The QCD phase diagram from analytic continuation

Sándor Katz

for the Wuppertal-Budapest collaboration

Institute for Theoretical Physics, Eötvös University

R. Bellwied, Sz. Borsányi, Z. Fodor, J. Günther, A. Pásztor, C. Ratti, K.K. Szabó

PLB 752 (2016) 175 & arXiv:1607.02493

ICHEP 2016

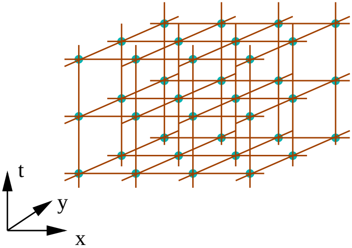
Outline

Introduction

- Determine QCD phase diagram and Equation of State (EoS) at finite μ_B
- Due to the sign problem direct lattice calculations are prohibited
- Use imaginary chemical potentials + analytic continuation
- Study systematic error coming from different ansatzes for continuation
- Continuum limit at physical quark masses

Lattice formulation

$$Z = \int dA_\mu d\Psi d\bar{\Psi} e^{-S_E}$$



- discretize spacetime $\rightarrow N_s^3 \cdot N_t$ lattice
- $S_E = S_g + S_f$ is the Euclidean action
- Finite $T \leftrightarrow$ finite temporal lattice extension

$$T = \frac{1}{N_t a}$$

- Continuum limit: $a \rightarrow 0 \Longleftrightarrow N_t \rightarrow \infty$

The sign problem

Fermions are Grassmann variables: not suitable for Monte-Carlo $\rightarrow$ integrate them out

$$Z = \int dA_\mu \, \det M e^{-S_g}$$

where S_g is the gauge action and M is the fermion matrix:

$$S_f = \bar{\Psi} M \Psi$$

Monte-Carlo requires positivity of $\det M$.

At $\mu_B > 0$ this does not hold $\rightarrow$ sign problem

Possible ways out:

reweighting, Taylor expansion in μ_B, analytic continuation in μ_B, use of canonical ensemble, complex Langevin, Lefshetz thimbles, etc.

At $\mu_B = 0$ or imaginary positivity is ensured by $M^\dagger(-\mu^*) = \gamma_5 M(\mu) \gamma_5$

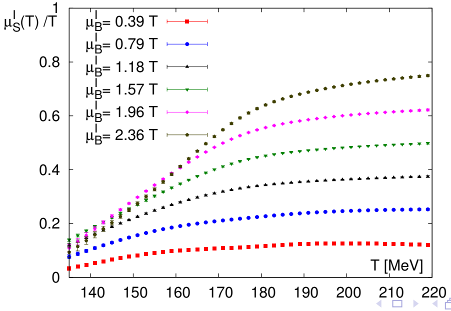
Use imaginary μ_B & analytic contiuation for the phase diagram & EoS

Lattice parameters

- Tree level Symanzik improved gauge action
- $N_f = 2 + 1 + 1$ 4-stout smeared staggered fermions with physical masses
- four lattice spacings $N_t = 8, 10, 12, 16$
- $\mu_B^{(j)} = iT\frac{j\pi}{8}\ , \quad j = 1, 2, 3, 4, 5, 6$
- temperature range $T = 135 - 210\ \mathrm{MeV}$
- T dependence mostly detemined by high statistics $\mu = 0$ data

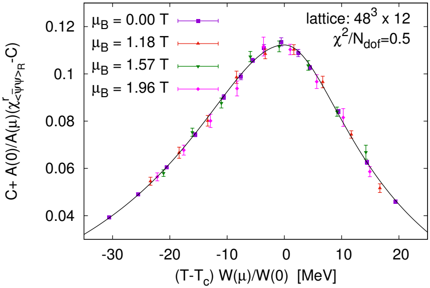
Strangeness neutrality

- On the lattice the input are the μ_u, μ_d, μ_s chemical potentials
- Alternative basis:
  $\mu_u = (\mu_B + 2\mu_Q)/3 \quad \mu_d = (\mu_B - \mu_Q)/3 \quad \mu_s = (\mu_B - \mu_Q)/3 - \mu_S$
- In the experimental setup $\langle n_S \rangle = 0$ and $\langle n_Q \rangle \approx 0.4 \langle n_B \rangle$
- this requires a non-trivial tuning of the bare chemical potentials



Analytic continuation for the phase diagram

- we need the μ dependence of the peak of the chiral susceptibility
- fit/interploate high statistics $\mu = 0$ data
- fit finite μ data with the same function with μ dependent peak position/width
- $\frac{\chi^r_{\bar\psi\psi}(\mu,T)}{m_\pi^4} = C + \frac{A(\mu)}{1+W^2(\mu)(T-T_c(\mu))^2+BW^3(\mu)(T-T_c(\mu))^3}$



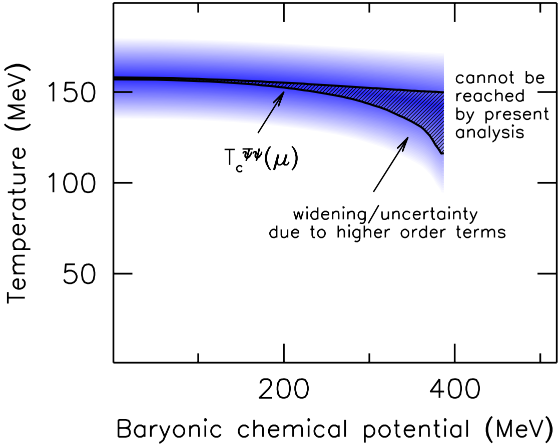
Analytic continuation for the phase diagram

- once we have the $T_c(\mu_B^{\mathrm{Im}})$ phase diagram, we can extract the curvature $\kappa = -\frac{1}{T_c(\mu_B=0)} \frac{dT_c(x)}{dx}$ where $x = \mu_B^2/T^2$
- result:

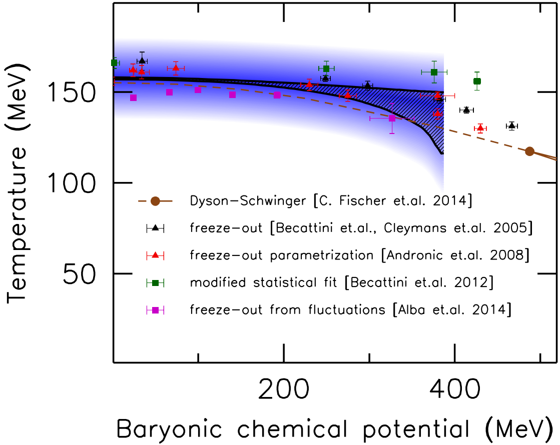
$$\kappa = 0.0149(21)$$

- higher orders: various ansätze for $T_c(\mu_B)/T_c(\mu = 0)$
  - $1 + ax$
  - $1 + ax + bx^2$
  - $(1 + ax)/(1 + bx)$
  - $(1 + ax + bx)^{-1}$
- systematic error from various interpolations, ansätze, cont. limit

The phase diagram

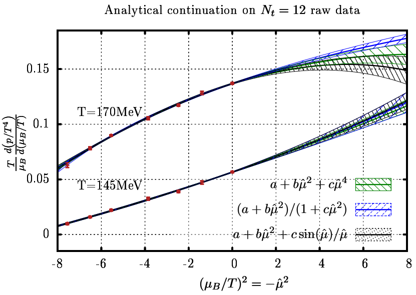


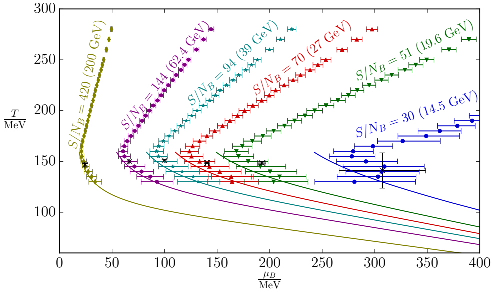
The phase diagram



Analytic continuation for the EoS

- similar strategy for the Equation of state
- various ansätze for analytic continuation of $dp/d\mu$
- fixed s/N_B trajectories have been determined





Summary

- Analytic continuation in μ_B to avoid the sign problem
- We determined the curvature from $T_c(\mu_B^{\mathrm{Im}})$: $\kappa = 0.0149(21)$
- Analytic continuation to get the phase diagram for real μ_B
- Same technique applied for the Equation of state
- Continuum results with physical quark masses