Game-theoretic power control in DS-CDMA wireless networks with successive interference cancellation

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A noncooperative game-theoretic power control framework for a wireless DS-CDMA uplink with imperfect successive interference cancellation is presented. Assuming a fixed cancellation ordering, a unique Nash equilibrium corresponding to the centralised solution is shown to exist, and simulation reveals significant utility improvement compared to traditional matched filter detection.

Introduction: Multiuser detection in wireless networks has attracted much attention [1] and of particular note is successive interference cancellation (SIC) [2], which is scalable and implementable. Under SIC detection, the reconstructed received signals of users detected first (often those with comparatively high received power) are cancelled from the composite signal to aid in the detection of subsequent (often weaker) users. Also of interest recently has been the application of game-theoretic models to wireless network resource allocation [3–6], which typically presume users to be selfish utility maximisers and to employ traditional matched filter detection. We herein extend the approach to a network with an SIC receiver at the base station (BS).

System model and constraints: We consider the uplink of a slotted, DS-CDMA network of N co-channel users with the common BS employing an SIC receiver. It is assumed that path gains to the BS are known and are quasi-static, at least over the time scale of the power control process.

We find in [7] an expression for each user's signal-to-interference-plus-noise ratio (SINR) that allows for imperfect SIC. Adapting the expressions in [7] to account for spreading gain, the SINR of user i, i.e. the ith user to be detected and cancelled, is given as follows (see Note below):

$$\Gamma_i = \frac{W}{R_i}\frac{Q_i}{\left(\sum_{k=i+1}^{N} Q_k + \sum_{k=1}^{i-1} \varepsilon_k Q_k\right) + N_0 W + I_{ex}} = \frac{W}{R_i}\frac{Q_i}{I_i} \tag{1}$$

where R_i is the bit rate of user i, Q_i is the received power of user i, W is the common bandwidth, $\varepsilon_k \in [0, 1]$ is residual power of user k after cancellation, N_0 is the noise power spectral density, I_{ex} is intercell interference, and I_i represent the total interference plus noise experienced by user i. (Note: Two types of indexing are needed in the present work. Standard subscripts refer to the cancellation ordering, whereas parenthetical subscripts (used below) refer to an arbitrary, *a priori* ordering of the users.)

We investigate two orderings of cancellation. First is the typical case in which users are detected and cancelled in decreasing order of received power. In this case, the SINR for user (i), $\Gamma_{(i)}$, is given by Γ_j if $Q_{(i)}$ is the jth largest received power. Second, given the set of users $\mathcal{N} = \{1, \ldots, N\}$, we introduce h: $\mathcal{N} \to \mathcal{N}$ which maps the arbitrary ordering of users into the ordering of cancellation. Thus, $\Gamma_{(i)} = \Gamma_{h((i))}$ in this simplified, fixed-ordering case.

System feasibility constraint and centralised power control with fixed cancellation ordering: Assuming no power constraints, it can be shown that given the SINR (1), a fixed cancellation ordering $h(\cdot)$, spreading gains $L_i = W/R_i$, and SINR targets γ_i, each user can obtain its target SINR if and only if:

$$\sum_{i=1}^{N} \frac{\gamma_i}{L_i + \gamma_i}\left(\prod_{j=0}^{i-1} \frac{L_j + \varepsilon_j \gamma_j}{L_j + \gamma_j}\right) < 1 \tag{2}$$

where $\gamma_0 \triangleq 0$, and ε_0 and $L_0 \in [0, \infty)$ are arbitrary. If (2) holds, then the unique power control solution for all $(i) \in \mathcal{N}$, call this $\mathbf{Q}^{pc}$, precisely satisfies the SINR targets and has elements

$$Q_i^{pc} = \frac{\gamma_i}{L_i + \gamma_i} \times \left(\prod_{j=0}^{i-1} \frac{L_j + \varepsilon_j \gamma_j}{L_j + \gamma_j}\right) \times \frac{N_0 W + I_{ex}}{1 - \sum_{i=1}^{N} \gamma_i/(L_i + \gamma_i)\left[\prod_{j=0}^{i-1}(L_j + \varepsilon_j \gamma_j)/(L_j + \gamma_j)\right]} \tag{3}$$

SIC game-theoretic power control game: Given the player set $\mathcal{N}$, the power control game is defined as follows.

1. The continuous action space, which correspond to the received power for each player, is given by $\mathcal{Q}_{(i)} \triangleq [Q_{\min}, Q_{\max}]$. The joint action space for the N users is $\text{X}_{i=1}^{N} \mathcal{Q}_{(i)}$, where X denotes the Cartesian product.
2. The utility function for each user is reasonably given, as in [3], as the ratio of the expected number of bits successfully transmitted and the transmit power. For user (i) then:

$$u_{(i)}(\mathbf{Q}) = u_{(i)}(Q_{(i)}, \mathbf{Q}_{(-i)}) = \frac{LRf(\Gamma_{(i)})g_{(i)}}{MQ_{(i)}} \tag{4}$$

where $f(\Gamma_{(i)}) = (1 - 2P_e(\Gamma_{(i)}))^M$ is the so-called *efficiency function* which is modulation dependent through the BER function $P_e(\cdot)$, R is the common bit rate, M is the packet size in bits, L is the number of information bits, $g_{(i)}$ is the path gain to the BS for user (i), $\mathbf{Q}_{(-i)}$ is the $(N-1)$-element vector of powers with $Q_{(i)}$ removed, and the SINR $\Gamma_{(i)}$ is given either by the dynamic detection ordering or by $\Gamma_{h((i))}$.

Each player $(i) \in \mathcal{N}$ solves the following optimisation problem:

$$\max_{Q_{(i)} \in \mathcal{Q}_{(i)}} u_{(i)}(Q_{(i)}, \mathbf{Q}_{(-i)}) \tag{5}$$

and we refer to this game as the SIC power control game (SIC-PCG). The standard equilibrium concept in noncooperative games is the Nash equilibrium [8].

SIC-PCG equilibrium analysis: With dynamic cancellation ordering, the utility function (4) exhibits a jump discontinuity at those points at which the received powers of two or more users are identical, a direct result of a corresponding discontinuity in the realised SINR. Consequently, traditional Nash equilibrium existence results based on fixed point theorems no longer apply [9]. Worse still, although there are results for games with discontinuous utility functions [9, 10], the requisite conditions on the utility function, namely quasi-concavity in $Q_{(i)}$ and upper semi-continuity in $Q_{(i)}$, are not satisfied by $u_{(i)}$. There are thus no (pure or mixed) Nash equilibrium existence results available for SIC-PCG with dynamic cancellation ordering.

With a fixed cancellation ordering, we find much more solid equilibrium guarantees. Specifically, much of the reasoning in Section IV of [3] may be applied. In this case, $u_{(i)}$ is continuous in $\mathbf{Q}$ and quasi-concave in $Q_{(i)}$, and, coupled with the compactness of the action space $\mathcal{Q}$, we may conclude that an equilibrium exists [8]. Furthermore, it is shown in [3] with matched filter detection, a unique equilibrium SINR $\tilde{\Gamma}_{(i)}$ that maximises the utility for user (i) exists, and $\tilde{\Gamma}_{(i)}$ solves

$$\frac{\partial f(\Gamma_{(i)})}{\partial \Gamma_{(i)}} \Gamma_{(i)} = f(\Gamma_{(i)}) \tag{6}$$

assuming that $\tilde{\Gamma}_{(i)}$ is feasible under (2). In SIC-PCG, (6) applies as well, and $\tilde{\Gamma}_{(i)} = \tilde{\Gamma}$ is identical for all users. We can then express the *best-response function* in similar fashion to [3]:

$$r_{(i)}(\mathbf{Q}_{(-i)}) = \frac{\tilde{\Gamma} I_{(i)}}{W/R} \tag{7}$$

i.e. presented with the measured interference $I_{(i)}$, user (i) sets its received power so that $\tilde{\Gamma}$ is precisely satisfied.

Theorem 1: Assuming a fixed cancellation order and system feasibility, the Nash equilibrium power vector that results from successive applications of (7) is unique and, if properly ordered, identical to $\mathbf{Q}^{pc}$ with target SINR $\tilde{\Gamma}$.

Proof. We know from [3] that the Nash equilibrium power vector resultant from repeated application of (7), call this $\mathbf{Q}^{g}_{(\cdot)}$ as it is ordered via the arbitrary ordering, is unique if the system is feasible, and we also know that at the equilibrium, each user will achieve the equilibrium

SINR $\tilde{\Gamma}$. Furthermore, given a cancellation order $h(\cdot)$, the power control solution $\mathbf{Q}^{pc}$ given in (3) exactly satisfies the target SINR $\tilde{\Gamma}$ and is itself unique. We conclude that for all $i \in \mathcal{N}$, $Q^{g}_{(i)} = Q^{pc}_{h((i))}$, and thus properly ordered, the received power vectors are identical. □

Note that the equilibrium existence result presented above is not dependent on the cancellation order mapping h and the cancellation order might be chosen to satisfy some system criterion or even be chosen randomly. In the simulation results, we consider the cancellation orderings that maximise or minimise the total utility in the cell.

Simulation: Fig. 1 demonstrates for a five-node network the per cent improvement in total cell utility over traditional matched filter detection at equilibrium against a common spreading gain. Perfect cancellation ($\varepsilon = 0$) and a moderate cancellation error ($\varepsilon = 0.3$) were tested. The dashed curve in each case represents the improvement gained by using the fixed detection ordering that yields the highest total utility, and the solid curve represents the fixed detection ordering that yields the lowest total utility. Fig. 1 shows that the greatest utility gains occur at lower spreading gains, cancellation error can degrade the utility significantly, and that even at the highest spreading gains tested an improvement of nearly 17% can be realised with perfect cancellation. The lower curve for $\varepsilon = 0.3$ reveals that with moderate cancellation error, the cancellation ordering that yields the lowest total utility can still realise a 5% improvement over matched filter detection.

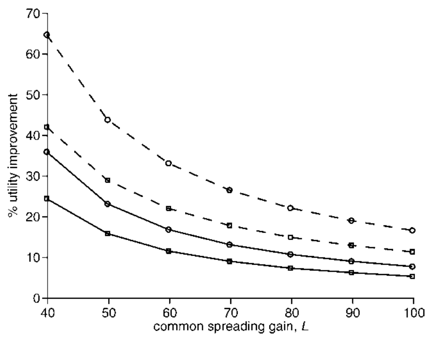

Fig. 1 *Per cent improvement in total cell utility at equilibrium over traditional $\varepsilon = 1$ case against a common spreading gain*

Results averaged over five runs; parameter values included $N = 5$, $L = 64$ bits, $M = 80$ bits, $R = 10$ kbits, $W = R \times L$, $N_0 W = -143.0$ dBm, $I_{ex} = 0$ W, $\tilde{\Gamma} = 4.57$, g values uniformly distributed between -110 and -90 dB, and BPSK modulation
- - ○ - - $\varepsilon = 0$, maximal-utility ordering
- - □ - - $\varepsilon = 0.3$, maximal-utility ordering
—○— $\varepsilon = 0$, minimal-utility ordering
—□— $\varepsilon = 0.3$, minimal-utility ordering

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