cm} (19)

where the soft estimate of the transmitted signal is $\hat{x}_n^{(2)} = \tanh(L_{det}^n(x_n^{(2)})/2)$ and the instantaneous variance is $v_n = 1 - \hat{x}_n^{(2)}$. It can be seen from above that only the a priori knowledge of $\hat{x}_n^{(2)}$ is needed in the derivation of the extrinsic information $\mathcal{L}_{det}^n(x_n^{(2)})$, which is gleaned from the outer DEC.

3) Decoding of the PANMC: The soft decoding of the PANMC is analogous to that of a Repeat Accumulate (RA) code [19]. As seen in Fig 2, it consists of the soft ACC decoder and soft combiner (COM). The PCM of the unity-rate ACC is defined in Eq. 11. When considering a non-systematic PANMC, the soft decoder’s graphical representation is shown in Fig 3. After inputting the soft output information of the DET to the PANMC decoder, the soft output of the ACC is forwarded to the soft combiner COM (L_{dec}^e) of all $\kappa$ versions of the differently interleaved information streams $[\pi_1(s), \pi_2(s), \ldots, \pi_\kappa(s)]$, which are then soft-combined and fed back to the ACC decoder ($L_{dec}^e$) for the sake of providing updated soft-information for the DET ($L_{det}^e$). When a systematic PANMC is employed, the soft-output of the ACC decoder provided for the COM block of Fig 2 corresponds to all $\kappa^n = (\kappa - \kappa^n)$ versions of the differently interleaved information streams $[\pi_{n+1}(s), \ldots, \pi_\kappa(s)]$. The rest of the soft-information related to the $\kappa^n$ versions of the differently interleaved information streams $[\pi_1(s), \pi_2(s), \ldots, \pi_{\kappa - 1}(s)]$ is directly fed to the soft-combiner block COM of Fig 2, which means that there is no ACC decoding block between the DET and COM blocks. After carrying out all the affordable iterations, the soft COM block of Fig 2 delivers its ultimate soft decision $\mathcal{L}_{com}^e$ concerning $\pi_i, i \in [1, K]$.

The soft-information delivered along each of the edges shown in Fig 3 obeys the classic sum-product algorithm [20], where variable nodes are denoted as circles and check nodes are denoted as squares. The message passed along edge $j$ from a variable node to a check node is given by:

$$\mathcal{L}_j^e = \sum_{i \in \pi_j} \mathcal{L}_i^e,$$  \hspace{1cm} (20)

while that passed along edge $j$ from a check node to a variable node is given by:

$$\mathcal{L}_j^c = 2 \tanh^{-1}\left[\sum_{i \in \pi_j} \tanh(\mathcal{L}_i^e/2)\right],$$  \hspace{1cm} (21)

where $d_c$ and $d_v$ denotes the check and variable degrees, respectively, i.e. the total number of edges connected to a check node or variable node.

Remarks: The soft decoding of a PANMC is similar to the conventional RA decoding [19], since both of them employ the sum-product algorithm and inherit quasi-random LDPC code-like properties [21]. The difference is that in a PANMC, a full decoding iteration comprises a three-stage process, namely the soft-information exchange across the DET $\Rightarrow$ ACC $\Rightarrow$ COM decoding chain, while conventional RA
decoding [19] performs a two-stage iteration denoted as ACC = COM. Because of this similarity, we will employ a RA code, when the SPC scheme is used as the PANC scheme’s benchmark in the next section. The main difference between employing a PANC and a SPC scheme from a decoding point of view is that the PANC arrangement benefits from the joint decoding of multiple information streams, while superposition coding performs single-stream channel-decoding, as seen in Fig 2.

III. PERFORMANCE EVALUATION

A. Assumptions and Parameters

Let us now quantify the achievable performance of the proposed coding schemes. We assume error-free Phase-I cooperation, which is achieved with the aid of using CRC during each Phase-I transmission and by ensuring that cooperation is only activated by a perfect CRC achieved with the aid of using CRC during each Phase-I transmission decoding, as seen in Fig 2.

Before comparing these two coding schemes, we firstly define our performance metric set $\gamma$, which consists of the achievable throughput $\eta$, the block error ratio $P_{bl}$, the delay $\tau$ and the complexity $\zeta$, i.e. we have $\gamma = \{\eta, P_{bl}, \tau, \zeta\}$. The system’s effective throughput $\eta$ may be defined as $\eta = R, NLM$, where $R_c$ is the channel coding rate, $N$ is the number of CSs, $L$ is the number of layers when the SPC scheme is employed, while we have $L = 1$ when the PANC scheme is considered. Finally, $F$ denotes the modulation scheme used, which is BPSK for both coding schemes. For the SPC scheme, we employ a regular rate $R_1$ systematic RA code as the outer channel code in conjunction with a rate $R_2$ repetition code in order to facilitate the multiple layers’ superposition. Thus the total code-rate becomes $R_c = R_1 R_2$. On the other hand, the code-rate $R_c$ of the PANC scheme is defined as the number of differently interleaved versions $K$, as introduced before.

Therefore, by setting the same system throughput $\eta$ and the same source information segment length of $N_i = 512$ symbols, resulting in a fixed delay $\tau$, we compare the two coding schemes in terms of their block error ratio $P_{bl}$ and associated complexity $\zeta$. The complexity $\zeta$ is simply quantified in terms of the number of iterations invoked. The total number of iterations of a SPC aided MSC scheme is the product of the number of DET = DEC iterations and the number of iterations within the RA code, while that of a PANC aided system is deemed to be proportional to the number of iterations invoked by the three-stage DET = ACC = COM decoder chain. The simulation parameters used are summarized in Table I.

B. Outage Bound Analysis

We now perform an outage bound analysis as a reference for the cooperative coding schemes proposed in Section II. Without loss of generality, we discuss the $N = 2$ MSC aided scenario. The maximum mutual information $I$ of an $N = 2$ MSC-aided multiple access channel is equal to the minimum amongst the individual source’s mutual information $I_1, I_2$ and the sum mutual information $I_s$, which is given by [22]

$$I = \min \{I_1, I_2, I_s\}.$$  \hfill (22)

where $I_1(g_i), I_2(g_i), I_s(g_i)\}$, more explicitly:

$$I_1(g_i) = \log_2 (1 + g_i \gamma)$$ \hfill (i = 1, 2) \\
$$I_s(h_s) = (1/2) \log_2 (1 + g_s \gamma).$$ \hfill (24)

Table II collects $g_i$ for different scenarios, where the upper bound (u.b) corresponds to the no cooperation scenario, while the lower bound (l.b.) corresponds to the cooperation bound [22], which has a max-flow min-cut interpretation. The outage probability of a fading channel is defined as the probability of having a mutual information between the received soft value and the decided symbol, which is less than the system’s target effective throughput $\eta$, formulated as:

$$p_{out} = \Pr \{I < \eta\}.$$ \hfill (25)

Finding the outage probability $p_{out}$ at the system’s target effective throughput $\eta$ and a given SNR per-bit $\gamma$ is equivalent to finding

$$p_{out} = \Pr \{g < g_0\},$$ \hfill (26)

where $g_0 = (2\gamma - 1)/\eta \gamma$ and $g = \min \{g_1, g_2, g_s\}$. The minimum outage probability $p_{out,min}$ at a given $\gamma$ value is achieved by letting $\eta \rightarrow 0$ and it is well known that $\lim_{\eta \rightarrow 0} (2\gamma - 1)/\eta = \ln 2$.

C. Simulation Results

Fig 4 and Fig 5 suggest that both of our proposed coding schemes are capable of approaching the outage probability bound at their corresponding system throughput $\eta$. When $N = K = 2$ sources cooperate in a cluster as characterized in Fig 4, the non-systematic PANC scheme employing $k_{PANC} = 20$ iterations performs within a small fraction of a dB from the SPC scheme, which requires a total of $k_{SPC} = 5 \times 20 = 100$ iterations, hence the former results in a significantly lower complexity. The same trend was also confirmed, when $N = K = 4$ sources cooperate in a cluster, as characterized in Fig 5. However, since the effective system throughput $\eta$ was doubled from half to unity, both schemes exhibited a slightly higher discrepancy w.r.t. the outage probability bound. Observe in Fig 5 that the systematic PANC performs better than its non-systematic counterpart and its performance is close to that of the more complex SPC system. If the affordable complexity is not an issue, then the SPC system may outperform the PANC system, as seen in Fig 5. Since the complexity imposed determines the total power consumption, the PANC scheme may be considered as being more power-efficient.

It was found in Fig 5 that the non-systematic PANC is unable to fully exploit the spatial transmit diversity gain provided by $N = 4$ CSs due to its randomly designed nature. This is particularly true, when the effective throughput becomes unity, which may be referred to as a ‘fully-loaded’ MSC-aided scenario. The systematic PANC, on the other hand, provides an additional diversity gain,
The powerful channel coded SPC scheme may be considered as a direct extension of our previous work [8], while the newly proposed PANC scheme may be referred to as a joint network coding and channel coding scheme. The simulation results of Fig 4 and Fig 5 demonstrate that both schemes are capable of performing close to the outage probability bound. When compared to the SPC arrangement, the novel PANC scheme exhibits a lower complexity at the cost of a slight performance degradation, while maintaining the same effective throughput and delay.

IV. CONCLUSION

In this paper, we proposed two different coding schemes in the context of energy efficient MSC, namely the SPC and PANC schemes. The powerful channel coded SPC scheme may be considered as a direct extension of our previous work [8], while the newly proposed

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