Pricing and Upgrade Strategies for Software Vendors in the Software-as-a-Service Market

Research-in-Progress

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Abstract

This study examines the competition between an independent software vendor (ISV) who offers a standard fixed-price software product and a cloud software vendor (CSV) who adopts a usage-based linear pricing scheme for its cloud-based software. Using a game theoretic approach, we set up a duopoly model to derive the optimal pricing and product feature choice decisions for both the ISV and the CSV in a market characterized with heterogeneous consumer preferences for software features, and different risk structures. Next, we plan to extend the duopoly model into a two-stage game and examine how the equilibrium outcomes change when we allow both vendors to exercise their software upgrade strategies. The results of our study will provide important implications to vendors and consumers in software markets where the Software-as-a-Service (SaaS) business model began to prevail and contribute to the growing literature on economics of cloud computing.

Keywords: Game theory, economics of information systems, cloud computing, Software-as-a-Service, SaaS, price analysis.

Introduction

Software-as-a-Service (SaaS) is a method of delivering software hosted in the cloud through the Internet to users (Katzan Jr et al. 2010) and has experienced dramatic growth in the past few years (Badidi 2013). According to market research firm IDC, the revenue of the SaaS industry is estimated to grow from $39 billion in 2013 to over $100 billion by 2018 (Mahowald and McGrath 2014). Usability, portability, and scalability are the factors that have been suggested as accelerators of the success of cloud computing services and applications (Ramachandran and Chang 2014). Unlike traditional independent software vendors (ISVs) who offer on-premise software through perpetual licensing, cloud software vendors (CSVs) allow customers to access their software through any PCs or even portable devices, thereby reducing costly investment in IT infrastructure. CSVs such as Salesforce.com and Concur charge a price based on actual functionalities used and the number of users, which converts high upfront software licensing cost into a variable usage cost with flexible payment options. Moreover, cloud applications allow vendors to aggregate demands for computing resources and enjoy savings from economy of scale on the cost front.
These unique benefits have enabled new and small businesses to adopt enterprise-level software that they would not have been able to afford in the traditional software market and opened up a new arena for software competition (Chen and Wu 2013).

Although SaaS is growing rapidly, on-premise software still holds a dominant position in the software industry due to its superior customizability and better control over system and data (GFI Software 2010). For example, ISVs such as Oracle and SAP offer their users a standard off-the-shelf version with an opportunity to customize the software to add more features, whereas CSVs generally do not have the resources to cater to each individual customer’s specific business needs. Furthermore, due to regulatory and security concerns, some businesses tend to stay away from SaaS products to avoid security breaches and service interruptions. In fact, some researchers have recommended that SaaS should only be used by small businesses while large businesses or mission critical processes should adopt on-premise software (Kateeb and Almadallah 2014). Given these distinct features, ISVs and CSVs often embrace completely different strategies to promote their products, leading to market dynamics that differ significantly from what we previously observed in the traditional software industry.

Therefore, this study attempts to examine the competitive strategies in such an emerging market and the associated market outcomes. Specifically, this research seeks to address the following research questions:

1. What are the optimal pricing strategies in a duopoly competition characterized with an ISV that offers a standardized fixed-price software product and a CSV that adopts a usage-based linear pricing scheme for its cloud-based software?

2. Given the ISV’s ability to provide in-house customization and the CSV’s cost advantage and flexibility in delivery mechanism, what are optimal software feature choice and product upgrade strategy for both software vendors?

3. What are the equilibrium outcomes (i.e. profits, market shares, social welfare) in the above settings, and their implications for software vendors, consumers, and the society?

Following the growing literature on the economics of cloud-based software (Choudhary 2007; Chen and Wu 2013; Ma and Seidmann 2008, 2015), this study uses game theory to analyze a software market characterized by heterogeneous consumers, an ISV that offers a standard but customizable product, and a CSV that charges users based on actual software usage and features requested. In deriving the optimal pricing and product feature choice strategies for software vendors, we focus on examining the situations where consumers differ in their software features requirements, and risk-bearing ability, and whether software vendors have the ability to upgrade their products in a future stage. The results of our study are expected to contribute to the growing interest in understanding the evolution of the increasingly popular cloud computing platform and provide important implications on how software vendors can leverage their unique product characteristics to gain competitive advantage in the emerging SaaS market.

**Literature Review**

Research in SaaS has taken different directions with most studies employing analytical modelling techniques to investigate various phenomena of interest. This study builds primarily on two streams of research: linear and non-linear pricing of information goods (i.e. usage-based pricing), and the economics on software quality choice and upgrade.

Usage-based pricing is the predominant pricing model in the SaaS industry. SaaS users have the option to pay for the usage of the software on a pay-per-use basis or on a monthly or yearly subscription based on either the features they choose and/or number of users. Such payment flexibility and ease of cost management have significantly increased consumers’ willingness to adopt SaaS and allow vendors to better differentiate their products to reach a larger market (Choudhary et al. 1998). Relative to the traditional perpetual licensing scheme, the SaaS delivery model has been found to improve profits for vendors and increase welfare (Gurnani et al. 2001), even in a market with potential for piracy (Jiang et al. 2007). However, prior research has also shown that the outcomes of fixed-price licensing and usage-based pricing depend heavily on the nature of the competition (Wu and Banker 2010), consumer preferences (Fishburn and Odlyzko 1999) and transaction cost (Sundararajan 2004). Given that the majority of the above studies adopt a monopoly framework to examine the equilibrium outcomes, we attempt to extend these models by introducing a dynamic framework into a duopoly competition, in
which an ISV optimizes its pricing and product feature choices with the anticipation that a CSV will adopt a usage-based pricing strategy, and both vendors have the option to upgrade their products in the second stage of the competition. Our model also departs from some of the recent studies on pricing strategies for SaaS (Chen and Wu 2013; Ma and Seidmann 2015) in that we not only examine software vendors’ pricing strategies, but also model their product feature choice as a strategic decision endogenously derived from consumer’s preference structures.

Prior studies have shown that CSVs differ from ISVs when it comes to software feature choice and software upgrade strategies which are common approaches used by vendors in the software market (Doğan et al. 2011). Due to the multi-tenancy capability and the cost-sharing mechanism inherent to the SaaS models (Dubey and Wagle 2007), CSVs generally have more incentives to invest in product development and perform software upgrades under the SaaS model (Choudhary 2007). For ISVs, product upgrade needs to be conducted for each individual client, preventing the same economy of scale enjoyed by the CSVs. However, ISVs have a significant advantage in providing customization to meet the unique business needs of their clients, often resulting in a better user experience for the client firms (Ma and Seidmann 2015). As such, the optimal effort invested by a vendor in the initial development of features often increases with the probability of future upgrades (Dogan et al. 2011). Our study attempts to capture these two important characteristics in an integrated model.1 Envisioning the benefits of customized product upgrade provided by the ISVs and the lower upgrade price offered by the CSVs, the consumers will strategically examine the tradeoffs between these two options and incorporate such considerations into their software choices, which in turn determines the vendors’ optimal product promotion strategies.

In addition to the above two aspects, potential service reliability and security issues facing cloud software customers will also affect a customer’s decision to choose the cloud platform over the traditional on-premise software (Ramachandran and Chang 2014). In light of the risks and uncertainty associated with the adoption of the cloud platform, organizations may choose to embrace the SaaS business model in a relatively small scale, or adopt the software incrementally (Kateeb and Almadallah 2014). Our model will explicitly incorporate the risk involved in using the cloud software application offered by the CSV and examine how the firms’ adoption decision changes as the risk level varies.

Model

We start our analysis by describing the characteristics of the software products and the market, as well as the strategies available for both the ISV and the CSV. Next, we derive the equilibrium in which the ISV and the CSV engage in a duopoly competition. Building on the results from these models, we plan to examine how the competitive landscape changes if we extend the static competition into a dynamic framework in which both vendors have the option to offer upgrades to their existing customers.

Market characterization

The market is characterized with \( N \) heterogeneous firms with varying demands for software features and functionalities (abbreviated to features hereafter). A firm’s demand for software features, \( f \), is assumed to be uniformly distributed on a unit interval \([0, 1]\) as represented by the horizontal axis in Figure 1.

![Figure 1. Firm’s Demand for Software Features and the Respective Utilities](image_url)

\[ f \]

For \( f < F \), the value derived from using the software is \( v(f) \).

For \( f \geq F \), the value derived from using the software is capped at \( v(F) \).

Features set by the software vendor

For \( f < F \), ISV charges \( p_i \)

and CSV charges \( \frac{f}{P_c}p_c \).

For \( f \geq F \), ISV charges \( p_i \)

and CSV charges \( p_c \).

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1 Please see Ma & Seidmann (2015) for a discussion on the market outcomes when firms have uncertainty on their transaction volume.
When either vendor develops their software products, they need to strategically determine what product features will be included in their applications (denoted by \( F \) in Figure 1) as software development cost often increases exponentially with the number of features included (Dogan et al. 2011). Let \( v(f) \) be the value derived from using a software vendor’s product. If a client firm has feature requirements less than what the software vendor’s product offers (i.e. for those with \( f < F \) in Figure 1), then a firm only derives a value based on the software feature it actually uses (\( v(f|F) = v(f) \)) because the extra features offers no additional value to the firm. However, if a firm has more feature requirements than the software offers (i.e. \( f \geq F \)), then the valued derived from using the software is capped at the feature level \( F \) as this is the maximum amount of features a firm can use (\( v(f|F = v(F) \)). Therefore we can derive the general form of the value function as \( v = v(\min(f, F)) \). Assuming that the value of the software is proportional to the amount of software feature the firm actually uses (e.g. \( v = \delta f \)), then without loss in generality, we can also normalize \( v(f) \) to be uniformly distributed on a unit interval \([0,1]\) and simplify the value function to be \( v(\min(f, F)) = \min(f, F) \). 

Based on this common market structure, we describe the strategies available to both the ISV and CSV. For ease of understanding, we also summarize the market segmentation when either vendor is a monopolist in the market. Then we build a baseline model to derive the equilibrium in a duopoly competition. Throughout the paper, we use lower case italic letters to denote the firm’s utility or preferences, and upper case italic letters to denote the variables for the ISV or CSV (differentiated through subscripts \( i \) or \( c \), respectively).

**ISV’s Strategies and Market Segmentation**

As discussed in the introduction, since the ISV does not have the ability (or cost advantage) to tailor their software products to the actual needs of the client firms, it will offer a standardized software product to its customers with a fixed set of features (i.e. by setting \( F_i=f \) in Figure 1), and charge all customers a fixed price \( P_i \), regardless of whether a firm’s actual software feature needs are below or above what the software offers. If a firm adopts the ISV’s software, they also incur a cost \( C_i \) to operate the software in house (i.e. costs of hardware, IT infrastructure, energy costs, etc.).\(^2\) Hence for customers with \( f < F_i \), they will buy the ISV’s standard software if and only if the utility of using the software exceeds the cost, that is:

\[
U_i = f - P_i - C_i \geq 0 \iff f \geq P_i + C_i \quad (1a)
\]

Similarly, for firms with \( f \geq F_i \), they will purchase the ISV’s standard software if and only if the utility of using the software exceeds the cost, that is:

\[
U_i = F_i - P_i - C_i - (f - F_i) \geq 0 \iff f \leq 2F_i - P_i - C_i \quad (1b)
\]

Note that since the software features are capped at \( F_i \), these firms only derive a value of \( F_i \) despite that they have greater feature needs. In this case, they are obviously worse off than a firm who pays the same price \( (P_f) \) and receives exactly what it needs (e.g. for a firm with \( f = F_i \)). To capture such a gap between what a firm demands and what it receives, we include a negative term \( (f - F_i) \) in (1b) to represent a firm’s disutility\(^3\) when they purchase the software but could not obtain their desirable software capabilities.\(^4\) Figure 2 illustrates the two possible forms of market segmentation that can arise when ISV is the only vendor in the market. When the ISV sets \( F_i < 1 \), it may attract both firms who demand fewer requirements than what the ISV’s software offers (region A2 in Figure 2), and firms whose feature requirements exceed the software capability \( F_i \) (region A3), as long as their net utility is greater than zero (e.g., either (1a) or (1b) is satisfied). In this case, there is a segment of the market that cannot afford the ISV’s software due to software and operation costs (region A1), and a possible segment that will forgo the consumption because of a large feature gap and the associated disutility\(^5\) (region A4). Alternatively, When the ISV set a \( F_i=1 \), it will attract all firms who derive a positive utility (region B2), and the rest of the market (region B1) will choose not to make a purchase.

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\(^2\) For ease of interpretation, we use \( C_i \) to capture both fixed and variable costs of running the software in house.  

\(^3\) An analogy of this specification is that when a blue collar worker and a millionaire both receive $100 for the same labor they exerted, the blue collar worker will obviously derive more utility out of these $100 than the millionaire.  

\(^4\) In the event a firm chooses not to purchase the software due to any negative utility resulted from the software feature gap, we consider this as a case where the firm will exit the market and receive a zero utility.  

\(^5\) This region may not exist if the ISV raises \( F_i \) to a high enough level to cover all the market to the right of \( F_i \).
When the CSV dominates the market, it will also determine an optimal level of software features just forgo the consumption. It is worth noting that since the CSV adopts a linear pricing strategy, as long as it is based on the number of users it expects to serve (so as not to waste any computing resources). Hence from the infrastructure (i.e. determining the number of servers, power supply, storage, etc.), it will choose an optimal infrastructure level offered by the cloud software (cloud) based software offers and sets the price market share of the CSV, and such a risk is often proportional to the number of features a firm uses on the cloud. To capture such a potential risk, we use an exogenous variable "healthcare, and government agencies) may face greater risk if they choose to adopt the cloud-based software, and such a risk is often proportional to the number of features a firm uses on the cloud. To capture such a potential risk, we use an exogenous variable \( r \) to measure the expected risk factor of the firms that demand high service reliability and security.

Based on these properties, we can identify the indifferent consumer that segments the market for the CSV. For firms with \( f < F_c \), they will adopt the CSV’s cloud software if and only if the utility exceeds the cost:

\[
U_c = (1 - r)\left( f - \frac{P_c}{F_c}\right) \geq 0 \iff r \leq 1 - \frac{P_c}{F_c} \tag{2a}
\]

Therefore, the adoption decision depends on the relative magnitude of risk to cloud software cost benefit ratio. In order for these firms to adopt the cloud software, the price charged by the CSV must be low enough to compensate for the expected loss due to reliability or security risk. Otherwise, these firms may just forgo the consumption. It is worth noting that since the CSV adopts a linear pricing strategy, as long as (2a) is satisfied, either all firms with \( f < F_c \) will adopt the cloud software, or they will all give it up.

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\( ^6 \) Even though from an ad hoc perspective, SaaS operation cost is rather fixed, when the cloud software vendor invests in the infrastructure (i.e. determining the number of servers, power supply, storage, etc.), it will choose an optimal infrastructure level based on the number of users it expects to serve (so as not to waste any computing resources). Hence from the ex-ante point of view, the operation cost reflects a vendor’s expected service capacity level and will increase with the number of expected users.

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**Figure 1: Market Division When ISV Dominates the Market**

CSV’s Strategies and Market Segmentation

When the CSV dominates the market, it will also determine an optimal level of software features \( F_c \) that its cloud-based software offers and sets the price \( P_c \) to maximize its profits. However, there are some important differences if a firm decides to adopt the CSV’s cloud-based software.

First, unlike the ISV, the CSV charges its customers based on the features they actually used, up to the level offered by the cloud software (\( F_c \)). Therefore, the price paid by the customer will be proportional to their actual software usage. Specifically, the CSV will charge a price \( P_c \) to consumers who demand the highest level of software capabilities \( F_c \). For customers whose feature requirements \( f \) is less than \( F_c \), a linear price scheme will be used and firms will pay a proportional price equal to \( \frac{f}{F_c} P_c \) (see Figure 1).

Second, as the software is hosted on the cloud, cloud vendor will bear most of the aggregate operation costs. Since such a cost is a function of the total number of users, we assume this aggregate cost function form to be: \( C_c - (MS_c \cdot N) \), where \( C_c \) is the average cost of serving a customer on the cloud platform, \( MS_c \) is the market share of the CSV, and \( N \) is the total number of firms that have adopted the CSV product. Due to economy of scale, we expect the average cost per firm to be a lot lower than the cost of operating the ISV software in house (\( C_c < C_i \)). On the consumer side, without loss in generality, we can assume that the marginal cost of using the cloud software is zero.

Third, as cloud software is not hosted in house, a firm may experience some service interruption when the server or the network experiences congestion, or in a worse situation, security breaches may occur and impose loss on the firms. Given such a possibility, firms in certain sectors (i.e. financial services, healthcare, and government agencies) may face greater risk if they choose to adopt the cloud-based software, and such a risk is often proportional to the number of features a firm uses on the cloud. To capture such a potential risk, we use an exogenous variable \( r \) to measure the expected risk factor of the firms that demand high service reliability and security.
For firms with \( f \geq F_c \), similar to the ISV’s case, they can only receive benefits up to level \( F_c \), and there is a disutility for not getting the software capabilities they actually need. Hence they will adopt the CSV’s product if and only if the utility of using the software exceeds the cost, that is:

\[
U_c = (1 - r)F_c - P_c - (f - F_c) \geq 0 \iff f \leq (2 - r)F_c - P_c. \tag{2b}
\]

We derive the equilibrium outcomes for these two monopoly cases and include them in the Appendix.

**A Duopoly Competition**

Based on the above description, we construct a duopoly model in which the ISV and CSV compete in the same market. Firms will base their decisions on the comparison of their utility for both products, along with conditions (1a) through (2b) identified in the previous section.

When both vendors are present in the market, there will be a couple different forms of market segmentation based on the level of features chosen by the ISV and CSV. We discussed each of these cases below.

**Case I: when \( F_c < F_i \)**

In this case firms with \( f < F_i \) will adopt the ISV’s software if and only if both (1a) and the following holds:

\[
U_i = f - P_i - C_i \geq U_c = (1 - r)f - \frac{f}{F_c}P_c \iff f \leq \frac{F_c(P_i + C_i)}{rF_c + P_c} \tag{3a}
\]

As shown in Figure 3, firms in region C2 will choose the ISV’s product over the CSV’s due to higher utility.\(^7\) Entering region C3 (\( F_c \leq f \leq F_i \)), based on (1a) and (2a), it is obvious that starting from \( f = F_c \), the utility of using the ISV’s product is increasing (as \( F_i \) is not reached yet) while the utility for the CSV’s product is declining (because the benefit is capped at \( F_c \)). Hence firms in region C3 will buy the ISV’s software too.

For firms with \( f > F_i \), will choose the ISV’s product if and only if both (1b) and (3b) hold:

\[
U_i = F_i - P_i - C_i - (f - F_i) \geq U_c = (1 - r)F_c - P_c - (f - F_c) \iff F_i - P_i - C_i \geq (1 - r)F_c - P_c \tag{3b}
\]

It can be seen that (3b) is not a function of the firm’s actual feature requirements. From (1b) and (2b) we can find that the utility of using either the ISV or the CSV product decreases at the same rate beyond \( f > F_i \). Hence as long as the boundary firm located at \( f = F_i \) prefers ISV, all firms in region C4 will choose ISV over CSV. It follows from our previous discussion that \( u_i(F_i) > u_i(F_c) > u_c(F_c) > u_c(F_i) \). Hence all firms in region C4 will adopt the ISV’s software. Now going back the region C1, when the risk factor \( r \) is large enough, it is also possible for firms to forgo the consumption of the cloud software. Hence we conclude that:

**Observation 1:** If the risk of using the cloud software is large enough, and the CSV sets a software feature level \( F_c < F_i \) then no firms will choose the CSV’s product and the ISV will dominate the market for firms with feature requirements between \( \frac{F_i(P_i + C_i)}{rF_c + P_c} \) and \( 2F_i - P_i - C_i \).

**Case II: when \( F_c > F_i \)**

In this case, firms with \( f < F_i \) will choose the ISV’s product when (1a) and (3a) both hold, and firms with \( f > F_i \) will adopt the ISV’s software if and only if both (1b) and (3b) both hold.

\(^7\) Note that if (2a) holds, then \( \frac{F_c(P_i + C_i)}{rF_c + P_c} \geq P_i + C_i \) always holds.
As before, since at \( f=F_i \), both vendors set their software capabilities at the same level. When this happens, equation (3a) and (3b) will be rewritten as

\[
\begin{align*}
U_i &= f - P_i - C_i \geq U_c = (1-r)f - \frac{f}{F} P_c \iff f \geq \frac{F(P_i + C_i)}{rF + P_c} \quad (3a') \\
U_i &= f - P_i - C_i \geq U_c = (1-r)f - P_c - (f - F) \iff P_i + C_i \leq rF + P_c \quad (3b')
\end{align*}
\]

As before, since at \( f=F_i \), both vendors set their software capabilities at the same level. When this happens, equation (3a) and (3b) will be rewritten as

\[
\begin{align*}
U_i &= f - P_i - C_i \geq U_c = (1-r)f - \frac{f}{F} P_c \iff f \geq \frac{F(P_i + C_i)}{rF + P_c} \\
U_i &= f - P_i - C_i \geq U_c = (1-r)f - P_c - (f - F) \iff P_i + C_i \leq rF + P_c
\end{align*}
\]

As shown in Figure 4, similar to Case I, firms in region D2 will choose the ISV's product over the CSV's due to higher utility. Entering regions D3 (\( F_i \leq f \leq F_c \)) and D4 (\( f>F_c \)), following the same argument presented in Case I, all firms will adopt the CSV's software. Therefore, we can conclude that:

**Observation 2:** If the risk of using the cloud software is relatively small, and the CSV sets a software feature level \( F_c > F_i \), then the ISV will only cover the middle market segment between \( \frac{F_i(P_i + C_i)}{rF_i + P_c} \) and \( F_c \), giving CSV the potential to cover the rest of the market.

**Case III:** When \( F_c=F_i=F \)

A third case arises when both vendors set their software capabilities at the same level. When this happens, equation (3a) and (3b) will be rewritten as

\[
\begin{align*}
U_i &= f - P_i - C_i \geq U_c = (1-r)f - \frac{f}{F} P_c \iff f \geq \frac{F(P_i + C_i)}{rF + P_c} \\
U_i &= f - P_i - C_i \geq U_c = (1-r)f - P_c - (f - F) \iff P_i + C_i \leq rF + P_c
\end{align*}
\]

As before, since at \( f=F \), the utilities for both products decrease at the same rate beyond \( f=F \), it follows that the market will be split at \( f = \frac{F(P_i + C_i)}{rF_i + P_c} \). However, when the risk of using the cloud software is smaller than the cost of running the ISV's software in house (e.g., \( rF_i+C_i \)), then as can be seen from (3a') and (3b'), if the CSV lowers the price such that \( P_i + C_i > rF_i + P_c \), then the CSV has the power to take over the market to the right of \( f=F \). At the same time, since \( P_i + C_i > rF_i + P_c \), it follows that \( f = \frac{F(P_i + C_i)}{rF_i + P_c} > F \), which squeeze the ISV's market share to zero. In this case, both vendors enter into a Bertrand price equilibrium in which they will push the price to as low as it can get (i.e., \( P_i = C_i \) and \( P_c = rF \)). When this happens, since the ISV cannot lower its price further, it has the incentive to increase the software features level \( F \) to the highest possible level, increasing the difficulty for the CSV to lower its price. If the CSV does not respond by increasing its standard software feature, then the competition falls into the situation described in case I. As such, when both vendors set the same feature level, then both vendors will set the feature at the highest level, namely, \( F=1 \). Therefore, we can conclude that:

**Observation 3:** If both vendors set the same feature level such that \( F_c=F_i=F \), and when the risk of using the CSV's cloud software is smaller than the cost of running the ISV's software in house (e.g., \( rF_i+C_i \)), then at equilibrium both vendors will set the optimal feature at \( F=1 \) in order for this market outcome to be emerge as a stable equilibrium.

Based on the derivation of market segmentations in these three possible cases, we can formulate the maximization problem for both vendors in each of the three cases. Specifically, in Case I, both the ISV and CSV will set the feature level \( F_i \) and \( F_c \), and price \( P_i \) and \( P_c \) to maximize their respective profits in:
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\[
\begin{align*}
\text{Max} \ & \prod_i \left( F_i - P_i - C_i + \frac{F_i \left( P_i + C_i \right)}{r F_i + P_i} \right) N P_i - D_i F_i^2, \\
\text{Max} \ & \prod_c \left( F_c(P_c + C_c) \right) \frac{F_c}{F_c} N(P_c - C_c) - D_c F_c^2, \\
\end{align*}
\]

subject to the constraints specified in (1a), (1b), (2a) and (2b). Note that since the CSV adopts a linear pricing scheme, the revenue should be calculated as the integral of the price function \( \frac{F_c}{F_c} \) over the market segment \([0, \frac{F_c}{F_c}]\). Recall that CSV also incurs a cost for serving all of its customers on the cloud platform. Hence the total software operation cost is calculated as the number of firms CSV serves multiplied by the average cost per customer \( C_c \) (\( C_c < C_i \)). The last term in both equations \( D_i F_i^2 \) and \( D_c F_c^2 \) are both convex functions that captures the fact that software development costs increase exponentially as more features are added.

In Case II, the maximization problems becomes:

\[
\begin{align*}
\text{Max} \ & \prod_i \left( F_i \frac{F_i \left( P_i + C_i \right)}{r F_i + P_i} \right) N P_i - D_i F_i^2. \\
\text{Max} \ & \prod_c \left( F_c(P_c + C_c) \right) \frac{F_c}{F_c} N(P_c - C_c) + (2 - r) F_c - P_c - F_c \right) N(P_c - C_c) - D_c F_c^2. \\
\end{align*}
\]

And in Case III, since both vendors will set the software features at the highest level, the equilibrium can be derived by solving the following ISV’s and CSV’s maximization problems:

\[
\begin{align*}
\text{Max} \ & \prod_i \left( 1 - P_i - C_i \right) N P_i - D_i F_i^2, \\
\text{Max} \ & \prod_c \left( F_c \right) N(P_c - C_c) - D_c F_c^2, \\
\end{align*}
\]

Research plan and Expected Output

Given the maximization problems describe in (4a) through (6b), we can derive the equilibrium outcome by simultaneously solving both vendors’ maximization problems. It is possible that not all three equilibria are sustainable. Upon solving the equilibrium prices and software feature levels, we can compare the vendors’ profits and derive the stable Nash equilibrium for this duopoly competition.

Based on the results of the baseline duopoly, we plan to extend the static competition into a two-stage dynamic game, in which both vendors have the option to provide software product upgrade to their customers. As discussed in the introduction, ISV and CSV differ in the way they provide software upgrades. Due to the fact that on-premise software runs in-house and customers are located at different physical locations, the ISV has to undertake significant efforts to help its client perform the upgrade, which gives its customers a higher utility due to the tailed customization. On the contrary, the CSV can perform the upgrade on the cloud platform and all of its customers can enjoy the benefits of the upgrade and share the costs of the upgrade. We expect the equilibrium outcomes to differ from what we obtain in the baseline model. Drawing on these results, we will also conduct comparative statics and derive consumer surplus and social welfare under these two settings. Through these analyses, we will be able to examine how these different settings and the relative strength of the exogenous parameters such as the software operating cost and risk factor affect the equilibrium outcomes, and discuss the implications for software vendors, consumers, and the society.
REFERENCES


Appendix

**ISV Monopoly Equilibrium**

When the ISV is the monopolist, based on the market share illustrated in Figure 2, it will set the feature level $F_i$ and price $P_i$ to maximize its profits:

$$\max_{F_i, P_i} \prod_i = (2F_i - P_i - C_i - (P_i + C_i))NP_i - D_iF_i^2. \quad (0 \leq \beta \leq 1, \text{ and } 0 \leq F_i \leq 1) \quad (7)$$

where the first term in the profit function is revenue of the ISV, calculated as the market share of the ISV, multiplied by the total number of firms in the market, and the price they pay, and the second term is a cost of developing a standard software application with features $F_i$. Following the literature on software development, we assume a convex functional form to capture the fact that software development costs increase exponentially as more functions and features are added.

Taking derivatives with respect to the strategic variables $F_i$ and $P_i$ yields the first order conditions (FOCs):

$$\frac{\partial \Pi_i}{\partial F_i} = 2NP_i - 2D_iF_i = 0. \quad (8a)$$

$$\frac{\partial \Pi_i}{\partial P_i} = (F_i^2 - 2P_i - C_i - 2P_i)N = 0. \quad (8b)$$

Jointly solving (8a) and (8b) yields the equilibrium feature level and price:

$$F_i^* = \frac{Nc_i}{N - 2D_i} \quad \text{where } N > 2D_i. \quad (9a)$$

$$P_i^* = \frac{D_iC_i}{N - 2D_i} \quad \text{where } N > 2D_i. \quad (9b)$$

And the ISV’s profit is given by:

$$\Pi_i^* = \frac{2ND_i^2c_i^2}{(N - 2D_i)^2} \quad \text{where } N > 2D_i. \quad (9c)$$

**CSV Monopoly Equilibrium**

When CSV is the monopolist, it will set the feature level $F_c$ and price $P_c$ to maximize its profits:

$$\max_{F_c, P_c} \prod_c = \int_0^{F_c} \left( \frac{f}{F_c} N(P_c - C_c) + [(2 - r)F_c - P_c]N(P_c - C_c) - D_cF_c^2 \right). \quad (10)$$

The first term in (10) is the profit from customers with $f < F_c$. The term in the brackets are the market size of the firms whose software feature demand is greater than what the CSV offers ($f \geq F_c$). The parameter $c_c$ is the cost of serving these customers, and $D_cF_c^2$ is a scalar the cost of developing the cloud software platform with features $F_c$, which is assumed a convex functional form.

Taking derivatives with respect to the strategic variables $F_c$ and $P_c$ yields the first order conditions (FOCs):

$$\frac{\partial \Pi_c}{\partial F_c} = \frac{NP_c}{2} - N(1 - r)(P_c - C_c) - 2D_cF_c = 0. \quad (11a)$$

$$\frac{\partial \Pi_c}{\partial P_c} = \frac{NF_c}{2} + N(F_c - rF_c - P_c) + N(P_c - C_c) = 0. \quad (11b)$$

Jointly solving (11a) and (11b) yields the equilibrium feature level and price:

$$F_c^* = \frac{2NC_c(1 - 2r)}{N(9 - 12r + 4r^2) - 16D_c} \quad \text{where } r < \frac{1}{2}. \quad (12a)$$

$$P_c^* = \frac{2C_c[N(3 - 5r + 2r^2) - 4D_c]}{N(9 - 12r + 4r^2) - 16D_c} \quad \text{where } r < \frac{1}{2}. \quad (12b)$$

And the CSV’s profit is given by:

$$\Pi_c^* = \frac{32ND_c^2C_c^2[N(3 - 5r + 2r^2)(1 - 2r) - 8D_c(3 - 5r + 2r^2)]}{(N(9 - 12r + 4r^2) - 16D_c)^2} \quad \text{where } r < \frac{1}{2}. \quad (12c)$$