Fast Global Image Registration Using Random Projections

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Outline

- **Problem statement:**
  - Image registration
  - Global image registration (overview of current approaches)

- **Key concepts:**
  - Manifold interpretation (Whitney’s embedding theorem)
  - Random projections (Johnson-lindenstrauss lemma)

- **New global image registration algorithm**
  - Ideally suited for large-scale problems: many to one

- **Preliminary results**
  - Comparison with multiresolution-type projections onto B-spline function spaces.
Image registration

- Find spatial transformation that relates images
- Biomedical imaging applications:
  - Data fusion
  - Motion & distortion correction
  - Tracking
  - Atlas-based segmentation
  - Computational anatomy
  - Super-resolution
  - …
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Mathematical Formulation:

- **Spatial transformation of an image:**
  - Fit continuous model to image data:
  - Define spatial transformation:
  - Resample it:

- **Seek transformation that best matches some target image**
  - Minimization Problem

\[
\tilde{s}(x) = \sum_{j \in \Gamma} b_j \phi_j(x) \quad x \in \mathbb{R}^d
\]

\[
f_P : \mathbb{R}^d \to \mathbb{R}^d
\]

\[
\tilde{s}_P = \tilde{s}(f_P(i)) \quad i \in \mathbb{Z}^d
\]

\[
p^* = \text{arg min}_p \|t - \tilde{s}_p\|^
\]
Key problem: local optima

- Example:
  - Rotation and shear
Typical approaches

- **Correlation → FFT, phase shift**
  - Translations

- **Multiple resolutions**
  - Coarse to fine. Hope to avoid local optima due to high-frequency components

- **Symplex-type methods:**
  - Independent from local derivatives

- **Multiple (random) initializations**
  - Stochastic searches

- **Exhaustive searches**
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- Exhaustive searches
Overview of new registration algorithm

- Based on exhaustive searches w.r.t. registration parameters
  - Cost function evaluation on reduced dimensions (teens instead of $N = Q^d$)
- For many to one image registration problems, articulated projections can be pre-computed
  - Nearest neighbor type of search
Manifold interpretation:

- Consider each image \( (N = Q^d \text{ pixels}) \) as a point in \( N \) dimensional space:
  \[
  \tilde{s} \in \mathbb{R}^N
  \]

- The set of spatially transformed images does not fill the entire space:
  \[
  \tilde{s}_p = \tilde{s}(f_p(i)) \in \mathcal{M}^K \\
  \dim(p) = K < N
  \]
Example

- It is not possible to transform, by translations, rotations and scaling, the image of a nucleus to an image of a actin filaments:
Whitney’s embedding theorem

**Theorem 2.1** [4] Let $\mathcal{M}$ be a compact Hausdorff $C^r$ $K$-dimensional manifold, with $2 \leq r \leq \infty$. Then there is a $C^r$ embedding of $\mathcal{M}$ in $\mathbb{R}^{2K+1}$.

- [Wakin, Baraniuk, ICASSP 06] Using mild assumptions, a randomly chosen projection from $N$ to $2K+1$ is also one to one (with some probability).
  - Choose projection matrix $P$ randomly (using a fixed random number generator)
  - Johnson-Lindenstrauss lemma can be used to obtain bounds on probability of errors.
Johnson-Lindenstrauss lemma

**Lemma 2.2** [Johnson-Lindenstrauss] Let $\Psi$ be a finite collection of points in $\mathbb{R}^N$. Fix $0 < \epsilon < 1$ and $\beta > 0$. Let $\mathcal{P}$ be a random orthoprojector from $\mathbb{R}^N$ to $\mathbb{R}^M$ with

$$M \geq \left(\frac{4 + 2\beta}{\epsilon^2/2 - \epsilon^3/3}\right) \ln(|\Psi|)$$

If $M \leq N$, then, with probability exceeding $1 - (|\Psi|)^{-\beta}$, the following statement holds: For every $s, t \in \Psi$,

$$(1 - \epsilon) \sqrt{\frac{M}{N}} \leq \frac{\|\mathcal{P}s - \mathcal{P}t\|}{\|s - t\|} \leq (1 + \epsilon) \sqrt{\frac{M}{N}}$$
Random projection

\[
\mathbf{c} = \mathbf{P} \mathbf{s}
\]

\[
\mathbf{M} = \begin{pmatrix}
P_{1,1} & P_{1,2} & P_{1,3} & \cdots \\
P_{2,1} & P_{2,2} & P_{2,3} & \cdots \\
P_{3,1} & P_{3,2} & P_{3,3} & \cdots \\
\end{pmatrix}
\]

\[
\mathbf{N} = \begin{pmatrix}
\mathbf{s}_1 \\
\mathbf{s}_2 \\
\mathbf{s}_3 \\
\vdots \\
\end{pmatrix}
\]
Random projection example

\[ \mathcal{P} \tilde{s}_p \]
Algorithm

- Consider the problem:
  - Registering many images to a template or atlas (affine)

- Training phase:
  - Compute random projections coefficients for some sampling of the parameter space.

- Registering an image to template
  1. Compute random projection
  2. Search for nearest neighbor

\[ \mathbf{c}_{p_i} = \mathcal{P} \tilde{t}_{p_i} \]

\[ d = \mathcal{P} s \]

\[ p^* = \arg \min_{p_i} \| \mathbf{c}_{p_i} - d \| \]

\[ p^* = \arg \min_{p} \| t - \tilde{s}_p \| \]
Experimental Results
Experiments

- Set of 87 cell nuclei images:
  - 382 x 512 pixels
- Registration:
  - Translation (x,y), rotation, scaling (x,y): 5 parameters
- Project to 20 dimensional space (from 382 x 512)
- Compare with orthogonal projection over B-spline function spaces (4 x 5 = 20)
  - Unser et al, IEEE TIP, 1995
Original data

Registered

B-spline

Random
Original data

Registered

B-spline

Random
Projection-based registration: B-splines vs. Random

\[ \| \mathbf{t} - \hat{S}_p \| \]
With gradient descent minimization

\[ \|t - \hat{S}_p\| \]
Gradient descent vs. RP + gradient descent

$$\| \mathbf{t} - \mathbf{s}_p \|$$
Details

- Mean error
  - B-spline projection: $6.96 \times 10^6$
  - Gradient descent: $5.45 \times 10^6$
  - Random projection: $4.81 \times 10^6$
    - All with gradient descent

- Direct comparisons
  - Number of times Rand. Proj. better than Grad. Des.: 69
  - Number of times Grad. Des. better than Rand. Proj.: 18
Computational complexity

- **Reduced from:**
  - Brute force nearest neighbor search:
    - $O(G^K N)$

- **To:**
  - Nearest neighbor search in reduced space:
    - $O(G^K M)$, with
    - $M \sim O(\ln(G^K))$

- **Could perhaps reduce further by reducing $G^K$ with**
  - Multiscale image manifold representations:
    - Rahman et al, Multiscale Modeling and Simulation, 2005
Summary

- **Key problem:** local optima in image registration
- **Random projection-based fast exhaustive search:**
  - Borrow ideas from compressed sensing [Baraniuk & Wakin, ICASSP 06]
- **Experimental results**
  - Outperforms multi-resolution-type B-spline approximations
  - Outperforms gradient descent alone
- **Computational complexity:**
  - $O(G^K M)$
  - $M \sim O(\ln(G^K))$
END

Thank you