A note on group decision-making based on concepts of ideal and anti-ideal points in a fuzzy environment

Ying-Ming Wang\textsuperscript{a,\*}, Ying Luo\textsuperscript{b}, Zhong-Sheng Hua\textsuperscript{c}

\textsuperscript{a} School of Public Administration, Fuzhou University, Fuzhou 350002, PR China
\textsuperscript{b} School of Management, Xiamen University, Xiamen 361005, PR China
\textsuperscript{c} School of Management, University of Science and Technology of China, Hefei 230026, PR China

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Abstract

In a very recent paper by Kuo et al. [M.S. Kuo, G.H. Tzeng, W.C. Huang, Group decision-making based on concepts of ideal and anti-ideal points in a fuzzy environment, Mathematical and Computer Modelling 45 (3–4) (2007) 324–339], a fuzzy multicriteria decision analysis method based on the concepts of ideal and anti-ideal points was presented. This note illustrates with a numerical example that the method presented by Kuo et al. is not correct and can evaluate more than one decision alternative as the best even if they are not Pareto optimal at all. The fundamental reason for this is because the closeness coefficient values adopted by the authors do not reflect the superiority or inferiority of decision alternatives and cannot be used for ranking purpose. Corrections to their method are therefore suggested.

Keywords: Fuzzy group decision-making; Fuzzy TOPSIS method; Alpha level set; Fuzzy weighted average

1. Introduction

Multicriteria decision-making (MCDM) often involves decision-makers (DMs)’ subjective judgments and preferences such as qualitative criteria ratings and the weights of criteria. These ratings and weights are usually difficult to be judged very precisely because of the existence of uncertainty, but can be easily characterized by linguistic terms such as good, poor, average or very important, important and so on, which are fuzzy in nature. To make a decision analysis under fuzzy environment, classic TOPSIS method proposed by Hwang and Yoon [9] has been extensively extended by many researchers to deal with fuzzy multicriteria decision-making problems.

For example, Tsaur et al. [17] transformed a fuzzy MCDM problem into a crisp one through centroid defuzzification and solved the nonfuzzy MCDM problem using the TOPSIS method (see also Benítez et al. [1] for the application in hotel industry). Chen and Tzeng [4] transformed a fuzzy MCDM problem into a nonfuzzy MCDM by using fuzzy integral. Instead of the use of distance, they employed grey relation grade to define the relative closeness of each alternative. Chu [5,6] and Chu and Lin [7] also changed a fuzzy MCDM problem into a crisp one and solved the crisp
MCDM problem using the TOPSIS method. Differing from the others, they first derived the membership functions of all the weighted ratings in a weighted normalization decision matrix using interval arithmetic of fuzzy numbers and then defuzzified them into crisp values using the ranking method of mean of removals. Chen [2] extended the TOPSIS method to fuzzy group decision-making situations by defining a crisp Euclidean distance between any two fuzzy numbers (see also Chen et al. [3] for the supplier evaluation and selection in supply chain management, Yang and Hung [21] for the application in plant layout design problem, and Wang and Chang [18] for the application in evaluating initial training aircraft under a fuzzy environment). Similar idea also appeared in Jahanshahloo et al. [10], but they utilized a slightly different normalization method which normalizes a set of triangular fuzzy numbers by transforming them into intervals using alpha level sets and then normalizing them in terms of interval arithmetic. Triantaphyllou and Lin [16] developed a fuzzy version of the TOPSIS method based on fuzzy arithmetic operations, which leads to a fuzzy relative closeness for each alternative. Wang and Elhag [19] proposed a fuzzy TOPSIS method based on alpha level sets, which is formulated as a nonlinear programming (NLP) problem and can derive exact fuzzy relative closeness. Kahraman et al. [12] presented a hierarchical fuzzy TOPSIS method, in which the hierarchical structure was unfolded and represented by an extended decision matrix, the positive and negative ideal points were determined by using the generalized mean for fuzzy numbers [14], and the distance between any two fuzzy numbers was defined as one minus the maximum membership of the intersection of the two fuzzy numbers.

Very recently, Kuo et al. [13] presented a fuzzy multicriteria decision analysis method based on the concepts of ideal and anti-ideal points and Li [15]’s definition of fuzzy preference relation between two fuzzy numbers with parabolic membership functions. Their method was claimed to be a good means of evaluation and more appropriate than other evaluation methods. This paper provides a numerical example to illustrate the fact that their method is not correct and can evaluate more than one decision alternative to be the best even if they are not Pareto optimal at all. The reason for this is analyzed and the closeness coefficient values adopted by the authors are found flawed and cannot be used for ranking purpose. The purpose of this paper is not to propose any new method for fuzzy group decision-making, but to give a note pointing out the problems with Kuo et al.’s method to avoid any possible misapplications and to bring forward the corrections to their method.

The rest of the paper is organized as follows. Section 2 gives a brief review of Kuo et al.’s method and Section 3 illustrates with a numerical example the failure of their method in identifying the best decision alternative. The reason is analysed in Section 4 and the corrections to their method are also suggested in this section. The paper is concluded in Section 5.

2. Kuo et al.’s fuzzy group decision-making method

Suppose a fuzzy multicriteria group decision-making problem with $n$ possible decision alternatives, $m$ criteria and $L$ decision-makers (DMs) can be concisely expressed in matrix format as:

$$\tilde{D} = \begin{bmatrix}
\tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1m} \\
\tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{x}_{n1} & \tilde{x}_{n2} & \cdots & \tilde{x}_{nm}
\end{bmatrix},$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_m],$$

where $\tilde{x}_{ij}$ is the fuzzy rating of alternative $A_i$ with respect to criterion $C_j$ and $\tilde{w}_j$ is the fuzzy weight of criterion $C_j$. Both the rating and the weight are assumed to be triangular fuzzy numbers denoted by $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ and are obtained by aggregating DMs’ fuzzy opinions through the equations below:

$$\tilde{x}_{ij} = \frac{1}{L} \left[ \tilde{x}_{ij}^{(1)} + \tilde{x}_{ij}^{(2)} + \cdots + \tilde{x}_{ij}^{(L)} \right] = \frac{1}{L} \sum_{l=1}^{L} \tilde{x}_{ij}^{(l)},$$

$$\tilde{w}_j = \frac{1}{L} \left[ \tilde{w}_j^{(1)} + \tilde{w}_j^{(2)} + \cdots + \tilde{w}_j^{(L)} \right] = \frac{1}{L} \sum_{k=1}^{K} \tilde{w}_j^{(k)},$$

where $\tilde{x}_{ij}^{(l)}$ and $\tilde{w}_j^{(k)}$ are the fuzzy rating and fuzzy weight provided by the $l$th DM. Based on the above information, Kuo et al.’s fuzzy group decision-making method can be summarized as follows [13]:
Step 1. Normalize fuzzy decision matrix $\tilde{D}$ by the following equations:

$$\tilde{r}_{ij} = \left( \frac{a_{ij}}{c^*_j}, \frac{b_{ij}}{c^*_j}, \frac{c_{ij}}{c^*_j} \right), \quad j \in \Omega_B,$$

$$\tilde{r}_{ij} = \left( \frac{a^*_j}{c_j}, \frac{a^*_j}{b_j}, \frac{a^*_j}{a_j} \right), \quad j \in \Omega_C,$$

where $c^*_j = \max_i(c_{ij})$, $a^*_j = \min_i(a_{ij})$, and $\Omega_B$ and $\Omega_C$ are the benefit and cost criteria set, respectively.

Step 2. Define the positive ideal point $A^* = (\tilde{r}^*_1, \tilde{r}^*_2, \ldots, \tilde{r}^*_m)$ and the negative ideal point $A^- = (\tilde{r}^-_1, \tilde{r}^-_2, \ldots, \tilde{r}^-_m)$ by the equations below:

$$\tilde{r}^*_j = \max_i(\tilde{r}_{ij}) \quad \text{and} \quad \tilde{r}^-_j = \min_i(\tilde{r}_{ij}), \quad j = 1, \ldots, m.$$

Step 3. Calculate the Hamming distance matrices $\tilde{H}^* = [\tilde{d}^*_i]_{n \times m}$ and $\tilde{H}^- = [\tilde{d}^-_i]_{n \times m}$ by the following equations:

$$\tilde{d}^*_i = \tilde{r}^*_i - \tilde{r}_i \quad \text{and} \quad \tilde{d}^-_i = \tilde{r}_i - \tilde{r}^-_i, \quad i = 1, \ldots, n; \quad j = 1, \ldots, m,$$

where $\tilde{d}^*_i$ and $\tilde{d}^-_i$ are triangular fuzzy numbers determined by fuzzy arithmetic and denoted by $\tilde{d}^*_i = (l_i^*, m_i^*, r_i^*)$ and $\tilde{d}^-_i = (l_i^-, m_i^-, r_i^-)$.

Step 4. Compute the fuzzy weighted Hamming distance matrices $\tilde{P}^* = [\tilde{p}^*_i]_{n \times m}$ and $\tilde{P}^- = [\tilde{p}^-_i]_{n \times m}$ by

$$\tilde{p}^*_i = \delta^*_i (\cdot) \tilde{w}_i = \left( \delta^*_1, \delta^*_2, \delta^*_3 / \gamma^*_i / \Delta^*_1, \Delta^*_2, \Delta^*_3 \right), \quad i = 1, \ldots, n; \quad j = 1, \ldots, m; \quad k = *, -,$$

where $(\delta^*_1, \delta^*_2, \delta^*_3 / \gamma^*_i / \Delta^*_1, \Delta^*_2, \Delta^*_3)$ are fuzzy numbers whose membership functions are parabolic and are defined by:

$$\mu_{\tilde{p}^*_i}(x) = \begin{cases} 
\frac{(-x)^2 + \sqrt{(\delta^*_2)^2 - 4\delta^*_1 \delta^*_1 (\delta^*_3 - x)}}{2\delta^*_1}, & \delta^*_3 \leq x \leq \gamma^*_i \\
- \frac{\Delta^*_1 - \sqrt{(\Delta^*_2)^2 - 4\Delta^*_1 \Delta^*_3 (\Delta^*_1 - x)}}{2\Delta^*_1}, & \gamma^*_i \leq x \leq \Delta^*_3 \\
0, & \text{otherwise}
\end{cases}$$

where

$$\delta^*_1 = (m^*_i - l^*_i) (w_{j2} - w_{j1}), \quad \delta^*_2 = m^*_i (m^*_i - l^*_i) + l^*_i (w_{j2} - w_{j1}), \quad \delta^*_3 = w_{j1} l^*_i,$$

$$\Delta^*_1 = (r^*_i - m^*_i) (w_{j3} - w_{j2}), \quad \Delta^*_2 = w_{j3} (r^*_i - m^*_i) + r^*_i (w_{j3} - w_{j2}), \quad \Delta^*_3 = w_{j3} r^*_i,$$

$$r^*_i = w_{j2} m^*_i, \quad i = 1, \ldots, n; \quad j = 1, \ldots, m; \quad k = *, -.$$

Step 5. Define the fuzzy weighted distance evaluation values $\tilde{p}^*_i$ and $\tilde{p}^-_i$ as:

$$\tilde{p}^*_i = \sum_{j=1}^{m} \tilde{p}^*_ij = (\delta^*_1, \delta^*_2, \delta^*_3 / \gamma^*_i / \Delta^*_1, \Delta^*_2, \Delta^*_3), \quad i = 1, \ldots, n,$$

where

$$\delta^*_g = \sum_{j=1}^{m} \delta^*_gj, \quad \Delta^*_g = \sum_{j=1}^{m} \Delta^*_gij \quad \text{and} \quad \gamma^*_g = \sum_{j=1}^{n} \gamma^*_gj, \quad g = 1, 2, 3; \quad i = 1, \ldots, n; \quad k = *, -.$$
Table 1
Fuzzy weights and fuzzy decision matrix for a virtual fuzzy MCDM problem

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(9, 10, 10)</td>
<td>(9, 10, 10)</td>
</tr>
<tr>
<td>A2</td>
<td>(7, 9, 10)</td>
<td>(7, 9, 10)</td>
</tr>
<tr>
<td>A3</td>
<td>(0, 0, 1)</td>
<td>(0, 0, 1)</td>
</tr>
<tr>
<td>Criteria weight</td>
<td>(0.4, 0.5, 0.6)</td>
<td>(0.4, 0.5, 0.6)</td>
</tr>
</tbody>
</table>

Step 6. Calculate the closeness coefficient value of each alternative $A_i$ ($i = 1, \ldots, n$) by:

$$CC_i = \mu_R(\hat{p}_i - (\hat{p}_i^*), 0) = \frac{\beta^+}{\beta^+ + \beta^-},$$
if $\delta_{3i} - \Delta_{3i}^+ < 0$, $\Delta_{3i}^- - \delta_{3i}^+ \geq 0$, $y_{i}^+ \geq y_i^*$,

$$\frac{\beta^+}{\beta^+ + \beta^-},$$
if $\delta_{3i} - \Delta_{3i}^+ \leq 0$, $\Delta_{3i}^- - \delta_{3i}^+ > 0$, $y_{i}^- \leq y_i^*$,

$$\frac{\lambda^+}{\lambda^+ + \lambda^-},$$
if $\delta_{3i} - \Delta_{3i}^+ \leq 0$, $\Delta_{3i}^- - \delta_{3i}^+ = 0$, $y_{i}^- = y_i^*$,

$$0.5,$$
if $\delta_{3i} - \Delta_{3i}^+ > 0$, $\Delta_{3i}^- - \delta_{3i}^+ = 0$, $y_{i}^- = y_i^*$,

$$0,$$
if $\delta_{3i} - \Delta_{3i}^+ > 0$, $\Delta_{3i}^- - \delta_{3i}^+ > 0$, $y_{i}^- \leq y_i^*$,

$$\frac{\beta^-}{\beta^+ + \beta^-},$$
if $\delta_{3i} - \Delta_{3i}^+ < 0$, $\Delta_{3i}^- - \delta_{3i}^+ < 0$, $y_{i}^- \leq y_i^*$,

where

$$\beta^+ = \left[\frac{1}{4} (\Delta_{1i}^- - \delta_{1i}^+ - \frac{1}{3} (\Delta_{2i}^- + \delta_{2i}^+ + \frac{1}{2} (\Delta_{3i}^- - \delta_{3i}^+))\right]$$
$$+ \left[\frac{1}{3} (\delta_{1i}^- - \Delta_{1i}^+)(1 - \mu_1^3) + \frac{1}{3} (\Delta_{2i}^- + \delta_{2i}^+)(1 - \mu_1^3) + \frac{1}{2} (\delta_{3i}^- - \Delta_{3i}^+)(1 - \mu_1^3)\right],$$

$$\mu_1 = \frac{-(\delta_{2i}^- + \Delta_{2i}^+ + \sqrt{(\delta_{2i}^- + \Delta_{2i}^+)^2 - 4(\delta_{1i}^- - \Delta_{1i}^+)(\delta_{3i}^- - \Delta_{3i}^+)}}{2(\delta_{1i}^- - \Delta_{1i}^+)},$$

$$\lambda^+ = \frac{1}{4} (\Delta_{1i}^- - \delta_{1i}^+ + \frac{1}{3} (\Delta_{2i}^- - \delta_{2i}^+ - \frac{1}{2} (\Delta_{3i}^- - \delta_{3i}^+))\mu_2^3,$$

$$\mu_2 = \frac{-(\Delta_{2i}^- + \Delta_{2i}^+ - \sqrt{(-\Delta_{2i}^- - \delta_{2i}^+)^2 - 4(\Delta_{1i}^- - \delta_{1i}^+)(\Delta_{3i}^- - \delta_{3i}^+)}}{2(\Delta_{1i}^- - \delta_{1i}^+)}.$$  

Step 7. Rank alternatives $A_i$, $i = 1, \ldots, n$, by their closeness coefficient values and select the alternative with the biggest closeness coefficient value as the best.

3. A numerical example

In this section, we offer a numerical example to show the fact that the above method can evaluate an inferior decision alternative as the best and is therefore not correct.

Consider a virtual fuzzy multicriteria decision-making problem with three alternatives $A_1$, $A_2$ and $A_3$ and two benefit criteria $C_1$ and $C_2$. The information on this fuzzy MCDM problem is presented in Table 1, where triangular fuzzy number $(9, 10, 10)$ represents linguistic assessment very good, $(7, 9, 10)$ Good, $(0, 0, 1)$ very poor. $A_1$ is both evaluated as very good on the two decision criteria, $A_2$ as good on both the criteria and $A_3$ as very poor on the both criteria. This is a straightforward decision-making problem and $A_1$ is obviously the best decision alternative. The ranking order of the three alternatives is $A_1 > A_2 > A_3$ without doubt.
We now turn to examining the problem using Kuo et al.’s fuzzy MCDM method. To do so, we first normalize the fuzzy decision matrix in Table 1. The normalized fuzzy decision matrix is shown in Table 2, from which the positive and negative ideal points can easily be determined as:

\[ A^* = [(0.9, 1, 1), (0.9, 1, 1)] \quad \text{and} \quad A^- = [(0, 0, 0.1), (0, 0, 0.1)]. \]

The corresponding Hamming distance matrices are shown in Tables 3 and 4, based on which the fuzzy weighted Hamming distance matrices are computed. The results are shown in Tables 5 and 6, from which the fuzzy weighted distance evaluation values of each decision alternative are obtained and shown in Table 7. Finally, the closeness coefficient values of the three decision alternatives are computed as:

\[ CC_1 = \mu_R(\tilde{p}_1^- (-) \tilde{p}_1^*, 0) = 1, \quad CC_2 = \mu_R(\tilde{p}_2^- (-) \tilde{p}_2^*, 0) = 1 \quad \text{and} \quad CC_3 = \mu_R(\tilde{p}_3^- (-) \tilde{p}_3^*, 0) = 0, \]

which lead to the conclusion that \( A_2 \) is as good as \( A_1 \) and both \( A_1 \) and \( A_2 \) can be selected as the best decision alternative. This conclusion is obviously incorrect and absurd. In the next section, we will analyze the reason why Kuo et al.’s method is incorrect and give suggestions for correcting their method.

### 4. Analysis to Kuo et al.’s method and suggestions

It is easy to find from the definition of closeness coefficient value that \( CC_i \) used by Kuo et al. was initially defined by Li [15] to characterize the fuzzy preference relation between two fuzzy numbers with parabolic membership functions and represents the degree to which one fuzzy number is bigger than the other one. More specifically, if \( \tilde{p}_i^- \) is greater than \( \tilde{p}_i^* \) and has no overlap with it, then \( CC_i = \mu_R(\tilde{p}_i^- (-) \tilde{p}_i^*, 0) = 1 \), which means \( \tilde{p}_i^- \) is greater than

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**Table 2**

Normalized fuzzy decision matrix for the virtual fuzzy MCDM problem

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(0.9, 1, 1)</td>
<td>(0.9, 1, 1)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(0.7, 0.9, 1)</td>
<td>(0.7, 0.9, 1)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(0, 0, 0.1)</td>
<td>(0, 0, 0.1)</td>
</tr>
</tbody>
</table>

**Table 3**

Hamming distance matrix \( \tilde{H}^* \) of the positive ideal point \( A^* \) to three decision alternatives

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(−0.1, 0, 0.1)</td>
<td>(−0.1, 0, 0.1)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(−0.1, 0.1, 0.3)</td>
<td>(−0.1, 0.1, 0.3)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(0.8, 1, 1)</td>
<td>(0.8, 1, 1)</td>
</tr>
</tbody>
</table>

**Table 4**

Hamming distance matrix \( \tilde{H}^- \) of the three decision alternatives to the negative ideal point \( A^- \)

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(0.8, 1, 1)</td>
<td>(0.8, 1, 1)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(0.6, 0.9, 1)</td>
<td>(0.6, 0.9, 1)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(−0.1, 0, 0.1)</td>
<td>(−0.1, 0, 0.1)</td>
</tr>
</tbody>
</table>

**Table 5**

Fuzzy weighted Hamming distance matrix \( \tilde{P}^* \)

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(0.01, 0.03, −0.04/0, 0.01, 0.07, 0.06)</td>
<td>(0.01, 0.03, −0.04/0, 0.01, 0.07, 0.06)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(0.02, 0.07, −0.04/0.05/0.02, 0.15, 0.18)</td>
<td>(0.02, 0.07, −0.04/0.05/0.02, 0.15, 0.18)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(0.02, 0.07, 0.32/0.5/0, 0.1, 0.6)</td>
<td>(0.02, 0.07, 0.32/0.5/0, 0.1, 0.6)</td>
</tr>
</tbody>
</table>
Table 6
Fuzzy weighted Hamming distance matrix $\tilde{P}^-$

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$0.02, 0.16, 0.32/0.5/0.1, 0.6$</td>
<td>$0.02, 0.16, 0.32/0.5/0.1, 0.6$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$0.03, 0.18, 0.24/0.45/0.01, 0.16, 0.6$</td>
<td>$0.03, 0.18, 0.24/0.45/0.01, 0.16, 0.6$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$0.01, 0.18, -0.04/0.01, 0.07, 0.06$</td>
<td>$0.01, 0.18, -0.04/0.01, 0.07, 0.06$</td>
</tr>
</tbody>
</table>

Table 7
Fuzzy weighted distance evaluation values of each decision alternative

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$\tilde{p}_i^-$</th>
<th>$\tilde{p}_i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$(0.04, 0.32, 0.64/1/0, 0.2, 1.2)$</td>
<td>$(0.02, 0.06, -0.08/0/0.02, 0.14, 0.12)$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$(0.06, 0.36, 0.48/0.9/0.02, 0.32, 1.2)$</td>
<td>$(0.04, 0.14, -0.08/0.1/0.04, 0.3, 0.36)$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$(0.02, 0.36, -0.08/0/0.02, 0.14, 0.12)$</td>
<td>$(0.04, 0.14, 0.64/1/0, 0.2, 1.2)$</td>
</tr>
</tbody>
</table>

Fig. 1. Fuzzy evaluation values of two decision alternatives.

$\tilde{p}_i^*$ to the degree of 100%; if $\tilde{p}_i^-$ and $\tilde{p}_i^*$ are exactly the same, then $CC_i = \mu_R(\tilde{p}_i^-(\tilde{p}_i^+, 0) = 0.5$, which means $\tilde{p}_i^-$ is indifferent to $\tilde{p}_i^*$; if $\tilde{p}_i^-$ is less than $\tilde{p}_i^*$ and has no overlap with it, then $CC_i = \mu_R(\tilde{p}_i^-(\tilde{p}_i^+, 0) = 0$, which means $\tilde{p}_i^-$ is greater than $\tilde{p}_i^*$ to the degree of zero; and in all other cases, the degree of $\tilde{p}_i^-$ over $\tilde{p}_i^*$ is determined by $\beta^+/\beta^-$ or $\lambda^+/\lambda^-$, depending upon whether $\gamma_i^-$ is greater than $\gamma_i^*$ or not. So, the closeness coefficient value $CC_i$ reflects only the degree to which $\tilde{p}_i^-$ is greater than $\tilde{p}_i^*$ and does not at all reflect whether the alternative $A_i$ is superior to the other alternatives or not. The use of the closeness coefficient values to rank decision alternatives is lack of theoretical evidence and incorrect. This is the reason why Kuo et al.’s fuzzy group decision-making method evaluates decision alternative $A_2$ to be as good as $A_1$ for the previous virtual example because for these two alternatives their fuzzy evaluation values $\tilde{p}_i^-$ are both greater than and have no overlaps with $\tilde{p}_i^*$, as shown in Fig. 1. Kuo et al.’s claim that ‘if the closeness coefficient value of an evaluation alternative is getting on for 1, then this alternative is the only alternative which has the shortest distance to the ideal point and the farthest distance to the negative ideal point’ (p. 332) is obviously not correct.

It is clear from the above analysis that what the closeness coefficient values compare are the magnitudes of the fuzzy weighted distance evaluation values of each alternative from the positive ideal point and the negative ideal point. Correct decision-making methods, in our opinion, should compare the overall assessment values of decision alternatives. The overall assessment value of each alternative may be defined as

$$\tilde{C}_i = \tilde{p}_i^--\tilde{p}_i^* \quad \text{or} \quad \tilde{C}_i = \tilde{p}_i^-/(\tilde{p}_i^* + \tilde{p}_i^-), \quad i = 1, \ldots, n.$$  

This is our suggested correction to Kuo et al.’s fuzzy group decision-making method.

When $\tilde{C}_i = \tilde{p}_i^- - \tilde{p}_i^*$ is defined as the overall assessment value of each alternative, it can be reduced to the following:

$$\tilde{C}_i = \tilde{p}_i^- - \tilde{p}_i^* = \sum_{j=1}^{m} \tilde{p}_{ij}^- - \sum_{j=1}^{m} \tilde{p}_{ij}^* = \sum_{j=1}^{m} \tilde{d}_{ij}^-()\tilde{w}_j - \sum_{j=1}^{m} \tilde{d}_{ij}^+()\tilde{w}_j = \sum_{j=1}^{m} (\tilde{d}_{ij}^- - \tilde{d}_{ij}^+)()\tilde{w}_j \quad = \sum_{j=1}^{m} (\tilde{r}_{ij}^- - \tilde{r}_{ij}^+ + \tilde{r}_{ij})()\tilde{w}_j = 2 \sum_{j=1}^{m} \tilde{r}_{ij}()\tilde{w}_j - \sum_{j=1}^{m} (\tilde{r}_{ij}^- + \tilde{r}_{ij}^+)()\tilde{w}_j.$$
In this case, the impact of the positive and negative ideal points $A^* = (\tilde{r}_1^*, \ldots, \tilde{r}_m^*)$ and $A^- = (\tilde{r}_1^-, \ldots, \tilde{r}_m^-)$ on the overall assessment value of each alternative is the same, which makes them have no or little impact on the final decision result. Accordingly, the comparison of decision alternatives comes down to the comparison of their fuzzy weighted assessment values $\sum_{j=1}^{m} \tilde{r}_{ij}^*(\cdot)\tilde{w}_j$, which is the fuzzification expression of the well-known simple additive weighting method [9] widely used in MCDM. An illustrate example can be found in Yang et al. [20].

When $\tilde{C}_i = \frac{\tilde{p}_i^-}{(\tilde{p}_i^- + \tilde{p}_i^+)}$ is defined as the overall assessment value (usually called the relative closeness in TOPSIS method) of each alternative, it can be further expressed as:

$$\tilde{C}_i = \frac{\sum_{j=1}^{m} \tilde{p}_{ij}^- / (\tilde{p}_i^- + \tilde{p}_i^+) = \sum_{j=1}^{m} \tilde{p}_{ij}^- + \sum_{j=1}^{m} \tilde{p}_{ij}^+}{\sum_{j=1}^{m} \tilde{d}_{ij}^*(\cdot)\tilde{w}_j / \sum_{j=1}^{m} (\tilde{d}_{ij}^- + \tilde{d}_{ij}^*)(\cdot)\tilde{w}_j}$$

which can be solved by using $\alpha$-level sets and Zadeh’s extension principle [22]. In particular, if the positive ideal point $A^*$ and the negative ideal point $A^-$ are respectively defined as $A^* = (1, \ldots, 1)$ and $A^- = (0, \ldots, 0)$, then the above equation can be further simplified as:

$$\tilde{C}_i = \frac{\sum_{j=1}^{m} \tilde{r}_{ij}^- (\cdot)\tilde{w}_j}{\sum_{j=1}^{m} \tilde{w}_j},$$

which is the fuzzy weighted average [8,11] investigated extensively in the literature.

For the previous example, it can be further computed from Table 7 that:

$$\tilde{C}_1 = \frac{\tilde{p}_1^-}{(\tilde{p}_1^- + \tilde{p}_1^+)} = (0.02, 0.46, 0.52/1/-0.02, -0.26, 1.28),$$

$$\tilde{C}_2 = \frac{\tilde{p}_2^-}{(\tilde{p}_2^- + \tilde{p}_2^+)} = (0.02, 0.66, 0.12/0.8/-0.02, -0.46, 1.28),$$

$$\tilde{C}_3 = \frac{\tilde{p}_3^-}{(\tilde{p}_3^- + \tilde{p}_3^+)} = (0.02, 0.56, -1.28/-1/-0.02, -0.28, -0.52).$$

Their pictures are presented in Fig. 2, from which it can be seen clearly that $\tilde{C}_1 > \tilde{C}_2 > \tilde{C}_3$, which represents $A_1 > A_2 > A_3$. This can be further verified by computing their fuzzy preference relations using the formula defined by Li [15] and adopted by Kuo et al. who utilized it to calculate the closeness coefficient values of decision alternatives:

$$\mu_R(\tilde{C}_1(\cdot)\tilde{C}_2, 0) = 0.5046 \quad \text{and} \quad \mu_R(\tilde{C}_2(\cdot)\tilde{C}_3, 0) = 1.$$

Although the above fuzzy preference relations give the ranking $\tilde{C}_1 > \tilde{C}_2$, the preference degree 0.5046 is questionable and not very convincing because it means $A_1$ is only very slightly better than $A_2$. Referring to Li’s paper [15], we could not find any derivation for the formula and therefore have no way to examine its correctness. But we do find that the formula is sometimes invalid. Consider the following two fuzzy numbers with parabolic membership functions:

$$\tilde{B}_1 = (\delta_{11}, \delta_{21}, \delta_{31}/\gamma_1/\Delta_{11}, \Delta_{21}, \Delta_{31})$$

$$= (-0.012, 0.615, -0.853/-0.251/0.029, -0.659, 0.38),$$
Due to the fact that:
\[
\delta_{31} - \Delta_{32} = -0.853 - 0.83 = -1.683 < 0, \\
\Delta_{31} - \Delta_{32} = 0.38 - (-0.351) = 0.731 > 0, \\
\gamma_1 - \gamma_2 = -0.251 - 0.229 = -0.46 < 0,
\]
\[
\mu_R(\tilde{B}_1(-)\tilde{B}_2, 0)
\]
will be determined by \(\lambda^+/(\lambda^+ + \lambda^-)\). To calculate \(\lambda^+\) and \(\lambda^-\), parameter \(\mu_2\) has to be first computed by:
\[
\mu_2 = \frac{\left(\Delta_{21} + \Delta_{22}\right) - \sqrt{(-\Delta_{21} - \Delta_{22})^2 - 4(\Delta_{11} - \Delta_{12})(\Delta_{31} - \Delta_{32})}}{2(\Delta_{11} - \Delta_{12})}.
\]
It is easy to verify that
\[
(-\Delta_{31} - \Delta_{22})^2 - 4(\Delta_{11} - \Delta_{12})(\Delta_{31} - \Delta_{32}) = -0.11845 < 0,
\]
which will be a complex number after performing a square root operation. Evidently, \(\mu_R(\tilde{B}_1(-)\tilde{B}_2, 0)\) cannot be determined in this situation. As such, when considering the following two fuzzy numbers with parabolic membership functions:
\[
\tilde{B}_3 = (\delta_{13}, \delta_{23}, \delta_{33}/\gamma_3/\Delta_{13}, \Delta_{23}, \Delta_{33}) \\
= (-0.013, 0.863, -0.271/0.579/0.025, -0.895, 1.45), \\
\tilde{B}_4 = (\delta_{14}, \delta_{24}, \delta_{34}/\gamma_4/\Delta_{14}, \Delta_{24}, \Delta_{34}) \\
= (-0.013, 0.863, -0.367/0.483/0.023, -0.77, 1.23),
\]
\[
\mu_R(\tilde{B}_1(-)\tilde{B}_2, 0)
\]
cannot be determined by \(\beta^+/(\beta^+ + \beta^-)\) because the parameter
\[
\mu_1 = \frac{(-\delta_{23} + \Delta_{24}) + \sqrt{(-\delta_{23} + \Delta_{24})^2 - 4(\delta_{13} - \Delta_{14})(\delta_{33} - \Delta_{34})}}{2(\delta_{13} - \Delta_{14})}
\]
is also a complex number due to the fact that
\[
(\delta_{23} + \Delta_{24})^2 - 4(\delta_{13} - \Delta_{14})(\delta_{33} - \Delta_{34}) = -0.2075 < 0.
\]
Therefore, \(\mu_1\)’s formula should be used very cautiously.

Finally, we point out here that normalization is absolutely unnecessary if all the criteria are assessed using linguistic terms because there is no dimensional unit in this situation [21]. Inappropriate normalization may cause rank reversal phenomenon.

5. Conclusion

In this paper we examined the fuzzy multicriteria group decision-making method developed by Kuo et al. with a virtual numerical example and showed that their method is lack of theoretical evidence and may evaluate an inferior decision alternative as the best. The closeness coefficient values adopted by them were found flawed. They do not reflect the superiority or inferiority of decision alternatives and cannot be used for ranking purpose. The fuzzy preference relation defined by Li [15] was also found invalid in some situations and should be used very cautiously. We point out these problems is to avoid any possible misapplications in the future.

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