Variable-basis topological systems versus variable-basis topological spaces∗

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Given a variety \( \mathbf{A} \) (with the dual category \( \text{LoA} \)), we introduced in [3] the category \( \text{LoA-Top} \) of variable-basis topological spaces over \( \text{LoA} \). Its objects are triples \((X, A, \tau)\), where \( X \) is a set, \( A \) is a localic algebra (an object of \( \text{LoA} \)) and \( \tau \) is a subalgebra of \( A^X \). Morphisms \((X, A, \tau)\) \( \xrightarrow{(f, \varphi)} (Y, B, \sigma) \) are \( \text{Set} \times \text{LoA} \)-morphisms \((X, A)\) \( \xrightarrow{(f, \varphi)} (Y, B) \) such that \( \varphi^{op} \circ p \circ f \in \tau \) for every \( p \in \sigma \) (the so-called continuity). Our definition subsumes the traditional lattice-valued approach of [2]. The motivation for the new concept was provided by the problem of doing fuzzy mathematics without order.

In [1] the authors consider a relation between topological systems in the sense of [4] and variable-basis topological spaces over the category of locales. Following the example one can introduce the category \( \text{LoA-TopSys} \) of variable-basis topological systems over localic algebras. Its objects are tuples \((X, A, B, \models)\), where \( X \) is a set, \( A \) and \( B \) are localic algebras and \( X \times B \xrightarrow{\models} A \) is a map (the so-called satisfaction relation) such that \( B \xrightarrow{\models(x, \_)} A \) is a homomorphism (a morphism of \( \mathbf{A} \)) for every \( x \in X \). Morphisms \((X, A, B, \models)\) \( \xrightarrow{(f, \varphi, \psi)} (X', A', B', \models') \) are \( \text{Set} \times \text{LoA} \times \text{LoA} \)-morphisms \((X, A, B)\) \( \xrightarrow{(f, \varphi, \psi)} (X', A', B') \) such that \( \varphi^{op}(\models'(f(x), b')) = \models(x, \psi^{op}(b')) \) for every \( x \in X \) and every \( b' \in B' \).

In this talk we will consider functorial relations between the two above-mentioned categories generalizing that of [1]. In particular, we will construct an embedding \( \text{LoA-Top} \xrightarrow{E} \text{LoA-TopSys} \) which has a right adjoint. We will also show that the category \( \text{LoA-TopSys} \) is topological over its ground category iff the respective underlying functor is an isomorphism that poses (the still unanswered) question whether the category \( \text{LoA-TopSys} \) is algebraic. Some intrinsic properties of \( \text{LoA-TopSys} \) (e.g., products) will also be considered.

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References


