

Dependence of kB^{-1} factor on roughness Reynolds number for barley and pasture

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Abstract

One of the surface energy balance components, the sensible heat flux, can easily be described for modelling and remote sensing purposes by means of the thermal roughness length (z_{ot}). This roughness length is usually expressed relative to the roughness length for wind speed (z_{ou}): $kB^{-1} = \ln(z_{ou}/z_{ot})$. In the past, the kB^{-1} factor has been taken as a constant having a value of ca. 2 in the case of homogeneous canopies. There is some theoretical evidence that kB^{-1} should depend on friction velocity (u_*) or roughness Reynolds number (Re_o). In this study, previously published barley and pasture data were re-evaluated. It was shown that the reciprocal of Stanton number, which is a part of kB^{-1} , could be expressed commonly for both surfaces. The re-evaluated kB^{-1} factor took the form: $kB^{-1} = 0.37Re_o^{0.3} - (1.2 \text{ or } 1.9)$ (for barley and pasture, respectively). © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

One of the surface energy balance components, the sensible heat flux, can easily be described by means of the roughness length concept. The evaporation term can then often be found as a residual. This is a simple one-layer canopy approach that contains the basic physics but keeps the number of needed parameters low. The approach is well justified for closed canopies where the soil contribution to fluxes is small; and it is particularly useful or unavoidable in atmospheric boundary layer studies where the canopy is merely a lower boundary, in regional and global modelling where only processes at large scales are of interest and resource-demanding computations are

required and also in remote sensing where airborne or satellite measured surface temperature provides a direct link to the sensible heat flux.

Since the momentum exchange is dominated by form drag but the heat transfer takes place by molecular diffusion then normally there is higher resistance to the heat exchange than to that for momentum, the difference being expressed via the kB^{-1} factor or alternatively by using different roughness lengths for wind speed (z_{ou}) and temperature (z_{ot}). The latter usually being smaller. Theory predicts that the logarithmic ratio of the two roughness lengths, $\ln(z_{ou}/z_{ot})$ or kB^{-1} , depends on the roughness Reynolds number, Re_o (Zilitinkevich, 1970). The roughness length z_{ou} is used as a characteristic length, and friction velocity u_* as a characteristic velocity to form Re_o as u_*z_{ou}/ν (ν is the viscosity). The relationship is given in the form of a power function: $kB^{-1} \sim Re_o^n$, which can be

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predicted theoretically but eventually still has to be confirmed experimentally.

The large dependence of kB^{-1} on Re_o has been shown for bluff-rough type of surfaces (Owen and Thomson, 1963; Chamberlain, 1968). Zilitinkevich (1970) proposed $n = 0.45$, but his later deductions gave $n = 0.5$ (Zilitinkevich, 1997). Alternatively, Brutsaert (1982) proposed a smaller $n = 0.25$, but with a larger multiplier, for this type of surfaces. Experimental verification of Brutsaert's formula was undertaken by Cahill et al. (1997), they kept $n = 0.25$ but had to adjust other parameters in the formula.

For vegetated surfaces the Re_o dependence has often been omitted and a constant value close to 2 has been suggested for homogeneous canopies (Garratt and Hicks, 1973; Heilman and Kanemasu, 1976; Garratt and Francey, 1978; Choudhury et al., 1986; Hicks et al., 1986; Mölder, 1997). However, theoretical models predict that even for vegetated surfaces a weak dependence on Re_o must exist. According to Brutsaert (1979, 1982), the bulk heat exchange coefficient between a leaf and the surrounding air, C_{if} , is a power function of u_* or of an alternative form of Reynolds number, $Re_* = u_* L_f / \nu$, where L_f is a characteristic dimension of leaves, and that introduces a Re_o dependence for kB^{-1} . Brutsaert (1979) discusses several possibilities to model $C_{if} \sim Re_*^{-m}$; depending on the nature of micro boundary layers on leaves and the degree of interaction between leaves the power m could vary between 0.2 and 0.5. His model, when calibrated against Chamberlain (1966) 'grass' data, makes use of $m = 0.25$. Because of non-linear equations, the power n for Brutsaert (1979) kB^{-1} values gets somewhat smaller, ca. 0.2. Note that Duynkerke (1992) used a model, similar to Brutsaert's, with $m = 0.4$. Thom (1972) suggested that $kB^{-1} \sim u_*^{1/3}$. Malhi (1996) found $n \approx 0.4$ when he applied a proper definition of aerodynamic surface temperature to savannah data. Jensen and Hummelshøj (1995) and McNaughton and Van Den Hurk (1995) present kB^{-1} as a function of Re_* raised to 1/3 and 0.5, respectively.

In Mölder (1997), kB^{-1} for two barley fields was found to be 2–2.5 for slightly unstable conditions. At larger instability levels kB^{-1} tended to much smaller values. Similar results were also found for a pasture field (Mölder et al., 1999). Here, the analysis was proceeded using the methodology outlined by Brutsaert (1979, 1982), where the kB^{-1} factor was expressed

through the roughness-sublayer Stanton number and the corresponding drag coefficient. The Stanton number showed a Re_o dependence, the data having the minimum scatter with $n = 0.5$. The purpose of this paper is to apply the same methodology as for pasture in Mölder et al. (1999) to the barley data from Mölder (1997) and to show that the scatter can be reduced even there. As the new analysis of the barley data as well as several results from the literature suggests $n = 0.3$, then all the data from Mölder (1997) and Mölder et al. (1999) are generalised with $n = 0.3$. Finally the degree of improvement in calculating the sensible heat flux from surface-air temperature difference using the Re_o -dependent kB^{-1} versus a constant kB^{-1} is discussed.

2. Materials and methods

The measurement site and equipment have been described in Mölder (1997) and Mölder et al. (1999). In short, the first paper presents measurements over two barely fields in Estonia: on the Tamme polder (58°16'N, 26°10'E; alt. 46 m) in 1989 and near the village of Tõravere (58°16'N, 26°27'E; alt. 70 m) in 1990. The second one describes measurements over a field of tall grass (pasture) in Lövsta near Uppsala (Sweden: 59°50'N, 17°48'E; alt. 5 m) in 1993. All the measurement periods coincided with active vegetation growth. Some data characterising the crops are summarised in Table 1. All the analysis is based mainly on profile measurements between 1 and 4 m. In the first two campaigns, fixed sensors were used, whereas reversing psychrometers were used in Lövsta. The infrared radiometer was the same, the Reemann type, in all studies. With this radiometer, the surface radiation temperature is measured as an average over nadir angles 0–45°, with an effective measurement angle of 30° (Mölder et al., 1999). Only daytime data have been analysed, but they may include both unstable and stable cases.

The methodology that was used in Mölder et al. (1999) was based mainly on ideas from Brutsaert (1979, 1982). The kB^{-1} factor can be expressed as a difference of two resistance terms:

$$kB^{-1} = kSt_o^{-1} - kC_{do}^{-1/2} \quad (1)$$

where St_o is the roughness-sublayer Stanton number,

Table 1
Height and aerodynamic characteristics of the studied crops

Canopy type, campaign	Canopy density	Canopy height, h (m)	Displacement height, d (m)	Relation for d	Roughness length, z_{ou} (m)	Relation for z_{ou}
Barley, Tamme-89	Very dense	0.8 (0.52 after lodging)	0.6 (0.4 after lodging)	$3/4h$	0.02–0.10	$0.3(h - d)$; $0.075h$
Barley, Tõravere-90	Moderately dense	0.3–0.8	0.2–0.55	$2/3h$	0.02–0.12	$0.3(h - d)$; $0.1h$
Pasture, Lövsta-93	Moderately dense near the ground, sparse in the upper part	0.5–1.0	0.1–0.25	$1/4h$	0.06–0.11	$0.15(h - d)$; $0.11h$

C_{do} the roughness-sublayer drag coefficient, and $k = 0.4$ the von Kármán constant. It can be shown that:

$$kSt_o^{-1} = k \frac{T_h - T_s}{T_*} \text{ and } kC_{do}^{-1/2} = k \frac{u_h}{u_*} \quad (2)$$

where T_h and u_h are respectively, temperature and wind speed at the top of the roughness sublayer, T_s the surface temperature, and T_* the scaling temperature.

As the roughness sublayer top matches with the overlaying atmospheric surface layer, the well-known semi-logarithmic profile equations for wind speed and air temperature

$$u = \frac{u_*}{k} \left(\ln \left(\frac{z-d}{z_{ou}} \right) - \Psi_u \left(\frac{z-d}{L} \right) + \Psi_u \left(\frac{z_{ou}}{L} \right) \right) \quad (3)$$

$$T = T_s + \frac{T_*}{k} \left(\ln \left(\frac{z-d}{z_{ot}} \right) - \Psi_t \left(\frac{z-d}{L} \right) + \Psi_t \left(\frac{z_{ot}}{L} \right) \right) \quad (4)$$

can be used to calculate T_h and u_h (Brutsaert, 1982). Here, Ψ_u and Ψ_t are the stability correction functions, L the Obukhov length, z the height, and d the displacement height.

The roughness sublayer height is identified with the canopy height, h . This assumption is a simplification but it does not seem to be crucial for kB^{-1} estimates (Brutsaert, 1979, 1982). In a roughness sublayer, the actual profiles of temperature and wind speed may deviate from semi-logarithmic profiles and introduce a bias in the estimates of kSt_o^{-1} and $kC_{do}^{-1/2}$. Since these biases are similar and that kB^{-1} is expressed through the difference of kSt_o^{-1} and $kC_{do}^{-1/2}$, they cancel each other out in kB^{-1} estimates.

The stability correction terms with z_{ou} and z_{ot} in (3) and (4) can be neglected and we can write for $z = h$:

$$k \frac{u_h}{u_*} = \ln \left(\frac{h-d}{z_{ou}} \right) - \Psi_u \left(\frac{h-d}{L} \right) \quad (5)$$

$$k \frac{T_h - T_s}{T_*} = \ln \left(\frac{h-d}{z_{ot}} \right) - \Psi_t \left(\frac{h-d}{L} \right) \quad (6)$$

According to Table 1, z_{ou} can be presented as $z_{ou} = k_2(h - d)$, where k_2 is a constant. This simplifies (5) to the form:

$$k \frac{u_h}{u_*} = \ln \left(\frac{1}{k_2} \right) - \Psi_u \left(\frac{h-d}{L} \right) \quad (7)$$

It is the first term on the right hand side of (6) that may be Re_o dependent. The stability correction term in (6) can be eliminated for a while by considering another temperature:

$$T_{h,n} = T_h + \frac{T_*}{k} \Psi_t \left(\frac{h-d}{L} \right) \quad (8)$$

and that allows us to write a more practical expression

$$k \frac{T_{h,n} - T_s}{T_*} = \ln \left(\frac{h-d}{z_{ot}} \right) = k_1 Re_o^n \quad (9)$$

for determination of k_1 and n when $T_{h,n} - T_s$ and T_* can be determined from experimental data.

Substitution of (5) and (6) into (1) and considering (7) and (9) and that the stability correction terms with $h - d$ almost cancel out each other, leaves us the final expression for kB^{-1} :

$$kB^{-1} = k_1 Re_o^n - \ln \left(\frac{1}{k_2} \right) \quad (10)$$

In this paper the measured profiles of wind speed and air temperature were fitted with (3) and (4) using the technique described by Zilitinkevich (1970)

and Mölder (1997). This technique includes the least squares method and an iteration procedure. When the parameters u_* , T_* and L were obtained, the analysis proceeded to calculate T_h from (6) and with its transformation to $T_{h,n}$ using (8). The difference $T_{h,n} - T_s$ was plotted against $(T_*/k)Re_o^n$. The values of n were varied from 0 to 1 in 0.1 steps. The ‘right’ value was obtained when the linear regression, forced through the origin, gave a minimum standard deviation.

3. Results and discussion

The barley data were re-evaluated using the methodology outlined above. The Tõravere-90 data cover a narrow range of T_* values and have too large scatter, therefore, they did not distinguish any particular value for n . For Tamme-89 data, which is a good data set, the best correlation between $T_{h,n} - T_s$ and $(T_*/k)Re_o^n$ was obtained with $n=0.3$ (Fig. 1).

Although $n = 0.5$ gave the minimum standard deviation of the regression line for the pasture data in Mölder et al. (1999), the standard deviation was not much higher for $n = 0.3$ or 0.4. Therefore, it is not excluded that $n = 0.3$ might also be a proper value for the pasture. Fig. 1 presents also the newly re-evaluated pasture data. It is clear that a common line can be applied for both data sets. The Tõravere-90 data do also

support this slope. It is quite striking that two different surfaces, although having some common features (dominant vertical orientation), can be described with a universal Stanton number:

$$kSt_o^{-1} = 0.37Re_o^{0.3} \quad (11)$$

Since different expressions, that relate z_{ou} to $h - d$, were found for barley and pasture (Table 1), the kB^{-1} formulas contain slightly different constant terms. The re-evaluated equations for kB^{-1} take the form:

$$kB^{-1} = 0.37Re_o^{0.3} - (1.2 \text{ or } 1.9) \\ \text{(barley and pasture, respectively)} \quad (12)$$

This finding is close to Brutsaert (1979, 1982) and Thom (1972) predictions. In field situations, it seems difficult to confidently determine the power of Re_o with accuracy higher than around ± 0.15 .

Although the presented analysis is a step from the simplest case of a constant kB^{-1} to a slightly more complicated case with kB^{-1} varying with u_* and z_{ou} , it is still based on many simplifications. Firstly, the surface radiation temperature may possess a directional dependence which is ignored in this study. Since the studied crops were practically closed ones and the radiometer used has a large opening angle, this phenomenon was not very crucial. Checks with an hand-held radiometer showed that radiative surface temperatures measured at different inclination angles above the barley at the Tamme site were within 0.3 K. Otterman et al. (1995) suggest to tilt a narrow-opening-angle radiometer to 50° from nadir to measure a representative surface temperature. Our radiometer ‘averaged’ radiation at angles from 0 to 45° and its effective temperature corresponded to the surface temperature ‘seen’ at 30° from nadir. Theoretical estimates for the pasture showed that this temperature should not deviate from the 50° temperature more than 0.5 K. However, temperatures measured with the large-opening-angle radiometer differed by 0.5–1 K from another, narrow-opening-angle radiometer being installed under a 45° nadir angle above the pasture. This difference is not significant considering that the sum of possible measurement errors is of the same order of magnitude. Nevertheless, the possibility that our results on kB^{-1} may be different for another set up of surface temperature measurements should be considered.

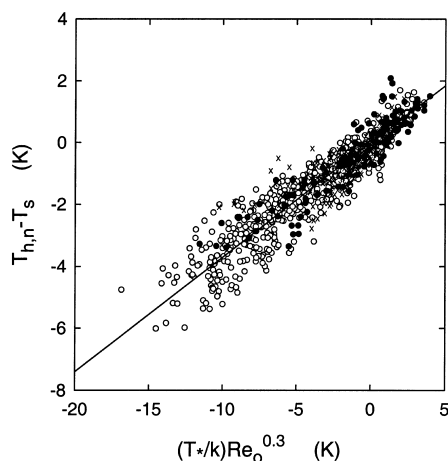


Fig. 1. The temperature difference $T_{h,n} - T_s$ vs. the scale $(T_*/k)Re_o^{0.3}$ (○) pasture, Lövsta-93; (●) barley, Tamme-89; (x) barley, Tõravere-90). The line is forced through the origin and has the slope of 0.37.

Secondly, the aerodynamic parameters d and z_{ou} may depend on more parameters than only on the canopy height h . The roughness density (or frontal area) is the relevant parameter most discussed in the literature (Verhoef et al., 1997). As our measurements were conducted under 1–2 months in the middle of on-going vegetation periods, it seemed reasonable to assume that the roughness density did not change considerably. Another complication might be that d and z_{ou} may depend on wind speed in the case of a flexible canopy (Monteith and Unsworth, 1990). On the other hand, observations by Baldocchi et al. (1983) and Jacobs and Van Boxel (1988) show no such dependence for soybean and corn crops, respectively. As our data on individual d and z_{ou} values are very scattered we cannot detect any dependence on wind speed reliably.

In this paper, the focus has been on finding a better parameterisation for the temperature profile. Results are good because: (i) we do not average individual $\ln(z_{ou}/z_{ot})$ values but study general trends in surface-air temperature differences versus a relevant scale (here $T_*Re_o^{0.3}$); (ii) we extrapolate temperature profiles down to the canopy height avoiding a much longer extrapolation to the z_{ou} level (in Mölder (1997) the difference $T(z_{ou}) - T_s$ was studied); (iii) inclusion of the $Re_o^{0.3}$ term reduces further the scatter in the data.

Next the question how much the new formulation of kB^{-1} can improve sensible heat flux calculations compared with the use of a standard $kB^{-1} = 2$ is investigated. Sensible heat fluxes (H_{model}) calculated from the surface radiation temperature and one-level air temperature and wind speed data (at 3.23 and 3.605 m, respectively) using two formulations for kB^{-1} are compared with the reference fluxes ($H_{profile}$) that are calculated using all available profile data (three temperature and five wind speed levels; no surface temperature involved). There is no dramatic change, but an improvement using a variable kB^{-1} is obvious (Fig. 2). The improvement is most notable for low friction velocities (shown with filled symbols), but the scatter is decreased for all other friction velocities, too. For $H_{profile} > 50 \text{ W m}^{-2}$, the correlation coefficient increased from 0.94 to 0.96 and the mean root square error decreased from 34 to 27 W m^{-2} . The slopes are practically equal to 1 in both cases, implying that $kB^{-1} = 2$ is also a good choice, not only for pasture as demonstrated here but for barley as well. Note that sensible heat fluxes are

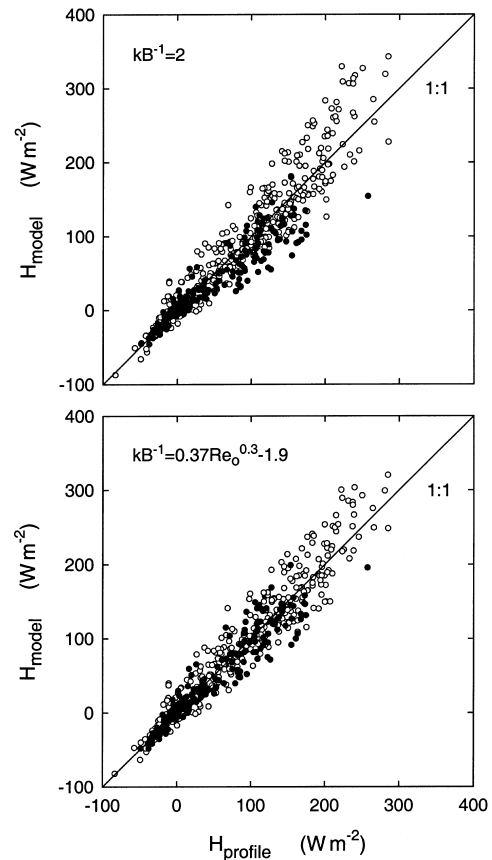


Fig. 2. Comparison of sensible heat fluxes calculated from surface radiation temperature and one-level air temperature and wind speed measurements using different formulations for kB^{-1} vs. fluxes calculated from all available profile data. Data over pasture at Lövsta site. Filled symbols distinguish the series with $0.1 < u_* < 0.3 \text{ (m s}^{-1}\text{)}$.

not merely a product of temperature gradients (T_*) but involve in an equal degree wind speed gradients (u_*) that increase the scatter in the data and mask the temperature-profile related processes.

4. Conclusions

Although the kB^{-1} factor has often been taken as a constant in the past, there is theoretical evidence that it depends on friction velocity or roughness Reynolds number even in the case of homogeneous

vegetated surfaces. Such a dependence is discovered easiest when the kB^{-1} factor is split into the roughness-sublayer Stanton number and the corresponding drag coefficient, and when that Stanton number is estimated by plotting the temperature difference $T_{h,n} - T_s$ against $(T_*/k)Re_o^n$. Re-evaluation of a barley data set gave $n = 0.3$. $n = 0.5$ has been previously published for tall grass (pasture) data, but the result was not much different for $n = 0.3$ or 0.4 . Applying $n = 0.3$ also to the pasture data set, both the barley and the pasture data collapse onto one and the same line. New equations for kB^{-1} take the form: $kB^{-1} = 0.37Re_o^{0.3} - (1.2 \text{ or } 1.9)$ (for barley and pasture, respectively). It has to be mentioned again that in field situations it is difficult to confidently determine the power of Re_o with an accuracy higher than around ± 0.15 . All the data sets give good results even with a constant $kB^{-1} = 2$, but the use of the new formulation reduces the scatter in the calculated sensible heat fluxes.

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