COMMON VALUES AND LOW RESERVE PRICES

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Abstract. I show that the benefit of a high reserve price in a common-values ascending auction is lower than in the observationally-equivalent private values setting. Put another way, when bidders actually have common values, empirical estimation based on a private values model will overstate the value of a high reserve price. Via numerical examples, I show that this same ranking typically applies to the level of the optimal reserve price as well, and often to the benefit of any reserve price, not just a high one. With common values, the optimal reserve price can even be below the seller’s valuation, which is impossible with private values.

1 Introduction

The workhorse model in auction theory – for both theoretical and empirical work – is the Symmetric Independent Private Values model. Among other things, this model suggests that a seller can always benefit from employing a reserve price strictly higher than her residual value for the unsold good, and often suggests that the profit-maximizing reserve price is fairly high – often much higher than reserve prices employed in practice. (Paarsch 1997 and Haile and Tamer 2003 are two examples.)

Subsequent theoretical work has shown, however, that a number of different deviations from the assumptions of the IPV model argue in favor of lower reserve prices. Correlation among bidder valuations (Quint 2008, Aradillas-López Gandhi and Quint 2013), uncertainty about the exact value distribution (Kim 2013), endogenous participation (Levin and Smith 1994, Samuelson 1985), and competition between sellers (Peters and Severinov 1997) have all been shown to reduce the reserve price that should be chosen by a profit-maximizing seller.
relative to the benchmark model – in some cases all the way down to the seller’s residual value.

All of these, however, maintain the assumption of private values. In this note, I compare the effect of a reserve price in ascending (English) auctions when bidders have private values, to its effect when bidder values are common, and bidders therefore face a winner’s curse.

Part of the challenge of this exercise is finding the right *ceteris paribus* comparison. In a common values setting, bidders’ valuations are inherently correlated, so comparing to a model with *independent* private values is potentially misleading. Instead, for a given common values environment, I define the private-values setting that would lead to identical bidding behavior in the absence of a reserve price.

I find that for an English auction in *any* interdependent or common values setting and a sufficiently high reserve price, both the likelihood of a sale and the expected revenue or profit are lower than under the analogous private values setting. (The same holds for *any* positive reserve price in a sealed-bid second-price auction.) I also explore a number of numerical examples, and find that the same revenue and profit rankings very often hold even outside of the range of reserves where the result is theoretically guaranteed, and that the optimal reserve price is similarly lower than it would be under private values.

My choice of comparison is meant to mirror the choice facing an empirical researcher. Given bid data from previous (reserve-free) auctions, the researcher could choose to rationalize the data via either a private- or a common-values model. My results suggest that when the true environment has common values but the researcher mistakenly assumes it has private values, her counterfactuals will be biased in a particular direction: they will overstate the benefit of any high reserve price (and likely any reserve price), and likely overstate the level of the profit-maximizing reserve.\(^2\)

One key feature of common value auctions, unlike private value auctions, is that bidders learn from each others’ bidding. This overturns the usual private-values result that reserve prices below the seller’s own valuation \(v_0\) are always dominated. Under common values, this need not be true, as a reserve price of \(v_0\) may truncate losing bids and thus reduce revenue from profitable sales. It’s even possible that two bidders who would not bid at a reserve of \(v_0\), would both bid at a lower reserve and then, seeing each other bidding, would both bid past \(v_0\), creating a profitable sale that would have been prevented by a reserve of \(v_0\). Thus, introducing a reserve price in an interdependent or common values setting has greater costs than it would in a similar private values setting, and these costs often seem to outweigh

\(^2\)The private values environment corresponding to a given common values setting will have affiliated values, so Quint (2008) implies that if the researcher assumed a common values setting actually had *independent* private values, the bias would be in the same direction, and even larger.
the benefits entirely. For many of the numerical examples I show in the paper, a reserve price of 0 maximizes revenue; when the seller’s valuation $v_0$ is fixed and positive, the profit-maximizing reserve price is sometimes below $v_0$, sometimes even 0. When the seller’s value is positively associated with the buyers’, this introduces an additional winner’s curse-type effect – the seller is more likely to retain the object when it is less valuable to him – which further decreases the value of a reserve price, and makes the optimal reserve price 0 in many cases. On the whole, relative to intuitions we have from private values settings, when bidder values are common or interdependent, this suggests that reserve prices should be used much more cautiously, if at all.

2 Closely Related Literature

Laffont and Vuong (1996) observe that in sealed-bid auctions, common and private values cannot be distinguished from one another purely from bid data, as the two models are observationally equivalent. I use their logic in comparing a common values setting to the corresponding private values setting that would lead to identical bidding.

Perhaps as a result of Laffont and Vuong’s finding, most of the empirical literature on auctions does not attempt to differentiate one model from the other empirically, and instead begins by assuming either one model or the other, but there are a few exceptions. Paarsch (1992) shows that the two models are distinguishable from each other under parametric distributional assumptions. For first-price auctions, Haile, Hong and Shum (2003) propose a test of common versus private values when there is exogenous variation in the number of bidders $N$, and Athey and Haile (2007), using ideas from Hendricks, Pinkse and Porter (2003), propose a test when there is a binding reserve price. For English auctions, Athey and Haile (2002) propose a test when there is variation in $N$, but also note that when $N$ is fixed and values are known to be common, the model is not identified from observed bids.

Vincent (1995) shows that in a common values setting, if the seller’s valuation is unknown but independent of the bidders’ valuation, the seller can sometimes benefit ex ante from using a secret reserve price. (The seller would still be tempted to deviate and announce the reserve price when it is low, however, and therefore needs to be able to commit to keeping it secret.) Several papers noted in the introduction show deviations from the standard IPV model (while maintaining the assumption of private values) which favor lower reserve prices than the IPV benchmark.
3 Second Price Auctions

While the focus of this paper is English (or ascending) auctions, much useful intuition can be gained from considering a simpler case: sealed-bid second price auctions. In this section, I show how second-price auctions under two different models of bidder valuations – one with interdependent values, and one with private values – which would generate the same bidding behavior in the absence of a reserve price, respond differently to the addition of a positive reserve price. In the next section, I will do the same for English auctions.

3.1 Two Models of Valuations

Model I – Interdependent Values

I use the standard interdependent values model with affiliated signals of Milgrom and Weber (1982). Fix $N$ the number of bidders. Let $X = \{X_1, \ldots, X_N\}$ be a set of signals drawn from a joint distribution which is symmetric and affiliated. Each bidder $i$ learns the realization $x_i$ of one signal $X_i$, but his valuation $V_i = u_i(X) = u(X_i, \{X_j\}_{j \neq i})$ depends (symmetrically) on the signals observed by the other bidders as well. As in Milgrom and Weber, $u : \mathbb{R}^N \rightarrow \mathbb{R}$ is assumed to be nonnegative, continuous, nondecreasing, strictly increasing in its first argument, and symmetric in its last $N-1$ arguments, with $\mathbb{E}u_i(X) < \infty$. Since the point of this paper is to contrast the interdependent case with the private values case, I assume $u_i$ has nondegenerate dependence on $\{X_j\}_{j \neq i}$, and will often refer to this model as the common values case, although pure common values ($u_i(X) = u_j(X)$) is not assumed.

For a given bidder $i$, define $v(x, y)$ as the expectation of his value $V_i$, conditional on the realization of his own signal and the highest signal of his opponents,

$$v(x, y) = \mathbb{E} \left\{ u(x, \{X_j\}_{j \neq i}) \middle| X_i = x, \max_{j \neq i} X_j = y \right\}$$

(Note that the expectation over $\{X_j\}_{j \neq i}$ is taken conditional on $X_i = x$, since the signals may be correlated.) In the absence of a reserve price, Milgrom and Weber establish that bidding in the symmetric equilibrium of the second price auction is

$$b_i(X_i) = v(X_i, X_i)$$
It’s straightforward to show that if one’s opponents play this strategy, a bidder could do no better even if he knew the highest bid submitted by his opponents.\footnote{The second price auction also has asymmetric equilibria, including some “collusive-looking” equilibria with low revenue; like the literature, I focus on the symmetric equilibrium, which is unique.}

**Model II – Private Values**

Laffont and Vuong (1996) observe that in second-price auctions without a reserve price, bid data cannot be used to distinguish common values from private values, because however bidders chose to bid under common (or interdependent) values, they could alternatively have been bidding their valuations in a private-values setting. Using this logic, we define the private-values setting that is “observationally equivalent” to the interdependent-values setting above: the signals \( \{X_i\} \) have the same distribution as above, each bidder observes a single signal \( X_i \), but his valuation is now

\[
V_i = v(X_i, X_i)
\]

regardless of the realization of the other signals. Without a reserve price, bidders have a weakly dominant strategy of bidding their values, and therefore \( b_i(X_i) = v(X_i, X_i) \), so the two models produce identical bidding behavior.

### 3.2 The Effect of a Reserve Price

The effect of introducing a reserve price \( r > 0 \) in a private-values setting is straightforward. Bidders continue to bid their valuations, as long as those valuations are above \( r \); if not, they do not bid. Thus, if no bidder has a valuation exceeding \( r \), the reserve price prevents a sale, and the seller retains the object; if one bidder has a valuation exceeding \( r \), then he pays \( r \) rather than the second-highest valuation.

In a common (or interdependent) values setting, the effect of a reserve price is more complicated. In particular, the highest reserve price a bidder is willing to meet is lower than the bid he would make in the absence of a reserve. This is because of the winner’s curse: a bidder willing to meet a reserve price \( r \) must be willing to win the object even if no other bidder is willing to bid \( r \). On the other hand, when contemplating raising ones bid, a bidder conditions on his bid being pivotal, i.e., on another competitor having an equally high signal, partly mitigating this curse.

Thus, in a second-price auction with reserve price \( r \) and common values, bidder \( i \) still
bids \( v(X_i, X_i) \) if he bids, but only bids if \( X_i \geq x^* \), where \( x^* \) is defined implicitly by

\[
  r = \mathbb{E} \left\{ u(x^*, \{X_j\}_{j \neq i}) \mid X_i = x^*, \max_{j \neq i} X_j < x^* \right\}
\]

(A bidder with signal \( X_i = x^* \) therefore loses whenever another bidder bids, and pays \( r \) for a prize whose expected value is \( r \) when no other bidder bids.) By iterated expectations, if we let \( X^{(k)} \) denote the \( k^{th} \) order statistic of \( \{X_1, \ldots, X_N\} \), the definition of \( x^* \) is equivalent to

\[
  r = \mathbb{E} \left\{ v(x^*, X^{(2)}) \mid X^{(1)} = x^* \right\}
\]

Since by definition \( X^{(2)} \leq X^{(1)} \), the right-hand side is strictly less than \( v(x^*, x^*) \) except in degenerate cases. Thus, a bidder willing to bid more than \( r \) in the absence of a reserve price, may still not be willing to bid at all with a reserve price of \( r \).\(^4\) On the other hand, in a private values setting, a bidder’s willingness to bid (in the absence of a reserve price) is exactly the reserve price he would be willing to meet. This leads to the following results:

**Theorem 1.** Fix a common values setting. For any reserve price \( r \)

1. the likelihood of a sale is lower than in the corresponding private values setting
2. expected revenue is lower than in the corresponding private values setting
3. if \( r \) is greater than the seller’s residual valuation \( v_0 \), expected profit is lower than in the corresponding private values setting

All theorems are proved in Appendix A. Theorem 1 says that if you had bid data from second-price auctions in a setting with common values, but chose to estimate a structural model under the assumption of private values and used it to evaluate a reserve price counterfactual, you would underestimate the reduction in sales that would follow from introducing a reserve price, and therefore overestimate the benefit of introducing a reserve.

### 4 English Auctions

Next, I turn to English auctions. It’s well know that with private values, English and second-price auctions are strategically equivalent. With common values, however, this is not the case, since bidders update their beliefs about their own valuations based on how their opponents bid.

\(^4\)This is the logic behind Vincent’s (1995) result that a secret reserve price can sometimes be beneficial: when the seller’s valuation is high and he therefore wants to set a high reserve price, two bidders might be deterred from bidding who might otherwise have bid above that price.
I’ll employ the same strategy for English auctions as I did for second price auctions: begin with a general interdependent values setting, imagine I observed bids in English auctions without reserve prices, use those bids to define an observationally-equivalent private values setting, and then compare the impact of introducing a reserve price across the two settings. Three things make this exercise more complicated for English auctions:

1. Since an English auction ends before the winner reveals his willingness to pay, I’ll need to make a decision about what his valuation should be in the private-values setting.

2. In a common values setting, bidders condition on each others’ behavior, so bids (and therefore imputed private valuations) are “more correlated” than information based on a single signal. This ends up implying that while high reserve prices are more likely to be met under the private values model, low but positive reserve prices may be more likely to be met under the common values model. (It will become clear why shortly.)

3. In a common values setting, a reserve price that does not set the price (because two or more bidders bid) still effects the price paid. If any losing bidders choose not to bid, their exact signals cannot be inferred by the bidder who eventually sets the price; via a “linkage principal” effect, this decreases expected price in the common values model, while it has no effect in a private values setting.

To address the first point, I imagine that after each auction, I was able to interview the winner and find out how high he had been planning to bid if his last opponent had not dropped out when he did, and use this as the winner’s valuation in the corresponding private values setting. (This is data one might conceivably have in settings where bidders use automated proxy bids, such as on eBay.) To address the second, I will define a threshold reserve $\tilde{r}$ above which I can unambiguously sign the revenue and profit rankings between the two models. I will also use numerical examples to show that even when $\tilde{r}$ is high (and therefore the theoretical result is weak), the same revenue and profit rankings very often hold for reserves below $\tilde{r}$. The third point simply adds steps to the proof of the result.

4.1 Two Models of Valuations

As in Milgrom and Weber (1982), I model English auctions as full information button auctions: the price starts low and rises continuously, bidders remain active until they choose to irreversibly drop out of the bidding, and bidders know who is currently active and at what price inactive bidders dropped out.
Model I – Interdependent Values

The model is the same as above. $N$ bidders receive affiliated signals $\{X_i\}$ and have valuations $V_i = u(X_i, \{X_j\}_{j \neq i})$. Symmetric equilibrium bidding is described by Milgrom and Weber (1982, sections 5 and 7).\(^5\) In the absence of a reserve price, equilibrium bidding can be briefly summarized as follows. At each point in the auction, the signals of the bidders who have already dropped out are correctly inferred by the remaining bidders. Given those signals, each bidder bids up until the price at which he would be exactly indifferent about buying the object if all his remaining opponents turned out to have signals matching his own.

Model II – Private Values

To create the observationally equivalent private values environment, I imagine we observe equilibrium English-auction bidding (with no reserve price) under the first model – including the price at which the winner planned to drop out – but interpret each bidder’s bid as his private value.\(^6\) Formally, let $i(k)$ denote the label of the $k^{th}$ highest signal, so that $X_{i(k)} = X^{(k)}$. Let $X^{(1)} \geq X^{(2)} \geq \ldots \geq X^{(N)}$ denote the order statistics of $\{X_i\}$, and $x^{(1)} \geq \ldots \geq x^{(N)}$ their realization. Then for a given realization $x$ of $X$, I define bidder valuations as

$$v_i(N) = u(x^{(N)}, x^{(N)}, x^{(N)}, \ldots, x^{(N)}, x^{(N)})$$

$$v_i(N-1) = u(x^{(N-1)}, x^{(N-1)}, x^{(N-1)}, \ldots, x^{(N-1)}, x^{(N-1)}, x^{(N)})$$

$$v_i(N-2) = u(x^{(N-2)}, x^{(N-2)}, x^{(N-2)}, \ldots, x^{(N-2)}, x^{(N-2)}, x^{(N-1)}, x^{(N)})$$

$$\vdots$$

$$v_i(2) = u(x^{(2)}, x^{(2)}, x^{(3)}, x^{(4)}, \ldots, x^{(N-2)}, x^{(N-1)}, x^{(N)})$$

$$v_i(1) = u(x^{(1)}, x^{(1)}, x^{(3)}, x^{(4)}, \ldots, x^{(N-2)}, x^{(N-1)}, x^{(N)})$$

Each bidder learns the realization $v_i$ of his own valuation $V_i$.

\(^5\)Bikhchandani, Haile and Riley (2002) show that a continuum of symmetric, separating equilibria exist, but they all lead to the same outcome. They also lead to the same values of $v_i(1)$ and $v_i(2)$ in the private values model described below, and therefore the same outcome in that model. Thus, the multiplicity is not important for our purposes, and I focus on the equilibrium described by Milgrom and Weber.

\(^6\)An alternative assumption about the winner’s private value would be to imagine we had an independent measure of the winner’s ex post surplus and use that, i.e., set $v_i(1)$ below equal to $u_i(1)(X)$. Under that assumption, Theorem 2 below would still hold under one change: modifying the definition of $\tilde{r}$ to be the highest crossing point of the CDFs $F_{U(1)}$ and $F_{R(1)}$, rather than $F_{V(1)}$ and $F_{R(1)}$. 

8
Observational Equivalence of the Two Models when \( r = 0 \)

By construction, a bidder’s private value \( v_i \) in the second model is exactly the price at which he would drop out on the equilibrium path under the first model in the absence of a reserve price. In addition, the winning bidder’s private value in the second model is the price at which he would have planned to drop out in the first model, had the second-highest bidder continued bidding.\(^7\) Thus, the two models generate the same equilibrium bids – and would still appear identical if we knew at what price the winner had planned to drop out.

4.2 The Effect of a Reserve Price

In the private values case, bidding in an auction with reserve price \( r \) is again straightforward: each bidder \( i \) bids if \( v_i \geq r \), and drops out at price \( v_i \).

In the common values case, the threshold signal \( x^* \) above which bidders are willing to bid in an English auction with reserve price \( r \) is the same as in a second price auction, and is defined (as above) by

\[
    r = \mathbb{E} \left\{ u(x^*, \{X_j\}_{j \neq i}) \mid X_i = x^*, \max_{j \neq i} X_j < x^* \right\}
\]

(Once again, a bidder with the threshold signal expects to be outbid if anyone else bids, and therefore must be indifferent when he is the only bidder willing to meet the reserve price.) Since bidders with signals below \( x^* \) do not reveal their signals through bids, bidders who do bid condition only on \( X_j < x^* \) for bidders who don’t bid; otherwise, equilibrium bidding among those who bid is the same, with each bidder dropping out when the price reaches the level at which he would in expectation be indifferent to winning, conditional on the information revealed by the bidders who have already dropped out, if all his remaining active opponents had received the same signal as him.

Define \( Rev_{CV}(r) \) and \( Rev_{PV}(r) \) as the expected revenue under the two models, as a function of the reserve price \( r \); and fixing the seller’s residual valuation \( v_0 \), define \( \pi_{CV}(r) \) and \( \pi_{PV}(r) \) as expected profits under the two models. My main theoretical result will be that \( Rev_{CV}(r) < Rev_{PV}(r) \) and \( \pi_{CV}(r) < \pi_{PV}(r) \) when \( r \) is sufficiently high. Since the observational equivalence described above implies \( Rev_{CV}(0) = Rev_{PV}(0) \) and \( \pi_{CV}(0) = \pi_{PV}(0) \), this means the benefit of a high reserve price is less under common values than under private values – or when the “truth” is common values, a private-values analysis will overstate the benefit of a high reserve price. After, I will explore several numerical examples,

\(^7\)As noted above, we could alternatively let the winner’s private value match his ex post surplus from winning in the first; with a change in the definition of \( \tilde{r} \), the results would go through.
under which these rankings often hold at all reserve price levels, and optimal reserve prices are lower under common values as well.

First, I need to formalize what it means for \( r \) to be “sufficiently high” for the results to hold. Consider three random variables,

\[
V^{(1)} = u(X^{(1)}, X^{(3)}, \ldots, X^{(N)})
\]

\[
U^{(1)} = u(X^{(1)}, X^{(2)}, X^{(3)}, \ldots, X^{(N)})
\]

\[
R^{(1)} = \mathbb{E}_{X^{(2)}, \ldots, X^{(N)}|X^{(1)}} \{ u(X^{(1)}, X^{(2)}, X^{(3)}, \ldots, X^{(N)}) \}
\]

and let \( F_{V^{(1)}}, F_{U^{(1)}}, \) and \( F_{R^{(1)}} \) denote their probability distributions.

Under the common values model, for a given realization \( x^{(1)} \) of \( X^{(1)} \), the expectation \( \mathbb{E} \{ u(x^{(1)}, X^{(2)}, X^{(3)}, \ldots, X^{(N)}) | X^{(1)} = x^{(1)} \} \) is the highest reserve price at which a bidder with signal \( x^{(1)} \) would be willing to bid under the common values model. Thus, \( F_{R^{(1)}}(r) \) is the probability that no bidder would be willing to bid given a reserve price of \( r \), and therefore the probability that a reserve of \( r \) would fail to be met under common values.

Under the private values model, \( u(x^{(1)}, x^{(1)}, x^{(3)}, \ldots, x^{(N)}) \) is the imputed valuation of the winning bidder, and thus the highest reserve price at which he would bid under the private values model. \( F_{V^{(1)}}(r) \) is therefore the probability that a reserve of \( r \) fails to be met under the private values model.

What will be crucial for us is that \( F_{V^{(1)}}(r) \leq F_{R^{(1)}}(r) \) for \( r \) sufficiently high. To see why, first note that \( U^{(1)} \) is a mean-preserving spread around \( R^{(1)} \), and thus, we would expect \( F_{U^{(1)}} \) to typically be above \( F_{R^{(1)}} \) at low values and below it at high values. On the other hand, since \( V^{(1)} \geq U^{(1)} \) for each realization of \( X \), \( F_{V^{(1)}} \leq F_{U^{(1)}} \) everywhere. Thus, we would expect \( F_{V^{(1)}}(r) \leq F_{R^{(1)}}(r) \) for high values of \( r \), while the rankings for low \( r \) are ambiguous.

I define \( \tilde{r} \) as the highest crossing point of these two CDFs; that is, \( \tilde{r} \) is defined such that \( F_{V^{(1)}}(r) \leq F_{R^{(1)}}(r) \) for every \( r > \tilde{r} \).

**Definition 1.** \( \tilde{r} \equiv \sup \{ r : F_{V^{(1)}}(r) > F_{R^{(1)}}(r) \} \).

Figure 1 illustrates the CDFs of \( V^{(1)} \), \( U^{(1)} \), and \( R^{(1)} \), and therefore how \( \tilde{r} \) is defined, for the two numerical examples from the next section: the “discrete \( \theta \)” example presented in Section 5.1, and the “continuous \( \theta \)” example presented in Section 5.4, each with \( N = 3 \) and \( N = 6 \). In each case, it’s easy to see that \( F_{U^{(1)}} \) crosses \( F_{R^{(1)}} \) just once, near the middle of the support, and that \( F_{V^{(1)}} < F_{U^{(1)}} \) everywhere. In the two examples with \( N = 3 \), the difference

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8While \( F_{R^{(1)}} >_{SOSD} F_{U^{(1)}} \) does not guarantee that the two CDFs cross only once, they do in all the numerical examples I’ve examined.
between $F_V(1)$ and $F_U(1)$ (the impact of changing one bidder’s signal) is large, making $F_V(1)$ below $F_R(1)$ on most of the range of valuations. (In the bottom-left pane, $F_V(1)$ and $F_R(1)$ never cross, so $\tilde{r} = 0$.) In the two examples with $N = 6$, one bidder’s signal has less effect, so $F_V(1)$ and $F_U(1)$ are closer together, making $\tilde{r}$ larger.

For reserve prices above $\tilde{r}$, revenue and profit rankings across the two models are clear:

**Theorem 2.** Fix a common values setting. For any reserve price $r \geq \tilde{r}$...

(i) the likelihood of a sale is lower than in the corresponding private values setting

(ii) expected revenue is lower than in the corresponding private values setting

(iii) if $r \geq v_0$, expected profit is lower than in the corresponding private values setting

Of course, the applicability of Theorem 2 depends on how restrictive the assumption $r \geq \tilde{r}$ is. In the next section, I examine a range of numerical examples, solved via simulation, to illustrate the level of $\tilde{r}$ as well as the profit curves $\pi_{CV}(r)$ and $\pi_{PV}(r)$. As Figure 1 suggests, when $N$ is small, $\tilde{r}$ tends to be close to 0, and so Theorem 2 is quite informative. (When $N = 2$, for example, $\tilde{r}$ is always 0.) When $N$ is large, $\tilde{r}$ tends to be high; in those instances,
Theorem 2 only applies to very high reserve prices, which may be unrealistic unless the seller’s own valuation $v_0$ is quite high.

However, the numerical examples also show that the revenue and profit rankings of Theorem 2 very often hold even for $r$ below $\tilde{r}$ – in most examples, for all $r$. We will also see that the profit-maximizing reserve price is typically lower under a common values model than under the corresponding private values model. In fact, with common values, the profit-maximizing reserve price is sometimes below the seller’s own valuation – which is impossible with private values.

5 Numerical Examples

In this section, I offer two numerical examples in which I explicitly calculate the level of $\tilde{r}$, compare $\pi_{CV}(r)$ to $\pi_{PV}(r)$, and compare the optimal reserve prices under the two models. In both examples, bidders have pure common values. (In Section 6.3, I’ll consider another example where valuations are a mix of common and private.) There is an underlying “state of the world” $\theta$ which is every bidder’s ex post valuation, and bidder signals are i.i.d. conditional on the value of $\theta$.9

5.1 Setup

For the first example, $\theta$ takes the values 0 and 1, with equal probability, and conditional on $\theta$, bidder signals $\{X_i\}$ are i.i.d. draws from a distribution $F(\cdot|\theta)$ on $[0,1]$, where

$$f(s|\theta) = \begin{cases} \alpha s^{\alpha-1} & \text{if } \theta = 1 \\ \alpha(1-s)^{\alpha-1} & \text{if } \theta = 0 \end{cases} \quad \text{and} \quad F(s|\theta) = \begin{cases} s^{\alpha} & \text{if } \theta = 1 \\ 1 - (1-s)^{\alpha} & \text{if } \theta = 0 \end{cases}$$

Note that for $\alpha > 1$, $f(\cdot|1)$ is increasing and $f(\cdot|0)$ is decreasing, so lower signals are more likely when $\theta = 0$ and higher signals when $\theta = 1$. A higher value of $\alpha$ means $f(\cdot|1)$ is more skewed toward high signals and $f(\cdot|0)$ more skewed toward low signals, so the signals are more informative about $\theta$ (and also more highly correlated with each other).

For the common values model, the bidders all have valuation $\theta$. The corresponding private values model is defined as in the previous section.

9In both examples in this section, the conditional distribution of bidder signals is increasing in $\theta$ via the strict MLRP, so bidder signals are affiliated, and $E(\theta|X)$ is increasing in $X$. Thus, if we think of valuations as $V_i = E(\theta|X)$ instead of $V_i = \theta$, both examples fit within the Milgrom-Weber framework.
5.2 Effect of Reserve Price on Bidding Behavior

For an illustration of how a reserve price effects bidding under the two models, we first consider bidding given a fixed realization of signals. Suppose that $N = 5$ and $\alpha = 2$, and that the realized signals are $X_1 = 0.82$, $X_2 = 0.65$, $X_3 = 0.5$, $X_4 = 0.35$, and $X_5 = 0.18$. (These are approximately the unconditional medians of each order statistic.) In an English auction with no reserve price, bidder 5 would drop out when the price reached

$$u(0.18, 0.18, 0.18, 0.18, 0.18) = E(\theta | X_1 = X_2 = X_3 = X_4 = X_5 = 0.18) \approx 0.0005$$

and the other bidders would correctly infer that $X_5 = 0.18$ from the price at which he dropped out. As a result, bidder 4 would drop out at a price of

$$u(0.35, 0.35, 0.35, 0.35, 0.18) \approx 0.0181$$

revealing $X_4 = 0.35$, and bidder 3 would drop out at

$$u(0.5, 0.5, 0.5, 0.35, 0.18) \approx 0.1057$$

revealing $X_3 = 0.5$. Bidder 2 would drop out at $u(0.65, 0.65, 0.5, 0.35, 0.18) \approx 0.2896$, and bidder 1 would be pleased, having been prepared to bid up to $u(0.82, 0.82, 0.5, 0.35, 0.18) \approx 0.7104$ had bidder 2 kept going that long. Thus, for the analogous private values setting, I consider $V_1 = 0.71$ and $V_2 = 0.29$.

Figure 2 contrasts the effects of adding a reserve price under the two models, given this particular set of realized signals. The blue (dashed) curve shows revenue under the private values model. When $r < 0.29$, bidders 1 and 2 both bid, so the reserve price does not bind; revenue equals bidder 2’s bid, which is his valuation, 0.29. For $r$ between 0.29 and 0.71, bidder 1 bids, and pays the reserve price. When $r > 0.71$, nobody bids, and revenue is 0.

The red (solid) curve shows revenue under the common values model, where the reserve price has more complicated effects.

- At $r \approx 0$, all five bidders bid, and the result is the same as without a reserve.
- The highest reserve at which bidder 5 is willing to bid is

$$E \{ u(X) | X^{(1)} = 0.18 \} = E \{ \theta | X_1 = 0.18, \{ X_1, X_2, X_3, X_4 \} < 0.18 \} \approx 0.00002$$

10Of course, this particular realization of signals is relatively unlikely – more typically, either all five signals will be higher (because $\theta = 1$), or all five will be lower (because $\theta = 0$). Still, this realization makes for a nice illustration of the effect of $r$. 
Thus, at reserves $r > 0.00002$, bidder 5 does not bid, and rather than learning $X_5 = 0.18$, the other bidders merely learn $X_5 < x^*$. Likewise, at $r > E(\theta|X^{(1)} = 0.35) \approx 0.001$, bidder 4 does not bid either; and at $r > E(\theta|X^{(1)} = 0.5) \approx 0.012$, even bidder 3 does not bid.

So for reserve prices $r \in (0.00002, 0.001)$, bidders 1 through 4 bid, and they condition on $X_5 < x^*$. For reserve prices $r \in (0.001, 0.012)$, bidders 1, 2, and 3 bid, and condition on $\{X_4, X_5\} < x^*$; and for $r \in (0.012, 0.09)$, only 1 and 2 bid, and condition on $\{X_3, X_4, X_5\} < x^*$. As the graph shows, this pushes revenue below what it would be under the private values model.

(Note that as $r$ increases within one of these intervals, revenue increases, since $x^*$ increases with $r$, and so the inference made about truncated signals below $x^*$ becomes less negative.)

- For reserve prices between 0.09 and 0.52, bidder 1 is the only bidder, and pays the reserve price. At $r > 0.52$, nobody bids, and revenue is 0.

For this particular realization of signals, every reserve price $r$ gives weakly less benefit under the common values model than under the analogous private values setting. This need not always be the case – for some signal realizations, certain reserve prices are more beneficial under common values – but Theorem 2 says that at least for reserve prices above $\tilde{r}$, those signal realizations which are less favorable under common values will dominate in expectation.
This example also demonstrates the magnitude of the winner’s curse. Bidder 2 received a signal $X_2 = 0.65$, which is nearly twice as likely when $\theta = 1$ as when $\theta = 0$. Nevertheless, when considering whether to bid at a given reserve price, he worries about winning when all other bidders had signals low enough to not bid. Although $E(\theta|X_2 = 0.65) = 0.65$, $E(\theta|X_2 = 0.65, \{X_1, X_3, X_4, X_5\} < 0.65) \approx 0.09$; so while bidder 2’s best guess on his own is that the prize is worth 0.65, he refuses to bid even at a reserve price of 0.1.

5.3 Effect of Reserve Price on Expected Revenue and Profit

Of course, our main interest is not the outcome for a particular realization of signals, but ex ante expected outcomes. Figures 3, 5, 6, and 7 compare expected auction outcomes at different reserve prices across the two valuation models for various parameterizations of this example. Outcomes were calculated via numerical simulation; details are given in Appendix B. For each chart, the $x$-axis is reserve price, ranging from 0 to 1 (the support of valuations), and the $y$ axis is expected profits (or expected revenue in the case of Figure 3). Outcomes under the common values model, $\pi_{CV}(r)$ (or $Rev_{CV}(r)$), and its maximizer $r^*_{CV}$, are shown in red; results for the corresponding private values model ($\pi_{PV}(r)$ or $Rev_{PV}(r)$ and its maximizer $r^*_{PV}$) are shown in blue. A dashed vertical line indicates the value of $\tilde{r}$, above which Theorem 2 applies; a green vertical line indicates the seller’s valuation, $v_0$.

For Figures 3, 5, and 6, the level of signal precision $\alpha$ is fixed at 2. Figure 3 shows expected revenue, as a function of reserve price, under the two different models, for various values of $N$. Some things to note:

- $\tilde{r}$ (the dashed black line) is 0 at $N = 2$, and then increasing in $N$. Thus, Theorem 3 covers the widest range of reserve prices when $N$ is small – exactly when reserve prices are most significant. Also note that the revenue-maximizing reserve price under the private values model, $r^*_{PV}$, is above $\tilde{r}$ for $N \leq 5$.

- While Theorem 2 only applies for $r \geq \tilde{r}$, the revenue ranking $Rev_{CV}(r) \leq Rev_{PV}(r)$ holds everywhere – at all $r$ both above and below $\tilde{r}$, for each $N$ considered in Figure 3.

- For $N \geq 3$, the revenue-maximizing reserve price under the common values model is $r = 0$, while the revenue-maximizing reserve price under the private values model is substantial.

- As $N$ grows, the effect of reserve price vanishes under private values, but not under common values. When $N = 20$, revenue under private values is almost perfectly flat over the entire range of possible reserve prices; but a reserve price of 0.16 (optimal under private values) would cause a 6% loss in revenue under common values.
Figure 3: Expected revenue as a function of $r$ for various $N$ ($\alpha = 2$)
Note that fixing $r$, as $N$ grows, expected revenue in the common values case is increasing toward 0.5, but gets there much more slowly than in the private values case. Figure 4 helps illustrate why, for the reserve price $r = 0.20$. In the common values case, as $N$ increases, $x^*$ increases as well, since the winner’s curse a bidder must account for gets more severe. The left pane shows that as $N$ increases, the likelihood of nobody bidding increases, driven by an increase in the likelihood of no bids when $\theta = 0$. (The likelihood of no bids when $\theta = 1$ is decreasing, but is already so low that the decrease has less effect.)

**Figure 4: Effect of $N$ on outcomes with fixed reserve price $r = 0.20$**

The right pane of Figure 4 shows expected revenue when $r = 0.20$ in the private values (blue) and common values (red) cases, as $N$ changes. The dashed gray line shows expected revenue (under either model) when no reserve price is used. The yellow line shows expected revenue under the common values model if bidders only bid when their signals exceed $x^*$, but those who do bid somehow learn the signals of the bidders who don’t – that is, expected revenue if we shut down the effect the reserve price has on revenue through the truncation of losing bids. Thus, the difference between the red and orange curves is the magnitude of this “bid truncation effect,” while the difference between the solid yellow and dashed gray lines is the revenue loss (or gain when $N$ is small enough) due to the more obvious effects of a reserve price – the tradeoff between the possibility of no sale and the higher price when exactly one bidder bids.

Figure 5 compares expected profit across the two models, assuming the seller’s valuation $v_0$ is 0.20. (Recalling that $E(V_i) = E(\theta) = \frac{1}{2}$ under the common values model, this means the seller’s valuation is 40% of the expected buyer valuation.) Things to note in Figure 5:

- Like in Figure 3 with revenue, the profit ranking $-\pi_{CV}(r) \leq \pi_{PV}(r)$ – holds nearly

---

11In the private values case, the top two valuations $V^{(1)}$ and $V^{(2)}$ are, approximately, the expected value of $\theta$ conditional on all the realized signals; as $N$ grows, these approach 1 with high probability when $\theta = 1$, and 0 when $\theta = 0$, so expected revenue quickly converges to 0.5 for any interior $r$. 

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Figure 5: Expected profit as a function of $r$ for various $N$ ($v_0 = 0.20, \alpha = 2$)
everywhere. The only exception is for \( N = 2 \) at some reserve prices below \( v_0 \), at which \( \pi_{CV}(r) > \pi_{PV}(r) \). (This is because a sale is more likely under the private values model, but sales at prices below \( v_0 \) are unprofitable.)

- With \( v_0 = 0.20 \), a strictly positive reserve is optimal under common values as well as under private values, but \( r_{CV}^* < r_{PV}^* \). Under both common and private values, the optimal reserve price is decreasing in \( N \);\(^{12} \) and as \( N \) grows, the increase in profits from setting \( r \) optimally (relative to setting \( r = 0 \)) gets small quickly under both models.

- With \( v_0 = 0.20 \), when \( N \) is large (\( N = 10 \) and \( N = 20 \)), the optimal reserve price under common values is strictly below \( v_0 \) – the seller benefits from setting \( r \) low enough to risk selling at a loss. (Under private values, the optimal reserve is always above \( v_0 \).)

Figure 6 considers the case \( N = 5 \) and \( \alpha = 2 \), and examines profit under different values of \( v_0 \). Some things to note in Figure 6:

- \( \tilde{r} \approx 0.41 \), so Theorem 2 applies to reserve prices above 0.41, a range which includes \( r_{PV}^* \) for all values of \( v_0 \), but only includes \( r_{CV}^* \) when \( v_0 \) is quite high.

- For each value of \( v_0 \), \( r_{CV}^* < r_{PV}^* \); and for each value of \( v_0 \) and every value of \( r \) (both above and below \( \tilde{r} \)), \( \pi_{CV}(r) \leq \pi_{PV}(r) \).

- When \( v_0 \) is small but positive, the profit-maximizing reserve price under the common values model is strictly less than \( v_0 \).

Also note that when \( v_0 = 0.05 \), the finding that \( r_{CV}^* = 0 \) is not an artifact of a coarse grid: even a reserve price of 0.00001 would reduce expected profit.\(^{13} \)

- Except when \( v_0 \) is quite high, the gain from setting the reserve price optimally under the common values model, relative to not using a reserve at all, is fairly small.

Figure 7 shows how expected profit varies with \( \alpha \), the precision of the bidders’ signals. Things to note:

- As individual signals get more precise, the effects of a reserve price get smaller.

- Across both values of \( N \) and all values of \( \alpha \) tested, \( \pi_{CV}(r) \leq \pi_{PV}(r) \) for all \( r \), and \( r_{CV}^* < r_{PV}^* \).

\(^{12}\)Under an independent private values model, \( r_{PV}^* \) would be the same across \( N \); but this need not hold when values are correlated, as they are here.

\(^{13}\)When \( N = 5 \), a reserve of 0.00001 would still require a bidder to have a signal of at least 0.16 to bid, and would therefore cause nearly 30% of bidders not to bid when \( \theta = 0 \).
Figure 6: Expected profit as a function of $r$ for various $v_0$ ($N = 5$, $\alpha = 2$)
Figure 7: Expected profit as a function of \( r \) for various \( \alpha \) and \( N \) \((v_0 = 0.20)\)

\( \alpha = 1.5 \)

\( \alpha = 2.0 \)

\( \alpha = 2.5 \)

\( \alpha = 3.0 \)
• $r_{CV}^*$ is below $v_0$ when signals are very precise (high $\alpha$).

Finally, Table 1 in Appendix C summarizes a few key measurements from each graph in Figures 3, 5, 6, and 7. The table shows the value of the optimal reserve price (in terms of the increase in expected profit relative to $r = 0$) for both the private and common value models, as well as the gain or loss under the common values model if the reserve price optimal under private values were used (again relative to no reserve). The patterns that emerge are these:

• First, the optimal reserve price nearly always gives less than half as much benefit under common values as under private values.

• And second, when $v_0$ is low, mistakenly using $r_{PV}^*$ when there are actually common values is worse than using no reserve price; although when $v_0$ is high, mistakenly using $r_{PV}^*$ is not so bad.

Aside from these, two patterns are consistent across Figures 3, 5, 6, and 7: the profit-maximizing reserve price is lower under common values in every case; and in every case, $\pi_{CV}(r) \leq \pi_{PV}(r)$ for every $r \geq v_0$, not just those above $\tilde{r}$. (The only case so far with $\pi_{CV}(r) > \pi_{PV}(r)$ anywhere is for some values of $r < v_0$ in the $N = 2$ case in Figure 5.)

However, it is worth noting that at least this last result does not hold universally – that is, it is not true that for every possible common values model and its corresponding private values model, $\pi_{CV}(r) \leq \pi_{PV}(r)$ for every $r \geq v_0$. For a counterexample to this possible conjecture, fix $N = 5$ and $\alpha = 2$, and modify the baseline example by letting the common value be $V_i = 1 + \theta$ rather than $\theta$ (and modify the private value analogue accordingly). For $v_0 = 0$ and $r$ below about 1.2, the common values model predicts higher profit than the private values model. Figure 8 illustrates this example.

Figure 8: Expected profit, $N = 5$, $\alpha = 2$, $v_0 = 0$, common value $V_i = 1 + \theta$
the private values model predicts a greater decrease in the likelihood of the sale; while for
any \( r \), it predicts a more favorable impact on price. By inflating valuations to \( 1 + \theta \) instead
of \( \theta \), this example increases bids by 1 as well, while leaving the seller’s valuation unchanged.
Thus, the value of the sales given up by setting a reserve is magnified, relative to the changes
in price when a sale occurs. Thus, for \( r \) just above 1, the reserve price hurts more under
private values than under common values.

Of course, Figure 8 also shows that this reversal is not so practically relevant, since both
models suggest that a nonbinding reserve \( r \leq 1 \) is optimal. I have not been able to find
an example yet where \( \pi_{CV} \) is above \( \pi_{PV} \) (or \( \text{Rev}_{CV} \) is above \( \text{Rev}_{PV} \)) at reserve prices that
are “better than” \( r = 0 \), nor an example where the profit-maximizing reserve price is higher
under common values than under private values. Still, I have not been able to show that
such cases are impossible.

5.4 An Alternative Model with Continuous \( \theta \)

To make sure that the effects found so far do not hinge on the discreteness of \( \theta \) and the
resulting extremeness of valuations, I consider a second example with continuous-valued \( \theta \).
This time, \( \theta \) is distributed uniformly on \([0, 1]\). Once again, bidder signals take values in \([0, 1]\),
and are i.i.d. conditional on the value of \( \theta \), this time with density

\[
f(s|\theta) = 1 + 4 \left( \theta - \frac{1}{2} \right) \left( s - \frac{1}{2} \right)
\]

Thus, when \( \theta < \frac{1}{2} \), \( f(\cdot|\theta) \) is decreasing, so lower signals are more likely; and when \( \theta > \frac{1}{2} \),
\( f(\cdot|\theta) \) is increasing, and higher signals are more likely.

One difference between this example and the previous one is that under this signal struc-
ture, signals are not “unboundedly strong”. Even a signal of \( X_i = 0 \) or \( X_i = 1 \) leaves a
chance of a wide range of possible \( \theta \), and as a result, \( E(\theta|X) \) does not have full support.
This means that for reserves below a certain level, no bidders will be deterred from bidding;
and for reserves about a certain level, nobody will bid.\(^{14}\) Another feature of this model is
that for \( N < 5 \), the CDFs of \( R^{(1)} \) and \( V^{(1)} \) never cross, so \( \tilde{r} = 0 \); Theorem 2 therefore applies
to all reserve prices when \( N < 5 \).

Figures 9 and 10 show expected revenue and profit for this example, for various values
of \( N \) and \( v_0 \). As noted above, \( x^* = 0 \) for a range of low reserve prices, so the first part of
each revenue or profit curve is perfectly flat, and a choice between \( r = 0 \) and a reserve price

\(^{14}\)The maximum reserve at which everybody bids with probability 1 depends on \( N \): it is \( r \approx 0.20 \) for
\( N = 3 \), \( r \approx 0.145 \) when \( N = 5 \), \( r \approx 0.115 \) when \( N = 7 \), and \( r \approx 0.085 \) when \( N = 10 \). The minimum reserve
at which nobody bids is \( r = \frac{2}{3} \) regardless of \( N \).
Figure 9: Expected revenue and profit, continuous-θ example

\begin{align*}
N &= 3 \\
N &= 5 \\
N &= 7 \\
N &= 10
\end{align*}
Figure 10: Expected revenue and profit, continuous-θ example (cont’d)

Profit, $v_0 = 0.20$

Profit, $v_0 = 0.35$

$N = 3$

$N = 5$

$N = 7$

$N = 10$
anywhere in this range is immaterial. The main takeaways from this example are similar to those from the discrete-θ example:

- \( \pi_{CV}(r) \leq \pi_{PV}(r) \) nearly everywhere.
  
  (For small \( N \) and \( v_0 \) sufficiently high (not shown), \( \pi_{CV} > \pi_{PV} \) for some \( r < v_0 \); and for \( N = 10 \), \( Rev_{CV} > Rev_{PV} \) for some positive reserve prices below \( \bar{r} \) (shown in figure 9, pane 7), but at reserves giving lower revenue than \( r = 0 \). Aside from these exceptions, \( \pi_{CV} \leq \pi_{PV} \) and \( Rev_{CV} \leq Rev_{PV} \) everywhere in every parameterization I’ve tried.)

- \( r_{CV}^* < r_{PV}^* \), except when both are 0

- \( r_{CV}^* \) is sometimes below \( v_0 \), and sometimes 0 even when \( v_0 > 0 \)

Thus, the main takeaway from the numerical examples is again that most of the time, the optimal reserve price, and the expected profit at any given positive reserve price, is lower under common values than under the corresponding private values model.

The last two parts of Table 1 in Appendix C summarize key measurements from Figures 9 and 10. In particular, they show that under this example, when \( v_0 \) is 0 or 0.10, reserve prices have no value; when \( v_0 = 0.20 \), a reserve price offers a modest (3 to 6%) profit increase under private values, but nothing under common values; and when \( v_0 = 0.35 \), reserve prices are valuable under either model, but about half as valuable under common values as under private values.

6 Extensions

6.1 When \( v_0 \) Depends On θ (the “Seller’s Curse”)

Up to now, I’ve assumed \( v_0 \) is constant – the seller has a fixed valuation for the unsold object. In many settings – for example, if the seller’s valuation is based on the same use as the buyers’ valuations, or on resale prospects – it is natural to think the seller’s valuation would vary along with the buyers’. With a positive reserve price, this introduces an adverse selection problem analogous to the winner’s curse: the seller is most likely to retain the object exactly when it is least valuable to him. This curse further reduces the benefit of a reserve price.

Figure 11 illustrates this problem, comparing \( \pi_{CV}(r) \) when \( v_0 = \beta\theta \) to the case where \( v_0 \) is fixed at \( \beta E(\theta) = \frac{1}{2}\beta \), looking at both the discrete-θ and continuous-θ examples explored in the previous section. At \( r = 0 \), of course, the two give the same outcome, since the object is always sold; but for any \( r > 0 \), expected profits (and therefore the benefit of a reserve
Figure 11: Expected profit, common values, fixed $v_0$ versus $v_0 = \beta \theta$

- $N = 3$, discrete $\theta$
- $N = 6$, discrete $\theta$
- $N = 3$, continuous $\theta$
- $N = 6$, continuous $\theta$

$v_0 = 0.2$ versus $v_0 = 0.4\theta$

$v_0 = 0.35$ versus $v_0 = 0.7\theta$
price) are substantially lower when the seller’s valuation depends on $\theta$. In most of these cases, $r = 0$ was not optimal when $v_0$ was fixed, but becomes optimal when $v_0$ depends on $\theta$.

It is worth noting one case – the third pane, corresponding to the discrete-$\theta$ example with $N = 3$ and a high seller valuation. While the value of a reserve price goes down significantly when $v_0$ is a function of $\theta$ rather than fixed, the optimal reserve price actually goes up. This is because when $v_0 = 0.7\theta$ and the reserve price is already being set reasonably high, the seller effectively “gives up” on selling when $\theta = 0$, and optimizes primarily for the case where $\theta = 1$ and his residual valuation is 0.7; the optimal reserve price turns out to be right around 0.7, although the expected profit is only slightly higher than from setting $r = 0$. (This optimal reserve is still below the seller’s optimal reserve under the analogous private values model with $v_0 = 0.35$, which is about 0.78.)

### 6.2 Seller’s Incentives to Disclose Information

If the seller’s valuation depends on $\theta$, it might also be natural to suppose that the seller, like the buyers, may have some private information about $\theta$.

**Verifiable Information**

If the seller’s information is verifiable, the result from Milgrom and Weber (1982, Theorem 18) extends to our setting, and the seller can always gain ex ante by committing to reveal his information (and adjusting the reserve price accordingly).\(^\text{15}\)

Numerical examples suggest that the seller is likely to gain from revealing his information even if he does not alter the reserve price based on that information. Consider the discrete example from earlier, with $\alpha = 2$ and $v_0 = \beta\theta$. Suppose now that the seller also receives a signal $S$ about $\theta$; for simplicity, suppose the seller’s signal is binary, and matches $\theta$ with probability $\frac{3}{4}$.\(^\text{16}\) Figure 12 compares the seller’s expected profit at each reserve price when he reveals $S$ (in blue) to when he does not reveal $S$ (in red). In all four examples – $N = 3$ and 6, $v_0 = 0.4\theta$ and 0.7$\theta$ – the seller’s expected profit is strictly higher at every reserve price when he reveals $S$. The increase in profits from revealing $S$ and setting reserve optimally ranges from 2% (when $N = 6$ and $v_0 = 0.4\theta$) to 22% (when $N = 3$ and $v_0 = 0.7\theta$).

\(^{15}\)The result from Milgrom and Weber says that for any reserve price $\hat{r}$ with threshold signal $x^*(\hat{r})$, the seller can do weakly better by revealing the value $s$ of his own information $S$ and then setting the reserve price $r(s)$ that, conditional on $s$, gives the same threshold signal $x^*(r(s)|s) = x^*(\hat{r})$ as before. While the result in Milgrom and Weber is for $v_0$ fixed, this new policy does not alter the set of signal realizations $(S, X)$ under which the object is sold, so the dependence of $v_0$ on $X$ has no effect and the result therefore carries over to this setting.

\(^{16}\)That is, $\Pr(S = 0|\theta = 0) = \Pr(S = 1|\theta = 1) = \frac{3}{4}$, and $\Pr(S = 1|\theta = 0) = \Pr(S = 0|\theta = 1) = \frac{1}{4}$. 

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In three cases, the optimal reserve price is \( r = 0 \), regardless of whether \( S \) is revealed or its value. However, in the case of \( N = 3 \) and \( v_0 = 0.7\theta \), the optimal reserve price is strictly positive in each case, and different across cases, so it bears a closer look. Figure 13 shows more detail for this case. In red is the seller’s expected profit if he does not reveal \( S \). In blue is expected profit if the seller reveals \( S \), but then still sets the same reserve price regardless of \( S \). In green is expected profit when \( S = 1 \) and this is revealed to bidders; in light blue is expected profit when \( S = 0 \) and this is revealed to bidders.

In this example, even though the optimal reserve price changes when \( S \) is revealed, and varies with \( S \), nearly all the gain in profit comes from the release of the information itself, not the subsequent adjustment of the reserve price. The red dot (and the dashed red line from the dot to the \( y \)-axis) indicates that when \( S \) is not revealed to bidders, the optimal reserve price is \( r = 0.705 \), and gives expected profit of 0.0646. Relative to this benchmark, the seller can increase profits by 21.4% by revealing the value of \( S \) and not adjusting the reserve price at all; by 21.7% by revealing \( S \) and changing the reserve to the new optimal one unconditional on the value of \( S \) (the blue dot, and the blue dashed line); or by 22.3% by revealing \( S \) and changing the reserve to the new optimal one for each realized value of \( S \). (This last option is indicated by the level of the horizontal black line, which is nearly
indistinguishable from the dashed blue line.) Thus, Milgrom and Weber guarantees the seller can gain by revealing his information and adjusting the reserve price accordingly; but this example suggests nearly all the gain comes simply from revealing the information, with or without the subsequent reserve price adjustment.

**Unverifiable Information**

Of course, this all assumes the seller’s information is verifiable. If it is unverifiable, the problem is much harder. The seller has a clear incentive to report $S = 1$ regardless of the truth; on the other hand, the seller’s choice of a reserve price can potentially serve as a (costly) signal of his information. (This is the informed principal problem – where a principal’s choice of a mechanism may reveal private information relevant to the agents participating in the mechanism.)

Jullien and Mariotti (2006) and Cai, Riley and Ye (2007) study the problem of seller signaling via reserve price, in a setting where the seller’s information is continuous. They focus, however, on the case where each buyer’s valuation depends only on his own signal and the seller’s, not the signals of the other buyers.\(^{17}\) Since it is not the main focus of this paper, I stick to the simpler case where the seller’s signal is binary. Continuing to work with the case of $N = 3$ and $v_0 = 0.7\theta$ explored above – since positive reserve prices are optimal in that case – I characterize all the pure strategy equilibria (both separating and pooling) of this setting in Appendix D. I find the following:

\(^{17}\)Cai, Riley and Ye show existence of a particular equilibrium in the more general case; but as noted in Lamy (2010), their comparative statics hold only in the case mentioned above.
• There is a continuum of separating equilibria, in which the seller sets reserve price $\hat{r}_0 = 0.648$ when $S = 0$, and reserve price $\hat{r}_1 \in [0.985, 0.991]$ when $S = 1$. The highest expected profit in any of these is $0.0317$—less than half—the profit the seller could earn if he had not learned $S$.

(What’s happening is that the effect of bidder beliefs on seller profit is quite strong, and a seller with $S = 1$ must dissipate nearly all the rents he would get from his “good” signal by setting a reserve high enough to avoid being imitated by a seller with $S = 0$.)

• There is a continuum of pooling equilibria, where the seller sets reserve price $\hat{r} \in [0.445, 0.949]$ regardless of $S$. This range includes the profit-maximizing reserve of 0.705. However, this optimal pooling equilibrium fails the “intuitive criterion” of Cho and Kreps (1987); and I believe the other pooling equilibria all fail it as well.

Thus, while there is a clear incentive for the seller to acquire (and reveal) verifiable information, there appears to be no incentive for him to acquire unverifiable information, since the best he could do is to replicate the outcomes he could get without that information, and at worst he could be forced into a signaling equilibrium yielding much lower profits.

6.3 One Final Example – a Simple Linear Model

Finally, I consider one other example, where bidder signals are independent and valuations have additive private and common value components. Let $\{X_i\}$ be distributed independently and uniformly on $[0, 1]$, and let

$$V_i = (1 - \lambda) \frac{N + 2}{2N} X_i + \lambda \frac{\sum_{j=1}^{N} X_j}{N}$$

That is, a bidder’s valuation is $1 - \lambda$ times his own signal, plus $\lambda$ times the average signal, so $\lambda = 0$ corresponds to pure private values and $\lambda = 1$ to pure common values. (The $\frac{N+2}{2N}$ term is a normalization to ensure that in the absence of a reserve price, expected revenue is constant as $\lambda$ changes, so that in some sense we’re comparing apples to apples. However, this example is not an instance of the general model presented in Section 4: the joint distribution of equilibrium bids changes with $\lambda$, so the $\lambda = 0$ case is not the “observationally-equivalent private values case” of other values of $\lambda$. This is a completely separate model being offered to show that the results seem to hold more broadly.)

Figure 14 illustrates this example, giving the expected revenue and profit curves for five values of $\lambda$ (0, 0.25, 0.5, 0.75, and 1) for several combinations of $N$ and $v_0$. 
Figure 14: Expected revenue and profit for linear-independent example

\( N = 3 \)

\( N = 6 \)

Expected revenue

Expected profit

Profit, \( v_0 = 0.20 \)

Profit, \( v_0 = 0.35 \)
In this example, the relevant rankings across models can all be established analytically. For a given reserve price \( r \), it’s straightforward to calculate that the bidding threshold \( x^* \), when it is interior, satisfies \( r = E(V_i|X_i = X^{(1)} = x^*) = \frac{N+2-\lambda}{2N} x^* \), and therefore that
\[
x^* = \frac{2N}{N + 2 - \lambda} r
\]
which is strictly increasing in \( \lambda \). From this, I show the following:

**Result 1.** In the linear-independent example, expected revenue at \( r = 0 \) is constant in \( \lambda \). For any \( r > 0 \) giving a nonzero probability of sale,

(i) the probability of a sale is strictly decreasing in \( \lambda \)

(ii) expected revenue is strictly decreasing in \( \lambda \)

(iii) if \( r \geq v_0 \), expected profit is strictly decreasing in \( \lambda \)

Further, the revenue- and profit-maximizing reserve prices are both strictly decreasing in \( \lambda \).

The calculation is shown in Appendix A4. Note that if signals were i.i.d. draws from any arbitrary distribution \( F \) rather than the uniform, the three numbered parts of this result would still hold, provided the private-value part of bidder valuations was normalized appropriately.\(^ {18} \)

## 7 Conclusion

When bidders have private values, a reserve price offers a seller a way to avoid unprofitable sales, but also a way to further increase profits, by trading off a lower likelihood of sale against a higher average price. In this paper, I show this tradeoff is generally less favorable when bidders have interdependent or common values. I offer theoretical results that a high reserve price is less likely to be met, and less profitable, when bidders have common rather than private values; and I offer simulation results showing that the same profit ranking very often holds for any positive reserve price, not just high ones, and that the profit-maximizing reserve is typically lower under common values as well. Put another way, when bidders have interdependent values, analysis based on the assumption of private values is likely to overestimate both the optimal reserve price and the benefit of setting it. Thus, common

\(^ {18} \)Like the \( \frac{N+2}{2N} \) term in the uniform case, the normalization would be to make expected revenue when \( r = 0 \) independent of \( \lambda \) for each realization of the second-highest signal. Specifically, this would mean defining valuations as \( V_i = (1-\lambda)h(X_i) + \lambda \frac{1}{N} \sum_{j=1}^{N} X_j \), where \( h(x) = \frac{1}{N} (2x + (N-2)E(X_j|X_j < x)) \).
values can be added to the list of departures from the standard workhorse model which would favor lower reserve prices – and which might help to explain the low reserve prices often observed empirically.

One reason for these findings is that beyond the usual tradeoff between likelihood of sale and minimum price, a reserve price also has an added cost under common values: it can reduce the expected price paid even when it does not bind, by concealing the bids of losing bidders. As a result, under common values, the profit-maximizing reserve price is sometimes below the seller’s own valuation, and sometimes 0 even when the seller’s valuation is positive. These effects are further magnified when the seller’s valuation is interdependent with that of the buyers. Thus, while a reserve price is often an effective tool to increase seller profits in environments where buyers are confident of their own willingness to pay (private values), this paper shows they should be used much more cautiously – if at all – when bidder values have a significant common component.
Appendix A – Proofs

A1 Proof of Theorem 1

Fix \( v_0 \). As in the text, define \( \pi_{CV}(r) \) and \( \pi_{PV}(r) \) as the seller’s expected profit at reserve price \( r \) under the two models, \( V^{(k)} \) the \( k^{th} \) highest valuation under the private values model, and

\[
v(x, y) = \mathbb{E}\{u(x, y, X^{(3-N)}) | X^{(1)} = x, X^{(2)} = y\}
\]

Under the private values model, expected profit is \( V^{(2)} - v_0 \) if \( V^{(2)} \geq r \) (so the winner pays the second-highest bid, equal to the second-highest valuation), \( r - v_0 \) if \( V^{(1)} \geq r > V^{(2)} \) (so the winner pays the reserve price), and 0 otherwise; so we can write it as

\[
\pi_{PV}(r) = \mathbb{E}_X \{1_{V^{(1)} \geq r > V^{(2)}}(r - v_0) + 1_{V^{(2)} \geq r}(V^{(2)} - v_0)\}
\]

The second line follows from the first because \( V^{(2)} \geq r \) implies \( V^{(1)} \geq r \), so \( 1_{V^{(1)} \geq r > V^{(2)}} = 1_{V^{(1)} \geq r} - 1_{V^{(2)} \geq r} \).

As for the common values case, the reserve price binds if \( X^{(1)} \geq x^* > X^{(2)} \), and does not bind if \( X^{(2)} \geq r \) (in which case the winner pays the second-highest bid \( v(X^{(2)}, X^{(2)}) = V^{(2)} \)), so

\[
\pi_{CV}(r) = \mathbb{E}_X \{1_{X^{(1)} \geq x^* > X^{(2)}}(r - v_0) + 1_{X^{(2)} \geq x^*}(v(X^{(2)}, X^{(2)}) - v_0)\}
\]

Thus,

\[
\pi_{CV}(r) - \pi_{PV}(r) \leq \mathbb{P}(X^{(1)} \geq x^*) (r - v_0) + \mathbb{E}_X \max\{0, V^{(2)} - r\} - \mathbb{P}(V^{(2)} \geq r) (r - v_0) - \mathbb{E}_X \max\{0, V^{(2)} - r\}
\]

Finally, I show that \( X^{(1)} \geq x^* \rightarrow V^{(1)} \geq r \), and therefore \( \mathbb{P}(X^{(1)} \geq x^*) \leq \mathbb{P}(V^{(1)} \geq r) \). Recall from the text that \( x^* \) solves

\[
r = \mathbb{E}_{X^{(2)} | X^{(1)} = x^*} v(x^*, X^{(2)})
\]
Due to affiliation, $\mathbb{E}\{v(x, X^{(2)})|X^{(1)} = x\}$ is increasing in $x$; so

$$X^{(1)} \geq x^*$$

$$\mathbb{E}_{X^{(2)}|X^{(1)}} \{v(X^{(1)}, X^{(2)})\} \geq r$$

$$\mathbb{E}_{X^{(2)}|X^{(1)}} \{v(X^{(1)}, X^{(1)})\} \geq r$$

$$v(X^{(1)}, X^{(1)}) \geq r$$

$$V^{(1)} \geq r$$

Thus, $\Pr(X^{(1)} \geq x^*) \leq \Pr(V^{(1)} \geq r)$. This means that in the common values setting, any reserve price $r$ is more likely to prevent a sale than in the private-values setting (part (i) of the theorem). If $r \geq v_0$, then

$$\pi_{CV}(r) - \pi_{PV}(r) \leq (\Pr(X^{(1)} \geq x^*) - \Pr(V^{(1)} \geq r)) (r - v_0) \leq 0$$

proving part (iii); repeating the argument with $v_0 = 0$ gives part (ii). \qed

### A2 Proof of Theorem 2

As in the second-price auction, profits in the private values case are

$$\pi_{PV}(r) = \Pr(V^{(1)} \geq r)(r - v_0) + \mathbb{E}_X \max\{0, V^{(2)} - r\}$$

As for the common values case, $x^*$ is the same as in the second-price auction,

$$r = \mathbb{E} \left\{ u(x^*, \{X_j\}_{j \neq i}) \left| X_i = x^*, \max_{j \neq i} X_j < x^* \right. \right\}$$

with a sale occurring if and only if $X^{(1)} \geq x^*$. If $X^{(1)} \geq x^* > X^{(2)}$, the sale occurs at price $r$. If $X^{(2)} \geq x^*$, then the sale occurs at the price where bidder $i(2)$ drops out; but that price depends on the realizations of the lower signals, and also depends on $x^*$, since bidder 2 will have learned (from equilibrium bidding) the signals of those bidders above $x^*$, but not those below $x^*$.

To capture all this in notation, for $k > 2$, let $Z^{(k)}$ denote a garbling of $X^{(k)}$ which is equal to $X^{(k)}$ if $X^{(k)} \geq x^*$, and equal to 0 otherwise. That is, $Z^{(k)}$ tells you exactly $X^{(k)}$ if $X^{(k)} \geq x^*$; but if $X^{(k)} < x^*$, $Z^{(k)}$ only tells you that $X^{(k)} < x^*$, not its exact value. Thus, the realization of $Z^{(k)}$ is exactly the information the other bidders learn about bidder $i(k)$’s signal from equilibrium bidding in the common-values case. Let $X^{(3-N)} = (X^{(3)}, \ldots, X^{(N)})$ and $Z^{(3-N)} = (Z^{(3)}, \ldots, Z^{(N)})$.

Finally, define

$$R(x, z^{(3-N)}) = \mathbb{E} \left\{ u(x, x, X^{(3-N)})|X^{(1)} = X^{(2)} = x, Z^{(3-N)} = z^{(3-N)} \right\}$$
For a given realization $x \geq x^*$ of $X^{(2)}$ and $Z^{(3-N)}$ of the “garbled” losing bids $Z^{(3-N)}$, this is the price at which the second-highest bidder will drop out, and therefore the price that is paid. Then we can write

$$\pi_{CV}(r) = \mathbb{E}_X \left\{ 1_{X^{(1)} \geq x^*, X^{(2)}(r - v_0)} + 1_{X^{(2)} \geq x^*} \left( R(X^{(2)}, Z^{(3-N)}) - v_0 \right) \right\}$$

$$= \mathbb{E}_X \left\{ 1_{X^{(1)} \geq x^*}(r - v_0) - 1_{X^{(2)} \geq x^*}(r - v_0) + 1_{X^{(2)} \geq x^*} \left( R(X^{(2)}, Z^{(3-N)}) - v_0 \right) \right\}$$

$$= \Pr(X^{(1)} \geq x^*)(r - v_0) + \mathbb{E}_X \left\{ 1_{X^{(2)} \geq x^*} \left( R(X^{(2)}, Z^{(3-N)}) - r \right) \right\}$$

$$= \Pr(X^{(1)} \geq x^*)(r - v_0) + \int_{x^*}^{\infty} \left( \mathbb{E}_{Z^{(3-N)|X^{(2)}=x}} \left\{ R(x, Z^{(3-N)}) \right\} - r \right) dF^{(2)}(x)$$

where $F^{(2)}(\cdot)$ is the distribution of $X^{(2)}$.

Next, I make a standard “linkage principle” argument that for any $x$, the “garbling” of losing bids due to the reserve price reduces the expected price paid conditional on $X^{(2)}$. To prove this, note that

$$\mathbb{E}_{Z^{(3-N)|X^{(2)}=x}} R(x, Z^{(3-N)}) = \mathbb{E}_{Z^{(3-N)|X^{(2)}=x}} \left\{ \mathbb{E}_{X^{(3-N)|X^{(1)}=x, X^{(2)}=x}} u(x, x, X^{(3-N)}) \right\}$$

$$\leq \mathbb{E}_{Z^{(3-N)|X^{(2)}=x}} \left\{ \mathbb{E}_{X^{(3-N)|X^{(2)}=x, Z^{(3-N)}} u(x, x, X^{(3-N)}) \right\}$$

$$= \mathbb{E}_{X^{(3-N)|X^{(2)}=x}} u(x, x, X^{(3-N)})$$

The inequality is because in the second line, the expectation over $X^{(3-N)}$ is conditional on all the different values $X^{(1)}$ could take conditional on $X^{(2)} = x$ – all of which are above $x$ – while in the first line, the expectation is taken conditional on $X^{(1)} = x$; the third line is simply iterated expectations. This means that

$$\pi_{CV}(r) = \Pr(X^{(1)} \geq x^*)(r - v_0) + \int_{x^*}^{\infty} \left( \mathbb{E}_{Z^{(3-N)|X^{(2)}=x}} \left\{ R(x, Z^{(3-N)}) \right\} - r \right) dF^{(2)}(x)$$

$$\leq \Pr(X^{(1)} \geq x^*)(r - v_0) + \int_{x^*}^{\infty} \left( \mathbb{E}_{X^{(3-N)|X^{(2)}=x}} u(x, x, X^{(3-N)}) \right) - r \right) dF^{(2)}(x)$$

$$= \Pr(X^{(1)} \geq x^*)(r - v_0) + \mathbb{E}_{X^{(2)}} 1_{X^{(2)} \geq x^*} \left( \mathbb{E}_{X^{(3-N)|X^{(2)}}} \left\{ u(X^{(2)}, X^{(2)}, X^{(3-N)}) \right\} - r \right)$$

$$= \Pr(X^{(1)} \geq x^*)(r - v_0) + \mathbb{E}_{X^{(2)}} 1_{X^{(2)} \geq x^*} \left( u(X^{(2)}, X^{(2)}, X^{(3-N)}) - r \right)$$

$$= \Pr(X^{(1)} \geq x^*)(r - v_0) + \mathbb{E}_{X^{(2)}} 1_{X^{(2)} \geq x^*} \left( V^{(2)} - r \right)$$

$$\leq \Pr(X^{(1)} \geq x^*)(r - v_0) + \mathbb{E}_{X} \max \{0, V^{(2)} - r\}$$
and so subtracting,

\[
\pi_{CV}(r) - \pi_{PV}(r) \leq \Pr(X^{(1)} \geq x^*) (r - v_0) + \mathbb{E}_X \max\{0, V^{(2)} - r\} \\
- \Pr(V^{(1)} \geq r) (r - v_0) - \mathbb{E}_X \max\{0, V^{(2)} - r\} \\
= (\Pr(X^{(1)} \geq x^*) - \Pr(V^{(1)} \geq r)) (r - v_0)
\]

Now, recall that \(x^*\) is the value of \(x\) solving

\[
r = \mathbb{E}_{\{X_j\}_{j \neq i}|X_i = x} \left\{ u(x, \{X_j\}_{j \neq i}) \left| \max_{j \neq i} X_j < x \right. \right\}
\]

and that due to affiliation, the right-hand side is strictly increasing in \(x\). Thus, \(X^{(1)} \geq x^*\) is equivalent to

\[
r \leq \mathbb{E}_{\{X_j\}_{j \neq i}|X^{(1)}} \left\{ u \left( X^{(1)}, \{X_j\}_{j \neq i} \right) \left| \max_{j \neq i} X_j < X^{(1)} \right. \right\} = R^{(1)}
\]
as defined in the text; so

\[
\pi_{CV}(r) - \pi_{PV}(r) \leq (\Pr(R^{(1)} \geq r) - \Pr(V^{(1)} \geq r)) (r - v_0) \\
= ((1 - F_{R^{(1)}}(r)) - (1 - F_{V^{(1)}}(r))) (r - v_0) \\
= (F_{V^{(1)}}(r) - F_{R^{(1)}}(r)) (r - v_0)
\]

Recall that \(\tilde{r}\) was defined in the text such that \(r \geq \tilde{r}\) implies \(F_{V^{(1)}}(r) \leq F_{R^{(1)}}(r)\), proving part (i). If \(r \geq \tilde{r}\) and \(r \geq v_0\), then \(\pi_{CV}(r) - \pi_{PV}(r) \leq (F_{V^{(1)}}(r) - F_{R^{(1)}}(r)) (r - v_0) \leq 0\) (part (iii)). Repeating the argument with \(v_0 = 0\) establishes \(\text{Rev}_{CV}(r) \leq \text{Rev}_{PV}(r)\) (part (ii)). \(\Box\)

### A3 A “Random” Extension to Theorem 2

**Theorem 3.** Suppose that \(\{V_i\}\) have bounded support; let \(\overline{V}\) denote the upper bound.

(i) For any \(x \geq 0\), expected revenue under a random reserve price \(r\) drawn uniformly from the interval \([x, \overline{V}]\) is lower under the common values model than under the corresponding private values model

(ii) For any \(x \geq v_0\), expected profit under a random reserve price \(r\) drawn uniformly from the interval \([x, \overline{V}]\) is lower under the common values model than under the corresponding private values model

Unlike in Vincent (1995), I don’t mean a secret reserve price, but one that is chosen at random and then announced to the bidders. I don’t mean to suggest here that picking a reserve price at random is a good idea; I merely intend Theorem 3 to reinforce the intuition that reserve prices are “typically” less advantageous under common values than under private values.
To prove Theorem 3, first note that for any distribution $F$ and any $v_0 \leq x$,

$$
\int_x^V (r - v_0) F(r) dr = \int_x^V \left( \int_{v_0}^r dy \right) F(r) dr = \int_x^V \int_{v_0}^x F(r) dy dr + \int_x^V \int_r^V F(r) dr dy \\
= \int_{v_0}^x \int_x^V F(r) dy dr + \int_x^V \int_y^V F(r) dr dy
$$

when we swap the order of integration. As noted in the proof of Theorem 2,

$$
\pi_{CV}(r) - \pi_{PV}(r) \leq (F_{V(1)}(r) - F_{R(1)}(r)) (r - v_0)
$$

for any $r \geq v_0$, and therefore for any $r \in [x, \bar{V}]$ when $x \geq v_0$; integrating,

$$
\int_x^V (\pi_{CV}(r) - \pi_{PV}(r)) dr \leq \int_x^V (F_{V(1)}(r) - F_{R(1)}(r)) (r - v_0) dr \\
\leq \int_x^V (F_{U(1)}(r) - F_{R(1)}(r)) (r - v_0) dr
$$

since as noted in the text, $F_{V(1)}(r) \preceq_{FOSD} F_{U(1)}(r)$. Thus,

$$
\int_x^V (\pi_{CV}(r) - \pi_{PV}(r)) dr \leq \int_{v_0}^x \left[ \int_x^V (F_{U(1)}(r) - F_{R(1)}(r)) dr \right] dy \\
+ \int_x^V \left[ \int_y^V (F_{U(1)}(r) - F_{R(1)}(r)) dr \right] dy
$$

Now, as noted in the text, $U^{(1)}$ is a mean-preserving spread around $R^{(1)}$, and therefore $\int_y^y F_{U^{(1)}}(t) dt \geq \int_y^y F_{R^{(1)}}(t) dt$ for any $y$ (including $y = x$). Integration by parts allows us to rewrite $\int_y^y F_{U^{(1)}}(t) dt$ as $\overline{V} - E(U^{(1)}) - \int_y^\overline{V} F_{U(1)}(t) dt$, and $\int_y^y F_{R^{(1)}}(t) dt$ as $\overline{V} - E(R^{(1)}) - \int_y^\overline{V} F_{R(1)}(t) dt$; since $U^{(1)}$ and $R^{(1)}$ have the same mean, this establishes that

$$
\int_y^\overline{V} F_{U^{(1)}}(t) dt \leq \int_y^\overline{V} F_{R^{(1)}}(t) dt
$$

for any $y$, so both terms above in square brackets are negative, giving

$$
\int_x^V \pi_{CV}(r) dr - \int_x^V \pi_{PV}(r) dr \leq 0.
$$

Repeating with $v_0 = 0$ establishes the revenue result. $\Box$
A4 Proof of Result 1

Let $\text{Rev}_\lambda(r)$ and $\pi_\lambda(r)$ denote expected revenue and profit, respectively, at reserve price $r$ given a value of $\lambda$. First, note that $\text{Rev}_\lambda(0)$ does not depend on $\lambda$. This is because given realizations $(x^{(1)}, \ldots, x^{(N)})$ of the signals, the second-highest bidder drops out at the price

$$E \left( V_i | X^{(1)} = X^{(2)} = x^{(2)}, X^{(3-N)} = x^{(3-N)} \right)$$

$$= (1 - \lambda) \frac{N}{2N} x^{(2)} + \lambda \frac{1}{N} (x^{(2)} + x^{(2)} + x^{(3)} + \ldots + x^{(N)})$$

Since, conditional on the realization $x^{(2)}$ of $X^{(2)}$, the types of the bidders who dropped out before him are independently uniform on $[0, x^{(2)}]$, this has expected value, given $x^{(2)}$, of

$$(1 - \lambda) \frac{N + 2}{2N} x^{(2)} + \lambda \frac{1}{N} \left( 2x^{(2)} + E \left\{ X^{(3)} + \ldots + X^{(N)} | X^{(2)} = x^{(2)} \right\} \right)$$

$$= (1 - \lambda) \frac{N + 2}{2N} x^{(2)} + \lambda \frac{1}{N} \left( 2x^{(2)} + (N - 2) \frac{1}{2} x^{(2)} \right)$$

$$= (1 - \lambda) \frac{N + 2}{2N} x^{(2)} + \lambda \frac{N + 2}{2N} x^{(2)} = \frac{N + 2}{2N} x^{(2)}$$

which does not depend on $\lambda$.

Second, as noted in the text, $x^* = \frac{2N}{N + 2 - \lambda} r$ is increasing in $\lambda$, so the probability of a sale, which is $\Pr(X^{(1)} > x^*)$, is decreasing in $\lambda$, giving part (i).

Third, note that conditional on a realization $x^{(2)} > x^*$ of $X^{(2)}$, the expected price at which the second-highest bidder drops out is independent of $x^*$. To see this, consider the case of $N = 3$. Given a realization of $X$, renumber the bidders such that $X_1 > X_2 > X_3$. When bidder 2 drops out, bidder 3 is already out. If he bid and then dropped out, the realization $x^{(3)}$ of $X^{(3)}$ would be inferred, and bidder 2 would drop out at price

$$B(x^{(2)}, x^{(3)}) = (1 - \lambda) \frac{5}{6} x^{(2)} + \lambda \frac{1}{3} (x^{(2)} + x^{(2)} + x^{(3)}) = \left( \frac{5}{6} - \frac{1}{6} \lambda \right) x^{(2)} + \frac{\lambda}{3} x^{(3)}$$

If bidder 3 did not bid because of a reserve price $r$, then bidder 2 takes the expectation of $B(x^{(2)}, x^{(3)})$ over the values of $X^{(3)}$ at which bidder 3 would not have bid. Importantly, conditional on $X^{(2)} = x^{(2)}$, the distribution of $X^{(3)}$ is uniform over $[0, x^{(2)}]$; so overall, the
expected price at which bidder 2 drops out, given \( X^{(2)} = x^{(2)} > x^* \), is

\[
\bar{B}(x^{(2)}) = \frac{x^*}{x^{(2)}} \mathbb{E}_{X^{(3)} < x^*} B(x^{(2)}, X^{(3)}) + \frac{x^{(2)} - x^*}{x^{(2)}} \int_{x^*}^{x^{(2)}} B(x^{(2)}, x) \frac{dx}{x^{(2)} - x^*}
\]

\[
= \frac{x^*}{x^{(2)}} \left( \left( \frac{5}{6} - \frac{1}{6} \lambda \right) x^{(2)} + \frac{\lambda x^*}{3} \right) + \frac{1}{x^{(2)}} \int_{x^*}^{x^{(2)}} \left( \left( \frac{5}{6} - \frac{1}{6} \lambda \right) x^{(2)} + \frac{\lambda x^*}{3} \right) \frac{dx}{x^{(2)} - x^*}
\]

\[
= \left( \frac{5}{6} - \frac{1}{6} \lambda \right) x^{(2)} + \frac{\lambda (x^*)^2}{6 x^{(2)}} + \frac{x^{(2)} - x^*}{x^{(2)}} \left( \left( \frac{5}{6} - \frac{1}{6} \lambda \right) x^{(2)} + \frac{1}{x^{(2)}} \frac{\lambda}{6} (x^{(2)})^2 \right)
\]

\[
= \frac{5}{6} x^{(2)}
\]

which does not depend on \( x^* \) (or \( \lambda \)) at all.\(^{19}\) The same holds for general \( N \): conditional on \( X^{(2)} = x^{(2)} > x^* \), the expected price at which the second-highest bidder drops out is \( \frac{N+2}{2N} x^{(2)} \), and does not depend on \( x^* \). Thus, if we let \( F_2 \) denote the CDF of \( X^{(2)} \) and \( B(X|x^*) \) denote the price at which the second-highest bidder will drops out on the equilibrium path, we can write expected profit as

\[
\pi_\lambda(r) = \mathbb{E}_X \left\{ 1_{X^{(1)} \geq x^* > X^{(2)}} (r - v_0) + 1_{X^{(2)} \geq x^*} (B(X|x^*) - v_0) \right\}
\]

\[
= \mathbb{E}_X \left\{ 1_{X^{(1)} \geq x^*} (r - v_0) + 1_{X^{(2)} \geq x^*} (B(X|x^*) - r) \right\}
\]

\[
= \mathbb{P}(X^{(1)} \geq x^*) (r - v_0) + \int_{x^*}^{1} (\mathbb{E}_{X|X^{(2)} = x} B(X|x^*) - r) \, dF^{(2)}(x)
\]

\[
= \mathbb{P}(X^{(1)} \geq x^*) (r - v_0) + \int_{x^*}^{1} (\bar{B}(x) - r) \, dF^{(2)}(x)
\]

where \( \bar{B}(x) = \frac{N+2}{2N} x \).

Now, if \( r \geq v_0 \), the first term is decreasing in \( x^* \). \( \bar{B}(x^*) = \frac{N+2}{2N} \frac{2N}{N+2-\lambda} - \frac{N+2}{N+2-\lambda} r > r \), so the second term is strictly decreasing in \( x^* \) (since the integrand is strictly positive at \( x = x^* \)). Since \( \bar{B} \) is independent of \( \lambda \), \( \lambda \) effects \( \pi_\lambda \) only through \( x^* \), which is strictly increasing in \( \lambda \); so \( \pi_\lambda(r) \) is decreasing in \( \lambda \) for \( r \geq v_0 \). That gives part (iii) of the result; repeating with \( v_0 = 0 \) gives part (ii).

\(^{19}\)This is in contrast to the affiliated case, such as in the proof of Theorem 2, where truncation of losing bids reduces expected revenue. When a losing bid is not observed, its expectation is taken by the second-highest bidder, conditional on \( X^{(1)} = X^{(2)} = x^{(2)} \). With affiliated signals, this is pessimistic relative to the truth that \( X^{(1)} > X^{(2)} = x^{(2)} \); but with independent signals, once we’re already conditioning on the value of \( X^{(2)} \), the distribution of the lower signals does not depend on the value of \( X^{(1)} \), and therefore this effect vanishes. Similarly, with independent signals, second-price sealed-bid auctions are revenue-equivalent to English auctions, while with affiliated signals, the latter are strictly revenue-superior.
To prove the last part of the result, let \( F^{(1)} \) denote the CDF of \( X^{(1)} \), and differentiate \( \pi_\lambda \) to get

\[
\pi_\lambda'(r) = (1 - F^{(1)}(x^*)) (r - v_0) + \int_{x^*}^1 (B(x) - r) dF^{(2)}(x)
\]

where \((x^*)'\) is the derivative of \( x^* \) with respect to \( r \). If we are in the range of \( r \) where \( x^* < 1 \), then \( x^* = \gamma r \), where \( \gamma = \frac{2N}{N+2-\lambda} \), and \((x^*)'\) is therefore equal to \( \gamma \). \( B(x^*) = r = \frac{N+2}{N+2-x^*} = \frac{N+2}{N+2-\lambda} r = \frac{\lambda}{N+2-\lambda} r \); so we can write this as

\[
\pi_\lambda'(r) = F^{(2)}(\gamma r) - F^{(1)}(\gamma r) - \gamma f^{(1)}(\gamma r)(r - v_0) - \gamma \left( \frac{\lambda}{N+2-\lambda} r \right) f^{(2)}(\gamma r)
\]

Since \( \{X_i\} \) are independently uniform, \( F^{(2)}(x) = N x^{N-1} - (N-1) x^N \) and \( F^{(1)}(x) = x^N \) (properties of order statistics), so

\[
\pi_\lambda'(r) = N(\gamma r)^{N-1} (1 - \gamma r) - \gamma N(\gamma r)^{N-1} (r - v_0) - \gamma \left( \frac{\lambda}{N+2-\lambda} r \right) N(1)(\gamma r)^{N-2} (1 - \gamma r)
\]

Dividing by \( N(\gamma r)^{N-1}(1 - \gamma r) \) preserves sign, so

\[
\pi_\lambda'(r) \overset{\text{sign}}{=} 1 - \frac{\gamma (r - v_0)}{1 - \gamma r} - \frac{\lambda(N-1)}{N+2-\lambda}
\]

This is strictly decreasing in \( r \), so \( \pi \) is strictly quasiconcave. As long as \( \gamma v_0 < 1 \) (or \( x^* < 1 \) at \( r = v_0 \)), this is strictly positive at \( r = v_0 \) (since the second term vanishes and the third term is strictly smaller than 1), and strictly negative as \( r \to 1 \) (since the denominator of the second term goes to 0 while the numerator remains positive), so \( \pi_\lambda'(r) \) has a unique maximizer characterized by \( \pi_\lambda'(r) = 0 \). And finally, recalling that \( \gamma = \frac{2N}{N+2-\lambda} \) is increasing in \( \lambda \), this last expression is strictly decreasing in \( \lambda \); so where \( \pi_\lambda'(r) = 0, \pi_\lambda'(r) < 0 \) for \( \lambda' > \lambda \), meaning that \( \arg \max \pi_\lambda'(r) < \arg \max \pi_\lambda(r) \), proving the final claim.

Also note that evaluating the integral in the expression for \( \pi_\lambda \), plugging in the expressions for \( B \), \( F_2 \), and \( F_1 \), and simplifying gives \( \pi_\lambda(r) = \)

\[
N(x^*)^{N-1}(1 - x^*)r + \frac{(N+2)(N-1)}{2N(N+1)} (1 - (N+1)(x^*)^N + N(x^*)^{N+1}) - (1 - (x^*)^N)v_0
\]

which was used for the charts in Figure 14. \(\square\)
Appendix B – Details of Simulations

For the first example (discrete \(\theta\)), note that by Bayes’ Law,

\[
\mathbb{E}\{\theta|X = x\} = \frac{\frac{1}{2} \prod_{i=1}^{N} \alpha x_i^{\alpha-1}}{\frac{1}{2} \prod_{i=1}^{N} \alpha x_i^{\alpha-1} + \frac{1}{2} \prod_{i=1}^{N} \alpha (1-x_i)^{\alpha-1}} = \frac{1}{1 + \prod_{i=1}^{N} \frac{(1-x_i)^{\alpha-1}}{x_i^{\alpha-1}}}
\]

(B1)

and

\[
\mathbb{E}\{\theta|X^{(1)} = x_1\} = \frac{\frac{1}{2} \alpha x_1^{\alpha-1} (x_1^{\alpha})^{N-1}}{\frac{1}{2} \alpha x_1^{\alpha-1} (x_1^{\alpha})^{N-1} + \frac{1}{2} \alpha (1-x_1)^{\alpha-1} (1-(1-x_1)^{\alpha})^{N-1}}
\]

= \frac{1}{1 + \frac{(1-x_1)^{\alpha-1}}{x_1^{\alpha-1}} \left(\frac{1-(1-x_1)^{\alpha}}{x_1^{\alpha}}\right)^{N-1}}

(B2)

and

\[
\mathbb{E}\{\theta|(X_1, \ldots, X_k) = (x_1, \ldots, x_k), \{X_{k+1}, \ldots, X_N\} < x^*\}
\]

= \frac{\frac{1}{2} \prod_{i=1}^{k} \alpha x_i^{\alpha-1} \prod_{i=k+1}^{N} \alpha (x^*)^{\alpha} \prod_{i=k+1}^{N} \alpha (1-x_i)^{\alpha-1} \prod_{i=k+1}^{N} (1-(1-x_i)^{\alpha})}{\frac{1}{2} \prod_{i=1}^{k} \alpha x_i^{\alpha-1} \prod_{i=k+1}^{N} \alpha (1-x_i)^{\alpha-1} \prod_{i=k+1}^{N} (1-(1-x_i)^{\alpha})}

(B3)

For the simulations, the following was done in Matlab:

1. 500,000 sets of simulated signal realizations were randomly generated, by picking \(\theta \in \{0, 1\}\) at random for each simulation, generating \(N\) independent uniform random variables \(\{\epsilon_i\}\), and letting \(X_i = F^{-1}(\epsilon_i|\theta)\).

2. For each set of realized signals, the highest two private valuations \(V^{(1)} = E(\theta|X = (x^{(1)}, x^{(1)}, x^{(3-N)})\} and \(V^{(2)} = E(\theta|X = (x^{(2)}, x^{(2)}, x^{(3-N)})\} were calculated via (B1).

3. For each \(r \in \{0, 0.005, 0.010, 0.015, \ldots, 0.995, 1.000\}\), \(x^*\) was calculated via Matlab’s numerical solver as the solution to \(E(\theta|X^{(1)} = x^*) - r = 0\) via (B2).

4. For each simulation and each \(r\), the outcome was calculated under each model by determining which bidders have signals above \(x^*\) (common values) or a valuation above \(r\) (private values) and calculating what price would be paid, if any (using (B3) for the common values case), and subtracting the seller’s valuation \(v_0\) in case of a sale. Revenue and profit curves were then produced by averaging across simulations, and optimal reserve prices were determined as the grid point giving the highest revenue/profit.

5. The same simulated valuations were used to calculate \(R^{(1)}\) and \(V^{(1)}\) as defined in the text, and \(\hat{r}\) was calculated as the highest crossing point of their empirical CDFs.
For the second example (continuous $\theta$), the simulations were done the same way, but calculation of $E(\theta|X)$ and $E(\theta|X^{(1)})$ were a bit more laborious. Letting $P$ denote the prior on $\theta$ and $p(X|t)$ the conditional density of $X$ given $\theta = t$, Bayes’ Law gives us

$$E(\theta|X) = \int_0^1 \theta \frac{P(\theta)p(X|\theta)}{\int_0^1 P(t)p(X|t)dt} d\theta = \frac{\int_0^1 tp(X|t)dt}{\int_0^1 p(X|t)dt}$$

since $P$ is uniform. So

$$E(\theta|(X_1, \ldots, X_k) = (x_1, \ldots, x_k), \{X_{k+1}, \ldots, X_N\} < x^*) = \frac{\int_0^1 t \prod_{i=1}^k f(x_i|t) \prod_{i=k+1}^N F(x^*|t)dt}{\int_0^1 \prod_{i=1}^k f(x_i|t) \prod_{i=k+1}^N F(x^*|t)dt}$$

Plugging in $f(x_i|t) = 1 + 4(t - \frac{1}{2})(x_i - \frac{1}{2})$ and $F(x^*|t) = x^* \left(1 + 4(t - \frac{1}{2})(\frac{1}{2}x^* - \frac{1}{2})\right)$ (calculated via integration), some algebra and a change of variables allow this to be rewritten as

$$E(\theta|(X_1, \ldots, X_k) = (x_1, \ldots, x_k), \{X_{k+1}, \ldots, X_N\} < x^*) = \frac{1}{2} + \frac{1}{4} \frac{\int_{-\frac{1}{2}}^{\frac{1}{2}} t \prod_{i=1}^N (1 + Ta_i)dt}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \prod_{i=1}^N (1 + Ta_i)dt}$$

where $a_i = x - \frac{1}{2}$ for $i = 1, 2, \ldots, k$ and $a_i = \frac{1}{2}x^* - \frac{1}{2}$ for $i > k$.

Setting $a_1 = x - \frac{1}{2}$ and $a_i = \frac{1}{2}x - \frac{1}{2}$, then, we integrated both numerator and denominator (separately for each $N -$ Matlab yielded closed-form integrals) to get an expression for $E(\theta|X^{(1)} = x)$, and found $x^*$ by solving $E(\theta|X^{(1)} = x) - r = 0$ numerically within Matlab. To calculate simulated bids, we simplified the last expression for $E(\theta|X)$ to

$$E(\theta|X) = \frac{1}{2} + \frac{1}{4} \cdot \frac{\frac{2^4}{3}A_1 + \frac{2^6}{5}A_3 + \frac{2^8}{7}A_5 + \frac{2^{10}}{9}A_7 + \ldots}{\frac{4}{3}A_2 + \frac{2^6}{5}A_4 + \frac{2^8}{7}A_6 + \frac{2^{10}}{9}A_8 + \ldots}$$

where

$$A_1 = \sum_i a_i$$

$$A_2 = \sum_{i<j} a_i a_j$$

$$A_3 = \sum_{i<j<k} a_i a_j a_k$$

$$A_4 = \sum_{i<j<k<l} a_i a_j a_k a_l$$

and so on, with $A_n$ therefore equal to $\prod_i a_i$ and $A_{n+1} = A_{n+2} = \ldots = 0$, and wrote code to calculate this within Matlab for each simulation. The simulations were then done just as in the discrete-$\theta$ case.
### Table 1: Summarizing Figures 3, 5, 6, 7, 9, and 10

| Figure 3 – discrete $\theta$, $\alpha = 2$, $v_0 = 0$ (expected revenue) |
|:------------------|:--|:--|:--|:--|:--|:--|:--|:--|
| $N$               | 2  | 3  | 4  | 5  | 6  | 8  | 10 | 20 |
| $\pi_{PV}(r_{PV}^*)/\pi_{PV}(0) - 1$ | 33.1% | 8.8% | 3.6% | 1.8% | 0.9% | 0.3% | 0.1% | 0.0% |
| $\pi_{CV}(r_{CV}^*)/\pi_{CV}(0) - 1$ | 2.2% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| $\pi_{CV}(r_{PV}^*)/\pi_{CV}(0) - 1$ | -13.0% | -14.9% | -14.8% | -14.1% | -12.3% | -11.6% | -10.9% | -8.3% |

| Figure 5 – discrete $\theta$, $\alpha = 2$, $v_0 = 0.20$ |
|:------------------|:--|:--|:--|:--|:--|:--|:--|:--|
| $N$               | 2  | 3  | 4  | 5  | 6  | 8  | 10 | 20 |
| $\pi_{PV}(r_{PV}^*)/\pi_{PV}(0) - 1$ | 146% | 63% | 47% | 41% | 38% | 35% | 34% | 33% |
| $\pi_{CV}(r_{CV}^*)/\pi_{CV}(0) - 1$ | 65% | 25% | 17% | 14% | 12% | 12% | 12% | 18% |
| $\pi_{CV}(r_{PV}^*)/\pi_{CV}(0) - 1$ | 44% | 14% | 6% | 5% | 4% | 4% | 7% | 13% |

| Figure 6 – discrete $\theta$, $N = 5$, $\alpha = 2$ |
|:------------------|:--|:--|:--|:--|:--|:--|:--|:--|
| $v_0$             | 0.00 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.50 |
| $\pi_{PV}(r_{PV}^*)/\pi_{PV}(0) - 1$ | 2% | 8% | 15% | 26% | 41% | 62% | 99% | * |
| $\pi_{CV}(r_{CV}^*)/\pi_{CV}(0) - 1$ | 0% | 0% | 1% | 6% | 14% | 27% | 49% | * |
| $\pi_{CV}(r_{PV}^*)/\pi_{CV}(0) - 1$ | -14% | -12% | -9% | -3% | 5% | 18% | 39% | * |

| Figure 7 – discrete $\theta$, $v_0 = 0.20$ |
|:------------------|:--|:--|:--|:--|:--|:--|:--|:--|
| $N$               | 3  | 3  | 3  | 3  | 6  | 6  | 6  | 6  |
| $\alpha$          | 1.5 | 2.0 | 2.5 | 3.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| $\pi_{PV}(r_{PV}^*)/\pi_{PV}(0) - 1$ | 38% | 63% | 58% | 49% | 27% | 38% | 36% | 34% |
| $\pi_{CV}(r_{CV}^*)/\pi_{CV}(0) - 1$ | 14% | 25% | 25% | 24% | 6% | 12% | 19% | 25% |
| $\pi_{CV}(r_{PV}^*)/\pi_{CV}(0) - 1$ | 3% | 14% | 18% | 20% | 1% | 4% | 11% | 19% |

| Figure 9 – continuous $\theta$, low $v_0$ |
|:------------------|:--|:--|:--|:--|:--|:--|:--|:--|
| $N$               | 3  | 5  | 7  | 10 | 3  | 5  | 7  | 10 |
| $v_0$             | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| $\pi_{PV}(r_{PV}^*)/\pi_{PV}(0) - 1$ | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| $\pi_{CV}(r_{CV}^*)/\pi_{CV}(0) - 1$ | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| $\pi_{CV}(r_{PV}^*)/\pi_{CV}(0) - 1$ | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | -0.4% |

| Figure 10 – continuous $\theta$, high $v_0$ |
|:------------------|:--|:--|:--|:--|:--|:--|:--|:--|
| $N$               | 3  | 5  | 7  | 10 | 3  | 5  | 7  | 10 |
| $v_0$             | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.35 | 0.35 | 0.35 |
| $\pi_{PV}(r_{PV}^*)/\pi_{PV}(0) - 1$ | 6.0% | 3.3% | 3.2% | 3.5% | 66% | 43% | 38% | 37% |
| $\pi_{CV}(r_{CV}^*)/\pi_{CV}(0) - 1$ | 0.3% | 0.1% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| $\pi_{CV}(r_{PV}^*)/\pi_{CV}(0) - 1$ | -0.5% | -1.7% | -1.2% | -0.7% | 26% | 22% | 19% | 18% |

* When $N = 5$, $\alpha = 2$, and $v_0 = 0.50$ in the discrete-$\theta$ example, $\pi_{PV}(0) = \pi_{CV}(0) < 0$, so the ratios are excluded because they’re meaningless.
Appendix D – Unverifiable Seller Information

Recall that we’re in the discrete \( \theta \) example, with \( N = 3, \alpha = 2, \) and \( v_0 = 0.7\theta; \) we’re supposing that the seller gets a signal \( S \) with \( \Pr(S = 0|\theta = 0) = \Pr(S = 1|\theta = 1) = 0.75 \) and \( \Pr(S = 1|\theta = 0) = \Pr(S = 0|\theta = 1) = 0.25, \) and can choose to condition the reserve price on the value of \( S, \) using reserve as a costly signal. I consider equilibria where the seller plays a pure strategy; there are two types of equilibria.

D1 Separating Equilibria

Here, a seller reveals his signal through his choice of reserve price, so bidders correctly infer the value of \( S \) on the equilibrium path. Let \( \hat{r}_0 \) denote the equilibrium reserve price set when \( S = 0, \) and \( \hat{r}_1 \) the reserve set when \( S = 1. \) To deter other reserve prices, I set off-equilibrium-path beliefs to \( \Pr(S = 0|r \not\in \{\hat{r}_0, \hat{r}_1\}) = 1. \) For \( (\hat{r}_0, \hat{r}_1) \) to be an equilibrium, a seller with \( S = 1 \) must prefer setting \( r = \hat{r}_1 \) (and the subsequent belief that \( S = 1 \)) to setting any other reserve price and being met with the belief that \( S = 0. \) A seller with \( S = 0 \) must prefer setting \( r = \hat{r}_0 \) and revealing that \( S = 0 \) to any other reserve price with the belief \( S = 0, \) and also to the reserve price \( \hat{r}_1 \) with the belief that \( S = 1. \) If we let \( \pi(r, p, q) \) denote the seller’s expected profit at reserve price \( r \) when \( S = p \) and the buyers believe that \( S = q, \) separating equilibrium requires

\[
\pi(\hat{r}_0, 0, 0) = \max_r \pi(r, 0, 0) \\
\pi(\hat{r}_0, 0, 0) \geq \pi(\hat{r}_1, 0, 1) \\
\pi(\hat{r}_1, 1, 1) \geq \max_r \pi(r, 1, 0)
\]

Figure 15 illustrates these constraints for our numerical example. On the left pane, the blue curve is \( \pi(r, 0, 0), \) which is maximized at \( r = 0.648, \) which the low-type seller must set in equilibrium. This gives expected profit of 0.0184. The red curve is \( \pi(r, 0, 1) \) – the low-type seller’s expected profit if he could convince the buyers that \( S = 1. \) The magenta (dashed) line shows that if \( \hat{r}_1 < 0.985, \) the low-type seller would choose to imitate the high type; so equilibrium requires \( \hat{r}_1 \geq 0.985. \)

Figure 15: IC constraints for separating equilibria

On the right pane, the blue curve is \( \pi(r, 1, 1) \) – the high-type seller’s payoff on the
equilibrium path. The red curve is $\pi(r, 1, 0)$ – the payoff a high-type seller could get by deviating to a reserve $r \neq \hat{r}_1$. If he chose to deviate, the red dot shows his optimal deviation would be to $r = 0.826$, giving expected profit of 0.0321. So $\hat{r}_1$ (combined with beliefs that $S = 1$) must give him a payoff higher than that. The magenta (dashed) line shows that this requires $\hat{r}_1 \leq 0.991$, and the solid black line shows $r = 0.985$.

So there is a continuum of separating equilibria, all with $\hat{r}_0 = 0.648$, and with $\hat{r}_1 \in [0.985, 0.991]$. The best of these ($\hat{r}_1 = 0.985$) gives the high type an expected payoff of 0.045. Thus, the best separating equilibrium gives expected profit of $\frac{1}{2}0.0184 + \frac{1}{2}0.045 = 0.0317$, about half the expected payoff the seller would get if he had not learned $S$ and set $r$ optimally. (As noted in the text, because buyer beliefs have such a strong effect on seller profit, nearly all the high type’s rents from his high signal are being dissipated by his need to set a reserve high enough that the low type won’t imitate it.)

**D2 Pooling Equilibria**

Second, I consider pooling equilibria. To support a pooling equilibrium with reserve price $\hat{r}$, I again assign beliefs $\Pr(S = 0) = 1$ if the seller sets any other (off-equilibrium-path) reserve price. For $\hat{r}$ to be a pooling equilibrium, two conditions must hold:

$$\pi(\hat{r}, 1, -) \geq \max_r \pi(r, 1, 0)$$
$$\pi(\hat{r}, 0, -) \geq \max_r \pi(r, 0, 0)$$

where $\pi(r, p, -)$ denotes expected profit to a seller with signal $S = p$ when buyers do not infer anything about $S$. Thus, the two conditions say that both buyer types prefer to pool at $r = \hat{r}$ (with buyers inferring nothing) to deviating to any other reserve price and having buyers infer that $S = 0$.

Figure 16 illustrates the two types’ incentive constraints for a pooling equilibrium. In the left pane, the blue (top) curve is $\pi(r, 0, -)$, and indicates the low type of seller’s equilibrium-path payoff at each potential value of $\hat{r}$; the red (bottom) curve is $\pi(r, 0, 0)$, and shows the profit he could get from deviating from $r = \hat{r}$. If he were to deviate, his optimal deviation would be to $r = 0.648$, giving expected profit 0.0184 when buyers believe $S = 0$; thus,
equilibrium requires $\pi(\hat{r}, 0, -) \geq 0.0184$. The magenta (dashed) line shows that this holds for any $\hat{r} \leq 0.949$.

The right pane shows the same exercise for the high type. Again, the blue (top) curve is the equilibrium payoff at various potential $\hat{r}$; the red is the payoff from deviating. In this case, the incentive constraint is satisfied for any $\hat{r} \in [0.445, 0.968]$. Intersecting the two, any reserve $\hat{r} \in [0.445, 0.949]$ is a pooling equilibrium. This includes the “optimal” pooling equilibrium, $\hat{r} = 0.705$, which gives expected profit of 0.0646.

However, Figure 17 shows that this optimal pooling equilibrium fails the “intuitive criterion” of Cho and Kreps (1987). In the left pane, the blue dot shows the high type’s equilibrium payoff in the optimal pooling equilibrium, which is 0.0657. The red curve is his expected payoff if buyers believed $S = 1$. The magenta dashed line shows that he would happily set any reserve price up to 0.974 if it would convince buyers that $S = 1$.

The right pane shows the same exercise for the low type. The low type’s equilibrium payoff at $\hat{r} = 0.705$ is 0.0635. The red curve, and magenta line, show that even with buyer beliefs $S = 1$, the low type’s expected payoff for $r > 0.93$ would be below 0.0635.

Thus, in a pooling equilibrium with $\hat{r} = 0.705$, a high type could set a reserve of, say, 0.95, and try to convince bidders that $S$ must be 1, because if $S = 0$, he wouldn’t have been willing to set such a high reserve even if he thought it would convince bidders that $S = 1$; hence, the pooling equilibrium fails the Cho-Kreps criterion.

While I haven’t done the same exercise for every possible pooling equilibrium, it appears they will all fail Cho-Kreps, because the right side of the red curve is so much steeper for the high type than for the low, making him “more willing” to set an extremely high reserve, thus making it likely that some such deviation would exist for every pooling equilibrium. (It is clear that any pooling equilibrium with $\hat{r} < 0.705$ would fail the condition, since the high type does worse and is therefore willing to go even higher with his deviation, while the low type does better on the equilibrium path.)
References


