Dynamical affinity in opinion dynamics modeling

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We propose here a model to simulate the process of opinion formation, which accounts for the mutual affinity between interacting agents. Opinion and affinity evolve self-consistently, manifesting a highly nontrivial interplay. A continuous transition is found between single and multiple opinion states. Fractal dimension and signature of critical behavior are also reported. A rich phenomenology is presented and discussed with reference to corresponding psychological implications.

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The paradigms of complex systems are nowadays being applied to an ample spectrum of interdisciplinary problems, ranging from molecular biology to social sciences. The challenge is to model the dynamical evolution of an ensemble made of interacting, microscopic constituents and infer the emergence of collective, macroscopic behaviors that are then eventually accessible for direct experimental inspection. Statistical mechanics and nonlinear physics provide quantitative tools to elucidate the key mechanisms underlying the phenomena under scrutiny, often resulting in novel interpretative frameworks. Agent-based computational models have been widely employed for simulating complex adaptive systems, in particular with reference to sociophysics applications. Within this context, opinion dynamics has recently attracted a growing interest clearly testified by the vast production of specialized contributions [1]. Peculiar aspects of its intrinsic dynamics make opinion formation a rich field of analysis where self-organization, clustering, and polarization occur.

Opinion dynamics models can be ideally grouped into two large classes. The first deals with binary opinions: Agents behave similarly to magnetic spins and just two states are allowed (up or down) [2]. Here social actors update their opinions driven by a social influence pressure, which often translates into a majority rule. Alternatively, opinions can be schematized with continuous variables, the latter being dynamically evolved as a result of subsequent interactions among individuals. In the celebrated Deffuant *et al.* [3] model, agents adjust their opinion as a result of random binary encounters whenever their difference in opinion is below a given threshold. The rationale behind the threshold ansatz reflects humans' natural tendency to avoid conflicting interests and consequently ignore the perception of incompatibility between two distant cognitions. In this respect, the threshold value measures the average *openness of mind* of the community.

In real life, the difference in opinion on a debated issue is indeed playing a crucial role. However, the actual outcome of a hypothetic binary interaction also relies on a number of other factors, which supposedly relate to the quality of the interpersonal relationships. Mutual *affinity* condensates in fact past interactions' history and contribute to select preferential interlocutors for future discussions. Previous attempts aimed at incorporating this effect resulted in static descriptions, which deliberately disregarded affinity's self-consistent evolution [4]. In this article we take one step forward by proposing a formulation where the affinity is dynamically coupled to the opinion, and consequently updated in time. Moreover, affinity translates in a social distance, a concept that is introduced here to drive preferential interactions between affine individuals. Macroscopically, the system is shown to asymptotically organize in clusters of agents sharing a common opinion, whose number depends on the choice of the parameters involved. Interestingly, a continuous transition is identified that separates the monoclustered from the fragmented phase. Scaling laws are also found and their implications discussed. Most importantly, our proposed theoretical scenario captures the so-called *cognitive dissonance* phenomenon, a qualitatively well documented theory in psychology pioneered by Leon Festinger in 1956 [5].

Consider a population of N agents, each bearing at time t a scalar opinion $O_i^t \in [0,1]$. Moreover, let us introduce the $N \times N$ time dependent matrix α^t, whose elements α_{ij}^t are bound to the interval [0,1]. Such elements specify the affinity of individual i versus j, larger numbers being associated to more trustable relationships. Both the opinions vector and the affinity matrix are randomly initialized at time $t=0$. At each time step t, two agents, say i and j, are selected according to a strategy that we shall elucidate in the forthcoming discussion. They interact and update their characteristics according to the following recipe [9]:

$$O_i^{t+1} = O_i^t - \mu \Delta O_{ij}^t \Gamma_1(\alpha_{ij}^t), \tag{1}$$

$$\alpha_{ij}^{t+1} = \alpha_{ij}^t + \alpha_{ij}^t [1 - \alpha_{ij}^t] \Gamma_2(\Delta O_{ij}), \tag{2}$$

where the functions Γ_1 and Γ_2 respectively read

$$\Gamma_1(\alpha_{ij}^t) = \frac{1}{2}\{\tanh[\beta_1(\alpha_{ij}^t - \alpha_c)] + 1\}, \tag{3}$$

$$\Gamma_2(\Delta O_{ij}) = -\tanh[\beta_2(|\Delta O_{ij}^t| - \Delta O_c)]. \tag{4}$$

Here, $\Delta O_{ij}^t = O_i^t - O_j^t$, while α_c and ΔO_c are constant parameters. For the sake of simplicity we shall consider the limit $\beta_{1,2} \to \infty$, which practically amounts to replacing the hyper-

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bolic tangent with a simpler step function profile. Within this working assumption, the function Γ_1 is 0 or 1, while Γ_2 ranges from -1 to 1, depending on the value of the arguments. Γ_1 and Γ_2 act therefore as effective switchers. Notice that, for $\alpha_c \to 0$, Eq. (1) reduces to the Deffuant *et al.* scheme [3]. To clarify the ideas inspiring our proposed formulation, we shall focus on specific examples. First, suppose two subjects meet and imagine they confront their opinions, assumed to be divergent ($|\Delta O_{ij}| \simeq 1$). According to Deffuant's model, when the disagreement exceeds a fixed threshold, the agents simply stick to their positions. Conversely, in the present case, the interaction can still result in a modification of each other beliefs, provided the mutual affinity α_{ij}^t is larger than the reference value α_c. In other words, individuals exposed to conflicting thoughts have to resolve such dissonance in opinion by taking one of two opposite actions: If $\alpha_{ij}^t < \alpha_c$, the agent ignores the contradictory information, which is therefore not assimilated; when instead the opinion is coming from a trustable source ($\alpha_{ij}^t > \alpha_c$), the agent is naturally inclined to seek consistence among the cognitions, and consequently adjust its belief. The mechanism here outlined is part of Festinger's cognitive dissonance theory [5]: Contradicting cognitions drive the mind to modify existing beliefs to reduce the amount of dissonance (conflict) between cognitions, thus removing the feeling of uncomfortable tension. The scalar α_{ij} schematically accounts for a larger number of hidden variables (personality, attitudes, behaviors, etc.), which are nontrivially integrated in an abstract affinity concept. Notice that the matrix α^t is nonsymmetric: Hence, following a random encounter between two dissonant agents, one could eventually update his opinion, with the other still keeping his own view. A dual mechanism governs the self-consistent evolution for the affinity elements; see Eq. (2). If two people gather together and discover they share common interests ($|\Delta O_{ij}^t| < \Delta O_c$), they will increase their mutual affinity ($\alpha_{ij}^t \to 1$). On the contrary, the fact of occasionally facing different viewpoints ($|\Delta O_{ij}^t| > \Delta O_c$) translates into a reduction of the affinity indicator ($\alpha_{ij}^t \to 0$). The logistic contribution in Eq. (2) confines α_{ij}^t in the interval [0,1]. Moreover, it maximizes the change in affinity for pairs with $\alpha_{ij}^t \simeq 0.5$, corresponding to agents which have not come in contact often. Couples with $\alpha_{ij}^t \simeq 1$ (0) have already formed their mind and, as expected, behave more conservatively.

Before illustrating the result of our investigations, we shall discuss the selection rule implemented here. First the agent i is randomly extracted, with uniform probability. Then we introduce a new quantity d_{ij}, hereafter termed *social distance*, defined as [10]

$$d_{ij}^t = \Delta O_{ij}^t (1 - \alpha_{ij}^t), \quad j = 1, \ldots, N, \; j \neq i. \tag{5}$$

The smaller the value of d_{ij}^t the closer the agent j to i, both in terms of affinity and opinion. A random, normally distributed, vector $\eta_j(0,\sigma)$ of size $N-1$ is subsequently generated, with mean zero and variance σ. The social distance is then modified into the new social metric $D_{ij}^{\eta} = d_{ij}^t + \eta_j(0,\sigma)$. Finally, the agent j which is closer to i with respect to the measure D_{ij}^{η} is selected for interaction. The additive random perturbation η is hence acting on a fictitious one-dimensional (1D) manifold, which is introduced to define the pseudo-particle (agent) interaction on the basis of a nearest neighbors selection mechanism. η is thus formally equivalent to a *thermal noise* [6]. Based on this analogy, σ is here baptized *social temperature* and sets the level of mixing in the community. Notably, for any value of σ, it is indeed possible that agents initially distant in the unperturbed social space d_{ij}^t mutually interact: Their chances to meet increase for larger values of the social temperature.

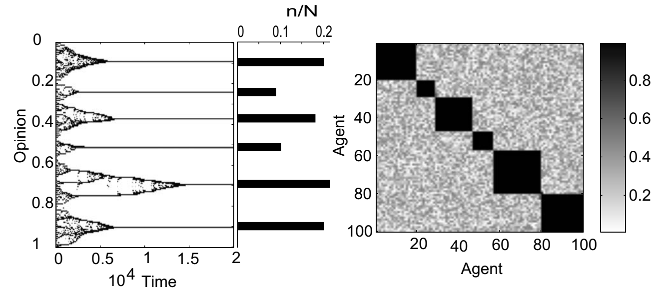

FIG. 1. Left: Typical evolution of the opinion vs time, i.e., number of iterations. Right plot: Final affinity matrix. Here $\sigma = 0.02$, $\Delta O_c = 0.5$, and $\alpha_c = 0.5$. Initial opinions are (random) uniformly distributed within the interval [0,1]. α_{ij}^0 is initialized with uniform (random) values between 0 and 0.5. Here, $\beta_1 = \beta_2 = 1000$.

Numerical simulations are performed and the dynamical evolution of the system monitored. Qualitatively, asymptotic clusters of opinion are formed, whose number depends on the parameters involved. The individuals that reach a consensus on the question under debate are also characterized by large values of their reciprocal affinity, as clearly displayed in Fig. 1. The final scenario results from a nontrivial dynamical interplay between opinion and affinity: The various agglomerations are hence different in size and centered around distinct opinion values, which cannot be predicted *a priori*. The dynamics is therefore significantly more rich, and far more realistic, than that arising within the framework of the original Deffuant *et al.* scheme [3], where cluster number and average opinions are simply related to the threshold amount. Notice that, in our model, the affinity enters both the selection rule and the actual dynamics, these ingredients being crucial to reproduce the observed self-organization.

To gain quantitative insight into the process of opinion formation, we run several simulations relative to different initial realizations and record the final (averaged) number of clusters, N_c, as function of the social temperature σ, for different values of the critical parameter α_c. Results of the numerics are reported in Fig. 2. All the curves are approximately collapsed together plotting N_c as function of the rescaled quantity $(\sigma\alpha_c)^{-1/2}$. A continuous phase transition is identified, above which the system is shown to asymptotically fragment in several opinion clusters. The proposed scaling is sound in terms of its psychological interpretation. When α_c gets small the barrier in affinity fades off and the agents update their beliefs virtually at any encounter. The imposed selection rule drives a rapid evolution toward an asymptotic fragmented state, by favoring the interaction of candidates that share a similar view (ΔO_{ij} small). This tendency can be counterbalanced by adequately enhancing the

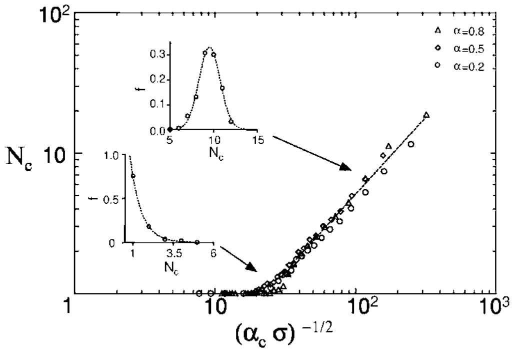

FIG. 2. Average number of clusters as function of the rescaled quantity $(\sigma\alpha_c)^{-1/2}$. A phase transition is found at $(\sigma\alpha_c)^{-1/2}\simeq 20$. Above the transition, histograms of the number of clusters are computed and enclosed as insets in the main frame: Symbols refer to the numerics; solid lines are fitted interpolation. Here, $\Delta O_c=0.5$. The variables O_i^0 and α_{ij}^0 are initialized as described in the caption of Fig. 1.

social mixing, which in turn amounts to increasing the value of $\sigma \propto \alpha_c^{-1}$. On the other hand, for large values of α_c the system is initially experiencing a lethargic regime, due to the hypothesized thresholding mechanism. Agents' opinions are therefore temporarily freezed to their initial values, while occasional encounters contribute to increasing the degree of cohesion (synchronization) of the community. As the affinity grows, the social metric D_{ij} becomes less sensitive to ΔO_{ij} and the system naturally flows toward an ordered (single-clustered) configuration. Notice that our system displays intriguing similarities with granular media that have been shown to develop analogous self-organization features. This entails the possibility of addressing the analysis of the observed structures within a purely statistical mechanics setting, where the balance between competing effects is explicitly modeled [7].

Aiming at further characterizing the process of convergence we have also analyzed the following indicators: The fractal dimension of the orbits topology and the distribution of opinion differences. First, we focused on the single-clustered phase (main plot in Fig. 3) and calculated the fractal dimension in the (O,t) plane, a parameter that relates to the geometrical aspects of the dynamical evolution. A standard box-counting algorithm is applied, which consists in partitioning the plan in small cells and identifying the boxes visited by the system trajectory. In this specific case, the space (O,t) is mapped into $[0,1]\times[0,1]$, and covered with a uniform distribution of squares of linear size l. The number of filled box N_b is registered and the measure repeated for different choices of l. In particular we set $l=2^{-n_b}$, where $n_b=1,2,\ldots$. For each n_b, N_b is plotted versus l, in log-log scale (see inset of Fig. 3): A power-law decay is detected, whose exponent $\gamma\simeq 1.57$ quantifies the fractal dimension. The orbits are also analyzed in the multiclustered regime and similar conclusions are drawn. In addition, every single cluster is isolated and studied according to the above procedure, leading to an almost identical γ. In Fig. 4 we also report the probability distribution function of $\delta O=|O_i^{t+1}-O_i^t|$. δO measures the rate of change of individuals' opinions. A power-law behavior is found which is an additional sign of the system's criticality.

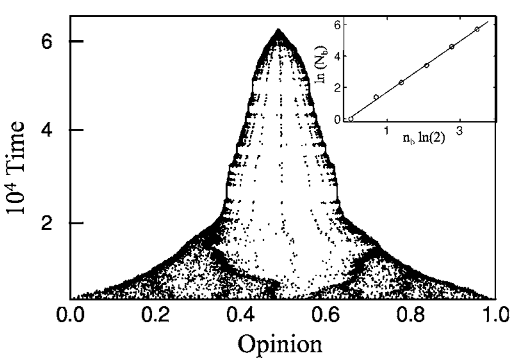

FIG. 3. Main plot: Typical evolution in the monoclustered phase. Inset: N_b vs $l=2^{-n_b}$ in log-log scale. For the choice of the parameters refer to the caption of Fig. 2

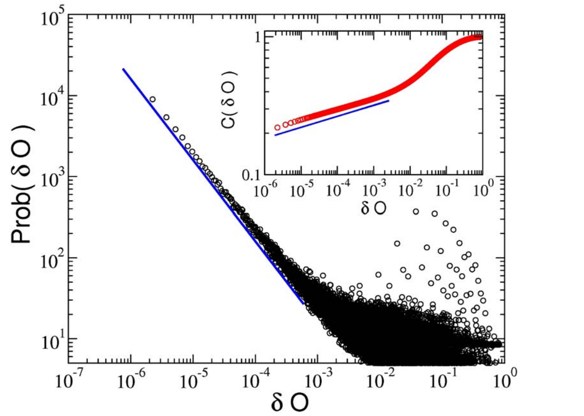

FIG. 4. (Color online) Main frame: Histogram of δO, as follows from the numerics ($N=100$, averaged over 1000 independent realizations), plotted in log-log scale (symbols). The solid line is a guide for the eye. Inset: Cumulative distribution of the differences δO, in log-log scale.

Finally, working in the relevant monoclustered regime, we also performed a dedicated campaign of simulations to estimate the convergence time, $T_c^\sigma(\alpha_c)$, i.e., the time needed to completely form the cluster under scrutiny. The experiments are conducted fixing the social temperature σ, and allowing α_c to span the interval $[0,\alpha_{\max}]$, where $\alpha_{\max}=\max_{i,j}\alpha_{ij}^0$. In Fig. 5 the rescaled convergence time $T_c^\sigma(\alpha_c)/T_c^\sigma(0)$ is plotted as a function of α_c, for various choices of σ. All the different curves nicely collapse together, revealing an interesting positive correlation between the relative convergence time and the threshold α_c. Again, this finding is certainly bound to reality: When α_c increases, individuals stick more rigidly to their opinion and changes happen only when encounters among neighbors occur. Instead, when reducing α_c large jumps in opinion are allowed which dynamically translate in a more effective mixing, hence faster convergence. To make this argument more rigorous, introduce $\mu'=\mu\{\tanh[\beta_1(\alpha_{ij}^t-\alpha_c)]+1\}/2$. A reduced dynamical formulation can be obtained by averaging out the dependence on $\alpha_{i,j}$ in Eq. (1), thus formally decoupling it from Eq. (2). This is accomplished, at fixed i, as follows:

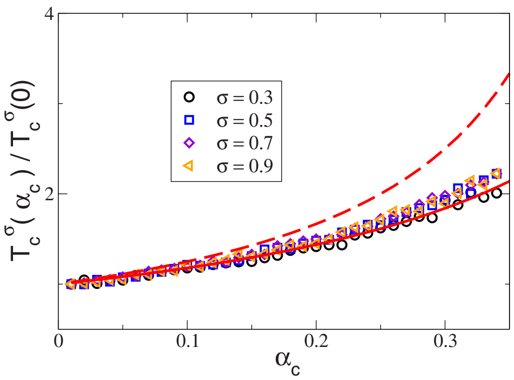

FIG. 5. (Color online) Rescaled convergence time $T_c^\sigma(\alpha_c)/T_c^\sigma(0)$ is plotted as function of α_c. Different symbols refer to different values of the social temperature σ; see legend. The dashed line stands for the theoretical prediction (7). The solid line is a numerical fit based on Eq. (7), where $\alpha_{\max}$ is replaced by the effective value α_{eff}=0.66 (see main text for further details).

$$\langle \mu' \rangle = \mu \int \Gamma_1(\alpha_{ij}^t) f_t(\alpha_{ij}^t) d\alpha_{ij}^t \simeq \mu \int_0^{\alpha_{\max}} \Gamma_1(\alpha_{ij}^0) f_0(\alpha_{ij}^0) d\alpha_{ij}^0 \simeq \mu \frac{\alpha_{\max} - \alpha_c}{\alpha_{\max}}, \tag{6}$$

where in the last passage we made use of the fact that $\beta_1 \to \infty$ and $f_0(\alpha_{ij}^0) = 1/\alpha_{\max}$ as it follows from the normalization condition. The function $f_t(\cdot)$ $[f_0(\cdot)]$ represents the affinity distribution of agents j versus i, at time t (at time zero). Within this simplified scenario, the time of convergence scales as $1/\langle \mu' \rangle$ [8] and therefore expression (7) immediately yields

$$\frac{T_c^\sigma(\alpha_c)}{T_c^\sigma(0)} = \frac{\alpha_{\max}}{\alpha_{\max} - \alpha_c}. \tag{7}$$

Relation (7) is reported in Fig. 5 (dashed line) and shown to approximately reproduce the observed functional dependence. A good agreement with direct simulations is found for small α_c. It however progressively deteriorates for larger α_c, due to nonlinear contributions. The latter can be incorporated into our scheme by replacing $\alpha_{\max}$ in Eq. (7) with an effective value α_{eff}, to be determined via numerical fit (solid line in Fig. 5). Such a value accounts for the system tendency to populate the complementary domain $1-\alpha_{\max}$ and results in an excellent agreement with the simulated data.

In this article we introduced a model for studying the process of opinion formation. This interpretative framework allows us to account for the affinity, which is an effect of paramount importance in real social systems. The model proposed here captures the essence of the cognitive dissonance theory, a psychological construction elaborated by Festinger in the late 1950s. Numerical investigations are carried on and reveal the presence of a phase transition between an ordered (single clustered) and a disordered (multiclustered) phase. Evidence of critical behaviors is provided, and the role of different parameters elucidated. We firmly believe that our formulation represents a leap forward in social system modeling, thus opening up new perspectives to reinforce the ideal bridge with the scattered psychology community.

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[9] The evolution of the quantities $O_j(t)$ and $\alpha_{ij}(t)$ is straightforwardly obtained by switching the labels i and j in the equations.
[10] The affinity can mitigate the difference in opinion, thus determining the degree of social similarity of two individuals. This observation translates into the analytical form here postulated for d_{ij}^t.