Nonlinear Diffusion Control of Spatiotemporal Chaos in the Complex Ginzburg Landau Equation

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(16 Jun 1997)

Abstract

The role of nonlinear diffusion terms in the stability of periodic solutions in the regime of spatio temporal chaos is studied. The stabilization of unstable plane waves in the Complex Ginzburg Landau equation in weakly chaotic regimes such as phase turbulence and spatiotemporal intermittency or in strong chaotic ones like defect turbulence is demonstrated.

Typeset using REVTEX

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I. INTRODUCTION

Under nonequilibrium conditions, a spatially extended system often undergoes a transition from a uniform state to a state with spatial variation, usually referred as pattern. Their formation is generally associated with nonlinear effects, which lead to qualitatively new phenomena such as the *Spatiotemporal Chaos* [1] (STC). Loosely, the term spatiotemporal chaos is commonly accepted to refer to a deterministic system that has irregular variation and is unpredictable in detail, both in space and in time. There are known examples of experimental systems, well characterized and precisely controlled [2,1] that show such a behavior. In most cases STC can be described within the context of weakly nonlinear theories since these states arise in the proximity of threshold. These theories are well developed in the form of so–called complex Ginzburg–Landau equations (CGLE) [3]. The CGLE is a prototypical equation for a complex field A that exhibit STC. It accounts for the slow modulations, in space and time of the oscillatory state in a physical system which undergoes a Hopf bifurcation [4]. The CGLE shows different types of STC [5] which have been profusely studied [6–10].

The control of the chaotic behavior of dynamical systems with few degrees of freedom has been successively tested in a number of systems [11]. The idea behind the control of chaos is to modify the dynamics of the system in such a way that a previously unstable state is now stable. Ideally only the stability is modified, not the state itself (i.e. if that state was a fix point or periodic orbit of the original system is still a fix point or periodic orbit of the modified system). The control of spatiotemporal chaos is a more complicated problem, and so, there is a wide variety of methods intended to control such chaotic behavior. There have been several attempts to achieve such control in the CGLE. For example, Aranson et. al. [12] stabilize an structurally unstable topological defect, whose analytical expression is known, by adding an extra term in the CGLE. The defect acts as a source of traveling waves which sweep all the other fluctuations to the system boundary. Stabilization of a plane wave extended through all the system has been achieved by adding time-delayed feedback terms to the CGLE. The feedback can be either local [13] (at each spatial point, the field at the same point at previous times is fed back) or global [14,15] (at each spatial point a term proportional to the integral of the field over the spatial variable is fed back). In both cases, the added terms vanish for the stabilized plane wave solution, so it is possible to stabilize precisely the same plane waves which are unstable in the original CGLE. However, the added global feedback terms do not preserve the phase invariance of the original CGLE.

Feedback is the most used approach for chaos control in spatially extended systems. It has been applied to a nonlinear drift-wave equation driven by a sinusoidal wave [16] and, in conjunction with a spatial filter it has also been applied to stabilize rolls and hexagonal structures in a model for a transversally extended three level laser [17] and to control filamentation in a model for wide aperture semiconductor lasers based on the Swift-Hohenberg equation [18].

In this paper we explore a different way to stabilize unstable periodic solutions based not in feedback terms but in nonlinear diffusion effects. Specifically, we show that stabilization of unstable plane wave solutions of the CGLE in the region of STC can be achieved adding a nonlinear diffusion (or diffraction) term. The added term preserves the intrinsic phase invariance of the CGLE equation and it vanishes when the stabilizing effect is achieved so any plane wave solution of the original CGLE is also solution of the modified equation. Nonlinear diffraction effects are present in optical systems where the Fresnel number is intensity dependent or in systems where the refraction index is intensity dependent, as for example in photorefractive materials.

In section II we briefly describe the parameter regions for which different chaotic behaviors have been found for the CGLE and we introduce the modified equation. Section III is devoted to the linear stability analysis of the plane wave solutions. We calculate for which parameter values the added term is able to stabilize plane waves in the STC regions of the CGLE. In section IV we show, integrating the equations numerically, that the stability of the plane waves when finite size perturbations are applied is in excellent agreement with the analytical prediction of the linear stability analysis. Finally we give some concluding remarks in Section V.

II. MODEL

The one dimensional CGLE [3,4,19,20] for a complex field $A(x,t)$, describes the slow dynamics of spatially extended systems close to a Hopf bifurcation,

$$\partial_t A = A + (1+ic_1)\partial_x^2 A - (1+ic_2)|A|^2 A\,. \tag{2.1}$$

We will assume periodic boundary conditions through all the paper. This equation admits plane wave solutions of the form

$$A_{pw}(x,t) = A_0 e^{i(kx-\omega t)}\,, \tag{2.2}$$

with amplitude $A_0 = \sqrt{1-k^2}$, $|k| < 1$ and frequency $\omega = c_2 + (c_1 - c_2)k^2$.

For $1 + c_1 c_2 > 0$ plane wave solution are linearly stable for wave numbers smaller than a limit value $|k| \le k_E$. For $|k| > k_E$, plane waves are unstable to phase perturbations (Eckhaus instability [21]). The limit value k_E is given by

$$q_E^2 = \frac{1 + c_1 c_2}{3 + c_1 c_2 + 2c_2^2}\,. \tag{2.3}$$

The stability range vanishes at $1 + c_1 c_2 = 0$ (the Benjamin–Feir–Newell (BFN) line), and no stable plane wave solution exist for $1 + c_1 c_2 < 0$.

Numerical work for L (length of the system) large [5–10,22] has identified regions of the parameter space displaying different kinds of regular and spatiotemporal chaotic behavior, leading to a "phase diagram" for the CGLE in the plane c_1–c_2. The five different regions, each leading to a different asymptotic phase, are shown in Fig. 1 as a function of the parameters c_1 and c_2. Two of these regions are in the BFN stable zone and the other three in the BFN unstable one. The NO CHAOS region, in the BFN stable zone, is a large region where the evolution ends in a plane wave with a wave number $|q| \le q_E$ for almost all the initial conditions. Also in the BFN stable zone there is the Spatio–Temporal Intermittency (STI) region [7]. Despite the fact that there exist stable plane waves, the evolution from random initial conditions is not attracted to them but to a chaotic attractor in which typical configurations of the field A consist of patches of plane waves interrupted by turbulent bursts. The modulus of A in such bursts typically touches zero quite often. Above BFN line, the evolution ends in a spatiotemporal chaos for almost every initial condition. The Defect Turbulence (DT) region is a strongly disordered region in which the modulus of A has a finite density of space-time zeros [6,7]. The Phase Turbulence (PT) [5,6,8,23–25] region is a weakly disordered one in which $|A(x,t)|$ remains away from zero. Nevertheless, under a particular type of initial condition it is possible to end in a ordered state [10,22]. Finally, the Bi-Chaos region is such that, depending on the particular initial condition, the system ends on attractors similar to the ones in regions of PT or DT, or in a new attractor in which the configurations of A consists of patches of phase and defect turbulence. A detailed description can be found in [5].

We consider a modification of CGLE in such a way that the plane wave stability region is increased. A way to do this is by changing a parameter of the system dynamically and proportionally to the deviation of the system from the state to be stabilized. We will show that stabilization of plane waves can be achieved by replacing the coefficient c_1 by $c_1 + \gamma(|A|^2/|A_{pw}|^2 - 1)$ where γ is a constant and $|A_{pw}|$ is the modulus of the plane wave to be stabilized. Notice that as the added term $\epsilon(|A|^2/|A_{pw}|^2 - 1)$ vanishes identically for $A = A_{pw}$, any plane wave A_{pw} that is a solution of (2.1) is also a solution of the modified equation. We are not changing the solution but we will change its stability. The added term also preserves the phase invariance of the solution of the original CGLE, $A \to Ae^{i\psi}$, with ψ being an arbitrary phase. The modified CGLE is then explicitly given by,

$$\partial_t A = A + [1 + ic_1 + i\gamma(|A|^2/|A_{pw}|^2 - 1)]\partial_x^2 A - (1 + ic_2)|A|^2 A\,. \tag{2.4}$$

From another point of view, eq. (2.4) can be rewrited as

$$\partial_t A = A + [1 + i\tilde{c}_1 + ic_{NL}|A|^2]\partial_x^2 A - (1 + ic_2)|A|^2 A\,, \tag{2.5}$$

with $\tilde{c}_1 = c_1 - \gamma$ and $c_{NL} = \gamma/|A_{pw}|^2$. In this way, the stabilizing added term can be seen explicitly as a nonlinear diffusive term in the CGLE.

III. STABILITY ANALYSIS

We study the stability of the plane wave solutions (2.2) of eq. (2.4) using a standard linearization procedure. Consider the time evolution of small perturbations in the amplitude and phase,

$$A(x,t) = (A_0 + \epsilon r(x,t))e^{i(kx-\omega t+\epsilon\phi(x,t))}\,, \tag{3.1}$$

where $r(x,t)$ and $\phi(x,t)$ are real perturbations in the amplitude and phase respectively and ϵ is a formal parameter to keep track of small numbers.

Substituting (3.1) in (2.4) yields to a polynomial in ϵ up to order ϵ^5. The terms of order ϵ^0 vanish identically. The first order terms yield the linearized equations for the perturbations

$$\partial_t r = 2A_0^2 r - 2A_0 k\partial_x\phi - 2c_1 k\partial_x r - c_1 A_0\partial_x^2\phi + \partial_x^2 r\,, \tag{3.2}$$

$$\partial_t\phi = -2c_2A_0 r + 2\gamma\frac{k^2}{A_0}r - 2c_1k\partial_x\phi + 2\frac{k}{A_0}\partial_x r + \partial_x^2\phi + \frac{c_1}{A_0}\partial_x^2 r\,. \tag{3.3}$$

We consider solutions of (3.2) and (3.3) proportional to $e^{\eta t+iqx}$, where for periodic boundary conditions q is real whereas η is in general a complex quantity. By substituting in (3.2) and (3.3) we obtain the dispersion relation

$$\begin{vmatrix} \eta + 2A_0^2 + q^2 + 2ic_1qk & 2iqk - c_1q^2 \\ c_1q^2 + 2c_2A_0^2 - 2iqk + 2\gamma k^2 & \eta + q^2 + 2ic_1qk \end{vmatrix} = 0\,. \tag{3.4}$$

The solutions of (3.4) are

$$\eta = -(A_0^2 + q^2 + 2ic_1qk) \pm \sqrt{u+iv}\,, \tag{3.5}$$

where u and v are polynomials

$$u = A_0^4 + 4q^2k^2 - 2c_1c_2A_0^2q^2 - c_1^2q^4 - 2\gamma c_1q^2k^2\,, \tag{3.6}$$

$$v = 4qk(c_1q^2 + c_2A_0^2 + \gamma k^2)\,. \tag{3.7}$$

The real part of η indicates the growth rate of the perturbations,

$$Re(\eta) = -A_0^2 - q^2 \pm \sqrt{\frac{u + \sqrt{u^2 + v^2}}{2}} \,. \tag{3.8}$$

We have two different branches [4,26,27] which are usually called "amplitude" and "phase modes" due to the fact that for a real Ginzburg-Landau equation the eigenvalues are related specifically to amplitude and phase perturbations. Although this is not the case for the CGLE, the names are still used.

The "amplitude modes" correspond to the negative sign of the square root in (3.8). For any value of c_1, c_2 and k, the growth rate, $Re(\eta)$ as function of the perturbation wavelength q is always negative and takes the value $Re(\eta) = -2A_0^2$ at $q = 0$. The added γ term modifies slightly the value of $Re(\eta)$ but it never changes its sign so these perturbations are always damped.

The "phase modes" are associated with the positive sign of the square root in (3.8). The growth rate vanishes identically at $q = 0$ for any value of the parameters c_1, c_2 and k, so all the plane wave solutions are marginally stable. The origin of this neutral stability is the phase invariance $A \rightarrow Ae^{i\psi}$ of the solutions of the equations (2.1) and (2.4). For q very large, the growth rate is negative and behaves as $-q^2$, so short wavelength perturbations are always damped. However long wavelength perturbations can grow destabilizing the original plane wave solution, to see this we expand (3.8) for small q.

$$Re(\eta) = Dq^2 + O(q^4) \,, \tag{3.9}$$

where

$$D = -1 - c_1 c_2 + 2(1 + c_2^2)\frac{k^2}{A_0^2} + \gamma \left(-\frac{c_1 k^2}{A_0^2} + 4\frac{c_2 k^4}{A_0^4} \right) + 2\gamma^2 \frac{k^6}{A_0^6} \,. \tag{3.10}$$

If this coefficient is positive, there is a range of long wavelength perturbations that grow. The condition $D < 0$ is necessary for stability but not sufficient, since the growth coefficient obtained from the full expression (3.8) can be positive for some q despite the coefficient D being negative. In this sense, in general, the requeriment $D < 0$ will give only an upper bound for the stability region in the c_1, c_2 and k space. However, for the values of c_1 and

c_2 considered in this work the requirement $D < 0$ gives a very good limit for the stability region.

For the unperturbed CGLE ($\gamma = 0$) the condition $D < 0$, leads to the standard Eckhaus instability limit: $|k| < k_E$ with k_E given by (2.3). For $\gamma \neq 0$ the first thing to notice is that independently of the value of the parameters c_1 and c_2 the added term never changes the stability of the homogeneous solution $k = 0$. This can be seen from the fact that in equations (3.6), (3.7) and (3.8), γ only appears in terms with powers of k. In general, the coefficient D depends on even powers of k up to the 6th power, so one has to solve a cubic equation to find explicitly the limits of the range of values of k for which plane waves are stable. In Figs. 2 - 4 we plotted this range as a function of the parameter c_1 for several values of γ and c_2.

Fig. 2 shows the stability region for $c_2 = -0.3$ and different values of γ as indicated in the figure caption. For plane waves with $k \neq 0$ the stability range clearly changes with the value of γ as displayed in Fig. 2 b)-d). For small γ (Fig. 2 b) the stability range is increased for large values of c_1 and slightly reduced for $c_1 \lesssim -0.5$, therefore, the added stabilizing term has the opposite effect for small c_1. Increasing the value of c_1 the last plane wave in losing stability is still the homogeneous solution as it was in the case $\gamma = 0$. For $\gamma > 0.6$ the shape of the stability range is strongly changed as can be seen in Fig. 2c)-d) for $\gamma = 0.7$ and $\gamma = 2$. Now there are plane waves with $k \neq 0$ which are stable for values of c_1 well above the BFN line, in the region of phase turbulence of the original CGLE (see Fig. 1).

Figs. 3 and 4 show the stability regions for $c_2 = -0.9$ and $c_2 = -2.1$ and several values of γ. As γ is increased the stability region changes its shape in a similar way as before but at larger values of γ. For $c_2 = -0.9$ it is possible to stabilize plane waves in the region of phase turbulence taking $\gamma \geq 2$, and for $c_2 = -2.1$ stabilization in the region of defect turbulence is possible for $\gamma \geq 4$.

Fig. 4c) ($c_2 = -2.1$ and $\gamma = 4$) shows an interesting intermediate shape. There are three stability regions so plane waves can exist below the BFN line and well above it, in defect turbulence, but not for values of c_1 just above the BFN line. Also there is no wave vector

k for which plane waves are stable both below and above the BFN line. As γ is increased the three regions coalesce and become a single one, as seen for $\gamma = 6$. This is a general behavior also observed at other values of c_2 for intermediate values of γ not shown in Figs. 2 and 3. The overall picture is as follows. For $\gamma = 0$ and a fixed c_2 the stability region in the k–c_1 plane is limited by a branch of eq. (2.3) (dashed line in Fig. 4) whose vertex corresponds to the BFN point. Decreasing the value of c_1 the width of the stability region $|k| < k_E$ increases and for $c_1 \to -\infty$, $k_E \to 1$. For any small $\gamma > 0$ there are three stability regions in the k–c_1 plane. From eq. (3.10) one can show that for $c_1 \to -\infty$ the limits of the central region approach the two vertical asymptotes $k_{A\pm} = \pm\sqrt{c_2/(c_2 - \gamma)}$ (dotted lines in Fig. 4c),d)). Two new stability regions ($D < 0$) appear symmetrically at very large values of c_1 and for values of $|k|$ between the vertical asymptotes and 1. The existence of these new regions, which broaden for $c_1 \to \infty$ and cover the intervals $k \in [-1, k_{A-}]$ and $k \in [k_{A+}, 1]$, implies that for any non-vanishing γ there will be always stable plane waves well above the BFN line. However, if γ is very small these regions are located at very large, and quite unrealistic, values of c_1. As γ is increased these regions extend to lower values of c_1 until they coalesce with the central region.

IV. NUMERICAL SIMULATIONS

We have performed numerical simulations of eq. (2.1) and (2.4) using a pseudospectral code with periodic boundary conditions and second-order accuracy in time. Spatial resolution was typically 1024 modes. Time step was typically $\Delta t = 0.001$. Since very small effects have been explored, care has been taken of confirming the invariance of the results with decreasing time step and increasing number of modes. The system size was always taken as $L = 512$. The details of the numerical method can be seen in Ref. [28]. We start from an initial condition which corresponds to a plane wave plus a small random perturbation

$$A(x, t = 0) = \sqrt{1 - k^2} e^{ikx} + \sigma \xi(x) \tag{4.1}$$

where $\xi(x)$ is a complex Gaussian random perturbation of zero mean and variance $\langle \xi(x)\xi^*(x')\rangle = 2\delta(x-x')$.

We have performed numerical simulations in different regions of the phase diagram (Fig. 1) to verify the results obtained from the linear stability analysis when finite size perturbations are applied. We have found a very good agreement between the prediction of the linear stability analysis and the numerical simulations. In the NO CHAOS region we have tested the stabilization of plane waves with wave vector $k > k_E$ by using small values of γ as predicted in Figs. 2 for $c_1 > -0.5$. With $\gamma = 0$, the perturbed unstable plane wave evolves towards another plane wave with wave vector $k < k_E$ whereas when the control term is added the initial perturbations are washed out by the dynamics and the system settles down to a plane wave with the initial wave vector. For small values of c_1, we have also tested cases where the added control term destabilizes an originally stable plane wave. In the modified CGLE, the initial plane wave evolves to another plane wave solution with a smaller wave number which is inside the stability range given by the linear stability analysis.

Stabilization is also possible in the different regimes of STC found in the CGLE, as characteristic examples we show the following results.

Fig. 5 shows the stabilization of a plane wave for parameter values $c_1 = 1.5$ and $c_2 = -0.9$. This corresponds to the phase turbulence regime (see fig. 1), where no plane waves are stable for the original CGLE. As predicted by the linear stability analysis (square points in Fig. 3a) and d)) a perturbed plane wave with $k = 0.5$ can easily be stabilized with $\gamma = 3$, while for $\gamma = 0$ the same initial condition decays in time $t = 80$ (approx.) to phase turbulence.

Fig. 6 shows a case of stabilization of a plane wave for $c_1 = 0$ and $c_2 = -2.1$, in the region of spatiotemporal intermittency. For the original CGLE, plane waves are stable in this region if $|k|$ is small enough, but if $|k| > k_E$ the initial perturbed plane wave evolves to an spatio temporal intermittent behavior [7]. The nonlinear diffusion term proved to be an effective way of supressing the evolution towards the disordered states (with defects and other localized structures), leading the system to a well behaved plane wave. The initial condition in this case was a perturbed plane wave with $k = 0.55$ ($\times$ points in Fig. 4a) and c)).

Finally Fig. 7, obtained for parameter values $c_1 = 1.5$ and $c_2 = -2.1$, shows stabilization of plane waves in the region of defect turbulence, where for the unperturbed CGLE there are no stable plane waves and the field A shows a strongly disordered STC state caracterized by the presence of defects. As predicted by the linear stability analysis a perturbed plane wave with $k = 0.55$ (square point in Fig. 4a) and d)) can be stabilized with $\gamma = 6$.

V. CONCLUDING REMARKS

We have stabilized unstable plane wave solutions in different parameter regions of the CGLE where spatiotemporal chaos exist. This have been done by adding a term to the CGLE which vanish for the stabilized plane wave, so that the stabilized plane waves are exactly the same unstable solutions of the original CGLE. The added term can be seen as a nonlinear diffusion and preserves the intrinsic phase invariance of the original equation. Although our method does not change the stability of the homogeneous solution $k = 0$, it is quite effective in stabilizing plane waves with non zero wave vector. We have calculated analytically the parameter regions where plane waves can be stabilized including regions of phase turbulence, spatiotemporal intermittency, bichaos and defect turbulence. We have studied numerically the stability of the plane waves when finite size perturbations are applied. The results are in excellent agreement with the analytical predictions of the linear stability analysis. Our analysis also shows that in general, in systems where nonlinear diffusion or diffraction effects are not negligible, these terms can change substantially the regions in parameter space for which plane waves are stable.

The basic purpose of this work was to show that stabilization of plane waves is possible by using a nonlinear diffusion term. The modified CGLE, however, can also have new kinds of solutions (chaotic or not) different from the ones of the original equation. From the analysis presented here one can draw some conclusions about the “phase diagram” of the modified CGLE. One is that the stability of the homogeneous solution is not changed, so it is only stable below the BFN line. Another is that for γ large enough, there are always stable plane waves with wave vector $k \approx 0.5$, so the BFN line is not any more the limit of stability of plane waves. Numerically we have observed phase and defect turbulence regimes in the modified equation when starting from random initial conditions above the BFN line. However we have not done a systematic exploration for different parameter values. A full "phase diagram" for the modified CGLE requires a very intensive numerical calculation which is beyond the scope of this work.

VI. ACKNOWLEDGEMENTS

Financial support from DGICYT (Spain) Projects No. PB94-1167 and No. PB94-1172 is acknowledged. R.M. also acknowledges partial support from the Programa de Desarrollo de Ciéncias Básicas (PEDECIBA, Uruguay), the Consejo Nacional de Investigaciones Científicas Y Técnicas (CONICYT, Uruguay), and the Programa de Cooperación con Iberoamérica (ICI, Spain).

The authors also acknowledge helpful discussions with M. San Miguel, E. Hernández-García, D. Walgraef and G. Huyet.

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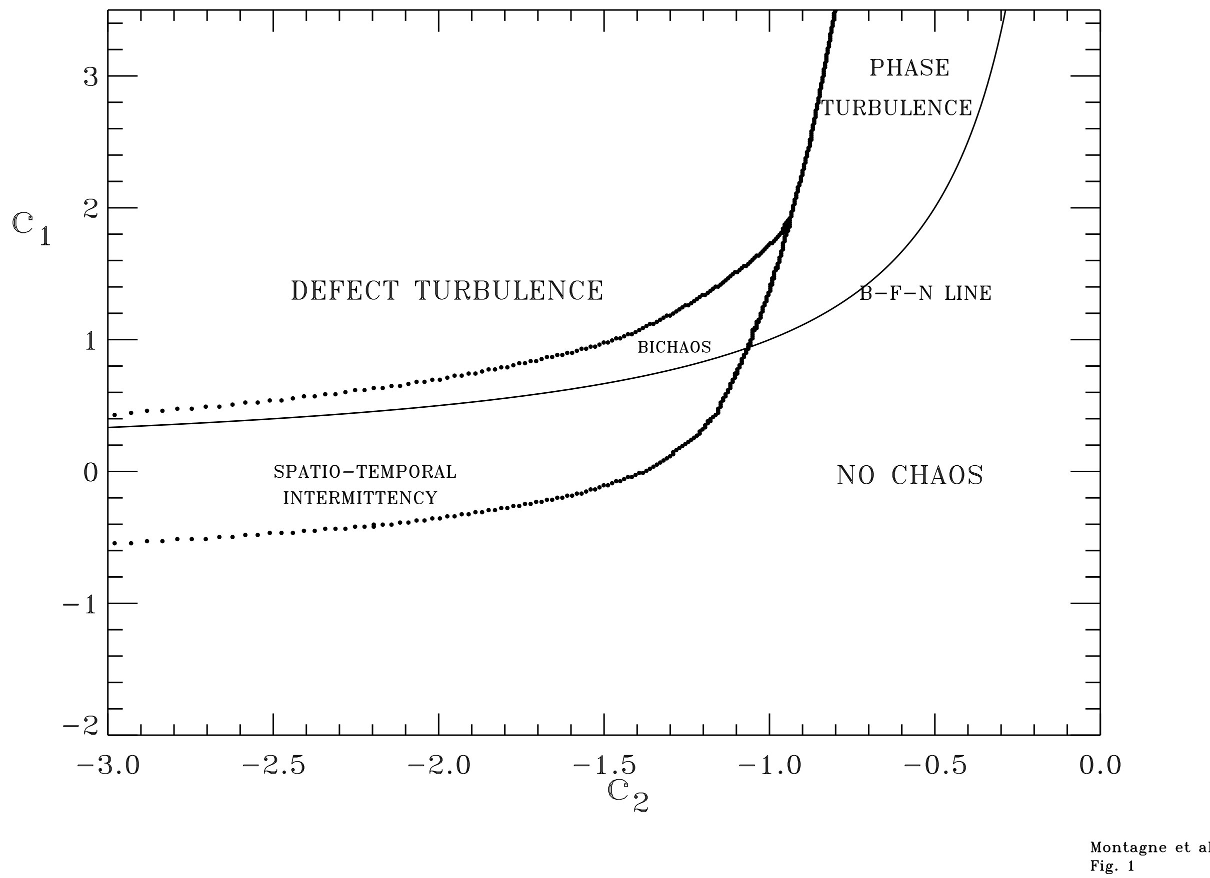

FIG. 1. Regions of the parameter space $c_1 - c_2$ for which the $d = 1$ CGLE displaying different kinds of regular and chaotic behavior. The analytically obtained line, the Benjamin-Feir–Newell line (BFN line) is also shown. The quantities plotted in all the figures of this paper are dimensionless.

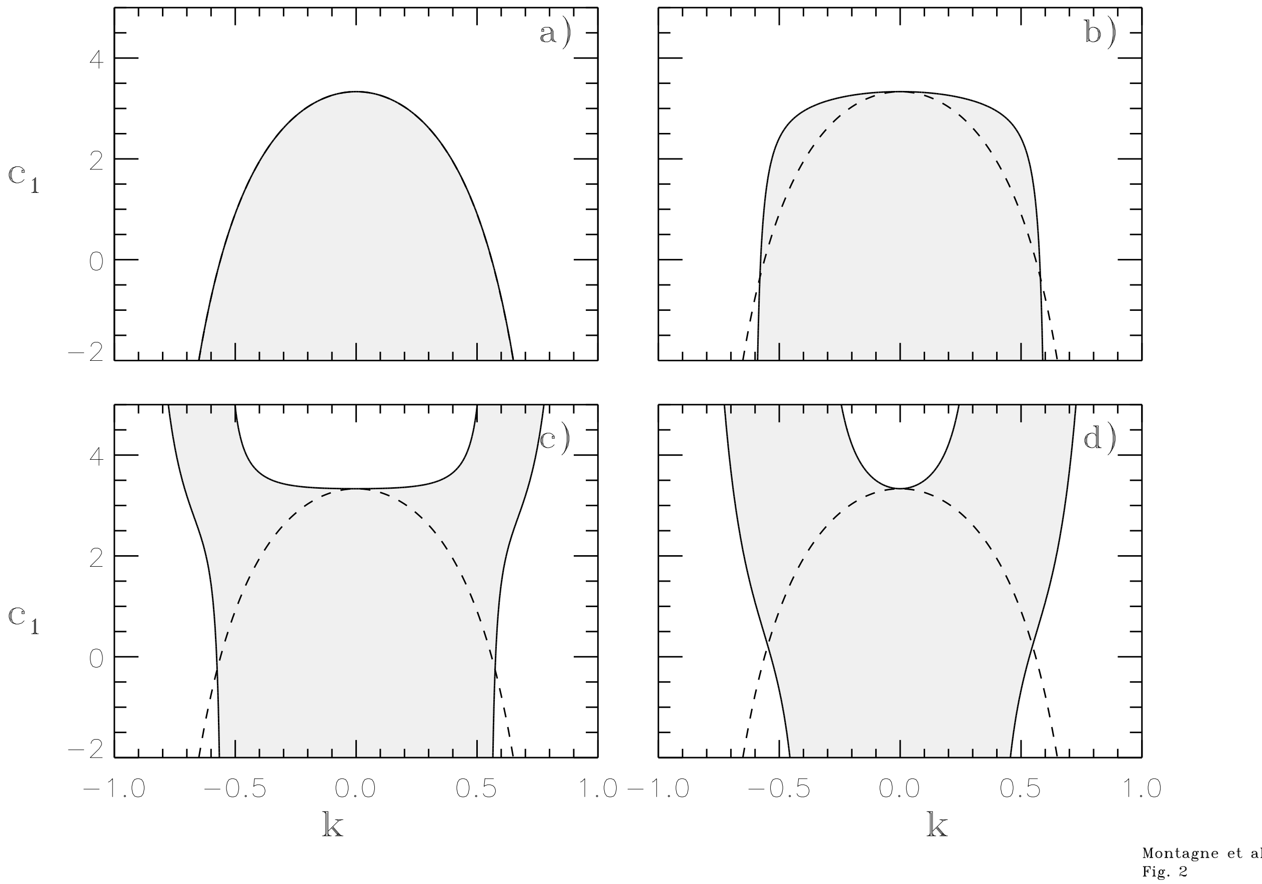

FIG. 2. Stability region for the plane wave (2.2) for $c_2 = -0.3$ for a) $\gamma = 0$, b) $\gamma = 0.5$, c) $\gamma = 0.7$, d) $\gamma = 2$. For comparison, the boundary of the stability region for $\gamma = 0$ is shown in figs. b)-d) as a dashed line.

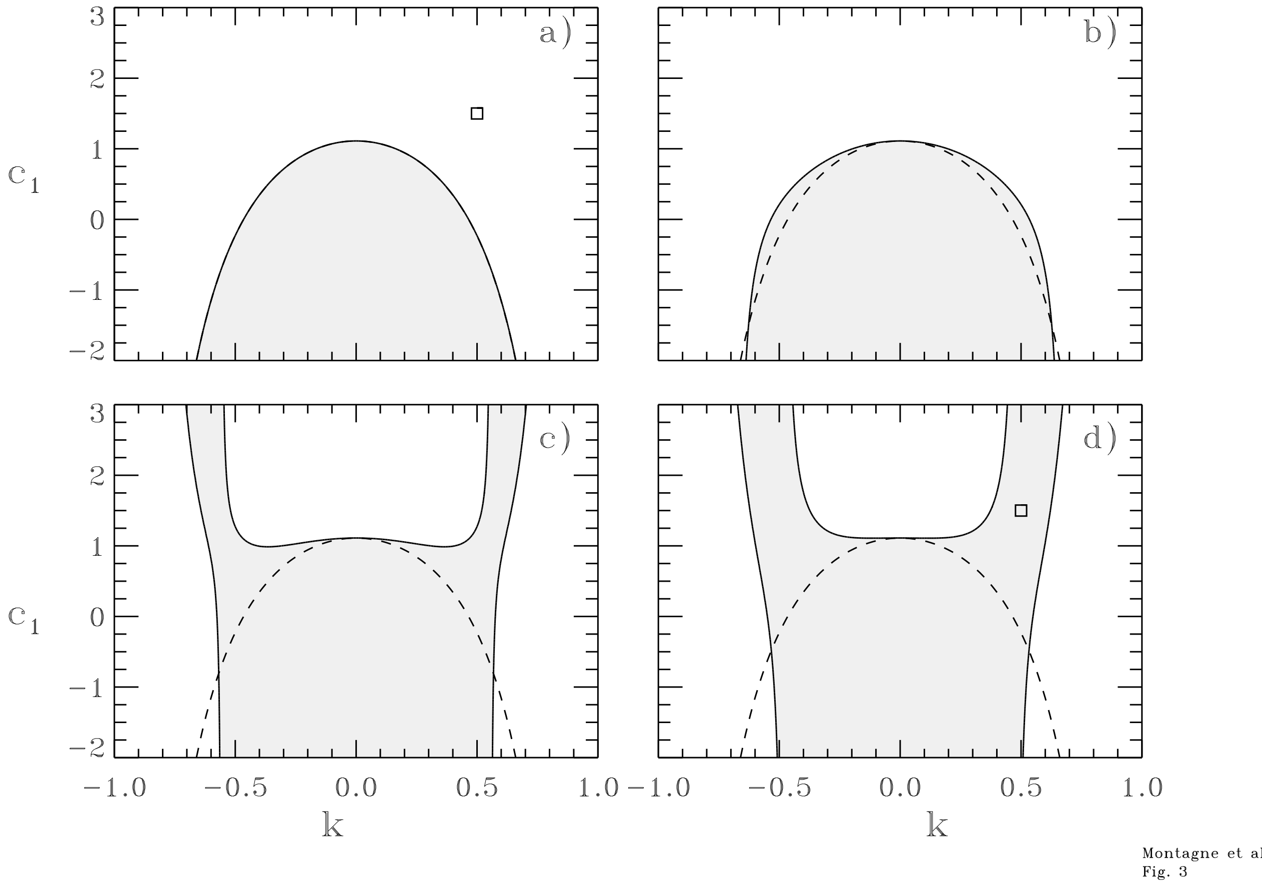

FIG. 3. Stability region for the plane wave (2.2) for $c_2 = -0.9$ for a) $\gamma = 0$, b) $\gamma = 1$, c) $\gamma = 2$, d) $\gamma = 3$. For comparison, the boundary of the stability region for $\gamma = 0$ is shown in figs. b)-d) as a dashed line.

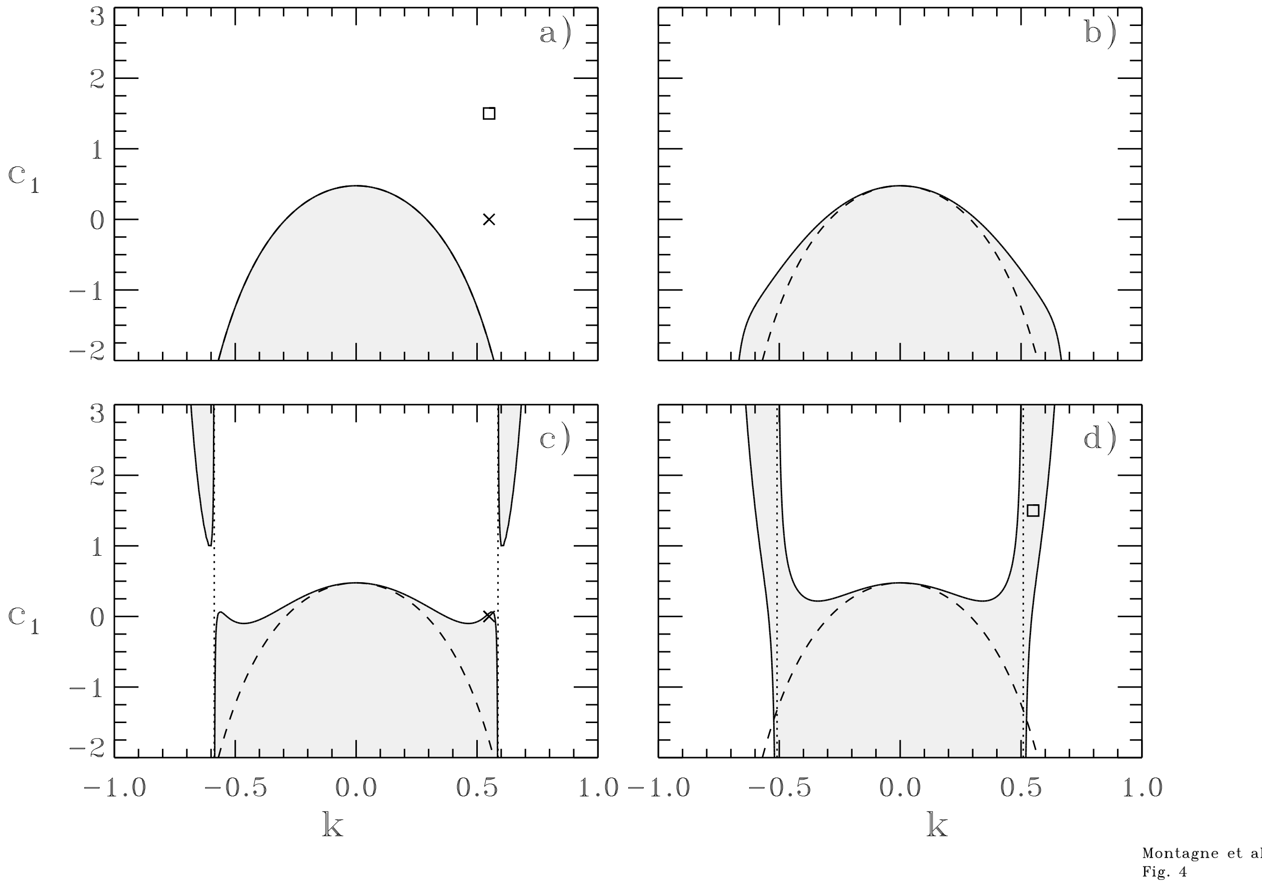

FIG. 4. Stability region for the plane wave (2.2) for $c_2 = -2.1$ for a) $\gamma = 0$, b) $\gamma = 2$, c) $\gamma = 4$, d) $\gamma = 6$. For comparison, the boundary of the stability region for $\gamma = 0$ is shown in figs. b)-d) as a dashed line. The vertical dotted lines show the asymptotic lines $k = k_{A\pm}$ (see text).

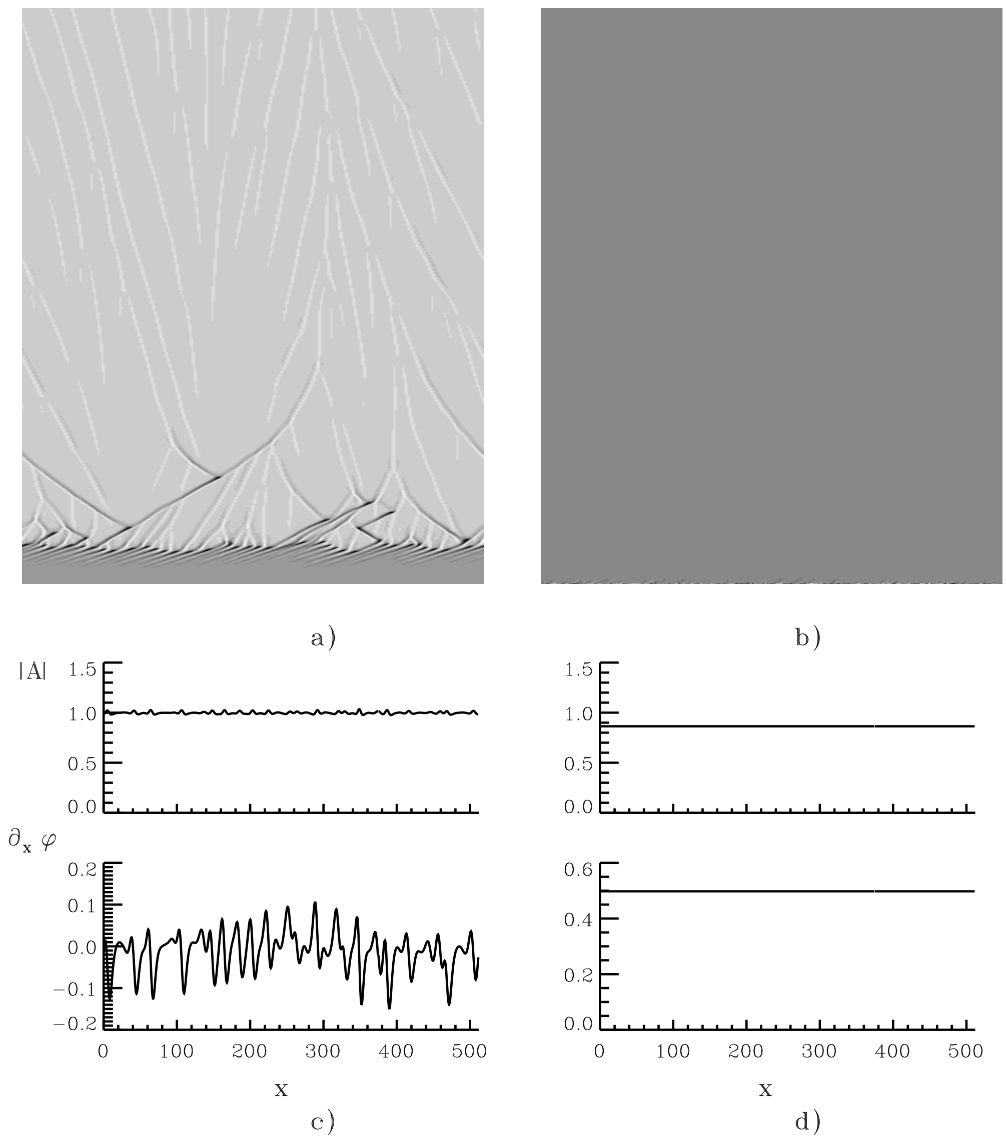

FIG. 5. Spatiotemporal evolution of the CGLE (2.4) for $c_1 = 1.5$, $c_2 = -0.9$ starting from a perturbed plane wave (4.1) with $k = 0.5$ and $\sigma = 0.007$. Figs. a) and b) show $|A(x,t)|$ with time running upwards from $t = 0$ to $t = 1000$ and x in the horizontal direction for $\gamma = 0$ and $\gamma = 3$ respectively. The absolute value of the field $|A(x,t_0)|$ and the phase gradient $\partial_x \phi(x,t_0)$ at $t_0 = 950$ are displayed in c) and d) for $\gamma = 0$ and $\gamma = 3$ respectively.

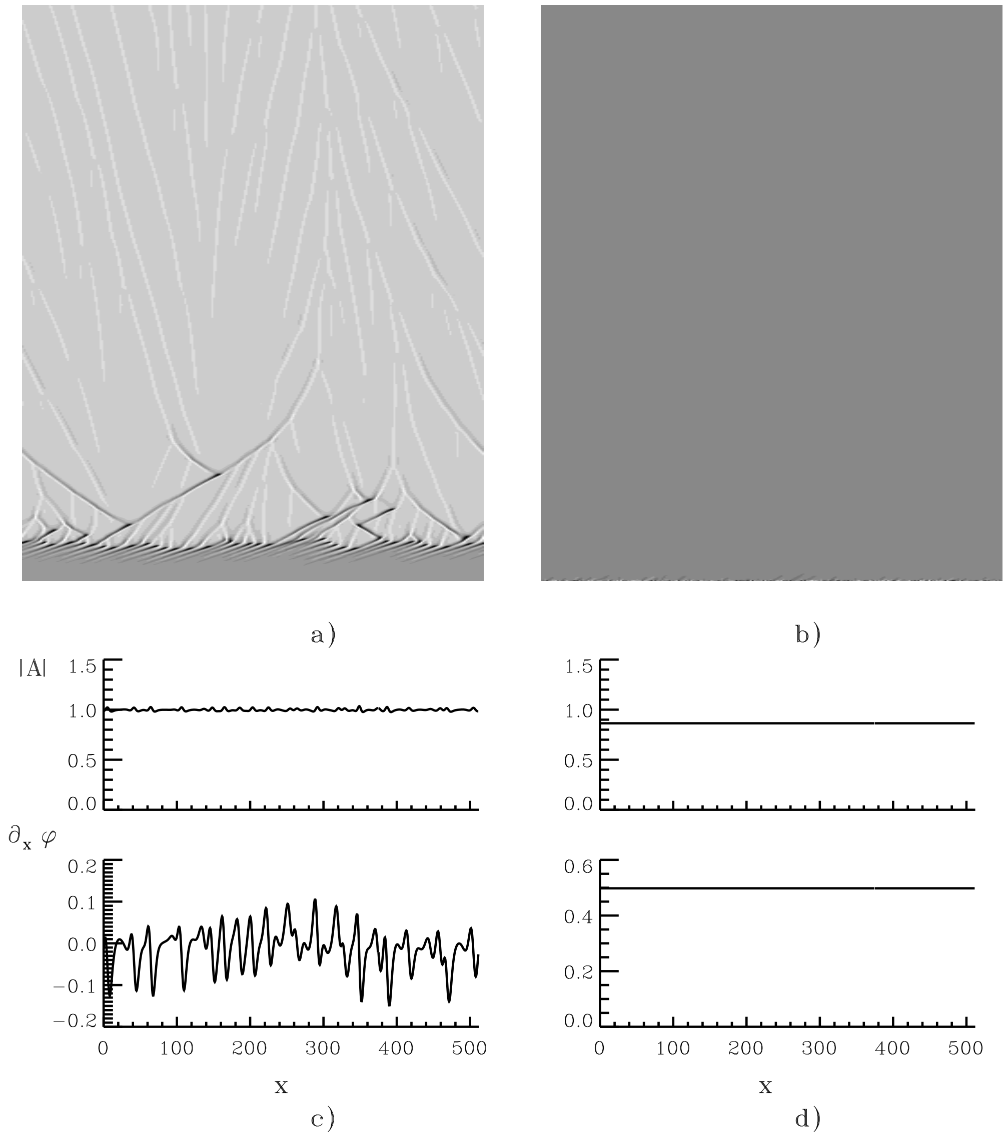

FIG. 6. Spatiotemporal evolution of the CGLE for $c_1 = 0$, $c_2 = -2.1$ starting from a perturbed plane wave with $k = 0.55$. Figs. a) and b) show $|A(x,t)|$ for $\gamma = 0$ and $\gamma = 4$ respectively. The values of $|A(x,t_0)|$ and $\partial_x \phi(x,t_0)$ at $t_0 = 950$ are displayed in c) and d) for $\gamma = 0$ and $\gamma = 4$ respectively. Other parameters are as in Fig. 5

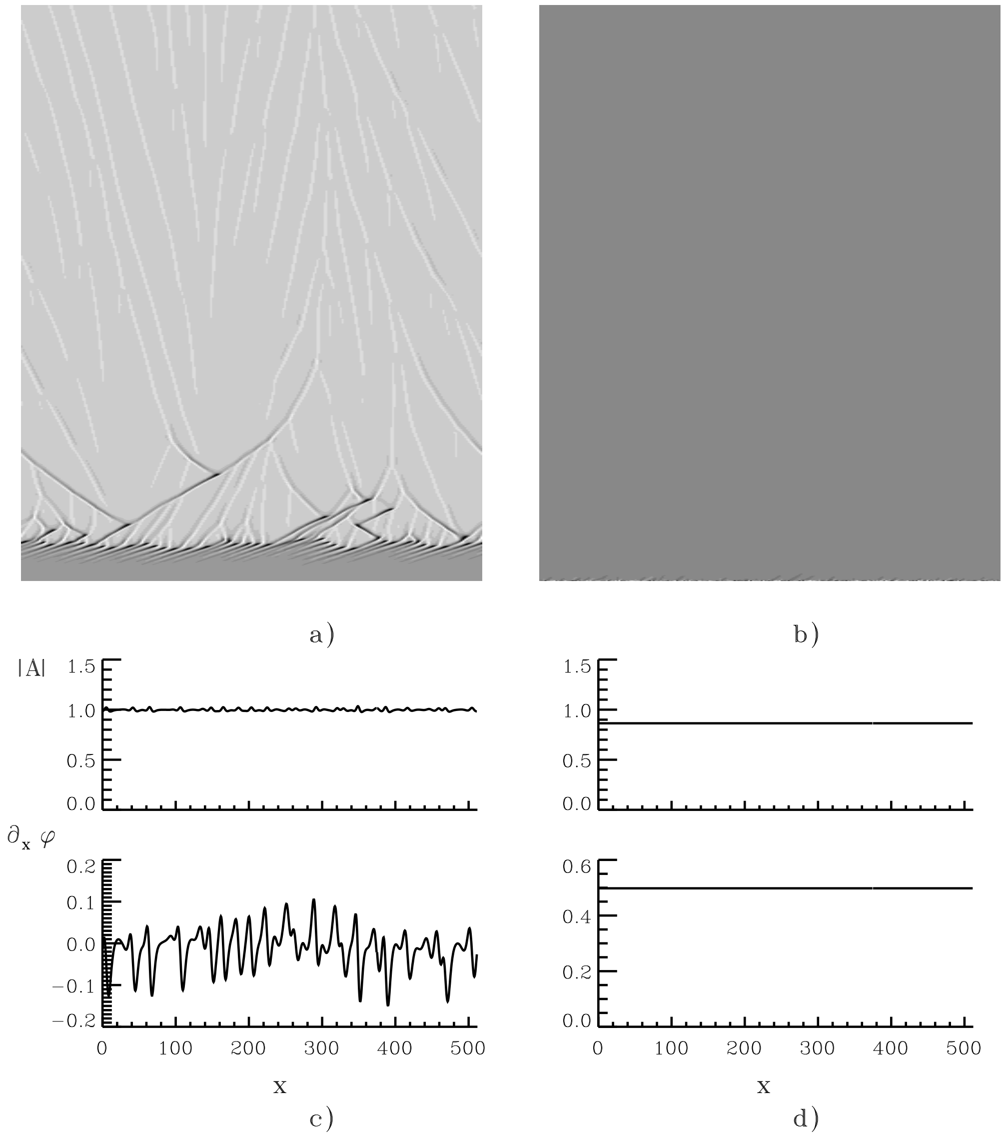

FIG. 7. Spatiotemporal evolution of the CGLE for $c_1 = 1.5$, $c_2 = -2.1$ starting from a perturbed plane wave with $k = 0.5$. Figs. a) and b) show $|A(x,t)|$ for $\gamma = 0$ and $\gamma = 6$ respectively. The values of $|A(x,t_0)|$ and $\partial_x \phi(x,t)$ at $t_0 = 950$ are displayed in c) and d) for $\gamma = 0$ and $\gamma = 6$ respectively. Other parameters are as in Fig. 5