Secrecy Rate Maximization for Information and Energy Transfer in MIMO Beamforming Networks

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Abstract—Consider a multiple-input multiple-output (MIMO) beamforming system, where a multi-antenna base station transmits information and energy simultaneously to a multi-antenna information receiver (IR) and a number of multi-antenna energy receivers (ERs). The ERs are assumed to possess dual functionality such that they can also decode information. This gives rise to the physical layer security issue of providing confidential information to the IR while ensuring efficient energy harvesting for the ERs. Hence, we employ an artificial noise (AN)-aided secrecy approach at the transmitter, where the AN improves the security by interfering with the information decoding at the ERs while, at the same time, providing them with wireless energy to harvest. In this paper, we address the problem of jointly optimizing the beamforming matrix for the IR and the covariance matrix of the AN such that the secrecy rate is maximized subject to energy harvesting constraints and a total power constraint. The corresponding optimization problem is a difference of convex functions (DC) programming problem, which is generally non-convex. Nevertheless, we propose an alternating optimization (AO) strategy to tackle this problem for the cases of a square and a non-square IR channel. The performance of the proposed algorithms is demonstrated by simulation results.

I. INTRODUCTION

Energy harvesting wireless systems, where terminals have access to external energy supplies to prolong their operation time, have recently drawn considerable attention [1]. A new energy supply source has emerged in form of energy-carrying radio signals that provide electromagnetic energy for receivers to harvest. Only very recently, this concept of energy transmission has been combined with wireless information transmission, termed simultaneous wireless information and power transfer (SWIPT) [2], [3]. In a typical SWIPT scenario, a base station (BS) transmits RF signals to a set of information receivers (IRs), which decode the information, and a set of energy receivers (ERs) that harvest the signal energy.

Due to the twofold purpose of information and energy transmission, the SWIPT concept gives rise to a new information security issue of providing confidential information to the IR while ensuring efficient energy harvesting for a group of ERs [4]. Specifically, a dual-functional ER, which has both information decoding and energy harvesting capabilities, can eavesdrop the confidential information intended for the IRs.

Without the energy transfer, several physical layer secrecy approaches [5]-[8] have recently been proposed and studied for multiple-input multiple-output (MIMO) networks. Additionally, the concept of transmitting artificial noise (AN) has been introduced to increase the secrecy rate in MIMO networks [9], [10]. Note that AN is not necessary in the single eavesdropper scenario, where a no-AN transmission with complex Gaussian signaling has been shown to achieve the secrecy capacity [5]. However, AN is very effective in improving the secrecy rate for multiple eavesdroppers. By designing the covariance matrix of the AN component, artificial interference can be directed towards the eavesdroppers to degrade their information reception and decoding abilities.

In the SWIPT beamforming scenario, the optimization problem is to design the transmit beamforming matrix by maximizing the secrecy rate under energy harvesting constraints and a total power constraint. However, this problem belongs to the class of difference of convex functions (DC) programming problems, which are generally non-convex [11]. The authors of [4] have recently considered an AN-aided SWIPT multiple-input single-output (MISO) beamforming system with a single-antenna IR and multiple single-antenna ERs, and proposed an optimal beamforming design for this scenario. In the SWIPT context, the use of AN has a dual purpose: It interferes with the ERs to improve the secrecy rate of the system while simultaneously carrying wireless energy for the ERs to harvest.

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In this paper, we study a more general MIMO beamforming system consisting of a BS, a single IR, and a group of ERs, all equipped with multiple antennas as depicted in Fig. 1. Assuming perfect channel state information (CSI), an AN-aided secrecy approach is employed at the BS. We develop an alternating optimization algorithm to address the non-convex secrecy rate maximization problem subject to energy harvesting constraints at the ERs and a total power constraint by jointly optimizing the beamforming matrix for the IR and the covariance matrix of the AN. For the presented algorithm, we consider the two cases of a square IR channel and a non-square IR channel, where the number of antennas at the BS exceeds that of the IR. We show that by additional precoding, the latter case can be reduced to the former such that the proposed algorithm is applicable in both scenarios. Finally, the performance is demonstrated via simulation results.

II. SYSTEM MODEL

Consider the MIMO broadcast system shown in Fig. 1, where a BS with $N_b$ antennas transmits a signal to a legitimate IR with $N_i$ antennas and to $K$ eavesdropping ERs each with $N_{e_k}$ antennas, where $k = 1, \ldots, K$. It is generally assumed that the ERs are positioned closer to the BS so that they can harvest enough energy. In order to prevent the ERs from decoding the confidential information intended for the IR, we employ an AN-aided transmit beamforming secrecy approach at the transmitter. Therefore, the $N_b \times 1$ transmit baseband signal can be expressed as

$$x = Ws + z,$$

where $W \in \mathbb{C}^{N_b \times d}$ denotes the beamforming matrix and $s \in \mathbb{C}^d$ is the transmit symbol vector comprising $d$ data streams to the IR. The symbol vector $s$ is assumed to contain independent identically distributed (i.i.d.) Gaussian random variables, i.e., $s \sim \mathcal{CN}(0, I_d)$. Moreover, $z \in \mathbb{C}^{N_b \times 1}$ is the AN component with $z \sim \mathcal{CN}(0, Z)$, where the covariance matrix is defined by $Z = \mathbb{E}\{zz^H\}$. Note that the purpose of the AN component is twofold as it interferes with the information reception of the ERs while, at the same time, providing them with wireless energy they can harvest.

The total transmit power at the BS is constrained by

$$\mathbb{E}\{x^Hx\} = \text{Tr}\left\{WW^H + Z\right\} \leq P,$$

where $P$ is the maximum allowable transmit power.

Assuming quasi-static full-rank flat-fading channels, the received signals at the IR and the $k$-th ER are given by

$$y_i = Hx + n_i \in \mathbb{C}^{N_i \times 1},$$

$$y_{e_k} = G_k x + n_{e_k} \in \mathbb{C}^{N_{e_k} \times 1}, \quad k = 1, \ldots, K,$$

where $H \in \mathbb{C}^{N_i \times N_b}$ and $G_k \in \mathbb{C}^{N_{e_k} \times N_b}$ represent the respective MIMO channels from the BS to the IR and from the BS to the $k$-th ER. Moreover, $n_i \in \mathbb{C}^{N_i \times 1} \sim \mathcal{CN}(0, N_i)$ and $n_{e_k} \in \mathbb{C}^{N_{e_k} \times 1} \sim \mathcal{CN}(0, N_{e_k})$ are the additive white Gaussian noise vectors at the IR and the $k$-th ER with the variances $\sigma_i^2$ and $\sigma_{e_k}^2$, respectively.

Based on the assumption that perfect CSI is available at the transmitter, we can maximize the achievable secrecy rate by jointly optimizing the transmit beamforming matrix $W$ and the AN covariance matrix $Z$. The secrecy rate for the transmission from the BS to the IR, where the $k$-th ER is taken into account has been shown to be [12]

$$R_{k}(W, Z) = C_{k}(W, Z) - C_{e_k}(W, Z),$$

where

$$C_{k}(W, Z) = \ln |I_{N_i} + (\sigma_i^2 I_{N_i} + HZH^H)^{-1} HWW^H H^H|$$

and

$$C_{e_k}(W, Z) = \ln |I_{N_{e_k}} + (\sigma_{e_k}^2 I_{N_{e_k}} + G_k ZG_k^H)^{-1} G_k W W^H G_k^H|.$$

To guarantee a sufficient energy transfer to each of the ERs, an energy harvesting constraint is employed, which, for the $k$-th ER, is given by [3]

$$\eta_k \text{Tr}\{G_k(WW^H + Z)G_k^H\} \geq E_k,$$

where $0 \leq \eta_k \leq 1$ is the energy harvesting efficiency and $E_k$ is the harvesting power target level at the $k$-th ER.

Then, the secrecy rate maximization problem for the SWIPT system under consideration can be formulated as follows:

$$\begin{align*}
R^*_k &= \max_{W, Z} \min_{k = 1, \ldots, K} R_{k}(W, Z) \\
\text{s.t.} \quad &\eta_k \text{Tr}\{G_k(WW^H + Z)G_k^H\} \geq E_k, \quad \forall k \\
&\text{Tr}\{WW^H + Z\} \leq P, \\
&WW^H \succeq 0_{N_b}, \quad Z \succeq 0_{N_b},
\end{align*}$$

where the goal is to maximize the worst secrecy rate among all the ERs.

The objective function in (6) contains a difference of concave functions. Thus, problem (6) belongs to the class of DC programming problems, which is generally non-convex. Specifically, problem (6) is not only jointly non-convex in $W$ and $Z$, but also non-convex in the variables $(W, Z)$ but also non-convex in either of the variables if the other one is fixed. The authors of [6] have recently proposed an alternating optimization approach for the transmit covariance design in MIMO wiretap channels without energy transfer. By introducing a new optimization variable, the original problem is decomposed into two separable convex problems that are solved in an alternating manner. Here, we extend this method to the case of simultaneous energy transfer in a MIMO beamforming system.

III. PROPOSED ALTERNATING OPTIMIZATION ALGORITHM

In this section, we present the alternating optimization algorithm to address the MIMO SWIPT secrecy maximization problem in (6). We consider the case of a square channel matrix $H$ with $N_b = N_i$ and a non-square channel matrix with $N_b > N_i$ that often occurs in practice and provide solutions for both scenarios.
A. Square Channel Matrix (N_b = N_i)

If the channel H is full-rank and square, i.e., N_b = N_i, according to d ≤ min{N_b, N_i}, we can transmit at most d = N_b = N_i data streams. Thus, the beamforming matrix W is also a square N_b × N_b matrix. By defining X \triangleq WW^H, we can express the secrecy rate maximization problem in (6) as

\[
\max_{X,Z} \min_{k=1,\ldots,K} R_{n_k}(X,Z) \quad (7)
\]

\[
\text{s.t. } \eta_k \text{Tr}\{G_k(X + Z)G_k^H\} \geq E_k, \ \forall k,
\]

\[
\text{Tr}\{X + Z\} \leq P,
\]

\[
X \succeq 0_{N_b}, \ Z \succeq 0_{N_b},
\]

which is still a DC programming problem and therefore non-convex. Note that if W is square, no low-rank constraint is required for the definition of X. The resulting rank of X after the optimization determines the optimal number of data streams in W. Similarly to [6], we introduce a new optimization variable to decompose the problem (7) into two separable convex problems that are solved in an alternating manner. To this end, we apply the following lemma [13]:

Lemma 1. For any matrix E ∈ C^{N \times N} with E ≥ 0, the following identities hold:

\[
\ln |E^{-1}| = \max_{S \succeq 0} \ln |S| - \text{Tr}\{SE\} + N, \quad (8)
\]

\[
- \ln |E^{-1}| = \min_{S \succeq 0} \text{Tr}\{SE\} - \ln |S| - N, \quad (9)
\]

where S ∈ C^{N \times N} and the optimal solution to the right-hand sides of (8) and (9) is S^* = E^{-1}.

Applying Lemma 1 to the objective function of (7) and using the epigraph form [14], we obtain an equivalent formulation of problem (7) as

\[
\max_{X,Z,S} \tau \quad (10)
\]

\[
\text{s.t. } C_1(X,Z,S_0) = \ln \|\sigma_e^2 I_{N_i} + H(X + Z)H^H\| - \text{Tr}\{S_0\},
\]

\[
C_{s_k}(X,Z,S_k) = - \text{Tr}\{S_k(\sigma_e^2 I_{N_i} + HZ_i H_i^H)\} + \ln |S_k|,
\]

where C_1(X,Z,S_0) and C_{s_k}(X,Z,S_k) are given by [6]

\[
C_1(X,Z,S_0) = \ln |\sigma_e^2 I_{N_i} + H(X + Z)H^H| - \text{Tr}\{S_0(\sigma_e^2 I_{N_i} + HZ_i H_i^H)\} + \ln |S_0|,
\]

\[
C_{s_k}(X,Z,S_k) = - \text{Tr}\{S_k(\sigma_e^2 I_{N_i} + G_k(X + Z)G_k^H)\} - \ln |S_k|.
\]

Note that we have applied the identities (8) and (9) from Lemma 1 to C_1(X,Z,S_0) and C_{s_k}(X,Z,S_k), respectively.

Although the reformulated problem (10) is still jointly non-convex in the variables (X,Z) and S_0, ℓ = 0, \ldots, K, one can show that it is now convex with respect to either (X,Z) or S_0 while the other variable is fixed. Hence, we can separate the problem into two convex optimization problems and perform alternating optimization. Particularly, we perform n = 1, 2, \ldots iterations until convergence over:

\[
X^{(n)}, Z^{(n)} = \arg \max_{X,Z,\tau} \quad \tau \quad (11)
\]

\[
\text{s.t. } C_1(X,Z,S_0^{(n-1)}) - C_{s_k}(X,Z,S_k^{(n-1)}) \geq \tau, \ \forall k,
\]

\[
\eta_k \text{Tr}(G_k(X + Z)G_k^H) \geq E_k, \ \forall k,
\]

\[
\text{Tr}\{X + Z\} \leq P, \ \ X \succeq 0_{N_b}, \ Z \succeq 0_{N_b},
\]

\[
S_0^{(n)} = \begin{cases} S_0 \geq 0_{N_b}, \ &\text{if } \ell = 0 \\ \arg \min_{S_0 \geq 0_{N_b}} C_{s_k}(X^{(n)},Z^{(n)},S_0), &\text{if } \ell = k. \end{cases} \quad (12)
\]

Algorithm 1: The alternating optimization (AO) algorithm for solving the problem (7)

1: Initialize n = 1, ϵ > 0, X^{(0)} = Z^{(0)} = (P/2N_b)I_{N_b};
2: while |R_{n_k}^{(n)} - R_{n_k}^{(n-1)}| > ϵ do
3: Solve (13) to obtain S_0 and S_{k}^{(n)} \forall k;
4: Solve (11) to obtain X^{(n)} and Z^{(n)};
5: n = n + 1;
6: end while
7: Output: X^{(n)}, Z^{(n)}.

It can be shown by using Lemma 1 that the second problem (12) has the closed-form solution:

\[
S_0^{(n)} = \begin{cases} (\sigma_e^2 I_{N_i} + HZH_i^H)^{-1}, &\text{if } \ell = 0 \\ (\sigma_e^2 I_{N_i} + G_k(X + Z)G_k^H)^{-1}, &\text{if } \ell = k. \end{cases} \quad (13)
\]

Therefore, in each iteration, we solve the problem (11) using interior point methods and (13) to eventually obtain the solutions X^* and Z^* after convergence. It should be mentioned that the applied alternating optimization procedure has been shown to converge to a Karush-Kuhn-Tucker (KKT) optimal point [6], i.e., each iteration step results in a non-decreasing objective value. From the eigendecomposition of X^*, we can subsequently extract the beamforming matrix W, which is formed by the eigenvectors corresponding to the dominant eigenvalues of X^*. As mentioned before, the rank of X^* and therefore the number of columns of W determine the optimal number of data streams for the transmission to the IR.

We summarize the above-developed alternating optimization procedure in Algorithm 1.

B. Non-Square Channel Matrix (N_b > N_i)

In this subsection, we consider the case, where the channel matrix H is not necessarily square. Specifically, we assume that N_b > N_i such that at most d = N_i data streams can be transmitted over the channel. Note that MIMO systems, where the BS has more antennas than the user terminals, often occur in practice. For such a scenario, the beamforming matrix W is of rank d and not square. Thus, Algorithm 1 is not readily applicable as using the definition X \triangleq WW^H in (7) would impose the rank constraint rank{X} ≤ N_i, which requires semidefinite relaxation (SDR) [15] to solve the resulting optimization problem. As it is well-known, dropping
the rank constraint and thereby relaxing the constraint set may only lead to an approximate solution of $W$ with a rank higher than $N_i$. Consequently, to obtain a solution of rank $N_i$, rank-1 randomization techniques are usually applied. Depending on the number $N_i$, rank-1 randomization [16], [17], rank-2 randomization [18] and rank-$r$ randomization [19] methods can be applied.

In order to avoid the rank relaxation problem, we here propose a different solution. Similarly to the popular concept of eigen-beamforming, we transmit the data into the eigenspace of the channel, which allows for a reduction of the effective channel to a square matrix of rank $N_i$. Afterwards, Algorithm 1 can be applied to solve the corresponding optimization problem.

Since we assume that perfect CSI is available at the BS, we can compute the truncated singular value decomposition (SVD) of the channel $H$ as

$$H = U_r \Sigma_r V_r^H,$$

where $r = N_i$ is the rank of $H$, $U_r \in \mathbb{C}^{N_i \times N_i}$ and $V_r \in \mathbb{C}^{N_i \times N_i}$ are the left and right singular vectors and $\Sigma_r$ contains the $N_i$ non-zero singular values on its diagonal. Thus, instead of (1), we now transmit the $N_i \times 1$ vector

$$\tilde{x} = V_r \tilde{w} + z,$$

where $\tilde{w}$ and $\tilde{z}$ are the new beamforming matrix and the new transmit symbol vector of size $N_i \times N_i$ and $N_i \times 1$, respectively. It is evident that by inserting (15) into the model (3a), the effective channel for the data vector $\tilde{w}$ to the IR becomes $HV\tilde{w} = U_r \Sigma_r W = \tilde{H}W$, where $\tilde{H} \in \mathbb{C}^{N_i \times N_i}$. Hence, we have reduced the original channel $H$ to the square channel $\tilde{H}$, which was already considered in Subsection A. As a consequence, we perform the optimization of $(W, Z)$ instead of $(W, X)$ and the definition $\tilde{X} \triangleq WW^H$ does not impose a rank constraint anymore. This implies that SDR and, potentially, the above-mentioned randomization techniques are not required.

Formulating the secrecy rate at the $k$-th ER according to (4) for the new transmit vector in (15) and following the steps in Subsection A, we can express the SWIPT secrecy maximization problem for $N_i < N_b$ similarly to (10) as

$$\max_{\tilde{x}, Z, \{S_k\}_{k=0}^{K-1}} \tau$$

s.t.

$$\tilde{C}_1(\tilde{X}, Z, S_k) - \tilde{C}_{\text{ek}}(\tilde{X}, Z, S_k) \geq \tau, \quad \forall k$$

$$\eta_k \text{Tr}(G_k(\tilde{V}_k X_k V_k^H + Z)G_k^H) \geq E_k, \quad \forall k$$

$$\text{Tr}(\tilde{V}_k \tilde{X}_k V_k^H + Z) \leq P,$$

$$\tilde{X} \geq 0_{N_i}, \quad Z \geq 0_{N_b},$$

where

$$\tilde{C}_1(\tilde{X}, Z, S_k) = \ln |\sigma^2 I_{N_i} + H (V_r X_r V_r^H + Z) H^H|$$

$$- \text{Tr}(S_k(\sigma^2 I_{N_b} + H Z H^H)) + \ln |S_k|,$$

$$\tilde{C}_{\text{ek}}(\tilde{X}, Z, S_k) = - \ln |\sigma^2 I_{N_b} + G_k Z G_k^H|$$

$$+ \text{Tr}(S_k(\sigma^2 I_{N_b} + G_k (V_r X_r V_r^H + Z) G_k^H)) - \ln |S_k|.$$

In analogy to problem (10), the problem (16) is convex with respect to either $(\tilde{X}, Z)$ or $S_k$ while the other variable is fixed. Thus it can be separated into two convex optimization problems. Subsequently, we can apply the same alternating optimization procedure as in the previous subsection and apply Algorithm 1 to solve problem (16).

### IV. Simulation Results

In this section, we provide simulation results that demonstrate the performance of the proposed AN-aided alternating optimization algorithm “AO-AN” to address the secrecy maximization problem in a SWIPT MIMO beamforming system. For comparison purposes, we include the non-AN version “AO-no-AN” of our proposed algorithm, which can be easily derived by using the same steps as in the derivation of the “AO-AN” algorithm. Furthermore, we also include the conventional “waterfilling” algorithm based on the SVD of $H$ that only allocates power across the IR channel irrespectively of the ERs.

In our simulations, the channels $H$ and $G_k$, $k = 1, \ldots, K$, are randomly generated and drawn from an i.i.d. complex Gaussian distribution (Rayleigh fading) with zero mean and unit variances. For simplicity, we set the noise powers $\sigma^n_\mathbf{e}$ and $\sigma^n_{\text{ek}}$, $\forall k$ to 0 dBm. The energy harvesting efficiency is $\eta_k = 50 \%$, $\forall k$ and all the ERs have the same target harvesting power level. The proposed algorithm is initialized with $\mathbf{X}^{(0)} = \mathbf{Z}^{(0)} = (P/2N_b)I_{N_b}$ and the tolerance level for the stopping criterion is set to $\epsilon = 10^{-5}$. The results are obtained by averaging over 100 independent Monte Carlo trials.

Fig. 2 illustrates the worst secrecy rate as a function of the transmit power $P$ for a setting of $N_b = 4$, $N_i = 3$, $K = 2$, and $N_{ek} = 2$. The target harvesting power level is given by $E_k = 0$ dBm, $\forall k$. It can be seen that the proposed algorithm AO-AN outperforms its non-AN counterpart AO-no-AN, which illustrates the gain obtained from an AN-aided transmission.
Both algorithms perform better than the waterfilling scheme that ignores the ERs.

In Fig. 3, we depict the worst secrecy rate as a function of the harvested power $E_k$, $\forall k$, where $N_h = N_i = 3$, $K = 3$, and $N_{ah} = 2$, $\forall k$ and $P = 10$ dBm. It is apparent that the secrecy rate decreases as the harvesting power grows, which highlights the trade-off between these two objectives. Again, the AO-AN algorithm performs significantly better than the AO-no-AN scheme, whereas the waterfilling method provides the worst performance. Note that the waterfilling scheme provides a constant curve as it is independent of the harvesting power.

V. CONCLUSION

In this paper, we have considered a MIMO SWIPT system, where a BS transmits both information and energy to a single IR and multiple ERs. Assuming that the ERs can decode the information intended for the IR, the physical layer secrecy issue of providing confidential information to the IR while still ensuring sufficient energy to be harvested by the ERs arises. Based on an AN approach at the transmitter, we have addressed the secrecy rate maximization by jointly optimizing the beamforming matrix for the IR and the covariance matrix of the AN subject to energy harvesting constraints and a total power constraint. Although this problem is a non-convex DC problem, we consider the two cases of a square and a non-square IR channel matrix and present an alternating optimization (AO-AN) procedure for both scenarios. The performance of the proposed algorithm has been demonstrated by simulations.

REFERENCES