A SUBSPACE CLUSTERING MODEL FOR IMAGE TEXTURE SEGMENTATION

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Abstract. We propose a novel image segmentation model, called the Semi-Supervised Subspace Mumford-Shah model, which incorporates subspace clustering techniques into a Mumford-Shah model to solve texture segmentation problems. While the natural unsupervised approach to learn a feature subspace can easily be trapped in a local solution, we propose a novel semi-supervised optimization algorithm that makes use of information derived from both the intermediate segmentation results and the regions-of-interest (ROI) selected by the user to determine the optimal subspaces of the target regions. These subspaces are embedded into a Mumford-Shah objective function so that each segment of the optimal partition is homogeneous in its own subspace. The method outperforms standard Mumford-Shah models since it can separate textures which are less separated in the full feature space. The method also has an increased robustness and convergence speed compared to existing subspace clustering methods. Experimental results are presented to confirm the usefulness of subspace clustering in texture segmentation.
1 Introduction

Texture may be defined as the variation of data at scales smaller than the scales of interest [12]. In segmentation, texture can be a useful cue for object recognition, but it can also be a nuisance since it creates many extra lines in the edge map. Most segmentation algorithms deal with texture by first transforming the intensity levels to high-dimensional feature vectors, followed by looking for regions which are homogeneous in the feature space. For example, texture features extracted by Gabor filters [4] or Laws’ filters [9] have been widely used [6, 10]. Numerous approaches for texture segmentation exist: clustering [6], region-growing [2], graph-partitioning [1] and optimization model-based [13]. Recent survey in this topic can be found in [5].

To search for homogeneous regions in the feature space, a reliable distance is crucial. Different textures are better captured by different features. Hence, a high-dimensional feature space is usually considered for texture segmentation. However, for each given pixel the number of relevant features is usually small, making the data very sparse. It is well-known in high-dimensional data analysis that distances computed in high-dimension are less reliable since contributions from irrelevant dimensions become significant. Moreover, in practice users may want to include all kinds of features and let the algorithm decide which set of features are relevant. A successful approach to alleviate this problem is to use subspace clustering techniques so that objects are projected onto their own relevant subspace only. Indeed, subspace clustering is a well-studied problem in data mining community [11]. However, none of the above mentioned texture segmentation approaches had considered the curse of dimensionality.

Among the various segmentation approaches, optimization model-based methods often give very promising results. For instance, in [13], Sandberg et al. considered the application of the vector-valued Mumford-Shah (MS) model to texture segmentation using Gabor features. The result showed that Gabor features are able to capture the characteristics of various kinds of textures. After some features have been selected (either manually or automatically by some criteria), all the selected features are used to describe all pixels. However, when different regions lay on different subspaces, this approach might fail to retrieve a good result. In this case, subspace clustering technique is indispensable.

Objective. In this paper, we propose the use of subspace clustering techniques for texture segmentation. Our first contribution is a novel model, called the Unsupervised Subspace Mumford-Shah model, which combines subspace clustering [7] and Mumford-Shah segmentation [3, 13] techniques. However the natural unsupervised learning of a feature subspace can easily be trapped in a local solution. Our second contribution is a novel semi-supervised algorithm which is much more robust for optimizing the proposed model. Experimental results are presented to confirm the usefulness of subspace clustering in texture segmentation.
2 Semi-Supervised Subspace Mumford-Shah Segmentation Model

In this section, we present the proposed semi-supervised model. To better explain the ideas behind, we first introduce an unsupervised version.

Let $\mathcal{C}$ be a set of curves partitioning the image domain $\Omega$ into $n$ mutually exclusive segments $\Omega_i$ for $i = 1, 2, \ldots, n$. Let $\mathbf{c} = \{c_{ij}\}$ and $\Lambda = \{\lambda_{ij}\}$ for $1 \leq i \leq n$ and $1 \leq j \leq m$ be two sets of scalars such that $\sum_{j=1}^{m} \lambda_{ij} = 1$ for each $i$ and $\lambda_{ij} \geq 0$. The Mumford-Shah image segmentation model in [13] and the $k$-means subspace clustering model in [7] can be naturally combined as follows:

$$F_{\text{USSMS}}(\mathcal{C}, \mathbf{c}, \Lambda) = \sum_{i=1}^{n} \sum_{j=1}^{m} \lambda_{ij} \int_{\Omega_i} |f_j(x, y) - c_{ij}|^2 dxdy + \gamma \sum_{i=1}^{n} \sum_{j=1}^{m} \lambda_{ij} \log \lambda_{ij} + \mu \cdot \text{Length}(\mathcal{C}).$$

We refer this model as the Unsupervised Subspace Mumford-Shah model (USSMS). This model inherits the essential features of the above two models—it enables cluster-wise feature selection and it regularizes the geometry of the segments in the spatial domain to avoid over-fitting. In this model, we seek for a segmentation of the image such that the $j$th feature value within the segment $\Omega_i$ is well-approximated by a constant $c_{ij}$, provided that the weight $\lambda_{ij}$ is “significant”. If $\lambda_{ij}$ is close to zero, then the determination of the $i$th segment is insensitive to the $j$th feature. Thus, a key feature of the model is that each segment can use a different subspace in computing the within-segment dispersion. Therefore, it opens up the possibility of using an optimal subset of features for each segment.

Unfortunately, this objective function is highly non-convex which is difficult to optimize. We found empirically that many spurious local minima can exist. We tried several standard optimization methods as well as with different initial conditions but their performance has been disappointing. The results often depend very much on the initial guess. Thus this model can be difficult to use in practice. To resolve this problem, we propose the following Semi-Supervised Subspace Mumford-Shah model (SSSMS) which amounts to the minimization of the following objective function:

$$F_{\text{SSSMS}}(\mathcal{C}, \mathbf{c}, \Lambda) = (1 - \beta) F_{\text{USSMS}}(\mathcal{C}, \mathbf{c}, \Lambda) + \beta \left[ \frac{|\Omega|}{\sum_k |\text{ROI}_k|} \sum_{i=1}^{n} \sum_{j=1}^{m} \lambda_{ij} F_{ij} + \gamma \sum_{i=1}^{n} \sum_{j=1}^{m} \lambda_{ij} \log \lambda_{ij} \right].$$

Here ROI$_k$ is the $k$-th user-specified region-of-interest (ROI) which serves as a sample of the $k$-th segment and

$$F_{ij} = \int_{\text{ROI}_i} |f_j(x, y) - \tilde{c}_{ij}|^2 dxdy$$

is the fitting error of the $j$th feature over ROI$_i$. The optimal fitting constant is given by

$$\tilde{c}_{ij} = \frac{\int_{\text{ROI}_i} f_j(x, y) dxdy}{|\text{ROI}_i|}.$$
The parameter $\beta \in (0, 1)$ is to control the degree of involvement of the ROI’s. In our implementation, the parameter $\beta$ is gradually decreased from 1 towards 0. Thus the ROI’s eventually drop out from the objective $F^{\text{SSSMS}}$ which converges to its unsupervised version $F^{\text{USSMS}}$. This can make the final weights depend on the image globally thus preventing any bias introduced by the ROI’s. One may think of this approach as using the ROI’s to specify an initial guess for the optimization of the unsupervised model $F^{\text{USSMS}}$.

Except for the introduction of the constant $|\Omega|/\sum_k |\text{ROI}_k|$ to normalize the difference between the size of the ROI’s and $\Omega$, the proposed objective function is essentially a convex combination of $F^{\text{USSMS}}(C, c, \Lambda)$ and $F^{\text{USSMS}}(\tilde{C}, \tilde{c}, \Lambda)$ where $\tilde{c} = \{\tilde{c}_{ij}\}$ and $\tilde{C}$ is the set of curves defining the ROI’s and is therefore determined. Roughly speaking, the idea of the model is that the given ROI’s (i.e. $\tilde{C}$) allow us to estimate the weights $\Lambda$ more accurately. Once the weights are in place, the problem essentially reduces to the original Mumford-Shah problem. Of course, the Mumford-Shah model itself can possess multiple local minima, but the problem is much more amenable. See [8] for global optimization of the Mumford-Shah model.

3 Results

3.1 Muscle image

In this test, we use a transmission electron microscopy image of rat myocardium with size $300 \times 300$. Our aim is to separate the nucleus in the upper part of the image from the striated muscle. To achieve this, we use 3 Gabor features of different orientations [4] and 4 Brodatz texture similarity measures (see [10] for details). The user specifies one ROI for each kind of cells. In Fig. 1, the results show that the standard Mumford-Shah (MS) model cannot find a clear boundary while the SSSMS model can. In the figure, we also report their percentage symmetric difference with respect to some manual segmentation.

![Figure 1: Segmentation of a rat myocardium image: SSSMS and MS models with 3 Gabor features and 4 Brodatz texture similarity measures as input features. $\mu = 0.5$ (SSSMS) and 1.5 (MS), $\gamma = 0.5$.](image)
3.2 Cuttlefish camouflage

In this test, we use a cuttlefish image with size 260 × 260 containing two infant cuttlefishes where one is located in the center and (part of) the other is located in the upper left corner of the image. To segment the image, we use 6 Gabor features of different orientations [4], 14 Laws’ Mask features [9] and 64 Brodatz texture similarity measures (see [10] for details). To optimize the performance of both the MS and the SSSMS models, we first select the top five channels which have the highest mean intensity differences between two selected ROI’s. Fig. 2 shows the segmentations using the SSSMS and the MS models and their percentage symmetric difference with respect to some manual segmentation. As the background of the image is quite close (in terms of color and texture) to the cuttlefishes, the texture descriptors exhibit much noise which hinders the separation between the cuttlefishes and the background. The results show that the use of subspace clustering allows an automatic feature selection where the noise components of each segment are ignored in the clustering process.

![Image showing segmentation comparison](image)

Figure 2: Segmentation of two cuttlefishes: SSSMS and MS models with 84 input features before feature selection process. μ = 0.1 (both SSSMS and MS), γ = 0.5.

4 Conclusion

In this paper, we propose a model which incorporates the concept of subspace clustering into the Mumford-Shah segmentation model and propose an improved semi-supervised optimization algorithm to optimize the model. The algorithm uses the ROI’s with a decreasing degree of involvement to avoid the problem of over-specification of the ROI’s. In our experiments, we show that even when the images have a very low contrast in textures, the algorithm can give much better results than the original Mumford-Shah model since irrelevant features are filtered which in effect increases the contrast between different segments.

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