New equations of state in simulations of core-collapse supernovae

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in collaboration with:

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New equations of state in simulations of core-collapse supernovae

Outline:
1.) introduction, EOS model, constraints
2.) EOS aspects in simulations at
   a.) sub-saturation densities
   b.) supra-saturation densities and high temperatures
3.) conclusions
Supernova EOS – Introduction

• EOS provides the crucial nuclear physics input for astrophysical simulations: thermodynamic quantities and nuclear composition

• plenty of EOSs for cold neutron stars

• challenge of the supernova EOS:
  – finite temperature: $T = 0 – 100$ MeV
  – wide density range: $\rho = 10^4 – 10^{15}$ g/cm$^3$
  – no weak equilibrium: $Y_e = 0 – 0.6$
  – EOS in tabular form, ~1 million grid-points ($T, Y_e, \rho$)
  – sub-saturation densities: nuclei/non-uniform nuclear matter/crust

• SN EOS: multi-purpose EOS
Supernova EOS – Introduction

• most used SN EOSs:
  • Lattimer & Swesty 1991 (LS): non-relativistic liquid drop model
  • H. Shen, Toki, Omayatsu and Sumiyoshi 1998 (STOS/Shen): relativistic mean-field (RMF), Thomas-Fermi approximation

• both models:
  – one representative nucleus: “single nucleus approximation”
  – no shell effects
  – only $\alpha$-particles of light clusters
# Available Supernova EOS

<table>
<thead>
<tr>
<th>Authors</th>
<th>EOS Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattimer &amp; Swesty (1991) (LS)</td>
<td>Skyrme interactions, compressible liquid-drop model, three different compressibilities, and tabulated variants</td>
</tr>
<tr>
<td>H. Shen et al. (1998) (STOS)</td>
<td>table for TM1, relativistic mean-field (RMF), Thomas-Fermi approximation</td>
</tr>
<tr>
<td>MH &amp; Schaffner-Bielich (2010) (HS)</td>
<td>tables for NL3, TM1, TMA, FSUgold, DD2, SHFo, SHFx: NSE, RMF, excluded volume</td>
</tr>
<tr>
<td>G. Shen, Horowitz, Teige (2010)</td>
<td>tables for NL3 and FSUgold: virial expansion, RMF, Hartree</td>
</tr>
<tr>
<td>Nakazato et al. 2008</td>
<td>quark matter with large $n_c$ added to STOS</td>
</tr>
<tr>
<td>Ishizuka et al. 2008</td>
<td>hyperons added to STOS</td>
</tr>
<tr>
<td>Sagert et al. 2009</td>
<td>quark matter with low $n_c$ added to STOS $\rightarrow$ explosions in 1D</td>
</tr>
<tr>
<td>H. Shen et al. 2010</td>
<td>lambdas added to STOS</td>
</tr>
</tbody>
</table>
EOS model: excluded volume NSE with interactions


- chemical mixture of nuclei and interacting nucleons in nuclear statistical equilibrium
- nuclei: theoretical and experimental nuclear mass tables
  - nuclear shell effects
  - ensemble of heavy nuclei and all possible light nuclei
- nucleons: full Fermi-Dirac statistics, various relativistic mean-field (RMF) interactions
- excluded volume effects:
  - interplay nuclei vs. unbound nucleons
  - smooth transition to uniform nuclear matter
- thermodynamic consistent & stable
EOS model: excluded volume NSE with interactions


• seven EOS tables for different RMF interactions:
  NL3, TM1, TMA, FSUgold, DD2, SHFo, SHFx
• three additional tables for TM1 with selection of light nuclei:
  \{\alpha\}, \{\alpha,d\}, \{\alpha,d,t\}
• abundances of all nuclei provided by program

http://phys-merger.physik.unibas.ch/~hempel/eos.html
Nucleons – non-linear RMF (TM1, TMA, FSUgold, NL3)

- relativistic mean-field model (RMF)
- interactions mediated via exchange of mesons and meson (self-) interactions

\[ L = \bar{\psi} (i \gamma^\mu \partial_\mu - M) \psi \]
\[ \quad + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - g_\sigma \bar{\psi} \sigma \psi \]
\[ \quad - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega \omega_\mu \omega^\mu + \frac{1}{4} g_4 (\omega_\mu \omega^\mu)^2 - g_\omega \bar{\psi} \gamma^\mu \psi \omega_\mu \]
\[ \quad - \frac{1}{4} R^a_{\mu\nu} R^{a\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{a\mu} - g_\rho \bar{\psi} \gamma_\mu \tau^a \psi \rho^{\mu a} - \Lambda \omega_\mu \omega^\mu \rho_\nu^a \rho^{av} \]

- alternative: density-dependent coupling constants (DD2)
- coupling constants fitted to experimental data
- well-established description of finite nuclei and nuclear matter

Sugahara & Toki 1994
Toki et al. 1995
Todd-Rutel & Piekarewicz 2005
Lalazissis et al. 1997
Typel 2005, Typel et al. 2010
EOS constraints – mass and radius measurements

- Bayesian analysis of observations of six NS (X-ray burster, low-mass X-ray binaries), Steiner et al. ApJ 2010
- Similar results from Chiral EFT (Hebeler et al. 2010)
- New SN EOS fitted to observations: Andrew Steiner (INT), Tobias Fischer (GSI Darmstadt)
- Compact PNS → more binding energy release in a SN
EOS constraints – mass and radius measurements

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[A. Steiner, MH & T. Fischer, in preparation]
EOS constraints – symmetry energy

- convergence of observational, experimental and theoretical constraints
- standard non-linear RMF in disagreement (TM1, NL3, TMA)
- two new RMF models/SN EOS: SHFo (optimal) SHFx (extreme)

- G: Gandolfi et al. 2012: quantum Monte-Carlo
- H: Hebeler et al. 2010: Chiral EFT, neutron matter

[Lattimer & Lim, arXiv:1203.4286]
### EOS constraints – saturation properties & maximum mass

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>TM1</td>
<td>0.146</td>
<td>-16.31</td>
<td>282</td>
<td>36.95</td>
<td>110.99</td>
</tr>
<tr>
<td>TMA</td>
<td>0.147</td>
<td>-16.03</td>
<td>318</td>
<td>30.66</td>
<td>90.14</td>
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<tr>
<td>FSUg 10d</td>
<td>0.148</td>
<td>-16.27</td>
<td>230</td>
<td>32.56</td>
<td>60.44</td>
</tr>
<tr>
<td>NL3</td>
<td>0.148</td>
<td>-16.24</td>
<td>271</td>
<td>37.39</td>
<td>118.50</td>
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<tr>
<td>DD2</td>
<td>0.149</td>
<td>-16.02</td>
<td>243</td>
<td>31.67</td>
<td>55.04</td>
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<tr>
<td>SHFo</td>
<td>0.158</td>
<td>-16.19</td>
<td>245</td>
<td>31.57</td>
<td>47.10</td>
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<tr>
<td>SHF 10d</td>
<td>0.160</td>
<td>-16.16</td>
<td>239</td>
<td>28.67</td>
<td>23.18</td>
</tr>
<tr>
<td>LS 180</td>
<td>0.155</td>
<td>-16.00</td>
<td>180</td>
<td>28.61</td>
<td>73.82</td>
</tr>
<tr>
<td>LS 250</td>
<td>0.155</td>
<td>-16.00</td>
<td>220</td>
<td>28.61</td>
<td>73.82</td>
</tr>
</tbody>
</table>

• span a broad range of possible RMF models
• provide a “best fit“ EOS

### references

<table>
<thead>
<tr>
<th>type of constraint</th>
<th>references</th>
</tr>
</thead>
<tbody>
<tr>
<td>compilation of measurements of isoscalar giant monopole resonances</td>
<td>Piekarewicz JPG 2010 [1]</td>
</tr>
<tr>
<td>measurement of Shapiro delay</td>
<td>Demostre et al. Nature 2010 [3]</td>
</tr>
</tbody>
</table>
EOS aspects at subsaturation densities

- heavy nuclei
Supernova simulations


• simulations by Tobias Fischer, GSI Darmstadt
  – general relativistic radiation hydrodynamics in spherical symmetry
  – three flavor Boltzmann neutrino transport

• weak reactions:
  – all light clusters treated as alpha-particles
  – only average heavy nucleus

\[ e^- + <A, Z> \leftrightarrow <A, Z-1> + \nu_e \]

\[ e^- + p \leftrightarrow n + \nu_e \]
\[ e^+ + n \leftrightarrow p + \bar{\nu}_e \]
\[ \nu + e^\pm \leftrightarrow \nu + e^\pm \]
\[ \nu + N \leftrightarrow \nu + N \]
\[ N + N \leftrightarrow N + N + \nu + \bar{\nu} \]
\[ \nu_e + \bar{\nu}_e \leftrightarrow \nu_{\mu/\tau} + \bar{\nu}_{\mu/\tau} \]

→ mainly sensitive to differences in thermodynamic quantities

Heavy nuclei

\( t_{pb} = -40 \text{ ms} \)

- systematic overprediction of \( A \) in STOS \( \rightarrow \) underprediction of e-captures (Bruenn `85 rates)
- indirect effect on \( X_p \) \( \rightarrow \) higher \( Y_e \) in outer layers
- correct description of nuclei as important as nuclear interactions
Heavy nuclei & e-captures

Hix et al. 03

Langanke et al. 03

• similar trends as in LMP rates, just due to the EOS
• wiggles in $Y_e$ due to shell effects
Nuclear composition in the SN

$M_B = 0.6 \, M_{\text{sun}}$

$log \rho [\text{g/cm}^3] = 9.20$
$T = 0.63 \text{ MeV}$
$Y_e = 0.444$
$t_{pb} = -265 \text{ ms}$
Nuclear composition in the SN

• detailed knowledge of nuclear composition
• based on nuclear structure calculations

→ allows more consistent treatment of e-captures
EOS aspects at subsaturation densities

- light nuclei
Light nuclei – animation from simulation

t_{pb} = -217.7 \text{ ms}
Light nuclei – postbounce evolution

- PNS core: nucleons
- PNS envelope & shock heated matter: light nuclei
- infalling matter: heavies, alphas, nucleons

compare also:
Sumiyoshi & Röpke PRC77 2008
Typel et al. PRC81 2010
Effect of light clusters on heating/cooling?

• S. Nakamura‘s talk
• Arcones et al. 2008: significant change of anti-neutrino energies ~1 MeV
• O‘Connor et al. 2007: A=3 breakup important for ν-energy loss
• difficult! more light nuclei ↔ less unbound protons
Light nuclei - measurement of symmetry energy

based on: [Natowitz et al.; 2010PRL104]

- free symmetry energy extracted from low-energy heavy ion collisions
- thermodynamic conditions: $T = 3 - 7$ MeV and $n_B = 1/100 - 1/20 \, n_B^0$
- cluster formation leads to increased symmetry energy

→ experimental evidence for appearance of light clusters in SN

$E_{\text{sym}}(n_B, T) = \frac{1}{2} \left( E_{\text{sym}}(n_B, T, Y_p = 1) + E_{\text{sym}}(n_B, T, Y_p = 0) \right) - E_{\text{sym}}(n_B, T, Y_p = 0.5)$

- $T$ [MeV] 3.3 3.3 3.6 4.2 4.7 5.3 6.2 7.5
- $n_B$ [10^{-3} \text{ fm}^{-3}] 2.1 1.7 2.3 3.8 4.7 4.9 5.5 6.4

[Typel et al.; 2010PRC81]
[Kowalski et al.; 2007PRC75]
Effect of light clusters on the EOS

- significant increase of entropy and internal binding energy
- light nuclei contribute to the symmetry energy $\rightarrow$ increase of $Y_e$ in beta-equilibrium

$\rightarrow$ non-trivial effect of LC on the EOS

$\rightarrow$ change of the structure of the proto-neutron star envelope
**EOS effect on SN dynamics**

- new EOS give most compact PNS up to 300 ms post-bounce
- complicated role of \( J, K, L \)
- low density EOS as important as nuclear interactions

[A. Steiner, MH & T. Fischer, in preparation]
EOS aspects at supersaturation densities
- time until black hole formation for a 40 $M_{\text{sun}}$ progenitor
Supernova simulations – EOS signal from black hole formation


- simulations by Tobias Fischer, GSI Darmstadt
- setup as for the 15 \( M_{\text{sun}} \) progenitor

- “failed supernova”: collapse to black hole
- high densities and temperatures
Neutrino signal

- $\mu/\tau$-neutrinos most sensitive to EOS because emitted from deeper layers
Time until black hole formation

• previous EOS studies for this progenitor

\[
\begin{array}{c|c|c}
\text{K [MeV]} & 180 & 282 \\
\text{M}_{\text{max}} [M_{\odot}] & 1.83 & 2.21 \\
\end{array}
\]


\[
\begin{array}{c|c|c}
180 & 220 & 282 \\
1.83 & 2.03 & 2.21 \\
2.21 & 2.72 & \\
\end{array}
\]


• possible conclusion: the compressibility K / the maximum mass M_{\text{max}} dictates „soft“ or „stiff“ behavior
Time until black hole formation

- unexpected: maximum mass of cold neutron stars and \( K \) are not directly correlated with \( t_{BH} \)
- neither correlated with \( J \) or \( L \)
- not found before, because only STOS and LS were available

\[
\begin{align*}
K \text{ [MeV]}: & \quad 180 \quad 220 \quad 245 \quad 230 \quad 239 \quad 318 \quad 282 \\
M_{\text{max}} \text{ [M}_{\odot}\text{]}: & \quad 1.83 \quad 2.03 \quad 2.06 \quad 1.74 \quad 2.13 \quad 2.02 \quad 2.21 \\
J \text{ [MeV]}: & \quad 28.6 \quad 28.6 \quad 31.6 \quad 32.6 \quad 28.7 \quad 30.7 \quad 37.0 \\
L \text{ [MeV]}: & \quad 73.8 \quad 73.8 \quad 47.1 \quad 60.4 \quad 23.2 \quad 90.1 \quad 111.0
\end{align*}
\]
Correlation of $t_{BH}$ with maximum mass

- TOV T = 0
- STOS(TM1)
- J1614-2230
- HS(TMA)
- SHFx
- HS(FSUgold)
- SHFo
- LS(220)
- LS(180)
Correlation of $t_{BH}$ with maximum mass

- maximum masses in the simulations are significantly increased (up to 0.6 $M_{\text{sun}}$) compared to $T = 0$
Correlation of $t_{\text{BH}}$ with maximum mass

- Maximum masses in the simulations are significantly increased (up to 0.6 $M_{\odot}$) compared to $T = 0$.
- Static $s = 4$ configurations reproduce the simulations.
- New correlation: $t_{\text{BH}}$ gives information about the finite entropy EOS.
- Degeneracy with progenitor star.


- Experimental/observational EOS constraints important.
- Extreme temperature effects.

→ Is there a simple explanation?
Specific heat

• specific heat in the degenerate limit as a guide-line:

\[ c_V = T \alpha_V = s \]

\[ \epsilon(T, n_B, Y_e) = \epsilon(T = 0, n_B, Y_e) + \frac{1}{2} T^2 n_B \alpha_V \]

\[ \alpha_V = \left( \frac{\pi}{3} \right)^{2/3} n_B^{-2/3} m^* \left[ Y_e^{1/3} + (1 - Y_e)^{1/3} \right] \text{ "level-density parameter"} \]


• change of pressure:

\[ p(T, n_B, Y_e) = p(T = 0, n_B, Y_e) + \Delta \epsilon \left( \frac{2}{3} - \frac{n_B}{m^*} \frac{\partial m^*}{\partial n_B} - \frac{2}{9} \frac{n_B}{Y_e^{2/3}} \frac{\partial Y_e}{\partial n_B} \right) \bigg|_{T=0} \]

\[ \Delta \epsilon = \frac{1}{2} T^2 n_B \alpha_V \]
Finite entropy effect in neutron stars

- beta-equilibrium, (here: without neutrinos):
  \[ Y_e = Y_e(n_B) \]
  \[ Y_e \sim \frac{4^3 E_{sym}^3}{3\pi^2 n_B} \]

- TOV eqs: only \( p(\varepsilon) \) relevant, not \( p(n) \)
- change of the pressure due to entropy at given energy density:

\[
\Delta p_\varepsilon = p(s, \varepsilon) - p(T = 0, \varepsilon)_{\text{at } n_0:}
= \frac{1}{2} s^2 n_B \frac{1}{\alpha_V} \left( \frac{2}{3} - \frac{n_B}{m^*} \frac{\partial m^*}{\partial n_B} - \frac{2}{9} Y_e^{2/3} \frac{\partial Y_e}{\partial n_B} - \frac{n_B}{\mu_B} \frac{\partial \mu}{\partial n_B} \right)
\]

\[ \alpha_V = \left( \frac{\pi}{3} \right)^{2/3} n_B^{-2/3} m^* \left[ Y_e^{1/3} + (1 - Y_e)^{1/3} \right] \]

J, L
K, J, L

<0
>0
>0
Finite entropy effect in neutron stars – effective mass?

<table>
<thead>
<tr>
<th></th>
<th>m*/m at n₀</th>
<th>M_max(T=0)</th>
<th>ΔM_max(s=4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS(220)</td>
<td>1</td>
<td>2.03</td>
<td>0.09</td>
</tr>
<tr>
<td>NL3</td>
<td>0.595</td>
<td>2.79</td>
<td>0.10</td>
</tr>
<tr>
<td>DD2</td>
<td>0.563</td>
<td>2.42</td>
<td>0.15</td>
</tr>
<tr>
<td>LS(180)</td>
<td>1</td>
<td>1.83</td>
<td>0.19</td>
</tr>
<tr>
<td>SHFo</td>
<td>0.761</td>
<td>2.06</td>
<td>0.21</td>
</tr>
<tr>
<td>SHFx</td>
<td>0.718</td>
<td>2.13</td>
<td>0.24</td>
</tr>
<tr>
<td>TM1</td>
<td>0.634</td>
<td>2.21</td>
<td>0.37</td>
</tr>
<tr>
<td>TMA</td>
<td>0.635</td>
<td>2.02</td>
<td>0.46</td>
</tr>
<tr>
<td>FSUgold</td>
<td>0.611</td>
<td>1.74</td>
<td>0.62</td>
</tr>
</tbody>
</table>

- • despite the same specific heat LS(180) and LS(220) differ
- • larger K in agreement with smaller ΔM for LS(220)
- • note: ΔM < 0 possible, i.e. softening, e.g. for LS(375) (O‘Connor & Ott 2011)
- • temperature effects tend to decrease with high M_max(T=0)

→ it’s not only m*₀ or αᵥ₀

\[ \Delta p_\epsilon = \frac{1}{2} s^2 n_B \frac{1}{\alpha_V} \left( \frac{2}{3} - \frac{n_B}{m^*} \frac{\partial m^*}{\partial n_B} - \frac{2}{9} \frac{n_B}{Y_e^{2/3}} \frac{\partial Y_e}{\partial n_B} - \frac{n_B}{\mu_B} \frac{\partial \mu_B}{\partial n_B} \right) \]

\[ \alpha_V = \left( \frac{\pi}{3} \right)^{2/3} n_B^{-2/3} m^* \left[ Y_e^{1/3} + (1 - Y_e)^{1/3} \right] \]

- • around saturation density: \[ \frac{\partial \mu_B}{\partial n_B} \bigg|_{n_B^0} = \frac{1}{n_B^0} \left( \frac{1}{9} (K + K_{sym}) + \frac{2}{3} L \right) \]
Conclusions

• new SHFo and SHFx EOS: fit to NS observations
• EOS tables and nuclear composition available for NL3, TM1, TMA, FSUgold, DD2, SHFo, SHFx: http://phys-merger.physik.unibas.ch/~hempel/eos.html

• the model for nuclei can be as important as the nuclear interactions

• could light nuclei influence the PNS cooling and/or the $\nu$-driven wind?

• black hole formation gives information about the finite entropy EOS
• temperature effects depend significantly on the EOS model

• complex role of the EOS, not “soft or stiff“
• individual nuclear matter properties are not sufficient to characterize the EOS

• EOS effect on explosions: $\rightarrow$ 3D simulations
New equations of state in simulations of core-collapse supernovae

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