Abstract—A Gaussian mixture model (GMM) based algorithm is proposed for video reconstruction from temporally-compressed video measurements. The GMM is used to model spatio-temporal video patches, and the reconstruction can be efficiently computed based on analytic expressions. The GMM-based inversion method benefits from online adaptive learning and parallel computation. We demonstrate the efficacy of the proposed inversion method with videos reconstructed from simulated compressive video measurements, and from a real compressive video camera. We also use the GMM as a tool to investigate adaptive video compressive sensing, i.e., adaptive rate of temporal compression.

Index Terms—Compressive sensing, Gaussian mixture model, online learning, coded aperture

I. INTRODUCTION

High-speed video cameras play an important role in capturing fast motion, of interest in many applications, from science to athletics. However, measuring high-speed video often presents a challenge to camera design. So motivated, compressive sensing (CS) [1, 2] has recently been employed to capture the information in high-frame-rate video using low-frame-rate compressive measurements [3, 4, 5].

In this paper we develop a Gaussian mixture model (GMM) based inversion method for video measured with a CS camera, of the type considered in [3, 4, 5]. The inversion is probabilistic, providing a posterior distribution [6] for the reconstructed video. Compared with other probabilistic algorithms [7, 8], the proposed method is computationally attractive, as it yields analytic CS inversion; further, the approach is readily amenable to parallel computation. In addition to considering CS video reconstruction, we also consider adapting the temporal compression rate.

A. Previous Work

Significant progress in video CS has been made with a single-pixel camera [9, 10, 11]. Veeraraghavan et al. [12] achieved compressive temporal superresolution by exploiting Fourier sparsity. More recently, Hitomi et al. [3] developed a video CS camera featuring a Liquid-Crystal-on-Silicon (LCoS) device; a dictionary-based inversion algorithm is used to reconstruct frames from a single coded exposure photograph. Reddy et al. [4] developed a programmable compressive high-speed camera with similar hardware, but used a different projection matrix. Optical flow [13] is used to estimate the motion of the objects in the video, leading to a computationally expensive reconstruction algorithm. Optical flow was also employed in [14].

We have recently developed a prototype Coded Aperture Compressive Temporal Imaging (CACTI) camera [5], which employs fast mechanical translation of a coding element. In CACTI, temporal frames of the continuous video are modulated at a high speed, by a shifted version of a binary code, and the detector sums the coded frames over a temporal window $\Delta_t$. Compared with [3, 4], CACTI benefits from easy implementation, low power, and low cost.

In this paper we apply the proposed CS inversion algorithm to data associated with the CACTI camera. However, the basic model and inversion method may be applied to any of the compressive video cameras discussed above.

B. Gaussian Mixture Model

The GMM is used widely in various learning tasks, including classification and segmentation [15, 16], as well as image denoising, inpainting and deblurring [17, 18]. Recent work has demonstrated that the GMM is also a powerful tool for image reconstruction in still-image CS [17, 19].

In this paper we extend the GMM to compressive video. Specifically, when performing CS inversion, the video’s pixel volume is divided into a set of space-time patches. All pixels within a patch are assumed drawn (at once) from a GMM. The covariance matrix of each mixture component defines a linear subspace (like in principal component analysis), and therefore the GMM representation is closely related to the union-of-subspace model [20, 21]. Within the GMM construction, each patch of data is effectively assumed drawn from one of the subspaces, from among the union of subspaces. The posterior distribution for the underlying data associated with the compressive measurement of the patch is shown here to also be an (updated) GMM; thus, the estimated video is a weighted average across multiple of the subspaces in the union. The connection of the GMM to a union of subspaces links the proposed approach to dictionary-learning-based CS inversion [3]. The advantage of using the GMM over dictionary-learning-based approaches is, as demonstrated below, that the CS inversion is analytic (the posterior distribution on the underlying patch pixels is represented by an updated GMM, with analytic parameter update).

In previous dictionary learning [3] and GMM-based CS inversion [19], it was assumed that the dictionary/GMM was learned offline. This assumes that one has access to an appropriate set of training data. To address this limitation, we propose an online-updated GMM algorithm, that adapts the...
GMM parameters as the reconstruction proceeds. We demonstrate that online updates of the underlying GMM mitigates the need for training data, as after sufficient data are observed the data under test drives the GMM, not prior training data.

C. Measurement Adaptivity

Studies have shown that improved CS performance can be achieved when projection/measurement matrices are designed to adapt to the underlying signal of interest \[22, 23\]. All of this previous work has been developed for CS measurement of a single image. After exhaustive study of this matter, we found that within the constraints imposed by the form of the compressive video cameras discussed above (e.g., positive codes), there is only modest gain accrued in trying to optimize the coding mask, relative to just employing random design. Variations of the methods in \[22, 23\] were examined in detail, and this was determined to not be a fruitful research direction (for video reconstruction under realistic hardware design constraints).

However, a second opportunity for adaptivity concerns the details of the designed video CS cameras considered in \[3, 4, 5\]. In each of these cameras, the high-speed video frames are coded with a mask, and then the coded frames over time window \(\Delta_t\) are summed to constitute a single measurement. The appropriate value for \(\Delta_t\) may vary depending on what is happening in the scene, and therefore there is an opportunity for adaptivity. We consider such adaptivity in the context of the framework proposed here.

D. Paper Organization

The remainder of the paper is organized as follows. Section II introduces the CACTI compressive video camera that we will use to motivate our simulated experiments, and it will also be a source of real data. GMM-based CS inversion for video developed in Section III. Section IV considers adapting \(\Delta_t\) based on the inferred complexity of the scene under test. An extensive set of experimental results are presented in Section V and Section VI. provides conclusions and suggestions for future work.

II. CODED APERTURE COMPRESSIVE TEMPORAL IMAGING (CACTI)

The CACTI measurement framework is summarized in Figure 1 and it is this system to which all experiments below are applied. The proposed modeling and inversion framework may also be applied to data collected by related compressive video cameras \(3, 4\). The distinction between CACTI and the cameras in \(3, 4\) is manifested in how the coding process (middle row of Figure 1) is implemented. In \(3, 4\) a Liquid-Crystal-on-Silicon (LCoS) device is employed to achieve per-pixel modulation, which affords greater coding flexibility than the proposed approach, but at greater implementation complexity. In CACTI, a single binary code is employed; this code is mechanically shifted via a pizeoelectronic translator. The code was designed by randomly drawing 1/0 values at every pixel value, with a 0.5 probability of a 1. The 0.5 probability was found to give best performance. In numerical experiments, we considered the case for which each of the codes on the second row of Figure 1 were drawn uniformly at random (not translating a single mask), to examine the performance degradation manifested by requiring the translation of the same shifted code (as actually implemented in CACTI). In those experiments, it was found that the CACTI design yielded almost as good CS recovery performance as the totally random design (each frame coded with a uniquely drawn random code). Therefore, the CACTI design manifests significant ease and simplicity of implementation, with minimal degradation in performance.

To explain the measurement process mathematically, consider a two-dimensional datacube, with one spatial dimension, \(s\), and time \(t\). In the actual camera the datacube is three-dimensional, with two spatial dimensions. Let \(x(s, t)\) represent the continuous time-space datacube that we wish to measure. The function \(T(s)\) represents the fixed binary code, which is a function of space dimension \(s\); the spatial translation of the code is characterized by the function \(\bar{S}(t)\). The discrete measurement at space-time point \((\bar{s}, \bar{t})\) is represented as

\[
g(\bar{s}, \bar{t}) = \int \int x(s, t) T(s - \bar{s}) T(t - \bar{t}) \text{rect} \left( \frac{s - \bar{s}}{\Delta_s} \right) \text{rect} \left( \frac{t - \bar{t}}{\Delta_t} \right) ds dt,
\]

where \(\text{rect}[(s - \bar{s})/\Delta_s]\) is a rectangular window function that has unit amplitude over spatial support \(\Delta_s\) centered at \(\bar{s}\), and is zero elsewhere (and similarly for the temporal rect function). In this setup the spatial coding rate on the mask \(T(s)\) could be different from that of the sampling system, denoted by \(\Delta_s\). In the actual CACTI camera we consider below, the size of the pixels in the code \(T(s)\) are the same as \(\Delta_s\), and CS decomposition is performed on the scale of \(\Delta_s\). This implies that there is no spatial compression in the system, with compression manifested in time, as detailed next.

The goal is to perform approximate recovery of \(x(s, t)\) for \(t \in [\bar{t} - \Delta_t/2, \bar{t} + \Delta_t/2]\), based upon measuring \(g(\bar{s}, \bar{t})\). The recovered \(x(s, t)\) is discretized in space and time. The spatial sampling rate is \(\Delta_s\), with the discrete spatial samples centered at all spatial measurement points \(\bar{s}\). Concerning the temporal sampling rate of the recovered data cube, consider 

\[x(s, n\delta_t)\]
for δ = Δ/T, and n = 1, . . . , T; these correspond to the underlying datacube at T discrete points in time. When we convert the inversion problem into a linear matrix, the question concerns choice of T, which defines the number of frames to be estimated from a single compressive measurement that integrates over time Δ.

Recall that the mask/code in CACTI is moving continuously as a function of time, as dictated by s(t). Therefore, we must choose T large enough such that the mask is approximately fixed in space over time period δ (to justify our conversion to a matrix equation, as discussed next). On the other hand, if T is made too large, the number of unknowns to be recovered may become excessive. To clarify these points, we next express the inversion problem explicitly in matrix form; for that we move to the actual three-dimensional datacube of interest.

Let X ∈ RNx×Ny×T represent the underlying discretized datacube to be recovered; the spatial sampling rate, that defines N_x and N_y, is specified by the hardware sampling rate Δ_x. As discussed above, T is a parameter that may be set, which impacts the accuracy of the approximate linear-equation representation of the system. Toward that linear equation, let Y ∈ RNx×Ny represent the two-dimensional discrete CACTI measurement, corresponding to one measurement window Δ_y. The relationship between the pixels of X and those of Y is specified as

\[ Y_{i,j} = \left[ A_{i,j,1} A_{i,j,2} \cdots A_{i,j,T} \right] X_{i,j,1,1,i,j,2, \cdots, i,j,T}', \]

where ' denotes the transpose of a vector, Y_{i,j} is component (i, j) of Y and X_{i,j,n} is associated with the same spatial pixels (i, j), and temporal bin n. The components \( A_{i,j,n} \in \{0,1\} \), as dictated by the binary mask, and it is assumed that the mask is approximately fixed in space over time period δ = Δ/T, as discussed above. Therefore, within the approximations specified, each pixel in Y constitutes a weighted sum of the corresponding pixels in the T frames of X, where the weights are binary.

Anticipating the inversion algorithm to be considered in the next section, we partition Y into a set of patches, where each square patch is composed of P^2 contiguous pixels. We may consider all possible overlapping patches, and let \( y_m \) represent the m-th patch. For each m, we write \( y_m = \Phi_m x_m \) where \( y_m \in \mathbb{R}^{P^2} \), \( x_m \in \mathbb{R}^{P^2} \), and \( \Phi_m \in \{0,1\}^{P^2 \times P^2} \), where the pixels in two-dimensional \( (y_m) \) and three-dimensional \( (x_m) \) settings have been “vectorized.”

For each m, the objective is to recover \( x_m \) based upon the measured \( y_m \). Since the dimension of the item to be inferred is much larger than that of the associated measurement, this inversion is ill-posed unless constraints or prior knowledge is imposed with respect to \( x_m \). This is done in the context of a Gaussian mixture model (GMM).

III. GAUSSIAN MIXTURE MODEL FOR VIDEO RECONSTRUCTION

A. GMM-Based Inversion

Consider a total of M two-dimensional measurement patches \( \{y_m\}_{m=1, M} \), each of which has an associated three-dimensional space-time signal \( x_m \) that we wish to recover. Each of the inversions are performed independently, and therefore there is significant opportunity for parallel acceleration. The final estimate of any pixel in \( X \) is its average from all recovered patches in which it resides. Each measurement may be expressed as

\[ y_m = \Phi_m x_m + \epsilon_m, \quad \forall \ m = 1, \cdots, M, \]

where we now consider measurement noise, and/or model mismatch error \( \epsilon_m \in \mathbb{R}^{P^2} \).

In the analysis that follows, we assume that the GMM representation for \( x_m \) corresponds to a \( P^2\)-T-dimensional signal, where we recall that \( T \) temporal frames are recovered for each CS measurement. We may alternatively assume \( x_m \in \mathbb{R}^{P^2 \times T_0} \), where \( T_0 = nT \), for \( n > 1 \). In this case multiple temporal CS measurements are employed to recover the underlying \( x_m \).

We assume \( x_m \) is drawn from a GMM, as

\[ x_m \sim \sum_{k=1}^{K} \lambda_k N(x_m| \mu_k, \Sigma_k), \]

where \( \mu_k, \Sigma_k, \) and \( \lambda_k \) are the mean, covariance matrix, and weight of the \( k \)-th Gaussian component, respectively \( (\lambda_k > 0) \) and \( \sum_{k=1}^{K} \lambda_k = 1 \). These parameters can be estimated by the expectation-maximization (EM) [24] algorithm, based on training video patches. One may also employ more-sophisticated approaches, such as nonparametric Bayesian models [19], to estimate the GMM parameters, from which an appropriate number of mixture components \( K \) can be inferred. Here we simply set \( K \), and the EM algorithm is employed. The GMM in (2) constitutes our prior for the underlying \( x_m \).

Concerning the measurement model, it is assumed that \( \epsilon_m \sim N(0, R) \), where \( R \) is a \( P^2 \times P^2 \) covariance matrix, and therefore

\[ y_m|x_m \sim N(y_m| \Phi_m x_m, R), \]

Applying Bayes’ rule results in the posterior

\[ p(x_m|y_m) = \sum_{k=1}^{K} \tilde{\lambda}_k N(x_m| \tilde{\mu}_k, \tilde{\Sigma}_k), \]

with

\[ \tilde{\lambda}_k = \frac{\lambda_k N(y_m| \Phi_m \mu_k, R + \Phi_m \Sigma_k \Phi_m^T)}{\sum_{l=1}^{K} \lambda_l N(y_m| \Phi_m \mu_l, R + \Phi_m \Sigma_l \Phi_m^T)}, \]

\[ \tilde{\Sigma}_k = (\Phi_m^T R^{-1} \Phi_m + \Sigma_k)^{-1}, \]

\[ \tilde{\mu}_k = \tilde{\Sigma}_k (\Phi_m^T R^{-1} y_m + \Sigma_k^{-1} \mu_k), \]

which is also a GMM [19]. The analytic inversion expression [4] results in efficient reconstruction of every patch.

The posterior distribution on \( x_m \) is of interest for characterizing confidence in the inversion. We often are interested in reporting a single estimate of the underlying signal, and for this we may employ the conditional expectation:

\[ \mathbb{E}(x_m|y_m) = \int x_m p(x_m|y_m) dx_m = \sum_{k=1}^{K} \tilde{\lambda}_k \tilde{\mu}_k \overset{\text{def}}{=} \tilde{x}_m. \]

We term this implementation of the above framework offline GMM-based inversion, as all patches use the same offline-learned GMM parameters for the prior (which assumes appropriate training videos).
B. Online-Updated GMM for Video Reconstruction

Though the GMM-based inversion based on an offline-learned GMM effectively reconstructs CS-measured video, this implementation does not explicitly exploit temporal dependence among patches, and it assumes appropriate training data $D_0$ (analogous to previous dictionary-learning based inversion, in which the dictionary was learned offline [3]). The inversion algorithm recovers video in patches, and the temporal length of each patch is $T_0$, recalling that $T_0$ is a multiple of $T$. Let $D_0$ represent the patches in the training data, and $D_l$ represents the recovered patches with support within time window $(l-1)T_0 < t \leq lT_0$, for $l = 1, \ldots, L - 1$. Based on $\{D_0, D_1, \ldots, D_{L-1}\}$, we wish to update the GMM prior, for use when performing inversion of CS-measured patches within the time window $(L-1)T_0 < t \leq LT_0$. Clearly the amount of data grows with increasing $L$, and therefore we wish to prune the data used for GMM learning, with an emphasis on the most recent data.

Assume that $M_l$ represents the total number of video patches we wish to train the GMM with. We may define $\pi_l \in \{0, 1\}$ with $\sum_{l'=0}^{L-1} \pi_l = 1$, and sample $\{\pi_l | M_l\}$ patches from $D_l$, uniformly at random, where $\lfloor \cdot \rfloor$ represents the nearest integer. By making $\pi_l < \pi_l + 1$, the model emphasizes more recent data over later data, and eventually the original training data $D_0$ becomes irrelevant.

The parameters $\{\pi_l\}$ are here defined by setting $\xi_l = (l+1)^\kappa$ and $\pi_l = \xi_l / \sum_{l'=0}^{L-1} \xi_{l'}$, where $\kappa \in [0, \infty)$ controls the decay rate of influence on the adapted GMM. A similar strategy has been used in online latent Dirichlet allocation (LDA) [25] and online beta process factor analyzer (BPFA) [26].

There are many ways in which such adaptivity may be performed, and by which the sets $\{D_0, D_1, \ldots, D_{L-1}\}$ may be defined. For example, a given batch of data $D_l$ may consider patches within time windows that are larger than $T_0$. Further, rather than adapting the GMM continuously, which may be expensive, the GMM may be updated after periodically specified intervals.

C. Connections to Existing CS Inversion Methods

1) Dictionary Learning: Dictionary learning has achieved success in image CS reconstruction [7,27,28] and has been recently applied to invert compressive video measurements [3]. In these methods, an overcomplete dictionary $W \in \mathbb{R}^{d_c \times I}$ ($I > d_c$) is considered for representation of the signal $x \in \mathbb{R}^{d_c}$, with a sparse vector of dictionary coefficients $f \in \mathbb{R}^I$, i.e., $x = W f + \epsilon$. The dictionary $W$ is dataset-specific, and its superiority over universal bases such as the discrete cosine transform (DCT) and the wavelet transform has been demonstrated in various image and video applications [3,7,17,27,28]. State-of-the-art dictionary learning methods include K-SVD [27], method of optimal directions (MOD) [29] and beta process factor analysis (BPFA) [7], among others. Extending these dictionary learning approaches, block-structured dictionary learning (BSDL) has recently been considered. In BSDL, the signal is assumed to be drawn from a union of a small number of disjoint subspaces. The recent BSDL methods, such as Block BP [20], Block OMP [21], group LASSO [30], block sparse dictionary optimization (BODO) [31] have yielded favorable results over standard dictionary learning methods. It is worth noting that most existing dictionary learning methods have high computational cost, in particular when compared with the GMM learning and reconstruction procedure. This computational problem is exacerbated in BSDL, as the block structure of dictionaries needs to be determined additionally [31].

We now illustrate that the GMM-based inversion method is related to dictionary learning, BSDL in particular. Recall [7] that the reconstruction $\hat{x}$ yielded by the GMM-based inversion method is expressed by $\hat{x} = \sum_{k=1}^{K} \hat{\lambda}_k \mu_k$. For convenience, we rewrite the mode $\mu_k$ as $\mu_k = \mathbf{\Sigma}_k (\mathbf{\Phi}_R^T R^{-1} y + \mathbf{\Sigma}_k^{-1} \mu_k)$, $E_k \Lambda_k^2 [I + \Lambda_k^2 (\mathbf{\Phi}_E^T R^{-1} \mathbf{\Phi}_E \Lambda_k^2)]^{-1} \Lambda_k^2 (\mathbf{\Phi}_E^T)^T R^{-1} y \overset{\text{def}}{=} f_{\text{init}}$, $E_k \Lambda_k^2 [I + \Lambda_k^2 (\mathbf{\Phi}_E^T R^{-1} \mathbf{\Phi}_E \Lambda_k^2)]^{-1} \Lambda_k^2 \mathbf{\Phi}_E^T \mu_k \overset{\text{def}}{=} \eta_k$, $E_k f_k + \eta_k$, in which the matrix inversion lemma and eigenvalue decomposition $\mathbf{\Sigma}_k = E_k \Lambda_k E_k^T$ are used.

When rewriting (10) as $\mu_k - \eta_k = E_k f_k$, the residual of $\mu_k$ and $\eta_k$ can be represented by a sparse representation $f_k$ on the base $E_k$. Herein, $E_k$ contains eigenvectors of covariance matrix of the $k$th Gaussian component. According to (9), the representation $f_k$ can also be written as $f_k = [\Lambda_k^{-1} + (\mathbf{\Phi}_E^T R^{-1} \mathbf{\Phi}_E)]^{-1} (\mathbf{\Phi}_E^T R^{-1} y)$, and this implies that $f_k$ is estimated by the weighted $\ell_2$-norm regularized regression,

$$f_k = \arg \min_f \left\{ \| y - \mathbf{\Phi} E_k f \|^2_2 + \sigma^2 f^T \Lambda_k^{-1} f \right\},$$

assuming $R = \sigma^2 I, \sigma \in \mathbb{R}$. Since eigenvalues contained in $\Lambda_k$ are dominated by a few elements, the solution $f_k$ in (11) is a sparse vector. It is easy to verify the sparsity of $f_k$ in (9).

Based on the above analysis, the proposed GMM-based inversion method resembles dictionary learning. After applying the derivation in (8) - (10) for all $\mu_k, k = 1, \ldots, K$, the dictionary $W$ can be set as a union of $K$ bases, i.e., $W = E_1 \cup \cdots \cup E_K$. However, rather than treat $K$ bases $E_1, \ldots, E_K$ equally like BSDL, GMM-based inversion uses the probabilistic weights $\lambda_1, \ldots, \lambda_K$ to specify the importance of the $K$ bases. These weights are involved in the inversion procedure as indicated by $\lambda_1, \ldots, \lambda_K$ in (4)]. Note that $\lambda_1, \ldots, \lambda_K$ are generally not sparse, while $\lambda_1, \ldots, \lambda_K$ are often sparse. This implies that a patch to be reconstructed can be drawn from a probabilistic union of a few of $K$ subspaces; each subspace is spanned by the leading eigenvectors of the corresponding component’s covariance matrix. The GMM-based inversion method simultaneously fulfills the following three tasks: dictionary learning, determination of block structure, and sparse signal representation. The associated computation is very fast due to the analytic inversion in (4).
2) Piecewise Linear Estimator: Piecewise Linear Estimators (PLE) \cite{17 32} constitute another recent GMM-based inversion method used for still-image compressive sensing. By dividing the image signal into a collection of $P \times P$ patches, PLE also models the image patch to be reconstructed by a GMM. Given the compressed measurement of each patch, the image patch can be reconstructed based on a Wiener filter, that also enjoys an analytic form.

Though the basic idea of PLE is similar to that of the proposed method, there are the following important differences between PLE and the proposed method: (i) PLE constrains that the reconstructed patch is drawn from one of the $K$ Gaussian distributions. Equivalently, PLE assumes one-block sparsity in BSDL. This hard assignment is different from our soft assignment as shown in \cite{7}. In the experiments of video CS, we observe that more than one Gaussian components often contribute to reconstructing a video patch. (ii) PLE employs a different initialization procedure, by exploiting the particular properties of images. Specifically, as claimed in \cite{17 32}, a 2D image patch often has certain patterns, such as dominant spatial directions. Thus, in the initialization of PLE, $K - 1$ Gaussian components correspond to $K - 1$ angles that are uniformly sampled from 0 to $\pi$. The PCA spaces of these $K - 1$ components are learned based on the synthetic directional image patches. The $K^{th}$ Gaussian component employing the DCT as the basis captures the isotropic image patterns. This initialization strategy is not straightforward to extend to the video case, as it is not clear how to define the dominant direction for a 3D video patch, especially when $T_0$ is large. PLE is also similar to BSDL in the sense that all $K$ Gaussian components are treated equally. If one writes the GMM prior of PLE in the form \cite{2}, it leads to $\lambda_1 = \lambda_2 = \cdots = \lambda_K$, which is not the case for the proposed GMM-based inversion method. (iii) In PLE, the GMM’s $K$ components are separately tackled in the procedures of initialization, parameters adaptation, and inversion. By contrast, in the proposed GMM-based inversion method, the $K$ Gaussian components are always jointly involved in these three procedures.

D. Computational Complexity

We analyze the computational complexity of the proposed GMM-based inversion method. As the online GMM method presented in Section III-B iterates over different time windows, we focus on the computational complexity within one of them. Also, we describe the computational complexity in two different scenarios: standard computation and accelerated computation.

1) Standard Computation: For convenience, we rewrite parameters of (4) as below:

$$\Sigma_k = \Sigma_k - \Sigma_k \Phi^T (R + \Phi \Sigma_k \Phi^T)^{-1} \Phi \Sigma_k; \quad (12)$$

$$\hat{\mu}_k = \Sigma_k \Phi^T (R + \Phi \Sigma_k \Phi^T)^{-1} y - \Phi \hat{\mu}_k + \mu_k. \quad (13)$$

The proposed GMM-based inversion method includes two parts: EM model learning and patch-based inversion. The EM algorithm is called once for every $T_0$ frames reconstruction. Particularly, in offline learning, it is only called once for the entire video reconstruction. For the part of patch-based inversion, the analytic form exists as shown in \cite{4}, and the main cost lies in matrix inversion for computing $A$ in \cite{5, 12} and \cite{13}, whose computational cost is $d_y^2 + d_x^2 \approx d_y^2$ using Cholesky factorization \cite{33}. Here, we assume the dimensions of $y$ and $x$ are $d_y$ and $d_x$ respectively. Note that the $\Phi$ involved in matrix multiplication can be computed very fast, since every column of $\Phi$ has at most only one nonzero element. A summary of the computational costs for standard computation is listed in Scenario 1 of Table I.

Note that when the covariance matrix $R$ and projection matrix $\Phi$ are translation-invariant with respect to patches, $A$ can be pre-computed and pre-stored. As a result, in “standard computation” section of Table I the first row in the “Patch-dependent” part are moved to the “Patch-independent” part and the total computational cost in the “Patch-dependent” part reduces from $O(K d_y^3 / 3 + K d_y^2 + K d_y d_x)$ to $O(K d_y^2 + K d_y d_x)$, which saves a lot of computation and memory.

2) Accelerated Computation: Using the derivation in \cite{8}, we can rewrite $\Sigma_k$ and $\hat{\mu}_k$ as

$$\Sigma_k = E_k A_k^R [I + A_k^R (\Phi E_k)^T R^{-1} \Phi E_k A_k^R]^{-1} A_k^R, \quad (14)$$

$$\hat{\mu}_k = E_k A_k^R [I + A_k^R (\Phi E_k)^T R^{-1} \Phi E_k A_k^R]^{-1} A_k^R (\Phi E_k)^T R^{-1} y \quad \equiv_{G}$$

$$+ E_k A_k^R [I + A_k^R (\Phi E_k)^T R^{-1} \Phi E_k A_k^R]^{-1} A_k^R \mu_k. \quad (15)$$

For convenience, denote the number of significant nonzeros eigenvalues in $\Lambda_k$ by $h_k$ ($h_k \ll d_x, d_y$).

As before, we categorize the computations into patch-independent and patch-dependant. Besides the EM algorithm, the patch-independent part also includes the eigen-decomposition operation for all $K$ Gaussian components,
\[ \Sigma_k = E_k A_k E_k^T \quad \forall k. \] In the patch-dependant part, though computations in \([5], \([14]\) and \([15]\) involve matrix inversion \(G\), the associated computational cost is significantly reduced as \(h_k\) (the number of significant eigenvalues in \(A_k\)) is much smaller than \(d_x\) and \(d_y\). A summary of the computational costs for accelerated computation is listed in Scenario 2 of Table [1]. Similar to standard computation, when \(R\) and \(\Phi\) are translation-invariant with respect to patches, \(G\) can be pre-computed and pre-stored. Consequently, the total computational cost in the “Patch-dependant” part in Scenario 2 reduces from \(O(K h_k^3 / 3 + K h_k d_x + K h_k d_y)\) to \(O(K h_k d_x + K h_k d_y)\).

IV. ADAPTIVE INTEGRATION WINDOW

With compressive video cameras of the type in \([3, 4, 5]\), it is relatively straightforward to adjust the integration window \(\Delta_t\). For a desired reconstruction fidelity, the maximum allowable \(\Delta_t\) is related to the complexity (e.g., velocity) in the associated motion within the scene. For periods of time with minimal temporal variation, \(\Delta_t\) may be made large, as there will not be substantial blur manifested in summing many frames over the window \(\Delta_t\). However, when the motion within the scene is substantial, it is desirable to make \(\Delta_t\) smaller. The value of \(\Delta_t\) defines the degree of compression, and therefore it is desirable to adapt \(\Delta_t\) based upon the complexity of the scene under test (i.e., make \(\Delta_t\) as large as possible, within a desired reconstruction-quality constraint).

Recall that \(T\) frames are recovered within time window \(\Delta_t\). We assume sampling rate \(\delta_t\) between consecutive frames, and therefore \(\Delta_t = T \delta_t\). Within the CACTI system, \(\delta_t\) is only employed within the software-based inversion for the underlying video, and therefore variable \(\delta_t\) may be examined within the context of inversion, given compressive video measurements. The mask in CACTI moves continuously, and therefore \(\delta_t\) is set with the goal of making the mathematics of the inversion accurate (the mask is approximately fixed in space over time \(\delta_t\)); by contrast, in \([3, 4]\) \(\delta_t\) is set by the hardware. We here assume \(\delta_t\) is fixed, and then by varying the integration window \(\Delta_t\) we also change the number of recovered frames, \(T = \Delta_t / \delta_t\), where \(T\) is assumed to be an integer.

Using the video CS camera of interest, here CACTI, we may constitute simulated compressive measurements, for variable \(\Delta_t\). Since the true underlying video is known, we may compute the quality of the reconstruction (e.g., peak signal to noise ratio (PSNR)) as a function of \(\Delta_t\). With a specified desired reconstruction quality, for each training video example we may specify the maximum associated \(\Delta_t\). Simulated data and associated reconstructions may be considered for many types of scenes, and for each a range of \(\Delta_t\) are considered; for each case the maximum \(\Delta_t\) is selected within the fidelity criterion.

After this process, we have access to a set of video clips, of variable length \(\Delta_t\), and compressive measurements over that window yielded the specified reconstruction fidelity. We would like to use this training data to design a means by which given compressive video, we may assign the appropriate compressive integration window \(\Delta_t\). Toward that end, we extract features from these video clips, and learn a mapping between the features and the associated appropriate compressive measurement window \(\Delta_t\). Once such a function is so learned, one may compute feature vectors “on the fly” based upon recent recovered video (after CS inversion), and map this feature vector to an associated updated integration window \(\Delta_t\).

We estimate the degree of motion within each video clip using the block-matching method, which has been employed in a variety of video codes ranging from MPEG1/H.261 to MPEG4/H.263 \([35, 36]\); the corresponding hardware has been well developed, and hence it may be done quickly in practice. Within each video clip we compute the maximum velocity within the scene. We then employ a standard Support Vector Machine (SVM) \([37]\) to map this feature (velocity) to \(\Delta_t\). To simplify the analysis, we consider a library of \(Q\) possible values for \(\Delta_t\), and the SVM does a multi-class mapping from the maximum velocity to the associated \(\Delta_t\) within the \(Q\)-dimensional library. The use of a finite library of values of \(\Delta_t\) is expected to simplify practical implementation, and it also simplifies inversion, as GMM signal models are only required for this finite set of \(Q\) integration windows \(\Delta_t\).

Within the experiments, the maximum velocity is computed based on the previous time window of length \(\Delta_t\), and this is then used to define the \(\Delta_t\) in the next time window of compressive measurements. Of course, in practice, the rate at which \(\Delta_t\) is adjusted is defined by hardware considerations, and the speed with which a method like the SVM can define an update for \(\Delta_t\).

In the above procedure, we define \(\Delta_t\) in terms of the maximum velocity anywhere in the video clip. However, in many situations, the moving objects within the scene are localized. This suggest that the value of \(\Delta_t\) may vary as a function of spatial subregion within the scene. This can be done in principle, but it complicates camera implementation, and therefore it is not considered here.

V. EXPERIMENTAL RESULTS

We demonstrate the proposed reconstruction method on a variety of scenes and present results in comparison with current leading reconstruction methods. All associated videos in the experiments can be accessed at http://people.duke.edu/~jj118/CACTI_GMM.htm.

A. Experimental Settings

For the proposed GMM-based inversion method, each coded measurement of size \(N_x \times N_y\) is partitioned into a collection of overlapping patches of size \(P \times P\), by horizontally and vertically sliding a \(P \times P\) block for \(v\) pixles, \(v \in [1, 2 \ldots, P]\). Accordingly, the number of patches ranges from \((N_x - P + 1) \times (N_y - P + 1)\) to \((N_x / P) \times (N_y / P)\) as \(v\) changes from 1 to \(P\). To balance simplicity and accuracy, we set \(v = P/2\) and \(P = 8\) throughout all experiments.

Regarding the measurement model, covariance matrix \(R\) for Gaussian distribution \(p(\epsilon) = N(0, R)\) is assumed to be a scaled identity matrix \(R = \sigma^2 I\) with \(\sigma = 10^{-3}\). In the GMM, the number of components \(K\) is simply fixed to 20 (although in practice this is an upper bound, and fewer than
For synthetic data, we use PSNR to evaluate the quality of every reconstructed frame, and finally the average of PSNR over all video frames is taken as the evaluation metric for video reconstruction. Mean square error (MSE) has also been used in evaluation, and it reflects similar patterns as PSNR in our experiments. Hence, the results on MSE are not reported. For real data, where ground truth is not available, the (subjective) visual quality is used to evaluate reconstruction.

B. Comparison with Other Methods

The proposed GMM-based inversion methods are compared with the following two leading reconstruction methods:

- Generalized Alternating Projection (GAP) [39] GAP is a generalized alternating projection method that solves the minimization problem:

\[
\min_f \|f\|_{\ell^2_1} \quad \text{s.t.} \quad HWf = \text{vec}(Y),
\]

where \( \|f\|_{\ell^2_1} = \sum_{k=1}^s \beta_k \sqrt{\sum_{i \in \mathcal{G}_k} f_i^2} \), vec(·) vectorizes a matrix inside (·), \( H \) is CACTI-specified projection matrix expressed as \( H = [H_1, H_2, \ldots, H_T] \) with \( H_t = \text{diag}(A_{1,t}, A_{2,t}, \ldots, A_{N_x,N_y,t}) \), \( t = 1, \ldots, T \), and \( W \) is the domain transform matrix that is imposed on the video signal \( X \) to achieve the sparse representation by \( f \), i.e., \( X = Wf \). In (16), \( f \) is partitioned into \( s \) non-overlapping groups \( \{G_1, G_2, \ldots, G_s\} \), and each group \( G_k \) is associated with a constant weight \( \beta_k \), \( k = 1, \ldots, s \). In the experiments, DCT and wavelet transform are used to yield sparse representation for \( X \), and the default settings on \( \beta_k, G_k \), \( \forall k \) are employed.

- Two-step iterative shrinkage/thresholding (TwIST) [38] TwIST solves the unconstrained optimization problem:

\[
\min_X \|\text{vec}(Y) - H\text{vec}(X)\|^2 + \tau \Omega(X) \quad (17)
\]

where \( \Omega(X) \) is the regularizer and \( \tau \) is the parameter to balance the \( \ell_2 \)-loss term and the regularizer term. Following previous CS inversion applied to still images and hyperspectral images [40], the total variation (TV) is used as the regularizer:

\[
\Omega(X) = \sum_{n=1}^T \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sqrt{(X_{i+1,j,n} - X_{i,j,n})^2 + (X_{i,j+1,n} - X_{i,j,n})^2}.
\]

The \( \tau \) is tuned in the range \([10^{-4}, 1]\). In [38], the developed two-step iterative shrinkage/thresholding optimization approach is used to solve (17).

C. Results on Synthetic Data: Unrelated Training Data

![Fig. 3. PSNR comparison of the offline and online GMM, TwIST [38], and GAP [39] for the traffic dataset (\( T = 8 \)). The corresponding reconstructed frames are displayed in Fig. 4.](image)

![Fig. 4. Reconstructed frames 89 to 96 for the traffic dataset (\( T = 8 \)) by TwIST, GAP and the online-updated GMM methods.](image)

The source code of TwIST may be downloaded from http://www.lx.it.pt/~bioucas/code.htm.
simulated by a random binary matrix with elements drawn from Bernoulli(0.5). The measurement is constituted by summing $T$ such coded frames. Note that in these simulations the mask is exactly fixed for each of these $T$ time points, while in the real experiments this is an approximation. A zero-mean white noise with standard deviation $\sigma = 10^{-3}$ is added to the simulated measurement, to represent model mismatch in the actual (real) data.

We synthesize three videos for reconstruction. They contain scenes denoted traffic, couple, and train. These three synthetic videos have different characteristics. The traffic and train is dominated by a moving foreground, while the couple video has more of a fixed background. The traffic video contains multiple moving objects, while the train video contains one moving object. All synthetic videos are of size $256 \times 256 \times 96$, and have temporal compression ratio $T = 8$.

For the proposed GMM-based inversion method, three-dimensional video patches are of size $8 \times 8 \times 24$, i.e., $T_0 = 3T$, ...

Fig. 5. (a). PSNR comparison of the offline and online GMM, TwIST, and GAP for the couple dataset ($T = 8$). (b). Reconstructed frames 49 to 56 by the online-updated GMM method.

Fig. 6. PSNR comparison of the offline and online GMM, TwIST, and GAP for the train dataset ($T = 8$).

Fig. 7. Reconstruction frames 17-24 (a), 65-72 (c) and their corresponding “error bars” (b,d) for the traffic dataset. Fig. 5 shows the corresponding PSNR values frame by frame.

Fig. 8. Illustration of the online-updated GMM priors for the time integration windows $(l-1)T_0 < t \leq lT_0$, $\forall l = 1, 4$. In each subfigure, the upper part shows all Gaussian components in 20 rows and each Gaussian component is shown in 24 ($T_0$) image patches size of $8 \times 8 (P \times P)$, and the lower part shows the weights of all Gaussian components.
and offline-training data are four arbitrary (distinct) videos, which contain the scenes of ducks, fountain, birds, and windbell (see example frames in Fig. 2), respectively. Note these training videos are all unrelated to the videos to be reconstructed.

1) Reconstruction Results: We plot PSNR curves to evaluate the reconstruction performance, and also display the reconstructed video of selected video frames. These results are shown in Figs. 3 for the data traffic, couple and train, respectively. Due to the space limitations, reconstructed frames by GAP and TwIIST for the couple and reconstructed frames by all methods for train are not shown here. Interested readers may refer to our aforementioned website for all the videos and frames. From the PSNR results shown in these figures, the online-updated GMM consistently performs best among all methods for these three synthetic datasets. Importantly, its PSNR is generally increasing as more frames are reconstructed. This vividly illustrates the merit of online learning.

The GAP algorithm generally performs as a runner-up among all methods, and TwIIST performs as well as the offline-learned GMM method. Visual quality evaluations on the reconstructed frames are consistent with the PSNR results. Specifically, from Fig. 4, the reconstructed frames by TwIIST has the bold artifacts among both foreground and background, while reconstructed frames by GAP looks better but with more blurring. The online-updated GMM can generally reconstruct the frames with sharper scenes than GAP and TwIIST (notice the small moving vehicles).

In addition, compared with GAP and TwIIST, the proposed GMM method can provide richer probabilistic information, such as confidence of reconstruction, or “error bars”. Specifically, the “error bar” is defined as the square root of variance of the estimator. Figure 7 shows the error bars of reconstruction provided by the online-updated GMM method as the variance of the estimator. Denoted by $\hat{\mu}_k$ and $\hat{\Sigma}_k$ respectively.

where $p(x_i|y_i)$ and $E(x_i|y_i)$ are given by (4) and (7), respectively. The diagonal elements of $\text{Cov}(x_i|y_i)$ are taken as the variance of the estimator. Figure 7 shows the error bars of reconstruction provided by the online-updated GMM method for the traffic dataset, at frames 17-24 and 65-72. From the figure, frames 65-72 not only have the better visual quality than frames 17-24, but also have lower error bars. This implies that as the inversion algorithm proceeds, the confidence of reconstruction increases. This again validates the efficacy of the online-updated GMM method. In addition, at each reconstructed frame, the foreground often corresponds to high “error bar” relative to the background. This is reasonable, as the foreground often has lower temporal redundancy.

2) Investigation on the online-updated GMM: In the online-updated GMM, the parameters of the GMM are updated once within every time integration window $(l - 1)T_0 < t \leq lT_0$ ($\forall l = 1, 2, \ldots$). Figure 8 demonstrates the updated priors at two integration windows. From the figure, it is clear that the patterns of the updated GMM components and the component weights are different after updating. For every GMM component, the pattern changes smoothly from the first frame to the last frame. This reflects that the GMM components in the prior contain the continuous motion information. This is crucial to successfully reconstruct foreground in a video.

Figure 9 illustrates the posterior distribution in the GMM-based inversion; an arbitrary patch in the traffic dataset is taken for an example. From Fig. 9(b), the GMM weights are very sparse, and only 2 of 20 Gaussian components have significant weights. However, the weights are not dominated by a single element. This justifies the soft assignment implied in the conditional expectation (7). From Fig. 9(a), the GMM components in the posterior also contain the smooth motion information. This is consistent with what we observed in the GMM prior.

D. Results on Synthetic Data: Related Training Data

We now evaluate the three reconstruction methods on two challenging tasks: high compression ratio and complicated motion. The first video is the same traffic dataset used above, but with higher compression ratio $T = 36$. The second video contains the NBA game scene, with $T = 8$, where a player has complicated movement during his dunk. These two videos are of size $256 \times 256 \times 216$ and $256 \times 256 \times 96$, respectively. For the proposed GMM method, we employ related data to train the GMM. Specifically, for the traffic video, the related training data are traffic videos having the same type of background but different foreground (moving vehicle). For the NBA game video, the related training data are videos containing similar NBA games (they generally do not contain the same background). It is worth noting that

\footnote{The four training videos can be downloaded from http://projects.cwi.nl/dyntex/database_pro.html}
utilizing related training data is often reasonable, since in some cases, like surveillance in the highway, we can first record some uncompressed videos and then use these videos as the training data to reconstruct the upcoming compressed videos. As the related training data are with high visual quality and strong relevance, the trained GMM is deemed to be good enough such that the online-updated GMM is not necessary. This has been verified in our extensive experiments. Thus, we adopt the offline-trained GMM to directly reconstruct the above two challenging videos; three-dimensional video patches of size $8 \times 8 \times T$ are used.

Figure 10 plots the PSNR curves against frame for the GMM, GAP and TwIST methods, for the highly-compressed traffic data, and displays the reconstructed frames 37 to 72 (corresponding to the second compressive measurement) by the proposed method. The reconstructed frames by GAP and TwIST are not shown here due to space limitations. From the figure, the proposed GMM method performs significantly better than GAP and TwIST in terms of PSNR and visual quality (please refer to our website). As far as the proposed method is concerned, its average PSNR value decreases 1.5 dB compared with the results in the case of $T = 8$ (shown in Fig. 3), while the reconstructed frames for $T = 36$ still maintain nearly comparable visual quality as those with $T = 8$. Note that GAP and TwIST do not utilize training data at all, and their generality is therefore undermined in this specific video challenge. On the other hand, the GMM results could be improved even further via online adaptivity to the video under test.

Figure 11 plots PSNR for the GMM, GAP and TwIST methods, for the NBA game dataset, and also display the reconstructed frames 81 to 88 (corresponding to the eleventh measurement) by the proposed method. From the figure, the proposed GMM method improves upon GAP and TwIST by 2.40 dB and 2.87 dB on average, respectively. Importantly, all three methods have PSNRs that decrease at frames 70 to 80, which correspond to the moment of the dunk. However, the proposed method still performs better than GAP and TwIST in this period. From the reconstructed frames, the proposed method can provide high visual quality (of course, this is subjective). Compared with the reconstructed frames by GAP and TwIST (please refer to our website), the proposed method maintains the highest fidelity in the foreground (the regions around the ball and the player). Note that defocusing exists in the ground truth (notice the audience in the video) and offline training data. As a consequence, the blur among audiences exists in the reconstructed frames for all reconstruction methods.

E. Results on Real CACTI Data

We now present results to demonstrate the efficacy of the proposed method on real data acquired by the CACTI camera we have built. This is a significant contribution, as the GMM-based inversion method has not been applied to reconstruct the videos for real data so far. As an effective reconstruction approach, in the following, we show the proposed method outperforms the GAP and TwIST methods for three real
datasets: chopper wheel, eye blink and hand lens. The chopper wheel dataset contains the scene that a letter “D” is placed at the edge of a chopper wheel rotating in front of the camera at an angular velocity of 15 blades per second. The eye blink dataset contains the scene of an eye blink from closed to open. The hand lens dataset contains the scene of an optical lens falling in front of a hand. These three videos are composed of 6, 8 and 5 compressed measurements, respectively. The CACTI camera captures data at 30 frames per second (fps). The integration window $\Delta t$ for each measurement is 33ms. The number of reconstructed frames from a single compressive measurement is $T = 14$, so each frame corresponds to $\delta_t = 2.36$ ms (420 fps). The measurement matrix $\Phi$ is provided by the details of the CACTI hardware.

The reconstruction results of these three datasets using the online-updated GMM method are displayed in Fig. 12 along with the TwIST and GAP results. The reconstructed frames by the three methods correspond to the sixth, fifth and third measurements for the chopper wheel, hand lens and eye blink datasets, respectively. Due to the difficulty of regenerating the temporal motion (the motion only lasts around 0.15s in real life), the reconstructions on these three datasets are evaluated solely by visual quality. Specifically, for chopper wheel, the online-updated GMM provides the clearest character “D” among the three methods. The character “D” exhibits ghosting due to ambiguities within the reconstructed solutions. While this ghosting effect exists for all three methods, the proposed method exhibits little ghosting and best preserves the video clarity. For eye blink, all three results can successfully show the process of the eye blink, from closed to open, while the proposed method appears to provide more complete process of the eye blink. For hand lens, all three methods can recover the falling process of the lens. TwIST does have clearer background than the other two results. However, the artifact in the simulation results now can also be observed from the red-box marked in frame 1. Only the GMM method can provide the unbroken edge (the small hole at right-bottom inside the red-box) of the lens. GAP still has the obvious blurring effect, which can be noticed from the red-box marked in frame 14.

F. Adaptive Integrative Window

We evaluate the proposed adaptive integration window $\Delta t$ on the simulated traffic data. Assuming $\delta_t$ is fixed, the adaptive integration window $\Delta t$ is equivalent to adapting the number of encoded frames $T$ in a measurement. The offline-trained GMM is used and the video patch is of size $8 \times 8 \times 12$. We begin the experiment with $T = 6$. As time progresses, the proposed method varies $T$ according to the estimated motion within each integration window. We artificially change the speed of the foreground during the 168 frames comprising

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Fig. 12. Left: the raw measurements of chopper wheel, eye blink and hand lens dataset ($T = 14$) acquired from CACTI. Right: the reconstructed frames from the raw measurement in the left by the online-updated GMM (a), TwIST (b) and GAP (c).
this video to evaluate the fidelity of the proposed method. Frames 1-60 (Fig. 13(b)) and 121-168 (Fig. 13(d)) play at the originally-captured frame rate, while the scene is frozen between frames 61-120 (Fig. 13(c)).

The proposed GMM updates \( T \) every 12 reconstructed frames (Fig. 13), as the same size of video patch is constantly used. From Fig. 13 we observe that the proposed method can effectively adapt \( T \) based on the estimated motion velocity in the reconstructed video. Specifically, the algorithm holds \( T \) at the initial value 6 for the first 72 frames, before increasing \( T \) to 12 when the scene motion stops. After the scene recovers to play at the originally-captured frame rate, \( T \) drops from 12 to 6. There are 12-frame delay of adapting \( T \) when the velocity is changed. This is reasonable, as the proposed method uses the reconstructed frames in the last \( T \) frames to predict the \( T \) for the current \( T \) frames as mentioned in Section IV.

The average \( T \) over all frames in the proposed method is 8.14. We also perform the evaluation of the reconstruction without adaptive \( T \), i.e., \( T = 8 \). The average PSNR values for the adaptive \( T \) and the fixed \( T \) are 26.851dB and 24.285dB, respectively.

![Graph showing PSNR values for different methods](image)

**Fig. 13.** Demonstration of adaptive temporal compression ratio on synthetic traffic data: (a) the plots of PSNR, adaptive CS ratio \( T \), estimated velocity from original videos and reconstructed videos against frame; (b,c,d) three typical measurements corresponding to different vehicle speeds.

### G. Computation Time

The proposed method and alternative methods are implemented in Matlab 2012(b), and the experiments are conducted on a PC with an Intel i5-2500 CPU running at 3.30 GHz and with 16GB RAM. The chopper wheel dataset is used to compare the running times of all three methods. Specifically, we record the running time of reconstruction from one measurement (i.e., 3936 overlapping patches) by each method. It is worth noting that the GMM-based inversion method can reconstruct all the patches in parallel. Therefore, it is reasonable to record the time of reconstructing a single patch for the proposed method, and the time is around 0.08 second.\(^5\) The time spent by TwIST and GAP in one measurement reconstruction are 42 and 2 seconds, respectively. Apparently, the proposed method is much faster than TwIST and GAP if it is implemented in parallel. As detailed in Section III-D, the computational cost of the proposed method can be further reduced if the projection matrix \( \Phi \) and covariance matrix \( \Sigma \) are translation-invariant with respect to the patches.

### VI. Conclusions

A GMM-based inversion algorithm is proposed to reconstruct high-speed videos from temporally compressed measurements. Extensive experimental results show that the proposed GMM-based inversion method can reconstruct video frames with greater fidelity than other methods, on both synthetic and real data. The proposed inversion algorithm enjoys the following advantages: i) **Online-updated GMM** improves the reconstructed video quality. Following this, the reconstruction becomes less-dependent on the offline training videos and more robust to different scenes and motions. Re-constructions of both real and synthetic coded videos show significant improvement for the online-updated GMM method. We found that for simple motions, the online-updated GMM method with even unrelated training videos can yield good reconstruction results. On the other hand, for fast and complicated motions, the offline-learned GMM with related training videos can yield promising results. In this case, the online-updated GMM is often unnecessary. ii) **Parallel reconstruction** using a GPU is applicable since the video is reconstructed patch by patch, and each of the inversions is performed independently. iii) The **analytic inversion expression results in efficient reconstruction of every patch.** iv) Confidence of reconstruction on every video pixel is provided by the proposed method’s probabilistic information “error-bars”.

Another contribution of this paper is the adaptive integration window. We utilize the reconstruction information of the proposed inversion method and learn a multiclass classifier to adapt \( T \) assuming fixed \( \delta_t \). Simulation results demonstrate that our method can adaptively change \( T \) subject to the motion of the objects in the scene while maintaining high-quality reconstruction performance. Future work lies in the implementation of adaptive integration window to the real CACTI system.

The GMM-based inversion algorithm developed in this paper can be straightforwardly extended to invert the hyperspectral datacubes from compressive measurements as discussed in [40]. Compared with the BPFA algorithm developed therein, the GMM-based inversion method enjoys fast reconstruction. Finally, this algorithm’s low order of complexity lends itself to future higher-dimensional imagery; in particular, we seek to extend the proposed inversion method to reconstruct 4D spatio-spectral-temporal data volumes from 2D compressed measurements in the near future.

### REFERENCES


