The method consists of solving for the roots of a polynomial equation in the controller without the use of a speed sensor [1]–[6]. However, many of these methods still require the value of $T_R$, which can change with time due to ohmic heating. That is, to be able to update the value of $T_R$ to the controller as it changes is valuable. The work presented here uses an algebraic approach to identify the rotor time constant $T_R$ without the motor speed information. It is most closely related to the ideas described in [7]–[12]. Specifically, it is shown that $T_R$ satisfies a polynomial equation whose coefficients are functions of the stator currents, the stator voltages, and their derivatives. A zero of this polynomial is the value of $T_R$. It is further shown that $T_R$ is not identifiable under steady-state operation because the system is not sufficiently excited.

The note is organized as follows. Section II introduces a space vector model of the induction motor. Section III uses this model to develop a algebraic equation that $T_R$ must satisfy. Section IV shows that in steady state, $T_R$ is not identifiable by either the proposed algebraic method or a standard linear least-squares method. Section V presents the experimental results, while Section VI gives the conclusions and future work. A preliminary version of this work appeared in [13].

II. MATHEMATICAL MODEL OF INDUCTION MOTOR

The starting point of the analysis is a space vector model of the induction motor given by (see, e.g., [14, p. 568])

$$
\frac{d}{dt} \dot{l}_S = \frac{\beta}{T_R} (1 - j n_{p} \omega T_R) \dot{w}_S - \gamma l_S + \frac{1}{\sigma L_S} u_S
$$

(1)

$$
\frac{d}{dt} \dot{w}_R = - \frac{1}{T_R} (1 - j n_{p} \omega T_R) \dot{w}_R + \frac{M}{T_R} \dot{l}_S
$$

(2)

$$
\frac{d\omega}{dt} = \frac{n_{p} M}{J T_R} \left( \dot{l}_S - \frac{\tau_L}{J} \right)
$$

(3)

where $\dot{l}_S \triangleq i_{S_a} + j i_{S_b}, \dot{w}_R = \psi_{R_a} + j \psi_{R_b}, \psi_{R_a}, \psi_{R_b}, \psi_{R_a}, \psi_{R_b}$ are the (two-phase equivalent) stator currents, $\psi_{R_a}, \psi_{R_b}$ are the (two-phase equivalent) rotor flux linkages, $R_S, R_R$ are the stator and rotor resistances, respectively, $M$ is the mutual inductance, $L_S$ and $L_R$ are the stator and rotor inductances, respectively, $J$ is the moment of inertia of the rotor, and $\tau_L$ is the load torque. The symbols $T_R = L_R / R_R$, $\sigma = 1 - (M^2 / L_S L_R)$, $\beta = M / \sigma L_S L_R$, $\gamma = (R_S / \sigma L_S) + (3 M / \gamma T_R)$ have been used to simplify the expressions. $T_R$ is referred to as the rotor time constant, while $\sigma$ is called the total leakage factor.

III. ALGEBRAIC APPROACH TO $T_R$ ESTIMATION

The idea of the approach is to solve (1) and (2) for $T_R$. However, (1) and (2) are only four equations while there are six unknowns, namely $\psi_{R_a}, \psi_{R_b}, \dot{w}_R, i_{R_a}, i_{R_b}, \psi_{R_a}, \psi_{R_b} / dt, \dot{w}_R / dt, \omega$, and $T_R$. Equation (3) is not used because it introduces the additional unknown $\tau_L$. To find two more independent equations, (1) is differentiated to obtain

$$
\frac{d^2}{dt^2} \dot{l}_S = \frac{\beta}{T_R} (1 - j n_{p} \omega T_R) \frac{d}{dt} \dot{w}_R - j n_{p} \beta \dot{w}_R \frac{d\omega}{dt} - \frac{\gamma}{\sigma L_S} \frac{d}{dt} u_S + \frac{1}{\sigma L_S} \frac{d}{dt} u_S
$$

(4)

Using the (complex-valued) (1) and (2), one can solve for $\dot{w}_R$ and $i_{R_a}$ in terms of $\omega$, $\dot{l}_S$, and $u_S$ and substitute the resulting expressions into (4) to obtain

$$
\frac{d^2}{dt^2} \dot{l}_S = - \frac{\beta}{T_R} (1 - j n_{p} \omega T_R) \left( \frac{d}{dt} \dot{w}_R + \frac{\gamma}{\sigma L_S} \dot{l}_S - \frac{1}{\sigma L_S} u_S \right) + \frac{\beta M}{T_R} (1 - j n_{p} \omega T_R) \dot{w}_R - \frac{\gamma}{\sigma L_S} \frac{d}{dt} u_S - \frac{1}{\sigma L_S} \frac{d}{dt} u_S
$$

(5)
Solving (5) for \(d\omega/dt\) gives

\[
\frac{d\omega}{dt} = \frac{-(1 - jn\omega T_R)\beta M}{jn\omega T_R^2} + \frac{1 - jn\omega T_R}{jn\omega T_R^2} \frac{\beta M}{T_R} (1 - jn\omega T_R) \frac{\gamma L_S}{\frac{4\pi^2}{\gamma L_S} + \gamma L_S}.
\]

The left-hand side of (6) is real, so the right-hand side must also be real. Note that (1) that \(dL_S^2/dt + \gamma L_S - \frac{u}{\sigma L_S} = (\beta / T_R) (1 - jn\omega T_R) \frac{\gamma L_S}{\frac{4\pi^2}{\gamma L_S} + \gamma L_S}\). so that the right-hand side of (6) is singular if and only if \(\frac{u}{\sigma L_S} = 0\). Other than at startup, \(\frac{u}{\sigma L_S} \neq 0\) in normal operation of the motor. Separating the right-hand side of (6) into its real and imaginary parts, the real part has the form

\[
\frac{d\omega}{dt} = a_2(u_S, u_S, i_S, i_S)\omega^2 + a_1(u_S, u_S, i_S, i_S)\omega + a_0(u_S, u_S, i_S, i_S) .
\]

The expressions for \(a_2(u_S, u_S, i_S, i_S), a_1(u_S, u_S, i_S, i_S),\) and \(a_0(u_S, u_S, i_S, i_S)\) are lengthy in terms of \(u_S, u_S, i_S, i_S\), and their derivatives as well as of the machine parameters including \(T_R\). As a consequence, they are not explicitly presented here. Appendix VII-B gives their steady-state expressions.

On the other hand, the imaginary part of the right-hand side of (6) must be zero. In fact, the imaginary part of (6) is a second degree polynomial equation in \(\omega\) of the form

\[
q(\omega) \triangleq q_2(u_S, u_S, i_S, i_S)\omega^2 + q_1(u_S, u_S, i_S, i_S)\omega + q_0(u_S, u_S, i_S, i_S)
\]

and, if \(\omega\) is the speed of the motor, then \(q(\omega) = 0\). The \(q_i\) are functions of \(u_S, u_S, i_S, i_S\), and their derivatives as well as of the machine parameters including \(T_R\). The expressions for \(q_2(u_S, u_S, i_S, i_S), q_1(u_S, u_S, i_S, i_S),\) and \(q_0(u_S, u_S, i_S, i_S)\) are lengthy and not explicitly presented here. (Their steady-state expressions are given in Appendix VII-A.) If the speed was measured, then (8) would be equal to zero and could then be solved for \(T_R\). However, in the problem being considered, \(\omega\) is not known. To eliminate \(\omega, q(\omega)\) in (8) is differentiated to obtain

\[
\frac{d}{dt} q(\omega) = (2q_2\omega + q_1)\frac{d\omega}{dt} + q_2\omega^2 + q_1\omega + q_0
\]

where \(dq(\omega)/dt \equiv 0\) if \(\omega\) is equal to the motor speed. Next, \(d\omega/dt\) in (9) is replaced by the right-hand side of (7) so that (9) may be written as

\[
\frac{dq(\omega)}{dt} = g(\omega)
\]

where \(g(\omega)\) is the third-order polynomial equation in \(\omega\) (with time-varying coefficients) given by

\[
g(\omega) \triangleq 2q_2\omega^2 + (2q_2 \omega + q_1)\omega + q_0 + q_2\omega^2 + q_1\omega + q_0
\]

for which the speed of the motor is one of its roots. Dividing \(g(\omega)\) in (10) by \(g(\omega)\), (g(\omega)) may be rewritten as \(q_2 \neq 0\) if \(\omega\) and the stator electrical frequency \(\omega_S\) are nonzero. See [6], [15]).

\[
g(\omega) = \frac{1}{q_2} (2q_2\omega^2 + q_2\omega + q_0)\omega + r_1(u_S, u_S, i_S, i_S)\omega + r_0(u_S, u_S, i_S, i_S)
\]

If \(\omega\) is equal to the speed of the motor, then both \(g(\omega) = 0\) and \(q(\omega) = 0\), and one obtains

\[
r(\omega) \triangleq r_1(u_S, u_S, i_S, i_S)\omega + r_0(u_S, u_S, i_S, i_S) = 0.
\]

This is now a first-order polynomial equation in \(\omega\) which uniquely determines the motor speed \(\omega\) as long as \(r_1\) (the coefficient of \(\omega\)) is nonzero. (It is shown in Appendix VII-C that \(r_1 \neq 0\) in steady state if \(q_2 \neq 0\).) Solving for the motor speed \(\omega\) using (14), one obtains

\[
\omega = -r_0/r_1.
\]

Next, replace \(\omega\) in (8) by the expression in (15) to obtain

\[
q_2\omega^2 + q_1\omega + q_0 \equiv 0.
\]

The expressions for \(q_i, r_i\) are in terms of motor parameters (including \(T_R\)) as well as the stator currents, voltages, and their derivatives. Expanding the expressions for \(q_0, q_1, q_2, r_0,\) and \(r_1,\) one obtains a twelfth-order polynomial equation in \(T_R\), which can be written as

\[
\sum_{i=0}^{12} C_j(u_S, u_S, i_S, i_S)T_R^i = 0.
\]

Solving (17) gives \(T_R\). The coefficients \(C_j(u_S, u_S, i_S, i_S)\) of (17) contain third-order derivatives of the stator currents and second-order derivatives of the stator voltages making noise a concern. For short time intervals in which \(T_R\) does not vary, (17) must hold identically with \(T_R\) constant. In order to average out the effect of noise on the \(C_i\), (17) is integrated over a time interval \([t_1, t_2]\) to obtain

\[
\sum_{i=0}^{12} \left( \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} C_j(u_S, u_S, i_S, i_S)dt \right) T_R = 0.
\]

The measured variables appear into the coefficients of (17) in a nonlinear manner, so that it would be difficult to quantify exactly how much noise is filtered out. However, assuming a sufficient frequency separation between the noise and the signal, one would expect that such filtering would help and the experimental results presented below bear this out.

\footnote{Given the polynomials \(g(\omega), q(\omega)\) in \(\omega\) with \(\text{deg } \{g(\omega)\} = n_3, \text{deg } \{q(\omega)\} = n_2\), the Euclidean division algorithm ensures that there are polynomials \(\gamma(\omega), \rho(\omega)\) such that \(g(\omega) = \gamma(\omega)q(\omega) + \rho(\omega)\) and \(\text{deg } \{\rho(\omega)\} \leq \text{deg } \{g(\omega)\} - 1 = n_3 - 1\). Consequently if, for example, \(\omega_0\) is a zero of both \(g(\omega)\) and \(q(\omega)\), then it must also be a zero of \(\rho(\omega)\).}
There are 12 solutions satisfying (18). However, simulation results have always given 10 conjugate solutions. The remaining two solutions include the correct value of $T_R$ while the other one was either negative or close to zero. The method is to compute the coefficients $(1/(t_2 - t_1))I(t)^2C dt$ and then compute the roots of $(18)$. Among the positive real roots is the correct value of $T_R$. Experimental results using this method are presented in Section V.

Remark: The expression (14) was used by the authors in [6], [20] (assuming $T_R$ is known) as a technique to estimate the speed of an induction motor for speed sensorless field-oriented control.

IV. IDENTIFIABILITY OF $T_R$ IN STEADY STATE

The goal of this section is to show that $T_R$ is not identifiable with the machine in steady-state because it is not sufficiently excited. We show this explicitly for the method proposed here and then show it explicitly for a linear least-squares formulation. The terminology “steady state” means the machine is running at constant speed and the voltages/currents are in steady state.

A. Algebraic Approach

The polynomial (18) is now considered with the machine in steady-state so that, in particular, the speed is constant. That is, $u_{sA} + ju_{sB} = L_s e^{j\omega_s t}$ and $i_{sA} + ji_{sB} = L_s e^{j\omega_s t}$ are substituted into (8) and (14) where $\omega_s$ is the electrical frequency. In steady state, the motor speed in (15) becomes (see Appendix VII-C and [15])

$$\omega = -\frac{r_0}{r_1} = \frac{\omega_s(1-S)}{n_p}$$

where $S \triangleq (\omega_s - n_0)/\omega_s$ is the normalized slip. Substituting the steady-state expressions for $q_2$, $q_1$, and $q_0$ from Appendix VII-A as well as the expression (19) for $\omega$ into (8), one obtains

$$q_2\omega^2 + q_1\omega + q_0 = \frac{n_p^2 T_R l_e^2 l_s^2 L_s (1 - \sigma)^2 (1 - S)}{\sigma (1 + S^2 \omega_s^2 T_R^2)}$$

$$+ \frac{n_p \omega (1 - \sigma)^2 (1 - S)}{\sigma (1 + S^2 \omega_s^2 T_R^2)}$$

$$+ \frac{(\omega_s (1 - S)}{n_p} - \frac{l_e^2 l_s^2 L_s (1 - \sigma)^2 (1 - S)}{\sigma (1 + S^2 \omega_s^2 T_R^2)} \equiv 0.$$
where $P \triangleq u_{sa}i_{sa} + u_{sb}i_{sb}$ and $Q \triangleq u_{sa}i_{sa} - u_{sb}i_{sb}$ are the real and reactive powers, respectively, whose steady-state expressions are given by (30) and (31) in the Appendix. Using (30) and (31) to replace $P$ and $Q$ in (25), one obtains
\[
\begin{align*}
\bar{D} & \triangleq D(t)W(t) \\
& = - \frac{L_s}{1 + S^2_s \omega_s^2 T_R^2} \left[ S^2_s \omega_s^2 T_R^2 \quad S\omega_s T_R \quad 1 \right] \quad (26) \\
\bar{Y} & \triangleq D(t)y(t) \\
& = -\omega_s \frac{L_s}{1 + S^2_s \omega_s^2 T_R^2} \left[ S\omega_s T_R \quad 1 \right].
\end{align*}
\]
That is, in steady state, $\bar{D} \triangleq D(t)W(t) \in \mathbb{R}^{n \times 2}$ and $\bar{Y} \triangleq D(t)y(t) \in \mathbb{R}^1$ are constant matrices. Further, it is easily seen that the determinant of $\bar{D} \triangleq D(t)W(t)$ is zero. Also,
\[
R_{DW} \triangleq \sum_{n=1}^{N} (D(nT)W(nT))^T (D(nT)W(nT))
= |L_x|^2 \sum_{n=1}^{N} W^T(nT)W(nT) = |L_x|^2 R_W.
\]
$R_{DW}$ is singular as $D(t)W(t)$ is constant and singular. It then follows that $R_W$ is also singular using steady-state data. Furthermore,
\[
R_{DWY} \triangleq \sum_{n=1}^{N} (D(nT)W(nT))^T (D(nT)y(nT))
= |L_x|^2 \sum_{n=1}^{N} W^T(nT)y(nT) = |L_x|^2 R_{YW}.
\]
Thus, $R_W$ and $R_{YW}$ are given by
\[
R_W = R_{DW} |L_x|^2 = N \bar{D}^T \bar{D} |L_x|^2
= N|L_x|^2 \left[ (1 - \sigma \omega_s^2 L_x^2) \quad S\omega_s T_R \quad S\omega_s T_R \quad 1 \right]
\]
\[
R_{YW} = R_{DWY} |L_x|^2
= N \bar{Y}^T \bar{Y} |L_x|^2
= \omega_s N|L_x|^2 \left[ (1 - \sigma \omega_s^2 L_x^2) \quad S\omega_s T_R \quad 1 \right] \quad (28)
\]
where again $\bar{D}$ and $\bar{Y}$ are from (26) and (27), respectively.

By inspection of (28) and (29), $K = [0 \quad \omega_s]^T$ is one solution to (24). The null space of $R_W$ is generated by $[-1/T_R \quad S\omega_s]^T$ so that all possible solutions are given by $[0 \quad \omega_s]^T + K[1/T_R \quad S\omega_s]^T$ for some $K \in \mathbb{R}$. In summary, solving (24) using steady-state data leads to an infinite set of solutions so that $T_R$ is not identifiable using the linear regressor (23) with steady-state data.

Remarks: There are a few ways to avoid the singularity problem in a real-time control application. For example, a small perturbation could be added to the speed reference. This type of technique has often been used for the adaptive control of insufficiently excited systems. A more interesting approach, however, would be to vary the flux reference while keeping the torque reference constant. The speed of the motor would not vary, but the voltages and currents would no longer be in sinusoidal steady-state, so that the speed and the rotor time constant would be identifiable. In [4], a linear regressor was obtained by assuming constant speed, but the data collected in [4] was not in sinusoidal steady-state (see [4, Figs. 7.1a and 7.1b]). In the identification method given in [16], the speed is assumed constant, but it requires the flux magnitude be perturbed by a small amplitude sinusoidal signal so it is also not in sinusoidal steady-state.

Fig. 1. Sampled two-phase equivalent voltages $u_{Sa}, u_{Sb}$.

Fig. 2. Sampled phase $b$ current $i_{sb}$ and its simulated response $i_{sb}^{sim}$.

V. EXPERIMENTAL RESULTS

To demonstrate the viability of the speed sensorless estimator (18) for $T_R$, experiments were performed. A three-phase, 0.5 hp, 1735 rpm (2 pole-pair) induction motor was driven by an ALLEN-BRADLEY PWM inverter to obtain the data. Given a speed command to the inverter, it produces PWM voltages to drive the induction motor to the commanded speed. Here a step speed command was chosen to bring the motor from standstill up to the rated speed of 188 rad/s. The stator currents and voltages were sampled at 10 kHz so that the sample period is $T_s = 0.001$ s. The real-time computing system RTLAB from OPAZ-RT with a fully integrated hardware and software system was used to collect data [17]. Filtered differentiation (using digital filters) was used for the derivatives of the voltages and currents. Specifically, the signals were filtered with a third-order Butterworth filter whose cutoff frequency was 100 Hz. The voltages and currents were put through a 3-2 transformation to obtain the two-phase equivalent voltages $u_{Sa}, u_{Sb}$, which are plotted in Fig. 1 and with the corresponding two-phase equivalent currents $i_{Sa}, i_{Sb}$ shown in Fig. 2.

Using the data $\{u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}\}$ collected between 0.84 and 0.91 sec ($T_s \triangleq 0.91 - 0.84 \approx 0.07$ sec is the batch data collection period), which includes the time the motor accelerates, the quantities $du_{Sa}/dt, du_{Sb}/dt, di_{Sa}/dt, di_{Sb}/dt, d^2i_{Sa}/d\tau^2, d^2i_{Sb}/d\tau^2$, $d^3i_{Sa}/d\tau^3$, $d^3i_{Sb}/d\tau^3$, $d^4i_{Sa}/d\tau^4$, $d^4i_{Sb}/d\tau^4$,.
The parameter $\gamma$, defined by (18), is used to compare with the measured (stator currents) outputs. Fig. 2 shows the sampled two-phase equivalent current $i_{S1}$ and its simulated response $i_{S1,\text{sim}}$. The phase $\phi$ current $i_{S1}$ is similar, but shifted by $\pi/(2n_p)$.) The resulting phase $b$ current $i_{S1,\text{sim}}$ from the simulation corresponds well with the actual measured current $i_{S1}$. Note that in (1) the parameter $\gamma = (R_s/\sigma L_s) + (3 M/T_R)$ also depends on $T_R$.

VI. CONCLUSION AND FUTURE WORK

This note presented an algebraic approach to the estimation of the rotor time constant of an induction motor without using a speed sensor. The experimental results demonstrated the practical viability of this method. Though the method is not applicable in steady state, neither is a standard linear least-squares approach. Future work includes studying an on-line implementation of the estimation algorithm and using such an online estimate in a speed sensorless field-oriented controller.

APPENDIX

STEADY-STATE EXPRESSIONS

In the following, $\omega_S$ denotes the stator frequency and $S$ denotes the normalized slip defined by $S = (\omega_S - n_{pH})/\omega_S$. With $u_{S1} + ju_{S1} = U_s e^{j\omega_S t}$ and $i_{S1} + ji_{S1} = I_s e^{j\omega_S t}$, it is shown in (19) that under steady-state conditions, the complex phasors $U_s$ and $I_s$ are related by

$$S \rho \approx R_s/\sigma \omega_s L_R = 1/\sigma \omega_s T_R$$

$$L_s = \frac{U_s}{R_s + j \omega_s L_s \left(1 + \frac{j \frac{1}{\sigma} S}{T_R}\right)}$$

and straightforward calculations (see [6], [15], and [20]) give

$$P \triangleq u_{S1} + u_{S1} = R_s \left(U_s \frac{L_s}{S}\right)$$

$$Q \triangleq u_{S1} + u_{S1} = I_s \left(U_s \frac{L_s}{S}\right)$$

A. Steady-State Expressions for $q_2, q_1$, and $q_0$

The steady-state expressions for $q_2, q_1$, and $q_0$ are (see [6], [15], and [20])

$$q_2 = n_p^2 T_R^2 |L_s|^4 \frac{\omega_s^2 L_S (1 - \sigma)^2 (1 - S)}{\sigma (1 + S^2 \omega_s^2 T_R^2)}$$

$$q_1 = n_p \omega_s |L_s|^4 \frac{L_S (1 - \sigma)^2 (1 - \omega_s^2 T_R^2 (1 - S)^2)}{\sigma (1 + S^2 \omega_s^2 T_R^2)}$$

$$q_0 = - |L_s|^4 \frac{\omega_s^2 L_S (1 - \sigma)^2}{{\sigma}^2 (1 + S^2 \omega_s^2 T_R^2)}$$

With $\omega \neq 0$ (equivalent to $S \neq 1$), it is seen that $q_2 \neq 0$. Conversely, $q_0 = 0$ if and only if $S = 1$ (i.e., $\omega = 0$). Also, if $\omega = 0$, then $S = 1$ and $q_1 = 0$.

B. Steady-State Expressions for $a_2, a_1$, and $a_0$

The steady-state expressions for $a_2, a_1$, and $a_0$ are (see [6], [15], and [20])

$$a_2 = - n_p^2 |L_s|^4 \frac{\omega_s (1 - \sigma)^2}{\sigma^2 (1 + S^2 \omega_s^2 T_R^2)}$$

$$a_1 = n_p |L_s|^4 \frac{2 \omega_s^2 (1 - \sigma)^2 (1 - S)}{\sigma^2 (1 + S^2 \omega_s^2 T_R^2)}$$

$$a_0 = - |L_s|^4 \frac{\omega_s^2 (1 - \sigma)^2 (1 - S)^2}{\sigma^2 (1 + S^2 \omega_s^2 T_R^2)}$$

and $\text{den} \triangleq n_p^2 T_R |L_s|^4 \left[\frac{(1 - \sigma)(1 + S^2 \omega_s^2 T_R^2) - S \omega_s^2 T_R^2}{\sigma T_R^2} \right]^2 + \left[\frac{(1 - \sigma) \omega_s}{\sigma (1 + S^2 \omega_s^2 T_R^2)} \right]^2$. (38)

Recall from Section III [following (6)] that $\text{den} = 0$ if and only if $|L_s|^4 = 0$.

C. Steady-State Expressions for $r_1$ and $r_0$

It is now shown that the steady-state value of $r_1$ in (12) is nonzero. Substituting the steady-state values of $q_2, q_1, q_0, a_2, a_1$, and $a_0$ (noting that $q_0 \equiv 0$ and $q_2 \equiv 0$ in steady state) into (12) gives

$$r_1 = - \frac{|L_s|^4}{1 + S^2 \omega_s^2 T_R^2} \frac{3}{4} \frac{n_p^4 (1 - \sigma)^2}{\sigma^4 L_s^3} \left[1 + \frac{T_R^2 \omega_s^2 (1 - S)^2}{1 + \omega_s^2 T_R^2} \right]$$

$$r_0 = \frac{|L_s|^4}{1 + S^2 \omega_s^2 T_R^2} \frac{3}{4} \frac{n_p^4 (1 - \sigma)^2}{\sigma^4 L_s^3} \left[1 + \frac{(1 + \omega_s^2 T_R^2) (1 - S)^2}{1 + \omega_s^2 T_R^2} \right]$$

where $\text{den}$ is given by (38). It is then seen that $r_1 \neq 0$ in steady state.

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Comments on “Optimizing Simultaneously Over the Numerator and Denominator Polynomials in the Youla-Kucera Parameterization”

Fikret A. Aliev and Vladimir B. Linar

Abstract—It is noted that the parameterization of the set of stabilizing regulators was first presented in a monograph by Linar V.B., Naumenko K.I., and Suntsev V.N.

These comment were prompted by the recent note [1] which, in its historical survey of parameterization of feedback systems, has overlooked reference [2]. We use this opportunity to re-iterate the fact that [2] was the first known publication that presented the parameterization of the set of stabilizing regulators, definitely before [3], as also acknowledged in [4] (see, for instance, the comment to [4, ref [29]]). It appears that the Youla–Bongiorno parameterization was rediscovered a couple of years later, but most probably without any knowledge of [2]. A discussion on parameterization can also be found in [5].

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Authors’ Reply

Vladimir Kučera

We would like to thank F.A. Aliev and V.B. Linar for their comments on [7], namely for bringing to our attention apparently the first publication [1] that presented a parametrization of all stabilizing controllers for a given plant.

The parametrization is obtained in [1] in the context of solving a linear-quadratic control problem with stability, in the frequency domain, applying the Wiener–Hopf approach. The free parameter represents a function to be varied in order to minimize the cost while assuring stability of the closed-loop system for any plant, stable or unstable, minimum phase or nonminimum phase. The exposition of the subject is elegant and instructive, showing why the parameter should be a linear combination of specific closed-loop transfer functions.

The setting of the best-known publication [2] on the parametrization result is the same: just the construction of the free parameter is slightly different, making full use of polynomial matrix fractions.

Reference [3] approaches the feedback system stability directly, in an algebraic manner, without any appeal to an optimization problem, to show that the set of stabilizing controllers for a given plant corresponds to the solution set of a Bezout equation. Since the solution set can be parametrized, the explicit controller parametrization immediately follows [4].

The algebraic nature of the parametrization result was further emphasized in [5]. A survey of research directions advanced by this fundamental result is presented in [6].