Performance Comparisons of Optimal Power Allocation over Nakagami-m and Rayleigh Fading Channels in Wireless Cooperative Systems

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Abstract—Cooperative wireless communication has been newly proposed in wireless communication systems for discovering the inherent spatial diversity in relay channels. The Amplify-and-Forward (AF) cooperation protocols with multiple relays have not been sufficiently examined; however, it has a low complexity in terms of application. Through this article, we evaluate a cooperative diversity technique whereby a source broadcasts some data to a destination with the assistance of multiple relay nodes with AF protocols, by taking into account the challenge of allocating power to be able to increase the total capacity of AF and also enhance resource utilization. We analyse the optimality of how much the power should really be allocated at the source as well as relays system by optimizing the symbol error rate (SER) performance in a useful method on the basis of Rayleigh and Nakagami-m fading. Firstly, we derive a closed-form SER formulation for MPSK signal making use of the idea of moment generating function (MGF) and some statistical approximations in high signal to noise ratio (SNR) for the system under studied. We therefore determine a tight corresponding lower bound, which converges to the identical limit the same as the theoretical upper bound, after that develops an optimal power allocation (OPA) strategy with mean channel gains over Rayleigh and also Nakagami-m fading. Secondly, we present an OPA scheme to show the performance after Nakagami-m fading to reduce the SER. Simulation results prove that our approach using Nakagami-m fading outperforms the (OPA) using Rayleigh fading scheme and is tight with the theoretical approximation based on the SER upper bound in high SNR for a different number of relays.

Index Terms—Wireless communication, amplify-and-forward, symbol error rate, Nakagami-m fading, power allocation.

I. INTRODUCTION

In wireless communications network, the channel links cannot simply confirmed continuously because of multipath fading. Relay channel has been proposed currently in a cooperative communication system for helping the transmission. In a relay technique, the information is delivered to the destination by all the relays. Therefore the opportunity of getting all the channel links to the destination drop is very low.

In this situation the system can be enhanced in terms of reliability by allowing the destination to detect the transmitted data. In recent times, a number of researchers are paying attention around the idea of the relay channel and have proven its advantage in wireless networks [1]-[2].

Various cooperation protocols of relaying have already been formulated to advance the communication system efficiency and also robustness in wireless systems. Commonly we now have Amplify-and-Forward (AF), Decode-and-Forward (DF), code cooperation and space-time cooperation [1]-[3]. In this work, we analyze power allocation in AF cooperation protocol for multiple relays in which the relay nodes in AF, amplify the received signal from the source with a parameter and forward it to the destination, the main advantage of AF is the fact that, AF has the advantages of lower complexity in terms of design and can attain full spatial diversity orders for improving network reliability [6]-[7].

Cooperative diversity performances can certainly be improved by considering the optimal power allocation approach for the system. In [4]-[7], the authors have shown some improvement about the symbol error rate with perfect channel state information at the transmitters and the receivers (CSI) and with knowledge of the mean channel from the transmitter respectively for a single relay system and multi-node relay over Rayleigh fading environment, using OPA technique. In [8], the authors presented an OPA scheme to show the performance analysis of a single relay system compared with the EAP scheme where power resources are equally distributed over-all nodes, which is not optimum in general[10]. In [11], the authors analyze the symbol-error-rate(SER) performance of Amplify-and-forward (AF) over Nakagami-m fading channel. In this work, we further extend the work in [13], for a multiple relays system for AF over Rayleigh fading and compared it with Nakagami-m fading.

In cooperative diversity, the Raleigh and Rician distribution are recently used. Although Nakagami model is often more versatile and has good performance in describing the amplitude of received signal after MRC, and fitting experimental results more generally than Rayleigh.
Practically, Nakagami-\(m\) has proven very useful due to an easy manipulation and a wide range of conformity of various approaches. In addition, its distribution can model different propagation conditions by varying its fading parameter. Moreover, it can provide more improvement and higher reliability in matching some experimental measurement in comparison with the other distributions.

Because of the problem of fading, the CSI is not always well known at the transmitter. As a result we apply for this work the fact that knowledge of the mean channel gain is considered at the transmitter. We first derive a closed-form SER formulation for MPSK signal using the moment generating function of the received SNR at the destination. Since the SER formulation is too complex, we after that find a tight lower bound, which converges to the identical limit as the theoretical upper bound in high SNR. Therefore making use of this result, we establish an OPA technique, by using Nakagami-\(m\) fading channel to minimize the SER and show by simulations the performance advancement of our new system when compared with that we actually did in [13]

The rest of the paper is organized as follows: In Section II, we describe the system model. In Section III, Nakagami-\(m\) fading model is presented. In section IV, we evaluate the SER for MPSK signal for our scheme by computing the moment generating function of the received instantaneous SNR at the destination. In section V we derive a tight lower and upper bound and applying OPA with SER then, we evaluate the comparisons between Nakagami-\(m\) and Rayleigh fading. In section IV, the simulation results are shown to validate the proposed scheme. Section V shows performance analysis and comparison evaluation. Finally, we finish this work by concluding in Section VI.

II. SYSTEM Model

A graphic diagram of the AF wireless cooperative communications studied in this paper is shown in Fig. 1. We assume that the system work under flat Nakagami-\(m\) fading channel with additive white Gaussian noise (AWGN) on top of pass loss, and the perfect CSI at the receiver is available and the main channel gains are known for the transmitter; signals are transmitted through orthogonal channels in case of TDMA, FDMA or CDMA system.

![Fig. 1. Communication scenario with source(S), n relays \((R_1, R_2, ..., R_n)\) and destination (D).](image)

The process of cooperative communication has two parts. In the first phase, the source broadcasts the information to the \(n\) relay nodes and the destination. The received signals at the destination (D) and the \(i^{th}\) relay (\(R_i\)) are respectively:

\[
Y_{SD} = h_{SD} x + n_{SD} \tag{1}
\]

\[
Y_{SR} = h_{SR} x + n_{SR} \tag{2}
\]

where \(x\) is the transmitted signal from the source, \(Y_q\) is the received signal from node \(i\) to node \(j\), \(n_j\) is the additive white Gaussian noise with variance \(N_0\) from node \(i\) to node \(j\) and \(h_i\) represents the fading channel coefficients from node \(i\) to node \(j\). The variable \(h_i\) is modeled as the independent zero-mean, circularly-symmetric complex Gaussian random variable with variance one.

In part 2, each relay amplifies the received signal from the source by a factor \(\beta_i\) and forwards it to the destination.

So the received signal at the destination from the \(i^{th}\) relay is given by:

\[
Y_{RD} = h_{RD} (\beta Y_{SR}) + n_{RD} = \beta h_i h_{RD} x + n_i \tag{3}
\]

where

\[
\beta_i = \sqrt{P_s/P_R/h_{SR}} \tag{4}
\]

\[
n_i = \beta_i h_{RD} n_{SR} + n_{RD} \tag{5}
\]

represent the amplifier gain and the equivalent noise of the \(i^{th}\) relay respectively. The power at the source and that at the \(i^{th}\) relay are respectively represented by \(P_s\) and \(P_R\). The variable \(h_{RD}\) is the fading channel coefficients from the \(i^{th}\) relay to the destination and is also modeled as the independent zero-mean, circularly-symmetric complex Gaussian random variable with variance one.

From these expressions, we derive the following relations:

\[
\gamma_S = \frac{P_s}{N_0} |h_{SD}|^2 
\]

\[
\gamma_R = \frac{P_s}{N_0} |h_{SR}|^2 + \frac{P_R}{N_0} |h_{RD}|^2 + 1 
\]

where \(\gamma_S\) and \(\gamma_R\) represent the instantaneous SNR of the direct and the \(i^{th}\) relay link respectively. \(\gamma_S\) is an exponential random variable with parameter \(\lambda_S = N_0/P_S \sigma_{SD}^2\) (\(\sigma_{SD}^2\) is the variance of \(h_{SD}\), and \(\lambda_R\) is the harmonic mean of two exponential random variables and can be approximated as an exponential random variable with parameter \(\lambda_R = N_0/P_S \sigma_{SR}^2 + N_0/P_R \sigma_{RD}^2\) (\(\sigma_{SR}^2\) and \(\sigma_{RD}^2\) are the variances of \(h_{SR}\) and \(h_{RD}\) respectively) at high SNR [10].
III. NAKAGAMI- \( m \) FADING CHANNEL MODEL

In the following, all links are assumed to be independent and Nakagami-\( m \) flat fading, \( h_{SD}, h_{SR} \) and \( h_{RD} \) are all described by a Nakagami-\( m \) distribution[14]. The probability density function (pdf) of \( h \) can be written as:

\[
p_h(h) = \frac{2^m \Gamma(m)}{\Gamma(m/2)} h^{2m-1} \exp\left(-\frac{m h^2}{\Omega}\right), \quad h \geq 0, \quad \Omega > 0
\]

where \( \Omega = E\{H^2\} \) is the power scaling parameter that controls the depth of fading envelope ranges from \((0.5 \rightarrow \infty) \), moreover \( \Gamma(\cdot) \) denotes the Gamma function and \( m \) is fading parameter. Furthermore, when \( m = 1 \) the Rayleigh model may consider, which mean that the Raleigh fading is a special case of Nakagami-\( m \) fading, at \( m \rightarrow \infty \) Nakagami-\( m \) meet (AWGN), which mean non-fading condition. The instantaneous SNR per symbol of channel \( \gamma \) has exponential distribution as:

\[
p_{\gamma}(\gamma) = \frac{1}{\Gamma(m)} \left(\frac{m}{\overline{\gamma}}\right)^m \gamma^{m-1} \exp\left(-\frac{m \gamma}{\overline{\gamma}}\right), \quad \gamma \geq 0
\]

where \( \overline{\gamma} = \Omega E_s/N_o \).

IV. EVALUATION OF THE SER

In this section, we evaluate the SER using MPSK signal. By making use of the maximum ratio combining (MRC) as in[4], [15], the instantaneous received SNR at the destination with the \( n \) relays is given by:

\[
\gamma = \gamma_S + \sum_{i=1}^{n} \gamma_R^i
\]

As in [9] It can prove that the average symbol error rate SER is given by integral that consist of the multiplication of pdf of fading, and Gaussian \( Q \)-function, as follows:

\[
P_{SER} = \int_0^\infty Q\left(b\sqrt{\gamma}\right)p_\gamma(\gamma)d\gamma
\]

\[
= \frac{1}{\pi}\int_0^\infty \exp\left(-\frac{b^2}{\sin^2 \theta}\right)p_\gamma(\gamma)d\gamma
\]

Now by taking Laplace transformation with respect to \( \gamma \) for (10) between brackets:

\[
\tilde{f}(s) = \tilde{p}(s) = \frac{1}{1+s\overline{\gamma}}
\]

After making integration the result becomes:

\[
\overline{p}(s) = \frac{1}{1+s\overline{\gamma}}
\]

By substituting Eq. (12) into Eq. (10), then the conditional SER with the SNR \( \gamma \) using MPSK signal is given as [16]:

\[
P_{SER} = \frac{1}{\pi} \int_0^{\pi\over2} \left[1 + \frac{b^2}{m \sin^2 \theta}\right]^{-m} d\theta
\]

where \( b = \sin^2 \left(\frac{\pi}{M}\right) \)

By averaging (13) over the distribution of \( \gamma_S \) and \( \gamma_R \), we obtain the following unconditional SER for our system:

\[
P_{SER} = \frac{1}{\pi} \int_0^{\pi\over2} \left[1 + \frac{b \Omega_{SD}}{m_{SD} \sin^2 \theta} \frac{1}{\Omega_{SD} b}\right]^{-m} \sin^2 \theta d\theta
\]

where \( M \) and \( M_{\gamma_S} \) denote the average symbol error rate and the corresponding Nakagami-\( m \) fading figures and channel variances (note that: in case of Rayleigh fading \( m_{SD} = m_{RD} = 1 \) and \( \Omega_{SD} = \Omega_{RD} = 1 \) whereas in Nakagami-\( m \), \( m_{SD} = m_{RD} = m \) and \( \Omega_{SD} = \Omega_{RD} = 1 \).

Equation (15) is an exact expression of the SER for our system over both Rayleigh (when \( m = 1 \)) and Nakagami-\( m \) fading and can be computed using numerical integration as in[9].
V. PERFORMANCE ANALYSIS AND COMPARISON EVALUATION

A. SER Lower and Upper bound Analysis

In this section, we derive a tight SER lower bound which converges to the same limit as a theoretical SER upper bound and show some performance analysis by applying OPA method. Using the fact that, $0 \leq \sin^2 \theta \leq 1$, we establish a tight SER lower bound as follows:

$$
C \left( \frac{1}{1 + \frac{1}{\Omega_{gb} b}} \right)^{m_{SD}} \prod_{i=1}^{n} \left( \frac{1}{1 + \frac{1}{\Omega_{b} b}} \right)^{m_{R_i}} \leq P_{\text{SER}} \leq C \left( \frac{1}{1 + \frac{1}{\Omega_{gb} b}} \right)^{m_{SD}} \prod_{i=1}^{n} \left( \frac{1}{1 + \frac{1}{\Omega_{b} b}} \right)^{m_{R_i}}
$$

(16)

where $C = \frac{1}{\pi} \left[ 1 + \frac{1}{M} \right] \sin 2M \pi + 2M + \pi \theta d\theta$.

The proof is given in Appendix. In high SNR, we have $1/\lambda_i \gg 1$ and $1/\lambda_R \gg 1$ (The effect of the 1’s in the denominator is not very significant). So, the obtaining SER lower bound (left term) converges to the same limit as a theoretical SER upper bound which is represented by the right term in (16).

For a two-relay system and QPSK signal. Assuming that $N_0 = 1$ and $P_i = P_{R_i} = P_{S_i} = P/3$, we represent in Fig. 2 the SER lower bound and the theoretical upper bound for both Rayleigh and Nakagami-$m$ fading. The figure also shown that the two bounds are tight in high SNR and the same results can be drawn using different power allocation method. Fig. 3 compares Rayleigh and Nakagami-$m$ fading with $m = 1, 2$ as in figure, we can see that at a given number of relays with $m=2$ the target symbol error rate in Nakagami-$m$ fading is lower than Rayleigh fading with the same average SNR. This result is important because it permits to bound the exact SER expression with two tight bounds.

To show the asymptotic performance of the system, we apply OPA to the tight SER lower bound. Considering that the variance of the noise is unit and replacing $\gamma_i$ and $\gamma_R$ by their values in (16), we then obtain the optimization problem using Nakagami-$m$ fading channel.

As we did in [13], we can obtain the exact OPA using Nakagami-$m$ fading as follows:

$$
\left( \begin{array}{c}
\min_{(P_i, \gamma_i, \gamma_R)} \left( C \left( \frac{1}{1 + \frac{1}{\Omega_{gb} b}} \right)^{m_{SD}} \prod_{i=1}^{n} \left( \frac{1}{1 + \frac{1}{\Omega_{b} b}} \right)^{m_{R_i}} \left( \frac{P_i + P_R}{P_i + P_R + bP_i P_R} \right)^{m_{R_i}} \right)^{m_{R_i}} \\
\right)
\right)

\left( P_i + \sum_{i=1}^{n} P_i = P \right)

(17)

$P$ represents the total limited power in the system. Setting $P_i = P_{R_i} = P_{S_i}$, the following function is formed after applying Lagrange multiplier approach into (17):

$$
Z(P_i, P_R, \lambda) = C \left( \frac{m}{m + bP_i} \right)^{m} \prod_{i=1}^{n} \left( \frac{m + P_i}{m + P_i + bP_i P_R} \right)^{m_{R_i}} - \lambda \left( 1 + \sum_{i=1}^{n} \frac{P_i - P}{P_S} \right)

(18)

Making use of the logarithm function in (18), we then derive the following relations:

$$
\frac{\partial Z}{\partial P_S} = C \left( - \frac{mb}{1 + bP_i} - \sum_{i=1}^{n} \frac{bP_i}{m + P_i + bP_i P_R} \right) - \lambda \frac{P}{P_S} = 0 \quad (19)
$$

$$
\frac{\partial Z}{\partial P_i} = C \left( \frac{mn}{m + P_i} - \sum_{i=1}^{n} \frac{mn + bP_i P_R}{m + P_i + bP_i P_R} \right) - \lambda = 0

\text{for } i = 1, 2, ..., n \quad (20)
$$

(20)

$$
\frac{\partial Z}{\partial \lambda} = -\left( 1 + \sum_{i=1}^{n} \frac{P_i - P}{P_S} \right) = 0 \quad (21)
$$

(21)

So now we have to solve the system formed by (19), (20) and (21). Assuming that all the relays have the same
power, it follows that \( P_1 = P_2 = \ldots = P_n \) and verifies an equation as follows:

\[
n (mn + 1) P_i^n + \left( (mn + 1) b + \frac{mn}{2} b^2 + b^2 \right) P_i^n + \left( 2mn b + (mn - b + b^2) \right) P_i^n + (mn b^2 + b^3) P_i^n + mn b^2 - mn b P_i - mn b^2 P = 0
\]  

(22)

where \( m \) is Nakagami-\( m \) fading figure.

Equation (22) is a polynomial relation and can be easily solved using MATLAB. The expression of \( P_i \) is a real positive value for different number of relays. And for a given number of relay, \( P_i \) is a positive increasing function less than one. This means that the power at the source is more than the one at each relay which is normal due to the second condition from relation (17). So making use of (21) and (22), we have:

\[
P_s = \frac{1}{1 + nP_i} P
\]  

(23)

Using the relation between \( P_s, P_r_i, \) and \( P_i \), we obtain:

\[
P_r_1 = P_r_2 = \ldots = P_r_n = \frac{n}{1 + nP_i} P
\]  

(24)

Relations (23) and (24) represent the optimum values obtained from OPA with multiple relays system.

- At given SNR value of the SER of Nakagami-\( m \) fading is smaller than Rayleigh fading.
- For \( m = 1 \) Nakagami-\( m \) fading becomes Rayleigh fading. Which mean that Rayleigh fading is a special case of Nakagami-\( m \) fading which can describe by the Nakagami-\( m \) distribution by an appropriate choice of relevant parameters.
- Except, \( m = 0.5 \) for any other values of \( m \) the performance of Nakagami-\( m \) is better than Rayleigh.

VI. SIMULATION RESULTS

This section presents the simulation results using MPSK signal to validate the mathematical expressions obtained above.

Fig. 5 compares the SER performance between the Rayleigh and Nakagami-\( m \) fading with fading parameter \( m = 2 \) for proposed OPA scheme with 4 relays according to the SNR. From the figure, we can see that the OPA scheme using Nakagami-\( m \) fading developed improves better than using Rayleigh fading for a given number of relays for the different modulation scheme (MPSK).

It can be seen also that increasing the number of fading parameter \( m \) decreases the SER as we presented in Fig. 6. Fig. 6 compares between Raleigh and Nakagami-\( m \) fading for different numbers of relays (\( n = 2, 3, 4, 10 \)). As in figure for given OPA power, we can see that
increasing the number of relays significantly decreases the SER.

Fig. 7 shows the average SER versus SNR with QPSK modulation with different fading parameters ($m = 1, 2, 3, 5, 10$) as in figure increasing the number of relays significantly decreases the SER.

![Fig. 7. Average SER for OPA scheme versus the average SNR over Nakagami-$m$ fading ($m = 1, 2, 3, 5, 10$) with 4 relays, for QPSK signal.](image)

VII. CONCLUSION

This work analyzed the performances of AF cooperative relaying in wireless communication systems with mean channel gains over the Nakagami-$m$ fading channel in high SNR. After establishing the expressions for the SNR and the SER using MPSK signal to the system under studied, we found a tight SER lower bound and proposed an OPA scheme to minimize the SER, and we compared it with Rayleigh fading that we did in [13]. Numerical results have shown the performance improvement of our scheme compared with Rayleigh fading, for a different number of relays.

APPENDIX: PROOF OF RELATION (10)

Since we have $0 \leq \left( \sin^2 \theta \right)^m \leq 1$, we can derive the following inequalities:

$$\left( \frac{b}{m \lambda_S} \right)^m \leq \sin^2 \theta + \frac{b}{m \lambda_S} \leq \left( \frac{1 + \frac{b}{m \lambda_S}}{m \lambda_S} \right)^m \Rightarrow$$

$$\sin^2 \theta \leq \frac{\sin^2 \theta}{1 + \frac{b}{m \lambda_S}} \leq \frac{\sin^2 \theta + \frac{b}{m \lambda_S}}{\sin^2 \theta + \frac{b}{m \lambda_S}} \leq \left( \frac{1 + \frac{b}{m \lambda_S}}{m \lambda_S} \right)^m \leq \left( \frac{b}{m \lambda_S} \right)^m \Rightarrow$$

$$\prod_{i=1}^{n} \left( \frac{\sin^2 \theta}{1 + \frac{b}{m \lambda_S}} \right)^m \leq \prod_{i=1}^{n} \left( \frac{\sin^2 \theta + \frac{b}{m \lambda_S}}{\sin^2 \theta + \frac{b}{m \lambda_S}} \right)^m \leq \prod_{i=1}^{n} \left( \frac{1 + \frac{b}{m \lambda_S}}{m \lambda_S} \right)^m \leq \prod_{i=1}^{n} \left( \frac{b}{m \lambda_S} \right)^m \Rightarrow$$

By integrating (27), we finally prove (16).

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