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PORTFOLIO INSURANCE AND BOND MANAGEMENT IN A VASICEK'S TERM STRUCTURE OF INTEREST RATES

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Portfolio Insurance and Bond Management in a Vasicek's Term Structure of Interest Rates

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ABSTRACT

Portfolio Insurance is a technique which aims to ensure a minimum value to a portfolio through the replication of a put option. The use of this technique to bond management has not been object of significant interest by the related literature due to the difficulty in obtaining closed form solutions for bond option pricing. In this research we demonstrate that portfolio insurance technique can be used as a bond management strategy, using a bond option pricing formula based on Vasicek model for the term structure of interest rates.

Introduction

Options bring to investors new possibilities for hedging financial risk. One of these possibilities consists in holding a portfolio composed by an asset and a put-option on it. This portfolio has a minimum value equal to the strike price of the option. The conventional technique on *portfolio insurance,* developed by Leland and Rubinstein (1981), consists in replicating a put-option through the combination, in adequate proportions, of a stock and the riskless asset.

According to *Black and Scholes* formula, the put's *delta* is negative:

$$\Delta = \partial P / \partial S_t = - N(-d_1) \qquad (1),$$

with values between -1, for a zero value of the underlying asset, and 0, for high values of the underlying asset. A riskless portfolio, V, can be created through the following combination of a put, P, and the underlying asset, S:

$$V = P+|\Delta| S \qquad (2).$$

Since

$$\partial V/\partial S= \partial P/\partial S + |\Delta| \partial S/\partial S = \Delta - \Delta =0 \qquad (3),$$

the return from this portfolio must be equal to the riskless interest rate, which means that it is equivalent to investing funds in the money market, MM:

$$MM = P+|\Delta| S \qquad (4).$$

According to equality (4), the put can be replicated by a combination of an investment in the money market and the short selling of the risky asset:

$$P = MM-|\Delta| S \qquad (5).$$

The synthetical put can be used to replicate the following portfolio

$$V^* = S+P \qquad (6),$$

which is then equivalent to:

$$S+P=MM+(1-|\Delta|) S \qquad (7).$$

According to equation (7), the portfolio insurance method consist of placing an amount equal to $P +|\Delta| S$ in the money market, and an amount equal to $1- |\Delta| S$ in the risky asset.

Portfolio Insurance is a dynamic procedure. Every change in S implies a change in the proportions of short term and long term asset in order to keep a hedged portfolio.

The strike price of the theoretical put can be chosen arbitrarily. Gallais-Hamonno and Berthon (1989), suggest that some care must be taken in the choice of that value. Since $|\Delta|$ varies inversely with S, it is more sensitive when S becomes lower than when it becomes higher. An high value for the strike price is not adequate to benefit from positive changes in S, while a low value for the strike price will make difficult the adjustment of the portfolio in case of a negative evolution for S. Another limitation that is often referred about conventional portfolio insurance, is that its capability to ensure a minimum value for a portfolio only exists for small changes in the price of the underlying asset.

Portfolio Insurance and Bond Management

Portfolio Insurance[1] as a tool for bond portfolio management has not received a great interest from the literature, mainly due to the difficulty in obtaining a closed form solution for bond option prices. In fact, there are only a few number of models for the term structure of interest rates with an analytic form for bond price volatility, the most known being the models of Vasicek (1977) and Cox, Ingersoll and Ross (1985).

In this article some simulations are presented which show that, in the case of a Vasicek's type term structure of interest rates, it is possible to implement the portfolio insurance method with successful results. For those simulations we have used the derivation of Jamshidian (1989) for a formula of call-options on bonds, based on Vasicek's term structure. In his formula, Jamshidian uses Vasicek's definition of bonds' return volatility:

$$\sigma_P = \sigma \frac{\left(1 - e^{-\alpha\tau}\right)}{\alpha} \qquad (8),$$

where, σ and α are respectively the variance of the stochastic process of the short term interest rate and its elasticity of return to the normal value, and τ is the term to maturity of the bond to which the volatility refers[2].

[1] Hereby represented by PI.
[2] In the case of a bond option, the volatility has to be referred to the forward price (the price at the option's expiration date).

Black and Scholes (1973) formula can be applied directly to obtain the following expression for the price of a call-option on a bond:

$$C(r,t,T,s,K) = P(r,t,s)N(h) - KP(r,t,T)N(h - \sigma_P) \qquad (9),$$

where C(r,t,T,s,K) denotes the price at time t, of a call-option with expiration at time T on a zero coupon bond with maturity at time s>T, with a strike price K. The current price of the underlying bond is P(r,t,s), and r is the short-term interest rate. In the same expression P(r,t,T) denotes the price of zero coupon bond with maturity at time T, and:

$$h = \frac{\log\left[P(r,t,s) / P(r,t,T)\right]}{\sigma_P} + \frac{\sigma_P}{2} \qquad (10).$$

Applying the put-call parity, the following expression is obtained for a put-option:

$$P(r,t,T,s,K) = KP(r,t,T)N(\sigma_P - h) - P(r,t,s)N(-h) \qquad (11),$$

and the delta of the put is:

$$\Delta = -N(-h) = N(h) - 1 \qquad (12).$$

The Simulations on Portfolio Insurance

In the simulations that we have made to evaluate the capability of PI technique to ensure a minimum value for a bond portfolio, the following values for the drift parameters of the stochastic process of the short term (riskless) interest rate[3] and the market price of risk:

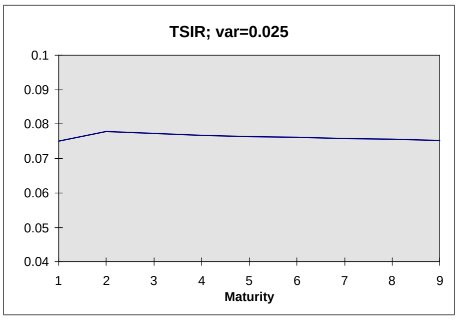
- elasticity of return to the normal value: α=0.2;
- normal value for the riskless interest rate: β=7.5%;
- market price of risk = 0.04.

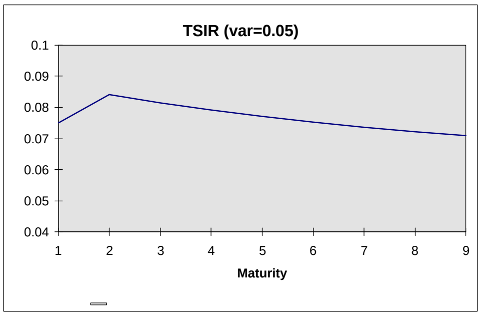
[3] The unit period in the present simulations is the year.

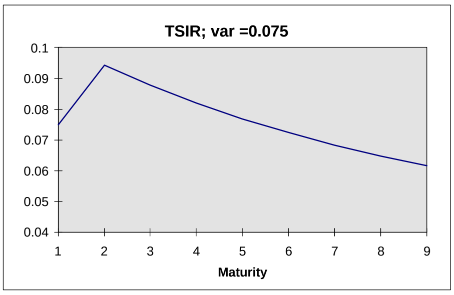
We have considered alternatively four values for the variance of this stochastic process: σ= 0.025, 0.05, 0.075 and 0.1. The use of different values for the variance allowed to put in evidence the influence of the volatility, not only on the price of the theoretical put, but also on the efficacy of the portfolio insurance strategy.

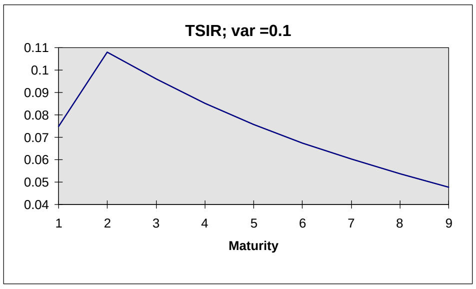
The term structure of interest rates

A different term structure of interest rates corresponds to each value of the variance σ. From the values of the parameters that have been chosen, four humped curves have resulted, which are represented in the following graphs.





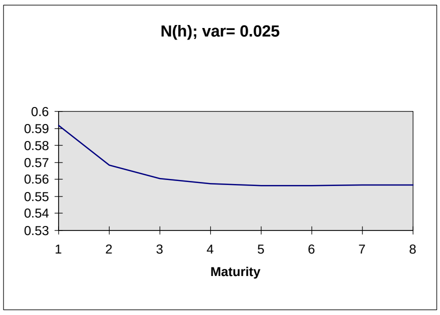


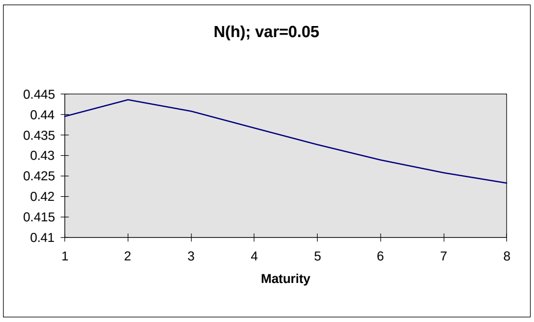


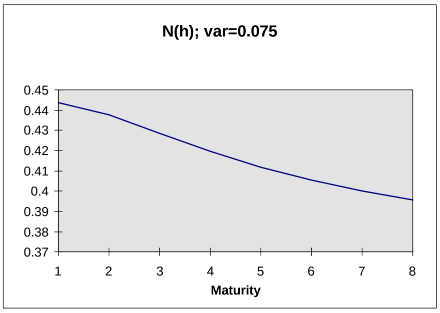
The deltas of the theoretical put-options

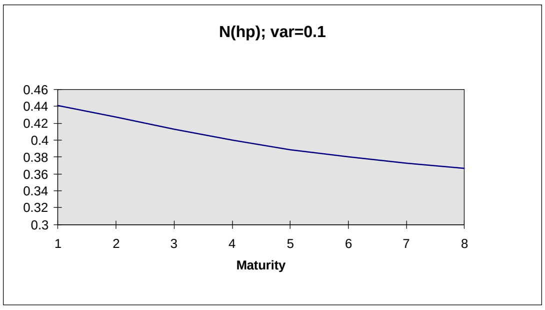
We have considered european options, at one year from expiration, on zero coupon bonds with maturities from one year to nine years, with exercise prices equal to 99% of the forward prices of the bonds at the expiration date. The starting value of the short-term interest rate was its long-term normal value, r_t=7,5%, in all the simulations.

The absolute values of the puts' deltas, N(h), by bond maturity, are represented in the following graphs (each graph is referred to a particular value of variance of the stochastic process of the sort-term rate).









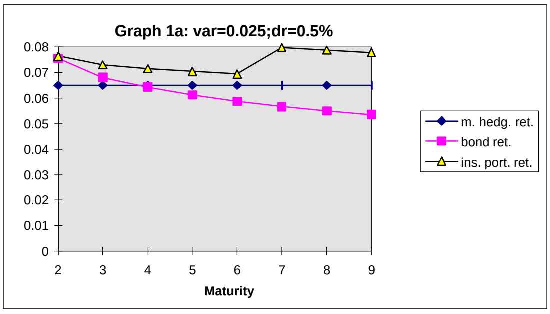
The portfolios' returns in the different simulations

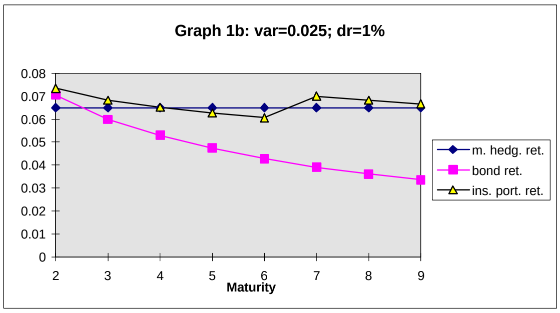
Two portfolios have been compared in each simulation:

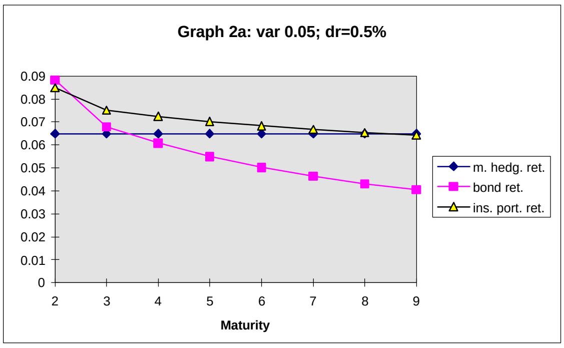
- one *single asset portfolio*, composed by a zero coupon bond with maturity between 2 and 9 years;
- one *insured portfolio*, composed by one bond with maturity between 2 and 9 years and the short-term (riskless) zero coupon bond, with proportions chosen according to equation (7).

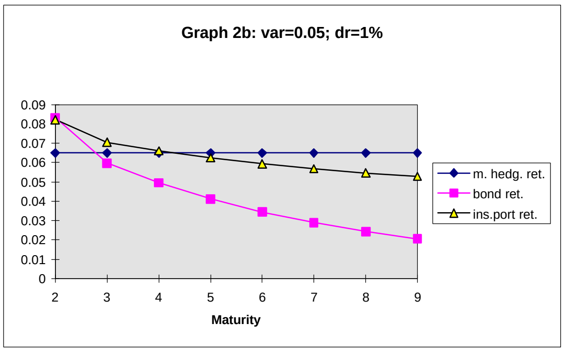
The one year holding period returns for the two portfolios have been computed, in reference to each of the four initial term structures, and considering alternatively the following changes in the short-term interest rate: Δr = +50 basis points, Δr = +100 basis points, Δr = -50 basis points and Δr = -100 basis points.

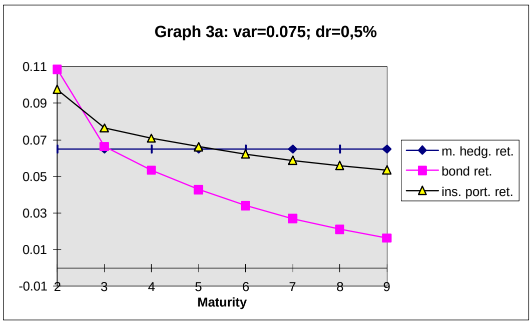
In each of the following graphs are represented those returns, by maturity of the bond included in the portfolio. Each graph is referred to an initial term structure (which corresponds to one particular value of the variance), and to one value for the change of the short-term interest rate. In each graph is also represented the minimum return that can be obtained from a hedged portfolio (a portfolio composed by a bond and a put option on the bond).

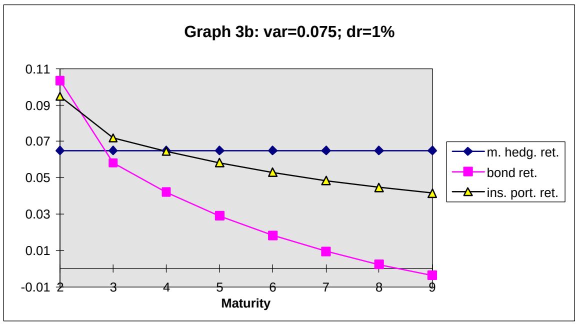

Graph 1a: var=0.025;dr=0.5%
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
2 3 4 5 6 7 8 9
Maturity
m. hedg. ret.
bond ret.
ins. port. ret.

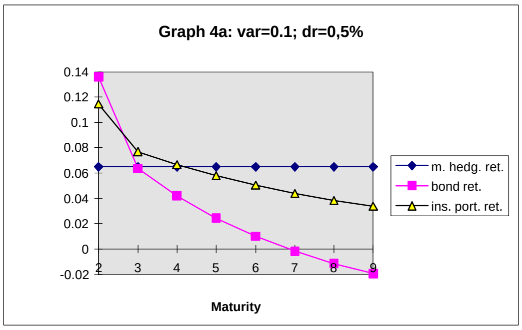

Graph 1b: var=0.025; dr=1%
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
2 3 4 5 6 7 8 9
Maturity
m. hedg. ret.
bond ret.
ins. port. ret.

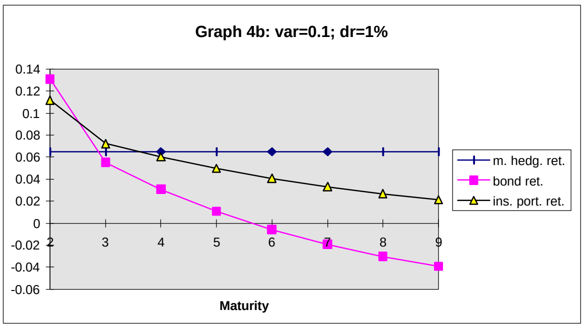

Graph 2a: var 0.05; dr=0.5%
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
2 3 4 5 6 7 8 9
Maturity
m. hedg. ret.
bond ret.
ins. port. ret.

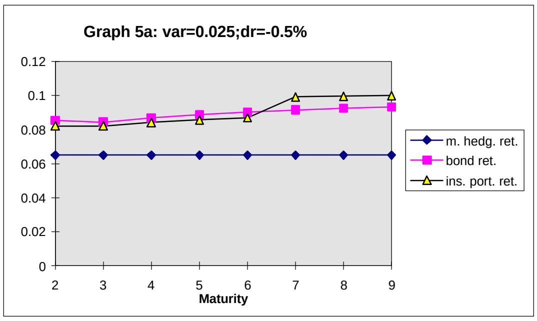

Graph 2b: var=0.05; dr=1%
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
2 3 4 5 6 7 8 9
Maturity
m. hedg. ret.
bond ret.
ins.port ret.

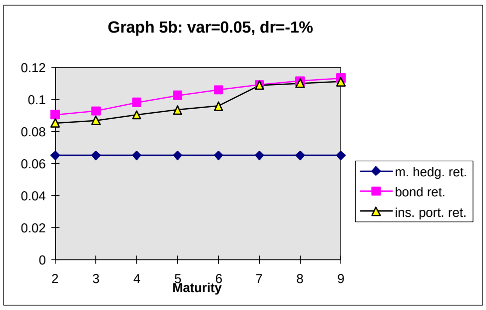

Graph 3a: var=0.075; dr=0,5%
0.11
0.09
0.07
0.05
0.03
0.01
-0.01
2 3 4 5 6 7 8 9
Maturity
m. hedg. ret.
bond ret.
ins. port. ret.

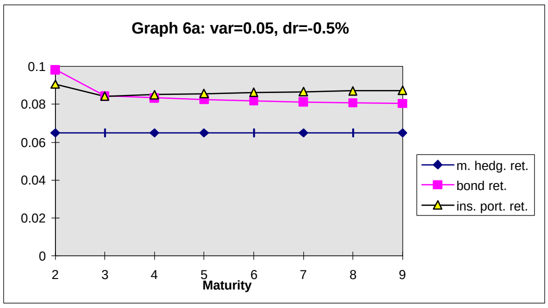

Graph 3b: var=0.075; dr=1%
0.11
0.09
0.07
0.05
0.03
0.01
-0.01
2 3 4 5 6 7 8 9
Maturity
m. hedg. ret.
bond ret.
ins. port. ret.

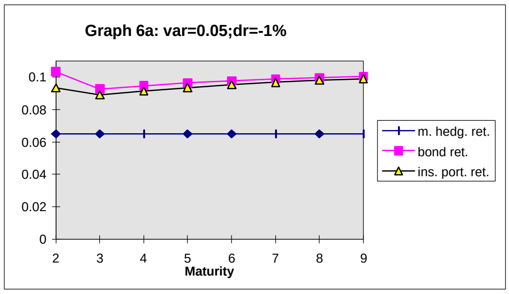

Graph 4a: var=0.1; dr=0,5%
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
2 3 4 5 6 7 8 9
Maturity
m. hedg. ret.
bond ret.
ins. port. ret.

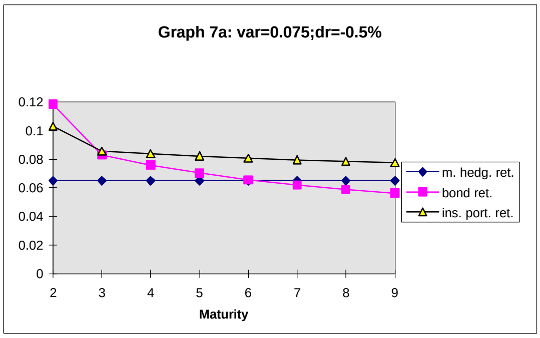

Graph 4b: var=0.1; dr=1%
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
2 3 4 5 6 7 8 9
Maturity
m. hedg. ret.
bond ret.
ins. port. ret.

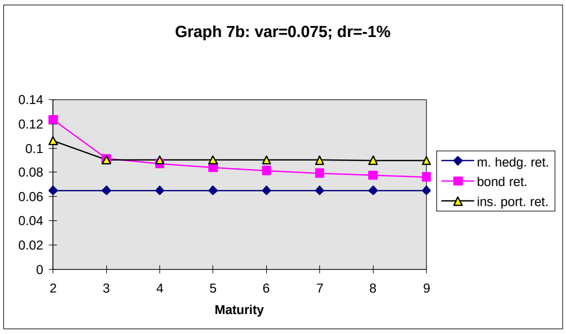


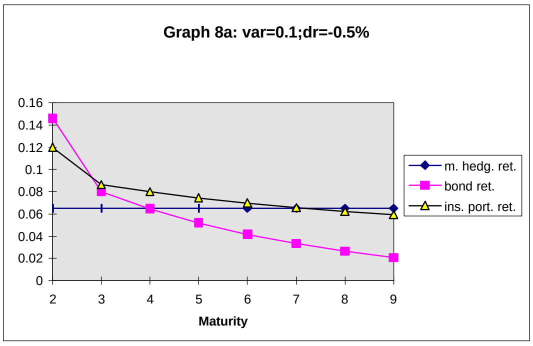


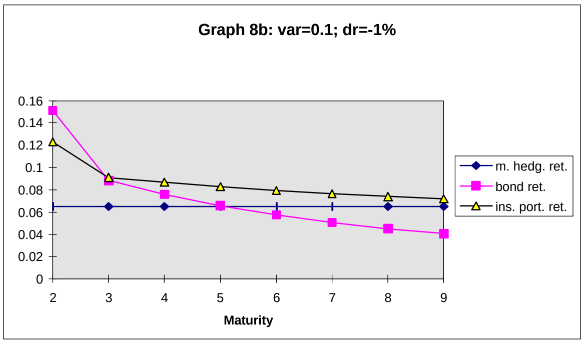












Analysis of the simulations results

For a positive change of 50 b.p. in the short-term rate, and variance $\sigma \leq 0.05$ (Graphs 1a) and 2a)), the return obtained from the PI technique exceeds the minimum return of a hedged portfolio. For higher values of the variance, and considering the same change in the short-term rate (Graphs 3a) and 4a)), only in the cases of bonds with maturities of 3 years or less, the return from the PI technique is superior to the minimum return of an hedged portfolio. For bond maturities over 3 years the PI returns were lower than the minimum return of a hedged portfolio. These results can be explained by the fact that, in general, the absolute value of the put's delta declines with maturity, which means that the portfolios containing longer-term bonds are less protected against positive shocks on the interest rate because they include a greater proportion of the risky bond. The same conclusion applies in the case of a positive shock of 100 b.p. over the short-term rate (Graphs 2b), 3b and 4b)). The only exception is the case of a variance equal to 0.025, Graph 2b) where there is a great proximity between the PI return and the minimum return, which can be explained by the fact that the put's delta absolute value is almost constant for maturities between 4 and 8 years. In almost all the cases of a positive variation in the short-term rate, PI provides a better return than the simple portfolio, the only exception being the case of two-year maturity bond. The fact that the PI procedure provides better results for the +50 b.p. variation in the short-term interest rate, than for the +100 b.p., confirms the hypothesis, generally accepted that the performance of this strategy is better for small variations on prices than for larger variations. On the other side, the fact that the performance of the portfolio insurance strategy varies inversely with the variance of the short-term rate stochastic process, is the result of the negative influence of that variance on the absolute value of the N(h) function.

In almost every cases of negative changes in the short-term interest rate, the return obtained from the portfolio insurance technique is superior to the minimum return of an hedged portfolio, with the exceptions of 8 and 9 year maturities, for σ=0.1 and Δr=-0.5%. For negative changes of the interest rate, the performance of the PI strategy is often superior to the performance of the single asset portfolio. This is clearer for higher values of σ.

Conclusion

In this research some simulations have been made with the aim of evaluating the capability of portfolio insurance to be used on bond management, compared with other traditional strategies. The results of the simulations demonstrate that this strategy has, in general, a better performance, than the “naïf” portfolio composed by a single bond.

The bonds' maturity, as well as the variance of the short-term rate stochastic process, are factors with influence on portfolio returns. Those factors influence negatively the put-options delta absolute value, thus reducing portfolios' sensibility to changes in interest rates. For this reason, portfolio insurance technique returns are, in general, better for short maturity than for long maturity bonds, and for lower variance cases than for higher ones.

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