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The Generalized Weibull-Exponential Distribution: Properties and Applications

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Abstract This paper defines a new distribution, namely, Generalized Weibull- Exponential distribution GWED. This distribution extends a Weibull-Exponential distribution which is generated from family of generalized T-X distributions. Different properties for the GWED are obtained such as moments, limiting behavior, quantile function, Shannon's entropy, skewness and kurtosis. Finally, analysis of several real data sets are carried out and thereafter compared the results with other distributions to illustrate the applications of the GWED.

Keywords Exponentiated Distribution, T-X Distribution, Shannon's entropy, Limiting behavior, Hazard Function

1. Introduction

Although the statistical distributions used to describe and interpret the phenomena, there are continuous motivations for developing these distributions to become more flexible or more fitting for specific real data sets. These new statistical distributions are called exponentiated distributions. The idea of exponentiated distributions were utilized to create new distributions. Cordeiro & Castro (2011) [5] extended many known distributions as normal, Weibull, gamma, Gumbel, and inverse Gaussian distributions. They expressed the ordinary moments of these new family of generalized distributions as linear functions of probability weighted moments of the parent distribution. Mudholkar & Srivastava (1994) [11] proposed the exponentiated Weibull distribution to analyze bathtub failure data. Gupta *et al* (1998) [8] introduced a class of exponentiated distributions which is based on cumulative distribution function as follows:

$$G(x)=[F(x)]^c \tag{1.1}$$

where $G(x)$ is the cumulative distribution function of the random variable X and c is an additional shape parameter.

The Weibull and exponential distributions are the most widely used in the reliability and survival studies. This is due to their simplicity and easy mathematical manipulations. In additions, the exponential distribution is one of the members for Weibull- X family as well. Let X be a random variable taken from exponential distribution with probability density function pdf and cumulative distribution function (CDF) respectively, $f(x)=\theta\exp(-\theta x)$, $x>0$ and $F(x)=1-\exp(-\theta x)$. Also, let T be a random variable taken from Weibull distribution with pdf is $r(t)=\frac{\alpha}{\gamma^{\alpha}}t^{\alpha-1}e^{-\left(\frac{t}{\gamma}\right)^{\alpha}}$, $t>0$. Then, the pdf of Weibull-Exponential distribution is

$$g(x)=\frac{c}{\gamma}\left[\frac{f(x)}{1-F(x)}\right]\cdot\left[\frac{-\ln(1-F(x))}{\gamma}\right]^{\alpha-1}\cdot e^{-\left(\frac{-\ln(1-F(x))}{\gamma}\right)^{\alpha}}$$

$$=\frac{c}{\gamma}\left[\frac{\theta\exp(-\theta x)}{1-\left(1-e^{-\theta x}\right)}\right]\cdot\left[-\ln\left(1-\left(1-e^{-\theta x}\right)\right)\right]^{\alpha-1}$$

$$\times e^{-\left(-\ln\left(1-\left(1-e^{-\theta x}\right)\right)\right)^{\alpha}} . \qquad ; x>0,\ \theta,\alpha,\gamma>0 \tag{1.2}$$

For the exponentiated T-X distribution, Alzaatreh *et al* (2013) [2] proposed a new method for generating many new distributions. It is called, the T -X family of distributions. It has a connection between the hazard functions and each generated distribution as a weighted hazard function of the random variable X. Alzaatreh *et al* (2013) [2] founded several known continuous distributions to be special cases of these new distributions. Akinsete *et al* (2008) [1] studied a four-parameter beta-Pareto distribution. They discussed various properties of the distribution and they founded the distribution to be unimodal and has either a unimodal or a decreasing hazard rate. Also, they obtained the mean, mean

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deviation, variance, skewness, kurtosis entropies, and they used the method of maximum likelihood to estimate the parameters. Carl *et al* (2013) [4] used five general methods of combination and variations of two historical periods. These combinations for generating statistical distribution approaches are the method of generating skew distributions, the method of adding parameters, beta generated method, transformed-transformer method and composite method.

The paper is organized as follows. In Section 2, The GWED will be defined. The properties for the new distribution as moments, limiting behavior, quantile function, Shannon's entropy, skewness and kurtosis are discussed in section 3. Finally, comparing of results for the GWED with other distributions via real data sets.

2. The GWED

Alzaghal A. *et al* (2013) [3] presented the pdf of the Exponentiated *T-X* distributions as follows

$$g(x)=\frac{c\alpha}{\gamma}\,\frac{f(x)\,F^{c-1}(x)}{1-F^{c}(x)}.\left\{\frac{-\ln\left(1-F^{c}(x)\right)}{\gamma}\right\}^{\alpha-1}\times\exp\left[-\left\{\frac{-\ln\left(1-F^{c}(x)\right)}{\gamma}\right\}\right]^{\alpha} \tag{2.1}$$

therefore, the pdf of the GWED is

$$g(x)=\frac{c\alpha}{\gamma}\frac{\theta e^{-\theta x}(1-e^{-\theta x})^{c-1}}{1-(1-e^{-\theta x})^{c}}.\left\{\frac{-\ln\left(1-(1-e^{-\theta x})^{c}\right)}{\gamma}\right\}^{\alpha-1}\times\exp\left[-\left\{\frac{-\ln\left(1-(1-e^{-\theta x})^{c}\right)}{\gamma}\right\}\right]^{\alpha}\;;x>\theta. \tag{2.2}$$

And the CDF is

$$G(x)=1-\exp\left[-\left\{\frac{-\ln\left(1-(1-e^{-\theta x})^{c}\right)}{\gamma}\right\}\right]^{\alpha}\;;\;x>\theta. \tag{2.3}$$

where (α,c) are the shape parameters and (θ,γ) are the scale parameters respectively.

In (2.3), note that, if $c=\alpha=1$, the pdf of the GWED reduces to the exponential distribution with parameter $\frac{\gamma}{\theta}$. Also, when $c=1$ and the GWED reduces to the Weibull distribution with parameters $\left(\frac{\gamma}{\theta},\alpha\right)$. The exponentiated exponential distribution produced when $\gamma=\alpha=1$ as well.

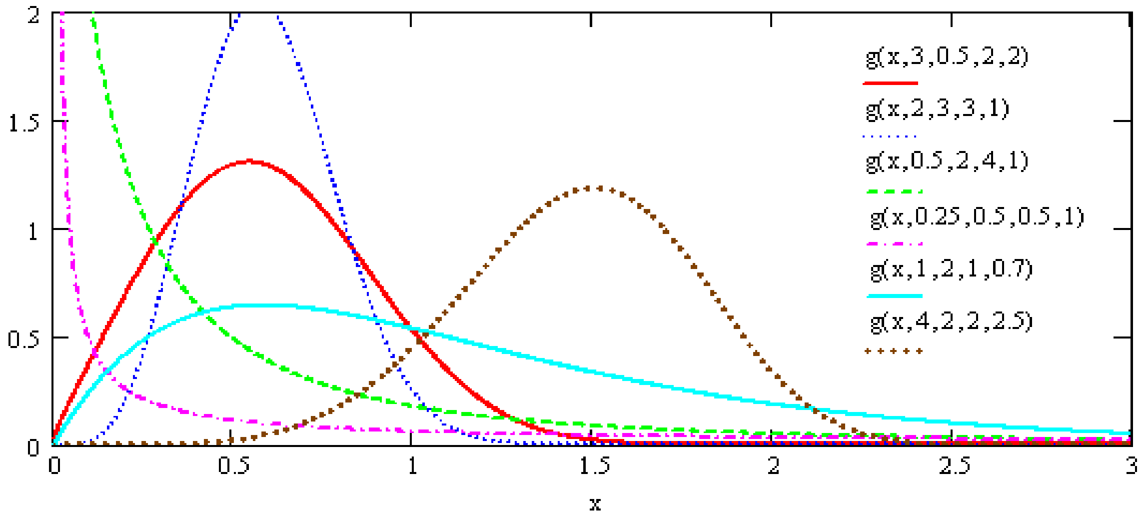

Figure (1). Different forms for pdf of the GWED with various values of the parameters (α,c,θ,γ) respectively

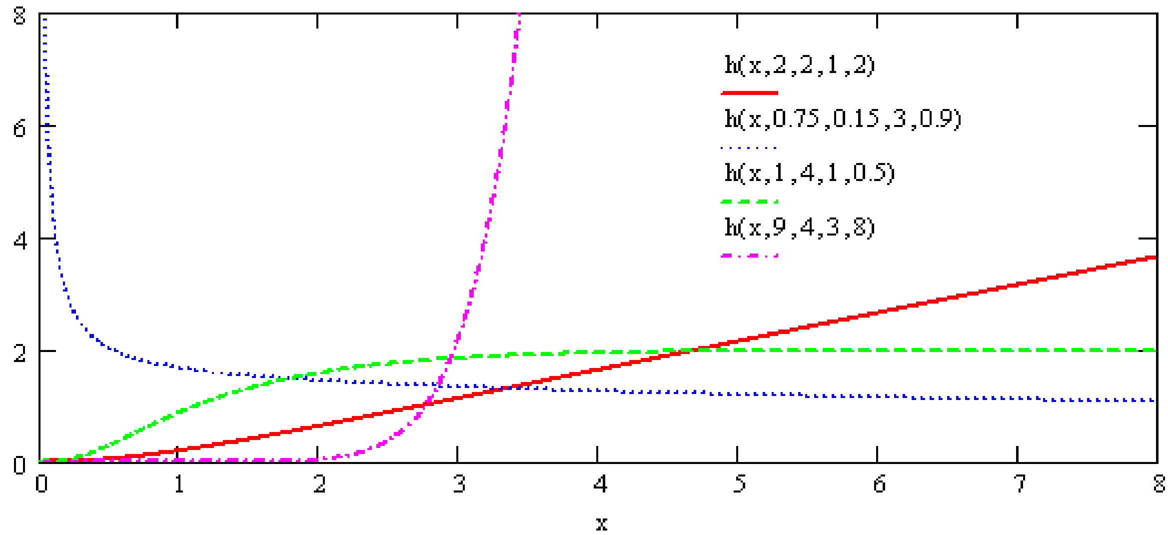

Figure (2). Different forms for hazard function of the GWED with various values of the parameters $(\alpha, c, \theta, \gamma)$ respectively

The survival function and the hazard function of the GWED respectively take the following forms:

$$R(x) = 1 - G(x) = \exp\left[-\left\{\frac{-\ln\left(1-(1-e^{-\theta x})^{c}\right)}{\gamma}\right\}\right]^{\alpha} \quad ; x > \theta \tag{2.4}$$

$$h(x) = \frac{g(x)}{R(x)} = \frac{c\,\alpha}{\gamma}\frac{\theta e^{-\theta x}(1-e^{-\theta x})^{c-1}}{1-(1-e^{-\theta x})^{c}} \cdot \left\{\frac{-\ln\left(1-(1-e^{-\theta x})^{c}\right)}{\gamma}\right\}^{\alpha-1} \quad ; x > \theta. \tag{2.5}$$

3. Properties of the GWED

There are some relations between the GWED and another distributions. These relations can be obtained by using the transform of variables. For example, let Y be an Exponentiated Weibull random variable with parameters $\left(\frac{1}{\gamma}, \alpha, c\right)$, then, using this transform $X = \alpha e^{Y}$, the result is the pdf for the GWED. Also, if Y be a standard exponential random variable with parameters $\left(\frac{1}{\gamma}\right)$, then, using $X = \theta\left[1-(1-e^{Y})^{1/c}\right]^{\frac{1}{\beta}}$ as a transform, the result is the pdf for the GWED. And using the transform $X = \alpha e^{\ln(Y)}$ be a random variable for the Exponentiated Frechet distribution to get on the pdf of the GWED.

3.1. Limiting Behaviors of PDF and Hazard Function

The limiting behaviors of the pdf for the GWED is given by the following lemmas.

Lemma 1: *The limit of GWED density is zero when* x *goes to infinity, and when* $x \to 0$*, the limit is given by:*

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} h(x) = \begin{cases} 0\,, & \alpha c > 1; \\ \dfrac{\theta}{\gamma^{\alpha}}, & \alpha c = 1; \\ \infty\,, & \alpha c < 1. \end{cases} \tag{3.1.1}$$

Proof. A result in (3.1.1) can be proofed as follows :

$$\lim_{x\to 0} g(x) = \frac{c\alpha\theta}{\gamma^{\alpha}} \lim_{x\to 0} \frac{e^{-\theta x}(1-e^{-\theta x})^{c-1}}{1-(1-e^{-\theta x})^{c}} . \left\{-\ln\left(1-(1-e^{-\theta x})^{c}\right)\right\}^{\alpha-1}$$

$$\times \exp\left[-\left\{\frac{-\ln\left(1-(1-e^{-\theta x})^{c}\right)}{\gamma}\right\}\right]^{\alpha} .$$

$$= \frac{c\alpha\theta}{\gamma^{\alpha}} \lim_{x\to 0}\left[\left\{-\ln\left(1-(1-e^{-\theta x})^{c}\right)\right\}^{\alpha-1}(1-e^{-\theta x})^{c-1}\right]$$

$$= \frac{c\alpha\theta}{\gamma^{\alpha}} \lim_{x\to 0}\left[\sum_{j=0}^{\infty}\left[\frac{(1-e^{-\theta x})^{cj}}{j}\right]^{\alpha-1}(1-e^{-\theta x})^{c-1}\right] \quad (3.1.2)$$

$$= \frac{c\alpha\theta}{\gamma^{\alpha}} \lim_{x\to 0}\left[\left[1+\frac{(1-e^{-\theta x})^{c}}{2}+\frac{(1-e^{-\theta x})^{2c}}{3}+\cdots\right]^{\alpha-1}(1-e^{-\theta x})^{c\alpha-1}\right]$$

When x goes to zero, the limit of the quantity between brackets $\left[1+\frac{(1-e^{-\theta x})^{c}}{2}+\frac{(1-e^{-\theta x})^{2c}}{3}+\cdots\right]^{\alpha-1}$ equals one. So, (3.1.2) reduces to:

$$\lim_{x\to 0} g(x) = \frac{c\alpha\theta}{\gamma^{\alpha}} \lim_{x\to 0}\left[(1-e^{-\theta x})^{c\alpha-1}\right] \quad (3.1.3)$$

If $\alpha c > 1$, then, (3.1.3) goes to zero, if $\alpha c < 1$, it goes to infinity and if $\alpha c = 1$, it reduces to $\frac{\theta}{\gamma^{\alpha}}$.

Lemma 2: *The limiting of the hazard function for the GWED when* x *goes to infinity is:*

$$\lim_{x\to\infty} h(x) = \begin{cases} 0 , & \alpha c > 1; \\ \frac{\theta}{\gamma}, & \alpha c = 1; \\ \infty , & \alpha c < 1. \end{cases} \quad (3.1.4)$$

Proof. The result comes by using a similar proof as in lemma 1.

3.2. Quantile Function and Shannon Entropy

Alzaghal *et al* (2013) [3] concluded the quantile function $Q(\lambda)$ and Shannon entropy η_x for the Exponentiated T-X distributions by using the CDF for the X and T distributions as follows:

$$Q(\lambda) = F^{-1}(1-e^{-R^{-1}(\lambda)})^{1/c} \quad (3.2.1)$$

And

$$\eta_x = E(-\ln g(x)) = -\int_0^{\infty} g(x)\cdot \ln g(x)\,dx$$
$$= -\ln(c) - E\left[\ln\left\{f\left(F^{-1}(1-e^{-T})\right)^{1/c}\right\}\right] + \frac{1-c}{c}E\left[\ln(1-e^{-T})\right] - \mu_T + \eta_T \quad (3.2.2)$$

So, the $Q(\lambda)$ and η_x of the GWED respectively are:

$$Q(\lambda) = -\ln(1-\left[1-e^{\left(\frac{-\ln(1-\theta\lambda)}{\gamma}\right)^{1/\alpha}}\right]^{1/c} \tag{3.2.3}$$

$$\eta_x = E(x) + \alpha\,\theta\left(\frac{1-c}{c}\right)\sum_{j=0}^{\infty}\frac{(-1)^{j+1}}{\gamma^{\alpha(j+1)}j!}\left(\sum_{n=0}^{\infty}\frac{\Gamma(\alpha(j+1))}{n^{\alpha(j+1)+1}}\right) + \frac{\gamma}{\theta}\left[\Gamma(1+\frac{1}{\alpha})+1-\frac{1}{\alpha}\right] + \ln(\frac{\gamma}{c\,\theta\,\alpha})+1 \tag{3.2.4}$$

And the quartiles of the GWED can be obtained by setting $\lambda = (0.25, 0.50, 0.75)$ in (3.2.3). Also, based on quartiles, the skewness and kurtosis for the GWED respectively are:

$$Sq = \frac{Q(3/4) - 2Q(1/2) + Q(1/4)}{Q(3/4) - Q(1/4)} \tag{3.2.3}$$

$$K = \frac{Q(7/8) - Q(5/8) + Q(3/8) - Q(1/8)}{Q(6/8) - Q(2/8)} \tag{3.2.4}$$

3.3. Moments

The k^{th} moments of the GWED can be obtained via the moment generating function which can be derived as follows:

$$M_x(t) = E(e^{tx}) = \int_0^{\infty} e^{tx}\, g(x)\, dx$$

$$= \int e^{tx}\frac{c\,\alpha}{\gamma}\frac{\theta e^{-\theta x}(1-e^{-\theta x})^{c-1}}{1-(1-e^{-\theta x})^c}.\left\{\frac{-\ln\left(1-(1-e^{-\theta x})^c\right)}{\gamma}\right\}^{\alpha-1} \tag{3.3.1}$$

$$\times \exp\left[-\left\{\frac{-\ln\left(1-(1-e^{-\theta x})^c\right)}{\gamma}\right\}\right]^{\alpha} dx$$

Let $u = \left[-\left\{\frac{-\ln\left(1-(1-e^{-\theta x})^c\right)}{\gamma}\right\}\right]^{\alpha}$, then, $dx = \frac{\gamma}{\alpha\, c\,\theta}\frac{\left(1-(1-e^{-\theta x})^c\right)}{\left[-\ln\left(1-(1-e^{-\theta x})^c\right)\right]^{\alpha-1} e^{-\theta x}(1-e^{-\theta x})^{c-1}}du$,

and the integral in (3.3.1) can be written as:

$$M_x(t) = \int_0^{\infty} e^{-u}\left[1-(1-e^{-\frac{\gamma}{\theta}u^{1/\alpha}})^{1/c}\right]^{-t} du$$

$$= 1 + \sum_{j=0}^{\infty}\frac{(t)_j}{j!}\left(\sum_{n=0}^{\infty}\frac{(-1)^j}{n!}\binom{j}{c}_n \sum_{m=0}^{\infty}\frac{(-1)^m n^m}{m!}\left(\frac{\gamma}{\theta}\right)^m \Gamma(1+\frac{m}{\alpha\,\theta}\right) \tag{3.3.2}$$

where $(t)_j = t(t+1)\cdots(t+j-1)$.

The result in (3.3.3) came by using the following series expansions:

$$\left[1-(1-e^{-\frac{\gamma}{\theta}u^{1/\alpha}})^{1/c}\right]^{-t} = \sum_{j=0}^{\infty}\binom{t+j-1}{j}(1-e^{-\frac{\gamma}{\theta}u^{1/\alpha}})^{j/c},$$

$$(1-e^{-\frac{\gamma}{\theta}u^{1/\alpha}})^{j/c} = \sum_{j=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{j}{c}\right)_n e^{-n\frac{\gamma}{\theta}u^{1/\alpha}}$$

and

$$e^{-\frac{\gamma}{\theta}u^{1/\alpha}} = \sum_{j=0}^{\infty}\frac{\left(n\frac{\gamma}{\theta}u^{1/\alpha}\right)^m}{m!}.$$

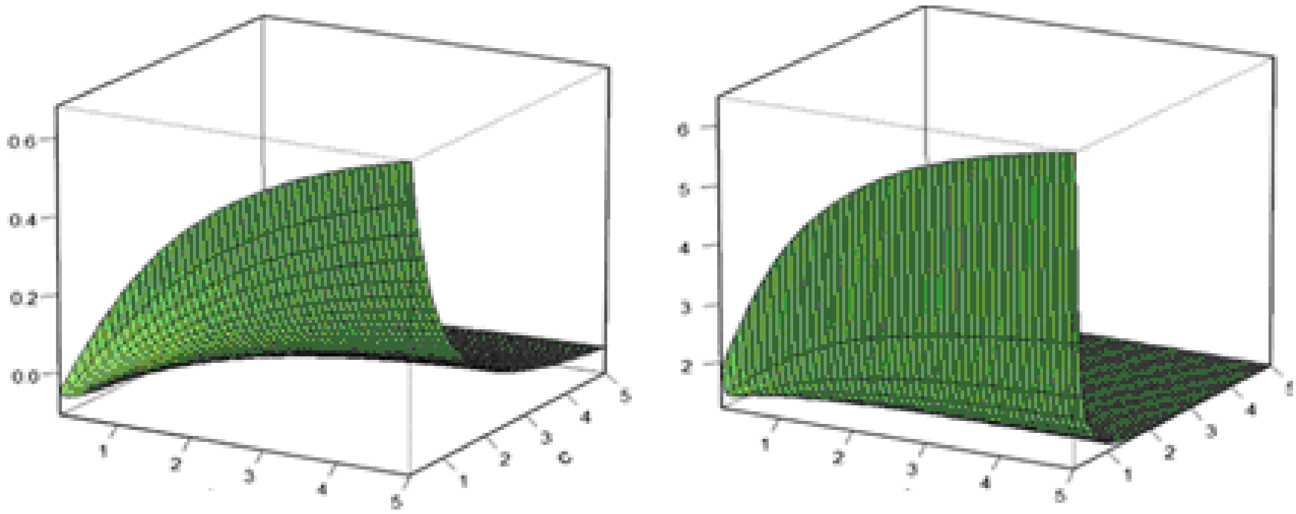

Figure (3). Skewness and Kurtosis of the GWED

Now, taking the k^{th} derivative of (3.3.2) and replacing $t=0$ to obtain the k^{th} moments. Therefore,

$$E(X^k) = \sum_{j=0}^{\infty}\frac{d^k}{dt^k}\frac{(t)_j}{j!}\left(\sum_{n=0}^{\infty}\frac{(-1)^j}{n!}\left(\frac{j}{c}\right)_n\sum_{m=0}^{\infty}\frac{(-1)^m n^m}{m!}\left(\frac{\gamma}{\theta}\right)^m\Gamma(1+\frac{m}{\alpha\theta}\right) \quad (3.3.3)$$

The mean of the GWED is:

$$E(X) = \sum_{j=0}^{\infty}\frac{1}{j}\left(\sum_{n=0}^{\infty}\frac{(-1)^j}{n!}\left(\frac{j}{c}\right)_n\left\{\sum_{m=0}^{\infty}\frac{(-1)^m n^m}{m!}\left(\frac{\gamma}{\theta}\right)^m\Gamma(1+\frac{m}{\alpha\theta}\right\}\right) \quad (3.3.4)$$

And the variance of the GWED is:

$$V(X) = \sum_{j=0}^{\infty}\frac{2\left(-\psi(1)+\psi(j)\right)}{j}\left(\sum_{n=0}^{\infty}\frac{(-1)^j}{n!}\left(\frac{j}{c}\right)_n\left\{\sum_{m=0}^{\infty}\frac{(-1)^m n^m}{m!}\left(\frac{\gamma}{\theta}\right)^m\Gamma(1+\frac{m}{\alpha\theta}\right\}\right) \quad (3.3.5)$$

Table (1) shows that the mean and variance of GWED are increasing when γ and c increase. And α, c are fixed the mean of GWED is increase while the mean and variance of GWED are decrease.

Table (1). Mean and variance for the GWED various values $\alpha,\gamma,,c$ and $\theta=2$ is fixed

		$\gamma=0.5$		$\gamma=1$		$\gamma=2$	
c	α	$E(X)$	$V(X)$	$E(X)$	$V(X)$	$E(X)$	$V(X)$
	1	0.2412	0.1946	0.7548	1.2546	1.7458	5.1247
	2	0.2178	0.1354	0.6351	0.8647	1.9542	3.9456
0.4	3	0.1567	0.1005	0.5419	0.2156	1.342	0.9421
	5	0.1379	0.0105	0.421	0.0459	1.3457	0.3481
	7	0.1201	0.0056	0.3457	0.0175	1.2942	0.1049
	1	0.3548	0.2873	0.9472	1.2381	1.9476	5.3164
	2	0.3248	0.1983	0.8641	0.9614	1.8621	3.9456
0.6	3	0.2458	0.1208	0.6478	0.1957	1.523	0.8712
	5	0.2486	0.9745	0.6254	0.0265	1.5142	0.3457
	7	0.2748	0.0248	0.6733	0.0472	1.5964	0.1259
	1	0.4784	0.4431	1.1033	1.6142	2.1305	5.401
	2	0.4532	0.3472	0.9334	0.9873	2.0168	3.9871
0.9	3	0.3942	0.1249	0.8421	0.3411	1.984	0.8142
	5	0.3998	0.0185	0.8287	0.0765	1.8421	0.3541
	7	0.2389	0.0005	0.8576	0.0125	1.8113	0.2113
	1	0.5483	0.4512	1.1269	1.4268	2.1568	5.6142
	2	0.5243	0.3588	1.0025	1.1023	2.015	4.1249
1.5	3	0.5001	0.1624	0.9421	0.3489	1.9421	0.9243
	5	0.4853	0.0426	0.9147	0.0942	1.8476	0.3429
	7	0.4652	0.0124	0.9349	0.0642	1.8554	0.2341
	1	0.9542	0.5246	1.6492	1.7648	2.7467	6.0012
	2	0.9248	0.3957	1.5381	1.3245	2.6104	5.5294
3	3	0.90001	0.158	1.4927	0.3487	2.4967	1.2034
	5	0.9231	0.0124	1.9641	0.0882	2.462	0.2899
	7	0.9365	0.1542	1.9948	0.0348	2.5964	0.1642
	1	1.5264	0.7241	2.3145	1.9324	3.2829	6.3481
	2	1.5103	0.5419	2.1426	1.5462	3.1955	5.1026
5	3	1.4651	0.1437	2.0988	0.4571	3.0458	1.9547
	5	1.4764	0.0542	2.9485	0.1028	3.1211	0.4261
	7	1.5813	0.0352	2.2314	0.1047	3.1734	0.1641
	1	1.8412	0.8104	2.6476	1.9875	3.8427	6.5347
	2	1.7417	0.6489	2.4517	1.5942	3.7265	5.0124
7	3	1.7966	0.2478	2.4283	0.9543	3.3491	2.0041
	5	1.8012	0.1006	2.4729	0.2381	3.4683	0.9423
	7	1.8426	0.0408	2.5378	0.0942	3.5018	0.2579

4. Application

Park *et al* (1964) [14] and Park (1954) [13] presented three frequency distributions related to adult numbers of Tribolium Confusum, Tribolium Castaneum Cultured at 24℃ and Tribolium Confusum strain. These data sets can be fitted via different distributions. For example, Exponentiated Weibull, Generalized Weibull, Lagrange-Gamma and Weibull-Exponential distributions which were presented by Mudholkar *et al* (1995) [11], Mudholkar *et al* (1996) [12], Famoya & Govindarajulu (1998) [6] and Alzaatreh *et al* (2013) [2] respectively. The main objective of this application is to compare the previously mentioned distributions with the GWED to provide an adequate distribution that best fits data.

Tables (2), (3) and (4) show relatively better fit of data for the GWED than for the other four distributions and corresponding p-value and skewness are the highest for the GWED among all the distribution considered.

Table (2). Calculated X^2 Values for Tribolium Castaneum Cultured at 24^o C

x-values	Observed	Generalized Weibull	Lagrange-Gamma	Exponentiated Weibull	Weibull-Pareto	GWED
		Expected				
20-	2	6.86	3.31	4.45	3.3	3.30124
30-	15	14.19	10.06	12.06	12.09	12.09124
40-	26	24.04	22.36	23.68	25.16	25.16124
50-	30	35.8	38.06	37.86	39.87	39.89354
60-	67	48.37	54.09	52.34	53.81	53.83354
70-	67	60.32	67.54	64.86	65.18	65.20354
80-	65	70.13	76.5	73.74	72.91	72.93354
90-	80	76.49	80.32	78.14	76.59	76.61354
100-	72	78.64	79.35	78.1	76.38	76.40354
110-	70	76.52	74.58	74.3	72.86	72.88354
120-	77	70.78	67.24	67.73	66.85	66.87354
130-	59	62.51	58.53	59.52	59.24	59.18753
140-	47	52.95	49.44	50.63	50.88	50.82753
150-	39	43.25	40.68	41.87	42.45	42.39753
160-	29	34.24	32.71	33.76	34.49	34.43753
170-	25	26.39	25.78	26.61	27.34	27.28753
180-	24	19.9	19.96	20.55	21.18	21.12753
190-	19	14.75	15.21	15.58	16.05	15.99753
200-	19	10.78	11.42	11.61	11.92	11.99206
210-	7	7.8	8.46	8.51	8.69	8.76206
220-	6	5.6	6.2	6.15	6.21	6.28206
230-	4	4.01	4.49	4.39	4.37	4.44206
240-	3	2.86	3.22	3.08	3.02	3.09206
250-	4	2.04	2.35	2.14	2.05	2.12206
26.-270	1	7.78	5.14	5.33	4.09	4.16206
Total	857	857	857	856.99	856.98	857.3092
parameters		$\hat{\alpha} = 0.3093$ $\hat{\sigma} = 120.83$ $\hat{\lambda} = -0.2963$	$\hat{r} = 7$ $\hat{\theta} = 0.0262$ $\hat{\lambda} = 0.24878$	$\hat{\alpha} = 1.6777$ $\hat{\sigma} = 87.0155$ $\hat{\theta} = 2.70992$	$\hat{c} = 6.9525$ $\hat{\theta} = 9.10652$ $\hat{\beta} = 0.3802$	$\hat{\beta} = 0.6281$ $\hat{\alpha} = 7.2357$ $\hat{c} = 4.2513$ $\hat{\theta} = 3$
χ^2		33.108	24.6894	23.882	21.2134	20.75338
df		20	20	20	20	20
p-values		0.1018	0.42279	0.4684	0.62611	0.653228

Table (3). Calculated X^2 Values for Tribolium Castaneum Cultured at 24^o C

x-values	Observed	Generalized Weibull	Lagrange-Gamma	Exponentiated Weibull	Weibull-Pareto	GWED
		Expected				
20-	0	0.47	0.02	0.05	0	0
30-	0	2.01	0.41	0.57	0	0
40-	3	5.96	2.91	3.33	2.37	2.9426
50-	9	14.15	11.38	11.81	12.82	11.3046
60-	39	28.48	29.57	29.3	32.97	36.2461
70-	53	49.96	57	55.45	59.04	56.9421
80-	77	77.04	87.64	84.72	85.11	77.63257
90-	105	104.29	112.87	108.87	105.46	101.3819
100-	135	123.36	126.05	121.41	116.18	117.3156
110-	114	127.42	125.27	120.42	116.08	114.8026
120-	113	115.77	112.93	108.36	106.54	109.34
130-	92	94.02	93.75	89.9	90.64	89.36257
140-	59	69.79	72.52	69.67	71.97	68.4291
150-	54	48.48	52.77	50.98	53.61	53.6842
160-	38	32.22	36.41	35.52	37.63	39.2549
170-	22	20.86	23.97	23.71	24.99	23.1824
180-	17	13.32	15.13	15.25	15.75	22.13642
190-	6	8.48	9.21	9.48	9.44	16.3479
200-	10	5.42	5.42	5.72	5.4	5.68731
210-	3	3.48	3.1	3.35	2.96	2.5497
220-	2	2.26	1.72	1.91	1.55	1.9176
230-	0	1.49	0.93	1.06	0.78	0.6423
240-	1	0.99	0.5	0.57	0.38	0.94215
250-	0	0.66	0.26	0.3	0.18	0.1167
26.-270	0	1.61	0.26	0.3	0.14	0.1049
Total	952	951.99	982	952.01	951.99	952.2662
parameters		$\hat{\alpha}=0.1838$ $\hat{\sigma}=118.86$ $\hat{\lambda}=-0.5831$	$\hat{r}=14$ $\hat{\theta}=0.06502$ $\hat{\lambda}=0.06502$	$\hat{\alpha}=1.8776$ $\hat{\sigma}=77.4141$ $\hat{\theta}=5.3097$	$\hat{c}=5.0587$ $\hat{\theta}=33.2082$ $\hat{\beta}=0.7473$	$\hat{\beta}=6.842$ $\hat{\alpha}=29.351$ $\hat{c}=0.7243$ $\hat{\theta}=4$
χ^2		23.02	17.22	14.26	15.26	13.894
df		14	14	14	14	14
p-values		0.0599	0.2448	0.4303	0.4303	0.496719

Table (4). Calculated X^2 Values for Tribolium Castaneum Cultured at 24^o C

x-values	Observed	Generalized Weibull	Lagrange-Gamma	Exponentiated Weibull	Weibull-Pareto	GWED
		Expected				
35-	5	4.15	2.29	3.06	2.83	2.68
40-	5	8.21	6.33	7.53	7.98	8.08
45-	14	14.74	15.30	15.33	16.38	16.582
50-	33	24.02	28.25	26.30	27.24	27.544
55-	40	35.36	41.83	38.48	38.54	38.919
60-	49	46.41	51.47	48.53	47.57	47.943
65-	44	53.48	54.12	53.16	51.84	52.101
70-	52	53.33	49.73	50.98	50.15	50.211
75-	44	45.63	40.65	43.10	43.14	42.985
80-	28	33.58	30	32.35	33	32.685
85-	29	21.53	20.24	21.71	22.42	22.05
90-	13	12.30	12.61	13.08	13.51	13.178
95-	9	6.42	7.31	7.11	7.21	6.966
100-	1	3.15	3.98	3.49	3.41	3.252
105-	1	1.48	2.05	1.55	1.42	1.339
110-115	1	4.19	1.84	2.24	1.36	0.486
Total	368	367.98	368	368	368	367.001
parameters		$\hat{\alpha}=0.1498$ $\hat{\sigma}=72.1687$ $\hat{\lambda}=-0.3299$	$\hat{r}=16$ $\hat{\theta}=0.1626$ $\hat{\lambda}=0.05351$	$\hat{\alpha}=3.1599$ $\hat{\sigma}=58.9881$ $\hat{\theta}=3.2394$	$\hat{c}=6.1612$ $\hat{\theta}=23.2825$ $\hat{\beta}=0.8657$	$\hat{\beta}=7.254$ $\hat{\alpha}=18.4752$ $\hat{c}=0.6248$ $\hat{\theta}=3$
χ^2		14.8	10.94	9.6	7.78	6.4281
df		9	9	9	9	9
p-values		0.0966	0.2797	0.3842	0.5562	0.5907

5. Conclusions

In this paper, The Generalized Weibull-Exponential Distribution has been defined and studied. various properties of GWED including moments, variance, quantal function, Shannon entropy, skewness and kurtosis have been obtained. Three real data were fitted for the GWED and compared with four known distributions. The results showed that the GWED is a relatively better model to fit data than the other four distributions and the GWED characterized with highest p-value and skewness among the other.

In simulation study, the mean and variance of GWED are increasing when the parameters γ and c increase. And α, c are fixed the mean of GWED is increase while the mean and variance of GWED are decrease.

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