Curved Folded Plate Timber Structures

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Summary

The objective of this paper is to show the feasibility of curved folded plate structures build with timber panels. The first part presents the method used to generate the geometry of the curved folded plate structures and the geometrical parameters of three prototypes as well as their detailing. The second part compares and discusses the building process and the result of the three prototypes.

Keywords: *Origami; folded plate structures; doubly corrugated surfaces; curved creases; timber panels.*

1. Introduction

Current research at the laboratory for timber construction IBOIS at EPFL focuses on shell structures. Two main families are investigated: shells made of linear elements, as timber rib shells, and shells made of surface elements, typically folded plate structures. With regard to timber rib shells, bent linear timber elements are used to create the shape of a shell. With folded plate structures, new timber derived products are used to create structures made of planar panels, where the panel simultaneously covers space and acts as load bearing element. Indeed, timber industry has developed in the last fifteen years new large-size timber panels. Composition and dimensions of these panels and the possibility of milling them with Computer Numerical Controlled machines shows great potential for folded plate structures. In the present paper we investigate to combine both approaches using bent timber panels for curved folded plate structures.

1.1 Goals

The objective of this paper is to investigate the feasibility of curved folded plate structures built with timber panels. Therefore, a series of empirical experimentations identifies the challenges of the construction of curved folded plate timber structures. In particular the geometry of the curved fold edge, the offset of the surfaces, the way to develop the plates and the detailing of the connection are discussed. Finally, this work deploys future potential of curved origami structures raising interest for further research in this domain.

1.2 Origami Structures in Architecture

Folded plate structures attract both architects and engineers for their structural, spatial and plastic qualities. Thin surfaces can be stiffened by a series of folds and thus not only cover space but also act as load bearing element. First folded plate structures are built with concrete. The challenge of large scale concrete shells is to diminish their dead weight and to make them very thin. To avoid the buckling of the shell, its inertia has to be augmented and this can be effectively done by a corrugation of the surface [1]. New composite materials like glass fibers inspired pioneers like Makovsky and Huybers to systematic research on folded plate geometries [2] [3]. Buri presents a method [4] to generate quadrilateral and triangular Origami meshes. Its advantage is its simple application by sketching a corrugation line and a cross section line to define the geometry of the mesh. The chapel St.Loup is the first timber folded plate structure realized by using this method [5].

1.3 Curved and Developable Structures

Already the students of Joseph Albers, head of the preliminary course at the Bauhaus, discovered the possibility to use curved folds in their paper folding exercises [6]. David Huffman, a computer scientist, was interested in folding [7] and experienced curved folding with a astonishing beauty [8]. Structural applications using curved folds have been proposed by Soykasap [9] for a lightweight space telescope. The geometry of the telescope is close to the geometry of the prototypes proposed in the present paper.

1.4 Motivations

The method used to generate planar folded plate structures is also capable to generate curved folded plate structures [4]. With regard to the fact that timber is an elastic material we were curious to investigate the possibility to build these geometries with bent timber plates. Compared to planar folded plate structures, curved folded plate structures would have the advantage of a better material continuity since their corrugation profile is smooth and not polygonal. This could lead to a better structural behaviour and a more economic construction since there are fewer connections to realize. The aesthetic and architectural qualities of curved folded plate timber structures would be most appealing.

2. Methods

2.1 Geometric Design of Origami Structures

The method considered for the geometric design of doubly corrugated surfaces has been presented in [4]. It is based on a corrugation profile and a cross section profile, which combined define the geometry of a folded origami figure in a three-dimensional Cartesian space as shown in figure 1. As an origami folding, such generated surfaces can be unfolded to a single, continuous, planar surface. This method has mainly been developed for the generation of origami patterns based on planar facets. For this purpose, discrete polylines - composed of a series of straight lines - are used as corrugation profile and cross section profile respectively. However, it has been mentioned that smooth curved corrugation profiles may be used for the design of curved origami structures.

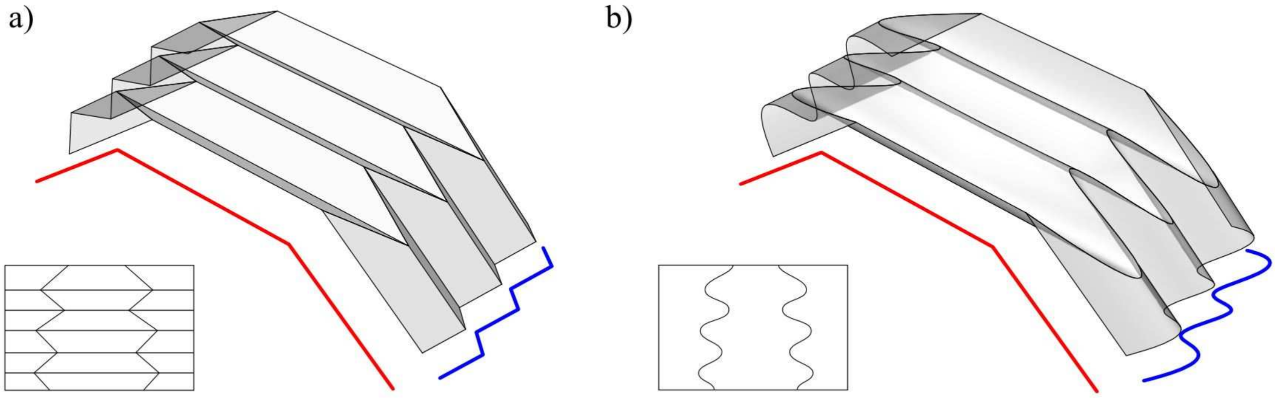

Fig. 1: Origami design based on a corrugation profile (blue) and a cross section profile (red)

Figure 1 a) shows the generation of a double corrugated origami surface that is defined by a zig-zag corrugation profile - shown in blue - and a cross section profile - shown in red. The resulting figure is composed entirely of plane elements. Figure 1 b) shows the generation of curved origami surface, which has been defined by a curved undulating corrugation profile. These two cases show that the difference between straight and curved origami figures depends on the geometry of the corrugation profile.

2.2 Curved Origami Elements

A series of three simple reduced scale models are proposed to analyze geometric and constructive problems involved with curved origami elements. The geometry of the models studied is first presented. The shape defining parameters (cf. figure 2) are given by a set of fixed and variable values.

Following constants have been established for the design of the three proposed specimens:

- The corrugation profile is a circular arc
- The cross section profile is composed of three straight line segments.
- The bending angles between the line segments are right-angled [$\varphi = 90°$]

The geometric dimensions [length/width/curvature radius] have been varied in function of material used as shown in table 1.

Specimen No.	Length [*L*]	Width [*W*]	Curvature Radius [*r*]	Material Thickness [*T*]	Weight [kg]
1	2200 mm	710 mm	2545 mm	Spruce 18 mm	19,5
2	2200 mm	510 mm	3450 mm	Spruce 18 mm	21,0
3	2390 mm	600 mm	3726 mm	Okume 8 mm	20,5

Table 1: Variable parameters used for the design of the curved origami elements

Reducing the variable design parameters in terms of bending curvature and length should allow analyzing better the influence of initial bending stress present in timber panels during assembly and service. The proposed origami elements are manufactured out of plane sheet material which is forced into its designed shape during the assembly process. Hereby, bending moments are induced to the components of the origami elements. The curved plywood components are pre-stressed, which has to a certain amount an influence on the control of the designed shape.

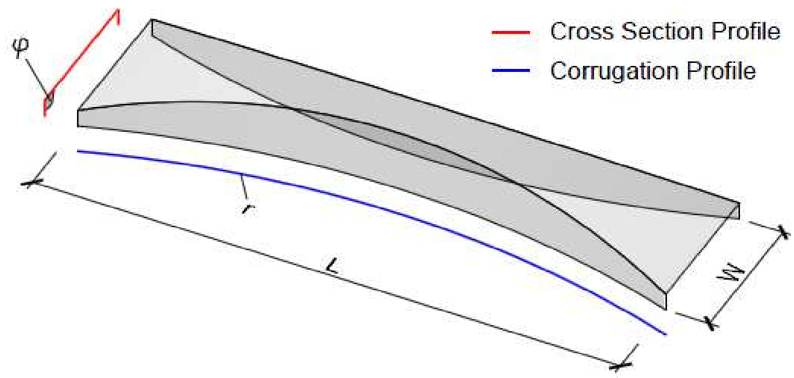

Fig. 2: Design parameters of the curved origami element.

2.3 Development and Cutting Patterns

The creation of cutting patterns for manufacturing is basically defined by unrolling the components of the origami elements. The component's surface is element of a truncated right circular cylinder with radius ***r***. It is a single-curved surface which can be unrolled about the axis ***A*** of the cylinder into a flat plane. For the development of the component's patterns, a true development method must be used - verifying that no distortion or stretching of the surface occurs. All dimensions in terms of curve length and surface area must be preserved. Principles of parallel line development, described in [10], verify these constraints and present a discreet way of unrolling the surfaces describing the origami element's components (cf. figure 3).

Let ***S*** be the surface describing one component of an origami element (cf. figure 4). The contour curves of the surface ***S*** are described by a series of cylindrical sections. The short sides of surface ***S*** are element the of straight parallel lines ***l***, which are defined by the intersection of the cylinder with a plane parallel to the cylinder axis ***A***. The long sides of the surface ***S*** are generally elliptic arcs - element of ***e*** - defined by the intersection of a plane oblique to the cylinder axis ***A***. If and only if the intersection plane of the cylinder is right angled to the axis of revolution, the intersection curve is a circular arc - element of ***c***.

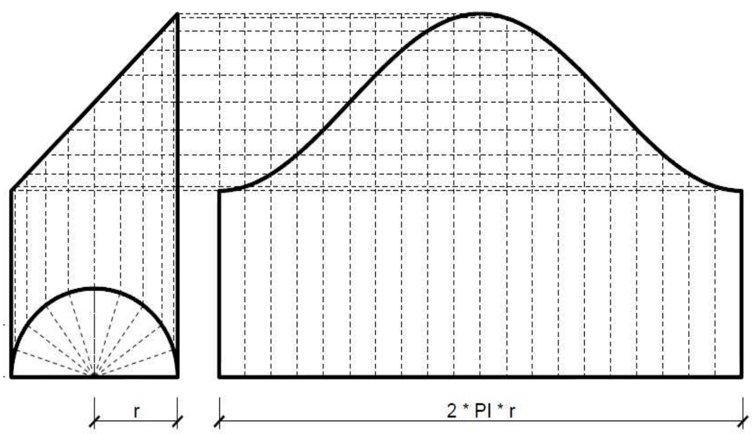

Fig. 3: Development of the truncated cylindrical surfaces by parallel lines.

Generally, a developed surface ***S'*** is delimited by two sine curve sections ***e'*** - that are images of the developed ellipse ***e*** - and two straight lines **l'** - images of the straight lines ***l***. If ***S*** is delimited by a circular boundary curve ***c***, a straight line ***c'*** results as a section of the developed circle ***c***.

Three circular surfaces of curvature radius ***r*** compose one origami element as used for the three specimens described above. Its respective developed surfaces ***S'*** define the base geometry of the cutting pattern used for the later production of a physical model of the specimen designed. This geometrical bases will further been refined with regards to material thickness and joining details used for the construction of the prototypes.

2.4 Material Thickness and Surface Offset

For the generation of cutting patterns of single-curved surfaces ***[S]***, a true development method has been defined in order to conserve the dimensions of the developed surfaces ***[S']***. While this constraint may be verified for the construction of thin surface models - such as for instance paper models - it is impossible to be met with regard to the use of thicker material - such as plywood. The construction of the prototypes with materials of a given thickness brings up the problem of the design of parallel offset surfaces. The offset distance is defined by the material thickness used for construction. Single curved cylindrical elements, which integrate thickness, are later subjected to the aforementioned development method.

The three proposed specimens are based on surfaces that are parts of right circular cylinders $\boldsymbol{C}$, defined by the cylinder axis $\boldsymbol{A}$ and the radius $\boldsymbol{r}$. The parallel offset surfaces are parts of right circular cylinders $\|\boldsymbol{C}$ defined by the axis $\boldsymbol{A}$ and $\boldsymbol{r_o}$; where $\boldsymbol{r_o} \neq \boldsymbol{r}$. The initial design surface $\boldsymbol{S} \in \boldsymbol{C}$ will be used as mid-surface for the generation of one exterior offset surface $\boldsymbol{S_e}$ and one interior offset surface $\boldsymbol{S_i}$. The normal distance between $\boldsymbol{S_e}$ and $\boldsymbol{S_i}$ equals the thickness $\boldsymbol{T}$ of the material chosen for a prototype. The offset surfaces are part of the cylinders $\boldsymbol{C_i}$ and $\boldsymbol{C_e}$ respectively defined as follows: $\boldsymbol{C_i}$: [**A**, $\boldsymbol{r_i}$] ; where $\boldsymbol{r_i} = \boldsymbol{r} - \boldsymbol{T}/2$. $\boldsymbol{C_e}$: [**A**, $\boldsymbol{r_e}$] ; where $\boldsymbol{r_e} = \boldsymbol{r} + \boldsymbol{T}/2$.

The problem of developing a set of offset surfaces $\{\boldsymbol{S_e}; \boldsymbol{S}; \boldsymbol{S_i}\}$ is related to the development of elastic bodies - volumes. In this case, true development conserving all dimensions is hence impossible. In case of the angle $\boldsymbol{\beta}$, which is initially right-angled $\boldsymbol{\beta} = \boldsymbol{\pi}/2$, the developed image $\boldsymbol{\beta'}$ does not conserve this value since $\boldsymbol{\beta'} > \boldsymbol{\pi}/2$. Similarly to the angle $\boldsymbol{\beta}$, the length of the timber panel used for the construction of a curved origami element provides three different values $\boldsymbol{\alpha r_i} \neq \boldsymbol{\alpha r} \neq \boldsymbol{\alpha r_e}$. As shown in figure 5, the length of the developed surface ***S'*** equals $\boldsymbol{\alpha r}$ where $0<\boldsymbol{\alpha}<2\boldsymbol{\pi}$. Thus, for production we need to define one unique value for the length of the component and one unique value for cutting angle $\boldsymbol{\beta}$. We will later discuss this topic with regard to detailing.

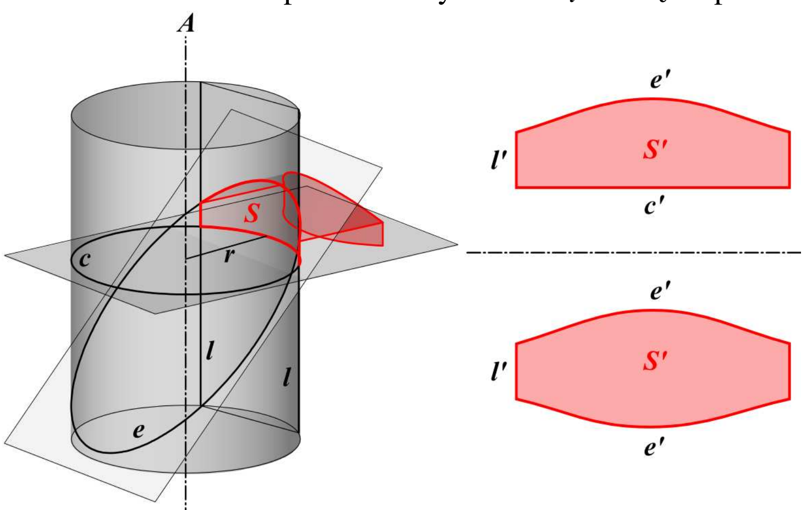

*Fig. 4: Circular surface **S** of an origami element and its developed image **S'**.*

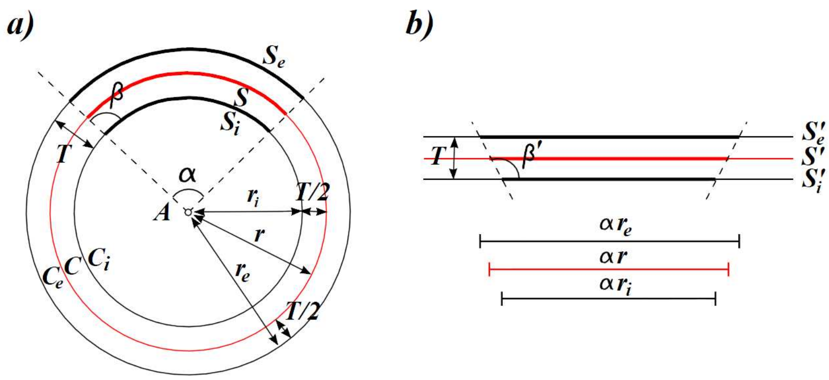

Fig. 5: Development of coaxial cylinders.

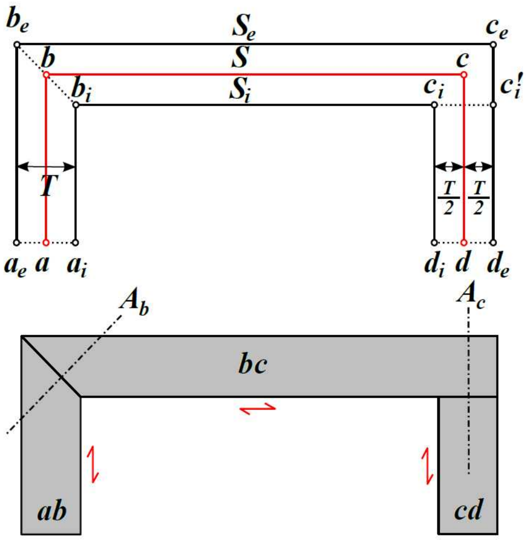

Fig. 6: Schematic cross sections through an origami element; side view.

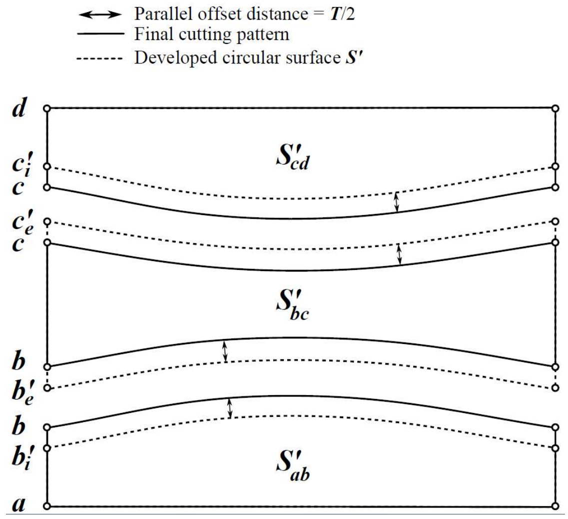

Fig. 7: Unrolled surfaces and cutting patterns

2.5 Detailing

The analysis of the joint geometry and the fasteners used is undertaken by means of cross sections. Figure 6 presents a cross section through a curved origami element. The section shows the three components {***ab***, ***bc***, ***cd***}. Basically, two joining geometries have been considered.

In corner ***b***, the two adjacent panels are connected by means of a miter joint. Each side of the adjacent panels is beveled according to their common bisector plane $\boldsymbol{b_e b_i}$. In corner ***c***, the top component ***bc*** and the side component ***cd*** finish by straight right-angled cuts. For corner ***b***, all the geometry data has been established in the sections above. The situation in corner ***c*** requires the definition of the edge $\boldsymbol{c'_i}$: the intersection line between the inner offset surface $\mathbf{b_i c_i}$ and the exterior offset surface $\boldsymbol{c_e d_e}$.

For this work, the joining detail of corner ***c*** has been chosen for the realization of the specimens. It presents following advantages:

- Greater ease of manufacturing of the panel's geometry: The panels do not need to be beveled; the cut-outs can be performed normal to the panel surface. Neither special tools nor 5 axis machining is required.
- Mechanic fasteners may be employed in a traditional way. Joining the panels by means of nails or self taping screws allows limiting greatly production and assembly time. Compared to beveled and miter joint connections, which often demand further machining in function of a given fastener (e.g. flat-dowel or spline).
- The dimensions of the cutting patterns can directly be base on the developed mid-surfaces {***S'***} by adding and subtracting half of the panel's thickness ***T***/2 respectively (cf. figure 7).

2.6 Prototypes

The design data of the tree prototypes has been described in the sections above. The most important variable parameters are summarized in table 1. The first two prototypes are made of 18 mm thick spruce plywood. They present single-layer origami elements. The difference between the first and the second specimen rely on different widths and curvature radii.

Fig. 8: Specimen number 1 (left) ; Specimen number 2 (right)

Specimen number 3 presents a double-layer curved origami element with an inner structure. Its inner and outer membranes have been realized in 8 mm thick Okume plywood. They are mounted onto a framework made of 40 mm square timber slats. The connection between the membrane surfaces is nailed while the surfaces itself are fastened to the slat structure by means of self tapping screws.

Fig. 10: Assembly sequence of specimen number 3.

3. Discussion

The assembly of the prototypes showed that the ratio between the thickness of the timber plates ***T*** and the curvature radius ***r*** of the corrugation profile strongly influence the ease of the montage and the accurateness of the final shape.

Due to its smaller curvature radius, specimen 1 has a ratio ***T/r*** = 1/140. The assembly of the plates needed a lot of force and prevented precise assembly. Stress induced by the curvature of the plates strongly deforms the final shape. The resulting element shows torsion and bending compared to its initial geometric design. The vertical panels ***ab*** and ***cd*** become strongly inclined whereas the cross section of the horizontal panel ***bc*** is curved instead of straight. Hence, the cross section opens up and becomes V-shaped.

Because of the observations made on specimen 1, specimen 2 has a greater curvature radius. The ratio ***T/r*** was increased to 1/190. Additionally, a base plate was used to fix the position of the vertical panels ***ab*** and ***bc***, which helped a lot with regards to assembly and form stability. However, the shape obtained showed still some deformations. The horizontal plate ***bc*** is curved. The base plate closes the cross section and keeps the panels ***ab*** and ***cd*** in a vertical position. For this specimen, the resulting object matches much closer the initial design and the U-shape of the cross section was maintained.

The ratio ***T/r*** of specimen 3 is 1/465. With this ratio, the panels could be bent without strong efforts and only little stress was induced. To keep the form of the vertical panels ***ab*** and ***bc***, a horizontal panel was used as falsework. Furthermore, the slats of the inner framework helped to keep the curvature of the bent panel constant. Temporarily, the horizontal panel ***bc*** was positioned relative to the vertical panels with nails. Then, panel ***bc*** was fixed to the framework by screws. The connection along the fold edge is realized by joining the slats of the framework with screws. This has the advantage that the screws can be placed with a sufficient distance to the edge of the slat. The panels themselves are too thin to be connected by mechanical fasteners which would be torn out when the structure is under load. Finally, the outer panels were fixed to the framework. The shape of the prototype corresponds to the designed geometry and is not deformed due to the stress induced by the bending of the panels. The prototype is very rigid and has the same weight as the other specimens.

4. Conclusions

The study shows the feasibility of curved folded plate structures with timber panels based on the form generation method using a curved corrugation profile and a polygonal cross section profile. The geometry of the unrolled surface can be described by a parallel line development. The use of the mid surface ***S'*** to define the geometry of the unrolled panels is accurate for the construction of right-angled prototypes.

The stress induced by bending the panels influences the shape of the prototypes. The internal strain depends on the material properties and on the ratio panel thickness to curvature radius ***T/r***. High initial stress strongly deforms the designed shape. From the experiences gained with the three prototypes, a ratio ***T/r*** of 1/250 or greater will be considered for further research. At scale of the built prototypes, the panel thickness is critical for the use of mechanical fasteners to connect the panels. The use of falsework allowed controlling the geometry of the prototypes during assembly.

The multi-layer specimen has shown several advantages. The thickness of the panels could be reduced resulting in a better ratio ***T/r***. This and the timber slats between the layers, which act as falsework, helped to precisely obtain the planed shape. The space between the panels is used to realize a strong connection of the panels along the curved fold edge. Multi-layer construction of curved origami structures may further allow the integration of technical layers such as thermal insulation, water-proofing, cladding, etc.

5. Future Work

In the future we see three fields of further development which are:

i. Extending the geometric design method for variable angles with multiple offset
ii. Further development curved panel construction systems
iii. Going towards big scale curved origami structures

The geometric design method used allows the design of curved origami structures with cross section profiles of variable bending angle φ. However, if $\varphi \neq 90°$, the method used for the generation of cutting patterns would need to be extended and generalized. Varying φ influences ultimately the geometry of the fold edge (curved crease) and its developed image. Combined with the problem of reverse folding parallel offset surfaces, a series of new questions arise within the scope of geometric design of curved origami.

For realization of curved origami structures following domains should further be developed: The montage of bigger structures demands the design of falswork and scaffolding allowing to control the border conditions. Depending on size and application, different materials have to be considered, demanding further development of fastening techniques.

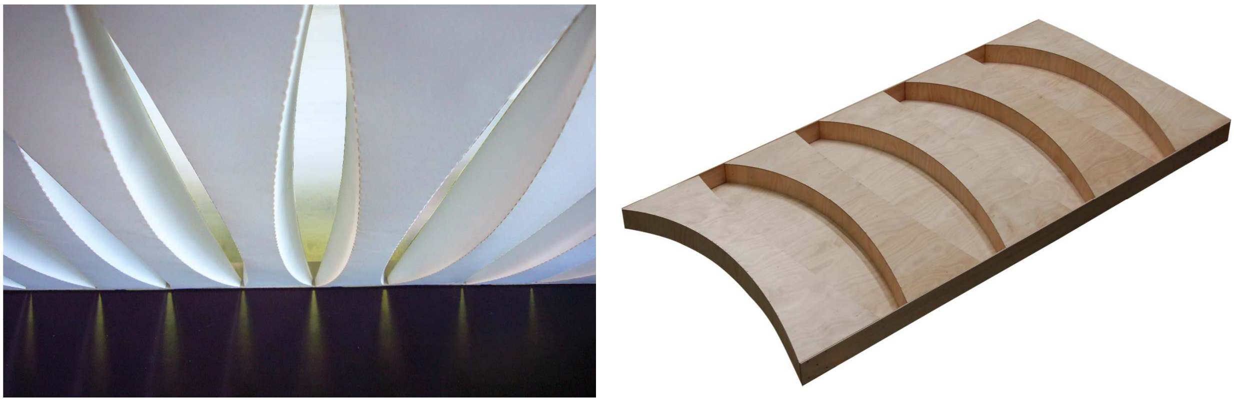

Fig. 11: Large span structures composed of multiple curved origami elements

For going big scale, several elements will be combined in order to cover large spaces. Early work-models of such structures are shown in figure 11. In parallel to these design studies, research on structural behavior of curved origami elements is currently undertaken at the laboratory for timber constructions IBOIS - EPFL. The results of this FEM-analysis compared to a series of bending tests, will ultimately allow to dimension and to build the first large scale curve origami elements.

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