

The Standard Model from Stable Intersecting Brane World Orbifolds

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Abstract

We analyze the perturbative stability of non-supersymmetric intersecting brane world models on tori. Besides the dilaton tadpole, a dynamical instability in the complex structure moduli space occurs at string disc level, which drives the background geometry to a degenerate limit. We show that in certain orbifold models this latter instability is absent as the relevant moduli are frozen. We construct explicit examples of such orbifold intersecting brane world models and discuss the phenomenological implications of a three generation Standard Model which descends naturally from an $SU(5)$ GUT theory. It turns out that various phenomenological issues require the string scale to be at least of the order of the GUT scale. As a major difference compared to the Standard Model, some of the Yukawa couplings are excluded so that the standard electroweak Higgs mechanism with a fundamental Higgs scalar is not realized in this set-up.

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1. Introduction

During the last years string theory has provided a lot of new insights into fundamental issues of theoretical physics, such as the relation between gauge theories and gravity or geometry, the quantum nature of black holes and the appearance of non-commutative space-time structures. Also concerning more phenomenological questions strings proved themselves to be rather fruitful, perhaps most notably in the context of string compactifications with large extra dimensions [1,2] or localized gravity on a four-dimensional domain wall [3,4]. D-branes and non-perturbative duality symmetries always played a key role in all these developments. Nevertheless, still it is a great challenge to derive the observed physics of the Standard Model of particle physics directly from strings.

Recently a class of string compactifications was investigated which comes relatively close to the goal of obtaining just the Standard Model from strings. These models are given by type I string compactifications on a six-dimensional torus T^6 with D9-branes where internal background gauge fluxes on the branes are turned on [5-12].¹ Thus, at tree level supersymmetry is only broken on the D-branes with the bulk still preserving some supersymmetry [20-23]. Turning on magnetic flux has the effect that the coordinates of the internal torus become non-commutative. In a T-dual picture one is dealing with D6-branes which wrap 3-cycles of the dual torus and intersect each other at certain angles, determined by the original gauge fluxes. In this way it was possible to construct string models with three generations of quarks and leptons and Standard Model gauge group $SU(3) \times SU(2)_L \times U(1)_Y$, where supersymmetry is broken on the branes by the gauge fluxes or, in the dual picture, by the different intersection angles.

Let us recall in slightly more detail the main features of these type I string models. Following the old ideas of [24] and [25] it was first described in [5] in a pure stringy language how type I compactifications with background fluxes or intersecting branes lead to a reduction of the gauge group, to chiral fermions and to broken supersymmetry on the branes. A nice geometrical feature of such models is that the number of chiral fermions which are localized at the intersection points of the D6-branes is simply determined by the corresponding topological intersection number of the branes. In this way a model with

¹ For alternative compactifications with D-branes in type I or type II string theory see [13-17]; a recent discussion of heterotic string compactifications with background gauge fluxes and their relation to type II compactifications with internal H-fluxes can be found in [18]. Non-supersymmetric string models with background RR fluxes were discussed in [19].

four generations of quarks and leptons and a Standard Model gauge group was obtained in [5]. Later it was shown in [11] how odd numbers of generations arise, in particular three, if one adds to the gauge fluxes also a quantized background NSNS B -field [26,27,28,8]. In the T-dual picture the torus is then no longer rectangular but tilted by a discrete angle. In [9,10] additional type II models with backgrounds of the form $T^{2d} \times (T^{6-2d}/\mathbb{Z}_N)$ were considered where the D(3+d)-branes wrap only d -cycles of the first torus and are point-like on the orbifold. In this way it is possible that the orbifold space, which is transversal to the branes, becomes large, whereas the large extra dimension scenario is in conflict with chirality for the case of T^6 compactifications [5]. Finally, in [12] a systematic analysis was provided how to obtain T^6 models with precisely three generations of quarks and leptons and just the Standard Model gauge group without any extension. In this context the mass generation for $U(1)$ gauge bosons due to flux-induced Green-Schwarz terms and the related issue of chiral $U(1)$ anomalies is very important. Furthermore the question how to avoid open string tachyons in a certain range of the toroidal background parameters and other phenomenological issues were also addressed.

For type I compactifications the main consistency restrictions considered so far come from the requirement of the absence of massless tadpoles in the Ramond-Ramond (RR) sector of the theory. Having no RR-tadpoles ensures the anomaly freedom in the effective field theory of the massless modes. As mentioned already, the absence of open string tachyons is another important constraint for model building, where however some ‘tachyons’ might be even welcome from the phenomenological point of view, namely those which contribute to the required spontaneous gauge symmetry breaking, in particular of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$. Hence, one may want to identify the standard Higgs field with a tachyon, and also those of other spontaneously broken local gauge groups, such as for instance $U(1)_{B-L}$.

In this paper we like to emphasize that all models considered so far are generically unstable due to the existence of NSNS closed string tadpoles. Specifically we will see that, already at the topology of the world sheet disc amplitude, the NSNS tadpoles related to the closed string moduli U^I , the complex structure deformations of T^6 , and those related to the closed string dilaton ϕ are non-vanishing. This means that in the induced effective potential these scalar fields do not acquire a stable minimum but show the typical runaway behaviour. The explicit form of the potential implies that the internal geometry is driven to a degenerate singular limit, where all D-branes finally lie on top of each other. As a result, space-time supersymmetry is reenforced. In addition, the dilaton tadpole drives

the theory to weak coupling. By T-duality this also disproves the existence of partial supersymmetry breaking in type I vacua after introducing magnetic fluxes into the toroidal $\mathcal{N} = 4$ compactification. Similar partial breaking from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ has been shown to be possible in heterotic and type II theories, albeit under very special circumstances only [29-32].

There are essentially two string theoretic methods to cure the problem of the NSNS tadpoles at least at the next to leading order. First one can employ the Fischler-Susskind mechanism [33,34], by which the back-reaction of the massless fields on the NSNS tadpoles is taken into account iteratively. As demonstrated in [35,36] solving the string equations of motions including the one-loop dilaton tadpole in general leads to warped geometries and non-trivial profiles of the dilaton and other scalar fields. Moreover, in the non-supersymmetric type I string theory discussed in [35,36] the phenomenon of spontaneous compactification occurred due to the NSNS tadpoles. Of course, at this state one is stuck again, as technically the non-linear sigma model in this highly curved backgrounds can not be solved exactly. If it could be solved, one would certainly detect a non-vanishing tadpole at the next order in the string coupling constant. Thus, one might hope that the non-supersymmetric string theory self-adjust its background order by order in string perturbation theory until eventually the true quantum vacuum with vanishing tadpoles to all orders is reached [37].

A second less ambitious approach to handle at least some of the tadpoles is simply by freezing the dangerous closed string scalar fields to fixed values. This can be achieved by performing appropriate projections in an orbifold theory. In the following we will focus on this second approach. In particular we will construct non-supersymmetric orbifold intersecting brane models, where the complex structure moduli of the torus are fixed, and hence there are no associated NSNS tadpoles. However, the dilaton tadpole will still survive, and it cannot be excluded that new tadpoles will be induced at higher orders in string perturbation theory. As noted in [38], in the M-theory context one can even contemplate on orbifold actions which freeze the size of the eleventh direction and therefore of the dilaton in the dual string theory. We will further outline the strategy how to obtain orbifold models with three generations of quarks and leptons and with Standard Model gauge group $SU(3) \times SU(2)_L \times U(1)_Y$ in this particular kind of background.

Our work will be organized as follows. In the next section we will review the main ingredients of the toroidal intersecting brane worlds. Next, in section three, we will extract for the toroidal case all NSNS tadpoles from the infrared divergences in the tree channel

Klein-bottle, annulus and Möbius strip amplitudes, and we will compute the corresponding scalar potential. In section four we will construct \mathbb{Z}_3 orbifold intersecting brane models which are free of geometric NSNS tadpoles, especially addressing the form of the massless spectrum and the question of anomaly cancellation. These results will be analyzed in chapter five to find models which come as close as possible to the Standard Model with three generations.

Unlike the previous toroidal constructions, where the Standard Model fermions originate from bifundamental open strings states, we will now be forced to realize the right-handed (u, c, t) -quarks in the antisymmetric representation of $U(3)$. With this assignment it is indeed possible to get models with three Standard Model generations. We will also discuss the open string tachyons of the theory to see whether the Standard Model Higgs and another Higgs breaking $U(1)_{B-L}$ can be realized as tachyons. It turns out that all models with an appropriate Higgs scalar descend from a GUT theory where $SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ can be unified into $SU(5) \times U(1)$ by a deformation which is marginal at tree level. The additional global symmetries prohibit the usual Yukawa couplings of the (u, c, t) quarks and the Standard Model Higgs doublets, so that the standard mass generation mechanism with fundamental scalar Higgs fields does not work. Furthermore, we analyze the unification behaviour of gauge couplings and the possibility of proton decay in the context of the $SU(5) \times U(1)$ GUT model. In an appendix we also include the results for similar six-dimensional models providing an extra consistency check for our formalism via the six-dimensional anomaly cancellation conditions.

2. Intersecting brane worlds

In this section we review the construction of generically non-supersymmetric open string vacua with D-branes intersecting at angles. Our starting point is an ordinary type I model, where for simplicity we consider only toroidal orbifold models, which we can write as

$$\frac{\text{Type IIB on } T^{2d}}{\{G + \Omega G\}}, \quad (2.1)$$

where G is a finite group acting on the $2d$ dimensional torus T^{2d} . In the following we restrict ourselves to the case that the closed string sector of the orientifold model (2.1) preserves some supersymmetry. Usually, tadpole cancellation requires the introduction of D-branes, which can be chosen to be BPS so that the open string sector preserves

the same supersymmetry as the closed string sector. However, RR tadpole cancellation alone does not require the open string sector to be supersymmetric. As shown in [5,7] for the toroidal case ($G = 1$), there exists the possibility of turning on various constant magnetic $U(1)$ fluxes on the D-branes without giving up RR tadpole cancellation. This breaks supersymmetry, and one faces the usual problems with non-supersymmetric string theories like tachyons, dilaton tadpoles, moduli stabilization and the cosmological constant problem.

Technically, it turned out to be more appropriate to describe such models in a T-dual language, where the new degrees of freedom are described in a purely geometric manner. Let us assume that the $2d$ -dimensional torus can be written as a product of d two-dimensional tori

$$T^{2d} = \bigotimes_{I=1}^d T_I^2, \quad (2.2)$$

where on each T_I^2 we introduce a complex coordinate $Z_I = X_I + iY_I$. Applying T-duality T_Y to the d Y_I -directions of the d two-dimensional tori, the orientifold model (2.1) is mapped to [39]

$$\frac{\text{Type II on } T^{2d}}{\{\hat{G} + \Omega\mathcal{R}\hat{G}\}}, \quad (2.3)$$

where \mathcal{R} is the reflection of the Y_I and \hat{G} is the image of G under T-duality $\hat{G} = T_Y G T_Y^{-1}$. For the case that $G = \mathbb{Z}_N$ the symmetry group acts on each torus by rotations

$$Z_I^L \rightarrow e^{2\pi i v_I / N} Z_I^L, \quad Z_I^R \rightarrow e^{2\pi i v_I / N} Z_I^R. \quad (2.4)$$

Supersymmetric models have been classified in terms of v_I in [40,41]. The T-dual action is then given by

$$Z_I^L \rightarrow e^{2\pi i v_I / N} Z_I^L, \quad Z_I^R \rightarrow e^{-2\pi i v_I / N} Z_I^R. \quad (2.5)$$

Thus, T-duality exchanges left-right symmetric actions with left-right asymmetric actions. Moreover, under T-duality $D9_a$ -branes with constant magnetic fluxes F_a^I are mapped to $D(9-d)$ -branes intersecting at relative angles [42]

$$\varphi_{ab}^I = \arctan(F_a^I) - \arctan(F_b^I). \quad (2.6)$$

They are wrapped around one-dimensional cycles on each T_I^2 , so that each brane a is specified by two coprime wrapping numbers (n_a^I, m_a^I) for each torus. Moreover, the $\Omega\mathcal{R}$ symmetry allows two inequivalent choices of the complex structure

$$U^I = U_1^I - i U_2^I = \frac{e_1}{e_2} = U_1^I - i \frac{R_1^I}{R_2^I} \quad (2.7)$$

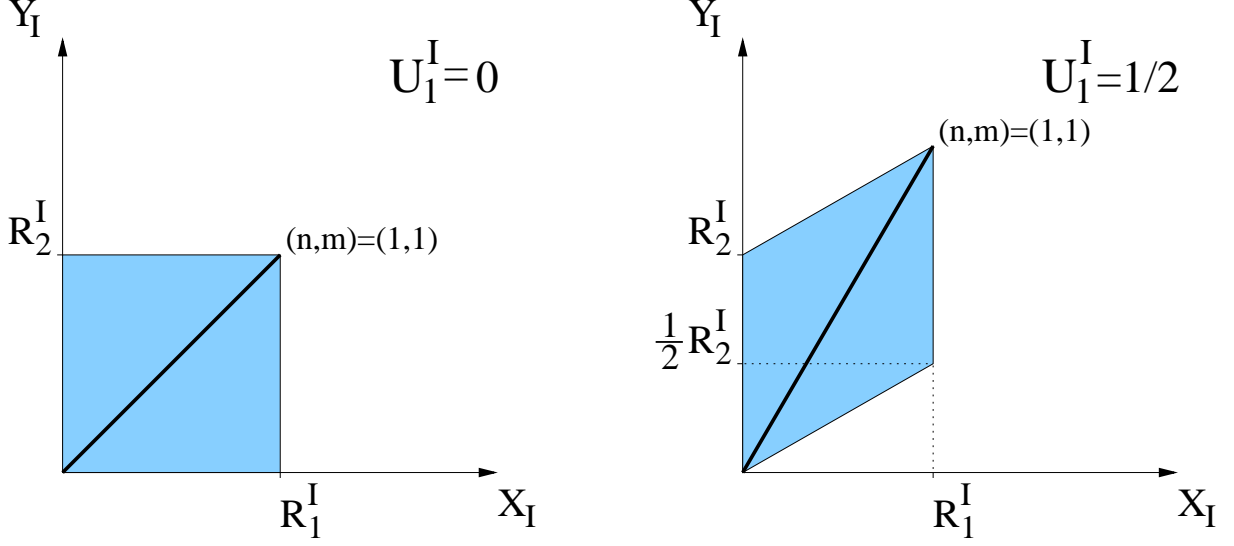


Figure 1

of each T^2 , $U_1^I = 0$ or $1/2$. The tori are depicted in figure 1.

In the T-dual picture with magnetic fluxes the tilt of the torus corresponds to turning on a discrete NSNS two-form B -field. One also has to take into account that for each brane D_a there must exist the mirror brane $D_{a'}$, which is its image under $\Omega\mathcal{R}$. For the purely toroidal case, often called type I' , the RR-tadpole cancellation conditions were derived in [5]. If one introduces K stacks of $D6_a$ -branes counted together with their $\Omega\mathcal{R}$ mirrors, then the four dimensional RR-tadpole cancellation conditions read

$$\begin{aligned}
\sum_{a=1}^K N_a \prod_{I=1}^3 n_a^I &= 16, \\
\sum_{a=1}^K N_a n_a^1 \prod_{I=2,3} (m_a^I + U_1^I n_a^I) &= 0, \\
\sum_{a=1}^K N_a n_a^2 \prod_{I=1,3} (m_a^I + U_1^I n_a^I) &= 0, \\
\sum_{a=1}^K N_a n_a^3 \prod_{I=1,2} (m_a^I + U_1^I n_a^I) &= 0.
\end{aligned} \tag{2.8}$$

This can be compactly written as

$$\sum_{a=1}^K N_a \Pi_a = \Pi_{O6} \tag{2.9}$$

where Π_a denotes the homological cycle of the wrapped $D6_a$ -branes and Π_{O6} the cycle of the orientifold planes along the X_I axes of all three T^2 s. In terms of the T-dual type I theory (2.8) refers to the cancellation of the D9-brane and O9-plane charges, respectively the vanishing of the three possible types of D5-brane charges. The tree-level massless spectrum consists of $\mathcal{N} = 4$ vectormultiplets in the gauge group

$$G = U(N_1) \times U(N_2) \times \dots \times U(N_K) \quad (2.10)$$

equipped with non-supersymmetric chiral matter in bifundamental, symmetric and antisymmetric representations of the gauge group¹. This chiral matter is localized at the intersections of two D-branes and therefore each state appears with a multiplicity given by the intersection number of the two D-branes. Taking these multiplicities into account, the RR-tadpole cancellation conditions guarantee the absence of gauge anomalies in the effective four-dimensional low energy theories.

The quite general form of the consistency conditions in terms of the wrapping numbers of the D-branes allows for a bottom-up approach to search systematically for features of the Standard Model in the class of these intersecting brane models. In particular, in [11] a left-right symmetric model with three generations and gauge group $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ was constructed. Moreover, in [12] the matter content of a three generation $SU(3) \times SU(2)_L \times U(1)_Y$ Standard Model was found, containing also right handed neutrinos and a slightly enlarged Higgs sector. In the first place the model had gauge group $U(3) \times U(2) \times U(1) \times U(1)$, but after analyzing mixed anomalies and the appropriate Green-Schwarz mechanism only the Standard Model gauge fields remained massless. The broken gauge symmetries including lepton and baryon number survived as global symmetries, thus guaranteeing the stability of the proton. Moreover, it was argued that at string tree level the radii of the three tori T^2 can be tuned in such a way that open string tachyons are absent. However, some of the tachyons are welcome, as they can serve as Higgs bosons for breaking the electroweak symmetry.

Even though from a phenomenological point of view the models look quite interesting, we will show in the next section that the string theory is highly unstable.

¹ More detailed information can be found in the papers [5,7,9,12].

3. NSNS tadpoles

So far, for toroidal intersecting brane worlds only the RR tadpole cancellation conditions were analyzed in detail. In this section we will compute the NSNS tadpoles and derive the effective scalar potential for the closed string moduli at open string tree level $e^{-\phi}$, which is next to leading order in string perturbation theory. For the purpose of a phenomenological application we perform the computation for four-dimensional models.

The massless fields in the NSNS sector are the four-dimensional dilaton and the 21 $\Omega\mathcal{R}$ invariant components of the internal metric and the internal NS-NS two form flux. In our factorized ansatz (2.2) only 9 moduli are evident, which are the six radions R_1^I and R_2^I related to the size of the internal dimensions and the two-form flux b_{12}^I on each T^2 . We extract the NSNS tadpoles from the infrared divergences in the tree channel Klein-bottle, annulus and Möbius-strip amplitudes, the open string one-loop diagrams. Adding up the latter three contributions leads to a sum of perfect squares, from which we can read off the disc tadpoles. The computation is straightforward and the relevant formulas can be found in [5,11]. By adding up all three contributions we get for the dilaton tadpole

$$\langle\phi\rangle_D = \frac{1}{\sqrt{\text{Vol}(T^6)}} \left(\sum_{a=1}^K N_a \text{Vol}(\text{D6}_a) - 16 \text{Vol}(\text{O6}) \right) \quad (3.1)$$

with

$$\text{Vol}(\text{D6}_a) = \prod_{I=1}^3 L^I(\text{D6}_a) = \prod_{I=1}^3 \sqrt{(n_a^I R_1^I)^2 + ((m_a^I + U_1^I n_a^I) R_2^I)^2} \quad (3.2)$$

and

$$\text{Vol}(\text{O6}) = \prod_{I=1}^3 L^I(\text{O6}) = \prod_{I=1}^3 R_1^I. \quad (3.3)$$

The result is simply the overall volume of the D6-branes and orientifold planes, the latter ones entering with a negative sign. It is just the effective four-dimensional tension in appropriate units. Intriguingly, the dilaton tadpole can be expressed entirely in terms of the complex structure moduli U_2^I of the three T^2 ,

$$\langle\phi\rangle_D = \left(\sum_{a=1}^K N_a \prod_{I=1}^3 \sqrt{(n_a^I U_2^I)^2 + \left((m_a^I + U_1^I n_a^I) \frac{1}{U_2^I} \right)^2} - 16 \prod_{I=1}^3 U_2^I \right). \quad (3.4)$$

One way to understand this is to realize that the boundary and cross-cap states only couple to the left-right symmetric states of the closed string Hilbert space. The complex structure

moduli are indeed left-right symmetric, whereas the Kähler moduli appear in the left-right asymmetric sector, i.e. D-branes and orientifold O6-planes only couple to the complex structure moduli. This is reversed in the T-dual type I picture, where the tadpole only depends on the Kähler moduli.

Besides the dilaton tadpole we also have three tadpoles for the imaginary parts of the complex structures, given by

$$\langle U_2^I \rangle_D = \frac{1}{\sqrt{\text{Vol}(T^6)}} \left(\sum_{a=1}^K N_a \Gamma^I(\text{D6}_a) L^J(\text{D6}_a) L^K(\text{D6}_a) - 16 \text{Vol}(\text{O6}) \right) \quad (3.5)$$

with $I \neq J \neq K \neq I$ and

$$\Gamma^I(\text{D6}_a) = \frac{(n_a^I R_1^I)^2 - ((m_a^I + U_1^I n_a^I) R_2^I)^2}{L^I(\text{D6}_a)}. \quad (3.6)$$

Analogous to (3.4) these tadpoles can also be expressed entirely in terms of the complex structure moduli U_2^I . Concerning type II models which have also been considered in similar constructions [9] one needs to regard extra tadpoles for the real parts U_1^I , which cancel in type I. All NSNS tadpoles arise from the following scalar potential in string frame

$$V(\phi, U_2^I) = e^{-\phi} \left(\sum_{a=1}^K N_a \prod_{I=1}^3 \sqrt{(n_a^I U_2^I)^2 + \left((m_a^I + U_1^I n_a^I) \frac{1}{U_2^I} \right)^2} - 16 \prod_{I=1}^3 U_2^I \right) \quad (3.7)$$

with

$$\langle \phi \rangle_D \sim \frac{\partial V}{\partial \phi}, \quad \langle U_2^I \rangle_D \sim \frac{\partial V}{\partial U_2^I}. \quad (3.8)$$

The type II potential would only change in erasing the term arising from the orientifold planes, and a third tadpole would appear due to $\langle U_1^I \rangle_D \sim \partial V / \partial U_1^I$. Note that this potential is leading order in string perturbation theory but already contains all higher powers in the complex structure moduli, though we have only computed their one-point function explicitly. One needs to be careful in interpreting it.

In field theory, the presence of a non-vanishing tadpole indicates that the tree-level value was not chosen at a minimum of the potential. Even if one can compute higher loop corrections formally, their meaning is very questionable, as we expect fluctuations to be large no matter how small the coupling constant may be. The theory is driven away to some distant minimum anyway, and perturbation theory around the unstable vacuum is impossible. As a second problem, the open string tachyons which are very often present in

non-supersymmetric string vacua would start to propagate at the open string loop level.¹ Thus it is mandatory to first shift to a minimum with vanishing tadpoles and without tachyons before taking perturbations into account.

In string theory the situation is even worse, as higher loop corrections cannot even be computed because of infinities. If there appears a massless tadpole at genus g in the string loop expansion, one encounters a divergence at genus $2g$ from the region in moduli space where a massless mode propagates along a long tube connecting two genus g surfaces. So either one finds a new vacuum by regarding the back-reaction of the massless fields along the lines of [33,34,35,36], or one uses a modification of the model where the tadpoles are absent. We shall pursue the latter strategy in the following chapter.

Actually, one could have anticipated the result (3.7) immediately, as the source for the dilaton is just the tension of the branes, to first order given by their volumes. The above expression is easily seen to arise from the Dirac-Born-Infeld action for a $D9_a$ -brane with constant $U(1)$ and two-form flux

$$\mathcal{S}_{\text{DBI}} = -T_p \int_{D9_a} d^{10}x e^{-\phi} \sqrt{\det(G + (F_a + B))} \quad (3.9)$$

including the Dp -brane tension

$$T_p = \frac{\sqrt{\pi}}{16\kappa_0} (4\pi^2\alpha')^{(11-p)/2}. \quad (3.10)$$

One can take all background fields to be block-diagonal in terms of the two-dimensional tori. There they take the constant values [39]

$$G^{ij} = \delta^{ij}, \quad (F_a^I)^{ij} = \frac{m_a^I}{n_a^I R_1^I R_2^I} \epsilon^{ij}, \quad (B^I)^{ij} = \frac{b^I}{R_1^I R_2^I} \epsilon^{ij} \text{ with } b^I = 0 \text{ or } \frac{1}{2}. \quad (3.11)$$

Integrating out the internal six dimensions, regarding that the brane wraps each torus n_a^I times, one only needs to apply the T-duality to arrive at (3.7) except for the negative contribution of the orientifold tension.

Due to the RR-tadpole cancellation condition and the triangle inequality, the only point where all four tadpoles vanish is at $U_2^I = \infty$. Interestingly, this proves the impossibility of a partial breaking of supersymmetry in $\mathcal{N} = 4$ vacua by relative angles between D6-branes, respectively magnetic fluxes on D9-branes.²

¹ For a discussion of the stability regions of intersecting D-branes with respect to the appearance of tachyons see [43].

² This possibility has been established in $\mathcal{N} = 2$ type II and heterotic vacua under certain rather special conditions [29,30,31,32].

The potential displays the usual runaway behaviour one often encounters in non-supersymmetric string models. The complex structure is dynamically pushed to the degenerate limit, where all branes lie along the X_I axes and the Y_I directions shrink, keeping the volume fixed. Put differently, the positive tension of the branes pulls the tori towards the X_I -axes. The typical runaway slope being set by the tension (3.10) proportional to the string scale, a ‘slow rolling’ does not appear to be feasible either. Apparently, this has dramatic consequences for all toroidal intersecting brane world models. They usually require a tuning of parameters at tree-level and assume the global stability of the background geometry as given by the closed string moduli. If at closed string tree-level one has arranged the radii of the torus such, that open strings stretched between D-branes at angles are free of tachyons, dynamically the system flows towards larger complex structure and will eventually reach a point where certain scalar fields become tachyonic and indicate a decay of the brane configuration.

Via T-duality the instability translates back into a dynamical decompactification towards the ten-dimensional supersymmetric vacuum. Thus, even if from a heuristic point of view toroidal intersecting brane world models look quite promising, the non-supersymmetric string theory is highly unstable.

4. Orbifold intersecting brane models

One way to avoid this runaway behaviour of the complex structure moduli is to freeze them from the very beginning. This can be achieved by dividing the toroidal model by an appropriate discrete symmetry. For instance, for the left-right symmetric orbifold \mathbb{Z}_3 acting as

$$\Theta : Z^I \rightarrow e^{2\pi i/3} Z^I \tag{4.1}$$

on all three complex coordinates, the complex structure on all three T^2 's is fixed to be either¹

$$U_{\mathbf{A}}^I = \frac{1}{2} + i\frac{\sqrt{3}}{2} \tag{4.2}$$

or

$$U_{\mathbf{B}}^I = \frac{1}{2} + i\frac{1}{2\sqrt{3}}. \tag{4.3}$$

¹ See [44] for a discussion of discrete parameters in type I vacua.

In [45,46], where this type of orbifold was considered for the first time, the torus (4.2) with Kähler modulus

$$T_{\mathbf{A}}^I = i \frac{\sqrt{3}}{2} R^2 \quad (4.4)$$

was called the **A**-torus and the torus (4.3) with

$$T_{\mathbf{B}}^I = i \frac{1}{2\sqrt{3}} R^2 \quad (4.5)$$

the **B**-torus. Note, that under T-duality these models are mapped to asymmetric type I orbifolds where the Kähler moduli are frozen. Vice versa, left-right symmetric type I orbifolds are mapped to asymmetric $\Omega\mathcal{R}$ orientifolds. Therefore, only for left-right symmetric $\Omega\mathcal{R}$ orientifolds with intersecting branes the disc scalar potential does not depend on the U^I , preventing the torus from shrinking to degenerate limits.

Thus, we are naturally led to consider the orientifold

$$\frac{\text{Type IIA on } T^6}{\{\mathbb{Z}_3 + \Omega\mathcal{R}\mathbb{Z}_3\}}. \quad (4.6)$$

This is precisely one example of the supersymmetric orientifolds with D6-branes at angles introduced in [45,46,47,48,49]¹. In the closed string sector this \mathbb{Z} -orbifold has Hodge numbers $(n_{2,1}, n_{1,1}) = (0, 36)$, where 9 Kähler deformations come from the untwisted sector and the remaining 27 are the blown up modes of the fixed points. As noted before, this manifold has frozen complex structure. Due to the \mathbb{Z}_3 symmetry we have three kinds of O6-planes located as indicated in figure 2, being identified under the orbifold action.

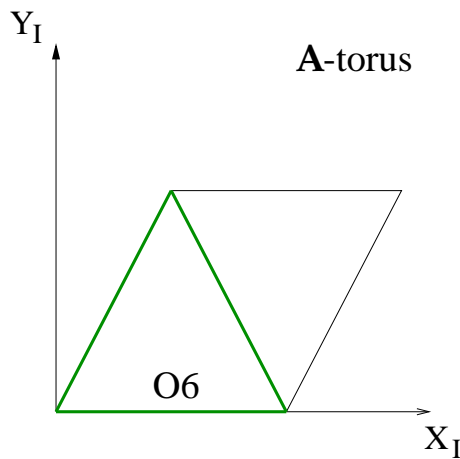


Figure 2

¹ In [50] extensions to $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold groups have been considered.

One can cancel the Klein bottle tadpoles locally by introducing four D6-branes on top of the O6-planes leading to a supersymmetric model with gauge group of rank $2 = 16 \cdot 2^{-3}$, in accord with the rank reduction normally encountered in type I vacua with NSNS B -field of rank 6. In the following we discuss the more general case, where one introduces D6-branes intersecting at angles into such a background and in particular determine the tadpole cancellation conditions and the chiral massless spectrum. In particular, we find that the phenomenological obstruction of the small rank can be lifted when putting branes at arbitrary angles on the orbifold space. In the dual flux picture this means that adding magnetic flux to the supersymmetric theory allows to have a larger gauge symmetry. This is very surprising in the first place, one would have naively expected the opposite to happen. But, effectively, a D9-brane with additional flux can carry less RR charge than without.

Note the difference compared to [51] where six-dimensional supersymmetric orientifolds of type I' have been combined with generic brane configurations on an extra T^2 . This latter compactification indeed suffers from the very small rank of the gauge group.

An individual D6_a-brane is again determined by three pairs of wrapping numbers (n^I, m^I) along the fundamental cycles

$$e_1^{\mathbf{A}} = e_1^{\mathbf{B}} = R, \quad e_2^{\mathbf{A}} = \frac{R}{2} + i\frac{\sqrt{3}R}{2}, \quad e_2^{\mathbf{B}} = \frac{R}{2} + i\frac{R}{2\sqrt{3}} \quad (4.7)$$

of each T^2 . Under the \mathbf{Z}_3 and $\Omega\mathcal{R}$ symmetry in general the branes are organized in orbits of length six. Such an orbit constitutes an equivalence class $[a]$ of D6_a-branes denoted by $[(n_a^I, m_a^I)]$. For the \mathbf{A} -torus the six branes contained in the equivalence class $[(n^I, m^I)]$ are given by

$$\begin{array}{ccccc} \Omega\mathcal{R} & \begin{pmatrix} n^I \\ m^I \end{pmatrix} & \xrightarrow{\mathbf{Z}_3} & \begin{pmatrix} -n^I - m^I \\ n^I \end{pmatrix} & \xrightarrow{\mathbf{Z}_3} & \begin{pmatrix} m^I \\ -n^I - m^I \end{pmatrix} \\ & \downarrow & & \downarrow & & \downarrow \\ & \begin{pmatrix} n^I + m^I \\ -m^I \end{pmatrix} & \xleftarrow{\mathbf{Z}_3} & \begin{pmatrix} -m^I \\ -n^I \end{pmatrix} & \xleftarrow{\mathbf{Z}_3} & \begin{pmatrix} -n^I \\ n^I + m^I \end{pmatrix} \end{array} \quad (4.8)$$

and for the \mathbf{B} -torus by

$$\begin{array}{ccccc} \Omega\mathcal{R} & \begin{pmatrix} n^I \\ m^I \end{pmatrix} & \xrightarrow{\mathbf{Z}_3} & \begin{pmatrix} -2n^I - m^I \\ 3n^I + m^I \end{pmatrix} & \xrightarrow{\mathbf{Z}_3} & \begin{pmatrix} n^I + m^I \\ -3n^I - 2m^I \end{pmatrix} \\ & \downarrow & & \downarrow & & \downarrow \\ & \begin{pmatrix} n^I + m^I \\ -m^I \end{pmatrix} & \xleftarrow{\mathbf{Z}_3} & \begin{pmatrix} n^I \\ -3n^I - m^I \end{pmatrix} & \xleftarrow{\mathbf{Z}_3} & \begin{pmatrix} -2n^I - m^I \\ 3n^I + 2m^I \end{pmatrix}. \end{array} \quad (4.9)$$

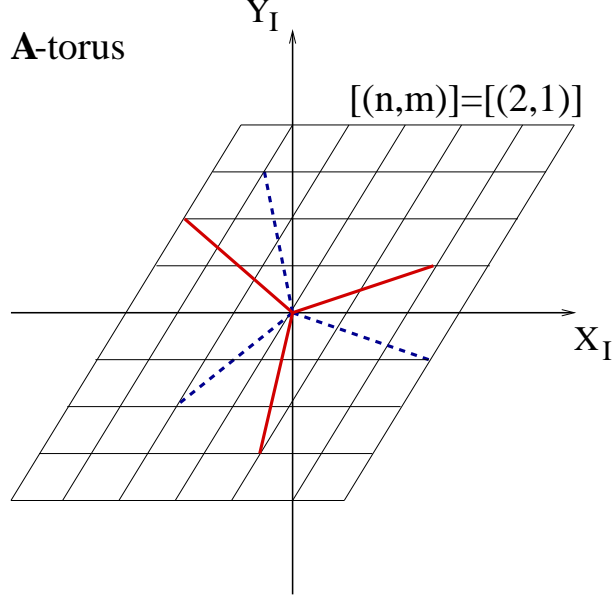


Figure 3

As an example for an orbit on a single A -torus the equivalence class $[(2,1)]$ is shown in figure 3. The solid lines represent the images under \mathbb{Z}_3 and the dashed lines the $\Omega\mathcal{R}$ mirror branes. Due to the relation

$$\Theta(\Omega\mathcal{R}) = (\Omega\mathcal{R})\Theta^{-1} \quad (4.10)$$

only untwisted sector fields couple to the orientifold planes. This is also clear from the fact that the orientifold planes are of codimension one on each T^2 and therefore can avoid a blown-up \mathbb{P}^1 from an orbifold fixed points. Similarly, the D6-branes can not wrap around the blown-up cycles to become fractional branes and thus are not charged under the twisted sector RR-fields. Thus, there are only untwisted tadpoles.

4.1. Tadpoles

Combining the results from [5,46] the computation of the Klein-bottle, annulus and Möbius strip amplitudes is a straightforward exercise and we will only present the salient features and results of this rather tedious computation. Since the complex structure is fixed we only get one RR and one NSNS tadpole cancellation condition. In the annulus amplitude all open string sectors contribute including those from open strings stretched between two

branes belonging to the same equivalence class. It turns out to be convenient to define the following two quantities for any equivalence class $[(n_a^I, m_a^I)]$ of $D6_a$ -branes

$$\begin{aligned} Z_{[a]} &= \frac{2}{3} \sum_{(n_b^I, m_b^I) \in [a]} \prod_{I=1}^3 \left(n_b^I + \frac{1}{2} m_b^I \right), \\ Y_{[a]} &= -\frac{1}{2} \sum_{(n_b^I, m_b^I) \in [a]} (-1)^M \prod_{I=1}^3 m_b^I, \end{aligned} \quad (4.11)$$

where M is defined to be odd for a mirror brane and otherwise even. The sums are taken over all the individual $D6_b$ -branes that are elements of the orbit $[a]$. The explicit expressions for $Z_{[a]}$ and $Y_{[a]}$ for the four possible tori, **AAA**, **AAB**, **ABB**, **BBB**, can be found in appendix A. If we introduce K stacks of equivalence classes $[a]$ of branes, then the RR-tadpole cancellation condition reads

$$\sum_{a=1}^K N_a Z_{[a]} = 2. \quad (4.12)$$

Note, that the sum is over equivalence classes of D6-branes. In fact, $Z_{[a]}$ is the projection of the entire orbit of $D6_a$ -branes onto the X_I axes, i.e. the sum of their RR charges with respect to the dual D9-brane charge. Therefore the appearance of $Z_{[a]}$ in the tadpole cancellation condition is very natural, simply meaning that the RR-charges of all D6-branes have to cancel the RR-charges of the orientifold O6-planes. If the $Z_{[a]}$ are all positive, as was the case in the supersymmetric solutions of [45,46], then (4.12) implies a very small rank of the gauge group. However, in the general case the $Z_{[a]}$ may also be negative so that also gauge groups of higher rank can be realized.

In the closed string NSNS sector all scalars related to the complex structure moduli are projected out under \mathbb{Z}_3 , so that only the dilaton itself can have a disc tadpole. This is indeed what we find from the tree-channel one loop amplitudes, as the only divergences there comes from the dilaton. The scalar potential for our model is

$$V(\phi) = e^{-\phi} \left(\sum_a N_a \prod_{I=1}^3 L_{[a]}^I - 2 \right) \quad (4.13)$$

with the lengths given by

$$L_{[a]}^I = \begin{cases} \sqrt{(n_a^I)^2 + (m_a^I)^2 + n_a^I m_a^I} & \text{for the } \mathbf{A}\text{-torus,} \\ \sqrt{(n_a^I)^2 + \frac{1}{3}(m_a^I)^2 + n_a^I m_a^I} & \text{for the } \mathbf{B}\text{-torus.} \end{cases} \quad (4.14)$$

Thus, similar to the toroidal case discussed in section 2, whenever the D-branes do not lie on top of the orientifold planes the dilaton tadpole does not vanish. In this way the *local* cancellation of the RR charge is in one to one correspondence with supersymmetric vacua and the cancellation of NSNS tadpoles. The only exception to this rule appears to be a parallel displacement of orientifold planes and D-branes, i.e. a Higgs mechanism breaking $SO(2N_a)$ to $U(N_a)$.

4.2. Massless spectrum

Having found the one-loop consistency condition the next step is to determine the massless spectrum and to see whether one can find phenomenologically interesting models. In the closed string sector at string tree level $\mathcal{N} = 1$ supersymmetry is preserved and we get the same massless spectrum of vector and chiral multiplets as in [46]. However, in the open string sector we break supersymmetry and get more interesting spectra. For the supersymmetric brane configurations the massless spectra for these kinds of orientifolds with D-branes at angles were always non-chiral, which is no longer true in the non-supersymmetric case.

In the following we discuss the most generic situation where all equivalence classes contain six different D6-branes, i.e. there are no dual D9-branes without any magnetic flux on their world volume. A string with both ends on the same individual brane in some equivalence class $[a]$ gives rise to an $\mathcal{N} = 4$ vectormultiplet in the gauge group $U(N_a)$. Open strings stretched between branes belonging to two different classes can break supersymmetry and give rise to chiral fermions in the bifundamental representations of the gauge groups. There are $36 = 6 \times 6$ different open string sectors of this kind. Due to the \mathbb{Z}_3 and $\Omega\mathcal{R}$ symmetry only 6 of them are independent. Thus, we can pick one brane, $D6_a$, from the first stack and determine the massless spectrum with all 6 branes $D6_{bi}$, $i \in \{1, \dots, 6\}$, from the second stack. Open strings between $D6_a$ and \mathbb{Z}_3 images of $D6_b$, i.e. $i \in \{1, 2, 3\}$, yield chiral fermions in the (\overline{N}_a, N_b) representation and open strings between $D6_a$ and mirror images, i.e. $i \in \{4, 5, 6\}$, in the second stack give rise to chiral fermions in the (N_a, N_b) representation. The multiplicity of these massless states is determined by the topological intersection number between the branes in question, where intersections with formally negative intersection number have flipped orientation leading to the conjugate representations.

In the end, only the net number of such fermion generations is relevant. For instance, let $D6_{b1}$ and $D6_{b2}$ be two different branes in the orbit $[b]$ and $D6_a$ another one in the orbit

[a]. Assume, that in the $D6_a$ - $D6_{b1}$ sector we have a chiral fermion $\psi_{a,b1}$ in the (\overline{N}_a, N_b) representation and in the $D6_a$ - $D6_{b2}$ sector a fermion $\psi_{a,b2}$ in the conjugate (N_a, \overline{N}_b) representation. In principle, these can pair up to yield a Dirac mass term with mass of the order of the string scale. Indeed, since in the $D6_{b1}$ - $D6_{b2}$ sector we get a massless scalar $H_{b1,b2}$ in the adjoint representation of $U(N_b)$, the three-point coupling on the disc diagram as shown in figure 4 exists.

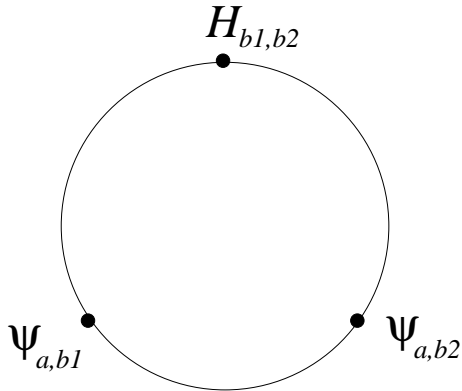


Figure 4

Giving a vacuum expectation value to the $SU(N_b)$ singlet in the adjoint of $U(N_b)$ leaves the gauge symmetry unbroken and gives a mass to the fermions. From the string point of view, this deformation is exactly the one studied in [52], which deforms the two intersecting branes of the orbit [b] into a single brane wrapping a supersymmetric cycle.

For the relevant net number of chiral left-handed bifundamentals one obtains the following simple expressions

$$\begin{aligned}
 (\overline{N}_a, N_b)_L &: Z_{[a]} Y_{[b]} - Y_{[a]} Z_{[b]}, \\
 (N_a, N_b)_L &: Z_{[a]} Y_{[b]} + Y_{[a]} Z_{[b]}.
 \end{aligned}
 \tag{4.15}$$

Thus, the combinations $Z_{[a]}$ and $Y_{[a]}$ can be interpreted as effective wrapping numbers.

Finally, we have the open strings stretched between different branes of the same equivalence class. Open string between the brane $D6_a$ and the three images under $\Omega\mathcal{R}\Theta^k$ give rise to chiral fields in the symmetric and antisymmetric representation. The net numbers of these massless fields are given by

$$\begin{aligned}
 (A_a)_L &: Y_{[a]}, \\
 (A_a + S_a)_L &: Y_{[a]} \left(Z_{[a]} - \frac{1}{2} \right).
 \end{aligned}
 \tag{4.16}$$

Finally, open strings between the brane $D6_a$ and its two \mathbb{Z}_3 images yield massless fermions in the adjoint representation

$$(\text{Adj})_L : 3^{n_{\mathbf{B}}} \prod_{I=1}^3 \left(L_{[a]}^I \right)^2, \quad (4.17)$$

where $n_{\mathbf{B}}$ counts the number of \mathbf{B} -tori in T^6 . This latter sector is $\mathcal{N} = 1$ supersymmetric, as the \mathbb{Z}_3 rotation alone preserves supersymmetry. Before going into the phenomenological details we first discuss the issue of gauge anomalies in the next section.

4.3. Anomaly cancellation

Since we have chiral fermions, there are potential gauge anomalies, which however should be absent due to the string theoretic one loop consistency of our models. For the spectrum shown in section (4.2) we obtain that the non-abelian gauge anomaly of the $SU(N_a)$ gauge factor is proportional to

$$\sum_{b \neq a} 2 N_b Z_{[b]} Y_{[a]} + (N_a - 4) Y_{[a]} + 2 N_a Y_{[a]} \left(Z_{[a]} - \frac{1}{2} \right), \quad (4.18)$$

which vanishes when we use the RR-tadpole cancellation condition (4.12) in the first term in (4.18). As usual, the abelian gauge anomalies do not cancel right away. Indeed the $U(1)_a - g_{\mu\nu}^2$ anomalies are proportional to

$$3 N_a Y_{[a]} \quad (4.19)$$

and the mixed $U(1)_a - U(1)_b^2$ anomalies are

$$2 N_a N_b Y_{[a]} Z_{[b]}. \quad (4.20)$$

In order to cancel these anomalies one has to invoke a generalized Green-Schwarz mechanism. It was pointed out in [9] that the relevant axions are among the untwisted sector RR-fields. Using the same notation, we are discussing the couplings in the T-dual type I language where angles are translated into fluxes. In ten space-time dimensions we have the RR fields C_2 and C_6 , $dC_6 = *dC_2$, with world-volume couplings

$$\int_{D9_a} C_6 \wedge F_a \wedge F_a, \quad \int_{D9_a} C_2 \wedge F_a \wedge F_a \wedge F_a \wedge F_a. \quad (4.21)$$

Upon dimensional reduction to four dimensions we only get one two-form, $B_2^0 = C_2$. Note, that the other three type I two-forms

$$B_2^I = \int_{T_J^2 \times T_K^2} C_6 \quad (4.22)$$

are projected out by the \mathbb{Z}_3 symmetry. The dual four-dimensional axion C_0^0 is given by

$$C_0^0 = \int_{T_1^2 \times T_2^2 \times T_3^2} C_6. \quad (4.23)$$

From the ten dimensional couplings (4.21) summed over an entire orbit of the symmetry group one obtains the four-dimensional couplings of the RR-forms to the gauge fields

$$\begin{aligned} N_a Y_{[a]} \int_{M_4} B_2^0 \wedge F_a, \\ N_b Z_{[b]} \int_{M_4} C_0^0 F_b \wedge F_b. \end{aligned} \quad (4.24)$$

Apparently, these couplings have precisely the right form to cancel the abelian gauge anomalies (4.19) and (4.20) via a generalized Green-Schwarz mechanism. In fact there is only one anomalous $U(1)$

$$F_{\text{mass}} = \sum_a (N_a Y_{[a]}) F_a \quad (4.25)$$

which becomes massive due to the first coupling in (4.24). It can be checked that the mixed $U(1) - G^2$ anomalies are also canceled by this generalized Green-Schwarz mechanism.

4.4. Stability

As the tadpole calculation (4.13) shows, after projecting out the complex structure moduli, the potential is flat for the remaining moduli at the leading order in the string perturbation theory. Only the dilaton still runs away to zero coupling. For the background geometry, we certainly expect to find corrections in higher loop diagrams, when untwisted and twisted Kähler moduli, decoupled only at leading order, can run around the loop.

Definite statements can hardly be made about the higher loop potentials, but at least qualitatively we can say something about the one-loop potential for the untwisted Kähler moduli $K_U^I = R_1^I R_2^I$. Since the closed string sector is supersymmetric the only dependence of the one-loop potential on K_U^I can arise from the annulus and the Möbius strip amplitudes. Since we assumed that no D6-branes lie along the X_I axes, the massless

strings in these sectors are localized at the intersection points of the respective D6-branes and O6-planes. They do not ‘see’ the global geometry of the torus and, hence, there is no explicit dependence of the Möbius strip amplitude on the radii.

Therefore, the only contribution can come from non-supersymmetric sectors in the annulus amplitude, which depend on the radii. These arise from open strings stretched between two intersecting D6-branes, which are parallel on one or two tori. In the directions of the latter tori there are both Kaluza-Klein and winding modes for the open strings. Since the KK modes scale like $1/(R^I)^2 \sim 1/K_U^I$ and the winding modes like $(R^I)^2 \sim K_U^I$ there is a good chance that the one-loop scalar potential stabilizes the untwisted Kähler moduli. Of course, in this argument we have assumed that the Fischler-Susskind mechanism does not qualitatively change the picture we derived from the flat tree-level background. Alternatively one can also proceed in a way analogous to the tadpoles associated to the complex structures U^I , namely considering orbifold groups where all radii are frozen, like the $\mathbb{Z}_3 \times \widehat{\mathbb{Z}}_3$ orientifold of [53,39].

Another issue concerns the existence of open string tachyons, which also may spoil stability at the open string loop-level. In general, the bosons of lowest energy in a non-supersymmetric open string sector can have negative mass squared. Here one has to distinguish two different cases. Either the two D-branes in question intersect under a non-trivial angle on all three two-dimensional tori or the D-branes are parallel on at least one of the tori T_I^2 . In the latter case one can get rid of the tachyons at least classically by making the distance between the two D-branes on the torus T_I^2 large enough. In the former case, it depends on the three angles, φ_{ab}^I , between the branes $D6_a$ and $D6_b$ whether there appear tachyons or not.

Defining $\epsilon_{ab}^I = \varphi_{ab}^I/\pi$ and let P_{ab} be the number of ϵ_{ab}^I satisfying $\epsilon_{ab}^I > 1/2$, to compute the ground state energy in this twisted open string sector one has to distinguish the following three cases

$$E_{ab}^0 = \begin{cases} \frac{1}{2} \sum_I |\epsilon_{ab}^I| - \max \{ |\epsilon_{ab}^I| \} & \text{for } P_{ab} = 0, 1 \\ 1 + \frac{1}{2} (|\epsilon_{ab}^I| - |\epsilon_{ab}^J| - |\epsilon_{ab}^K|) & \text{for } P_{ab} = 2 \text{ and } |\epsilon_{ab}^I| \leq \frac{1}{2} \\ -\max \{ |\epsilon_{ab}^I|, 1 - |\epsilon_{ab}^J|, 1 - |\epsilon_{ab}^K| \} & \text{for } P_{ab} = 3. \end{cases} \quad (4.26)$$

In order for a brane model to be free of tachyons, for all open string sectors $E_{ab}^0 \geq 0$ has to be satisfied. Since in the orbifold model each brane comes with a whole equivalence class of branes, and the angles between two branes do not depend on any moduli (like in the

toroidal case), freedom of tachyons is quite a strong condition. We will discuss this point further for the concrete three generation models in section 5. We shall actually find that, even though tree-level stability is a strong condition, it can be satisfied in particular cases.

Even if classically we can avoid tachyons by moving parallel branes far apart, at quantum level effective potentials for these open string moduli are generated which might spoil the stability of the configuration by pulling the branes together until a tachyon reappears. Without knowing the precise scalar potential definite statements can not be made.

5. Three generation models

In this section we will try to solve the consistency equations for the \mathbb{Z}_3 orbifold to search for models which come as close as possible to the Standard Model, respectively a moderate extension. Amazingly, even in this fairly constrained orbifold set-up it is not too difficult to get three generation models with $SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ or $SU(5) \times U(1)$ gauge group and Standard Model matter fields enhanced by a right-handed neutrino. Only when it comes to the Higgs sector and the Yukawa couplings we encounter some deviations from our Standard Model expectations.

In [9,11,12] three generation intersecting brane worlds were always realized on four stacks of D6-branes with gauge group $U(3) \times U(2) \times U(1) \times U(1)$ and chiral matter only in the bifundamental representations of the gauge factors. It turns out that such a scenario is not possible in our orbifold case for the following reason. Requiring that there does not exist any matter in the antisymmetric representation of $U(3)$ forces $Y_1 = 0$. Due to (4.15), this in turn implies that we get the same number of chiral fermions in the $(\mathbf{3}, \mathbf{2})$ and in the $(\bar{\mathbf{3}}, \mathbf{2})$ representation of $U(3) \times U(2)$ leading to an even number of left-handed quarks. Thus, employing only bifundamental fields is not sufficient.

5.1. Extended Standard Model

We are forced to realize the right-handed (u, c, t) -quarks in the antisymmetric representation of $U(3)$, which, accidentally, is the same as the anti-fundamental representation $\bar{\mathbf{3}}$. Moreover, requiring that there does not appear any chiral matter in the symmetric representation of $U(3)$ and $U(2)$ forces us to have $Z_3 = Z_2 = 1/2$. After some inspection

one realizes that the best way to approach the Standard Model is to start with only three stacks of D6-branes with gauge group $U(3) \times U(2) \times U(1)$ and

$$(Y_1, Z_1) = \left(3, \frac{1}{2}\right), \quad (Y_2, Z_2) = \left(3, \frac{1}{2}\right), \quad (Y_3, Z_3) = \left(3, -\frac{1}{2}\right). \quad (5.1)$$

Note, that this choice indeed satisfies the RR-tadpole cancellation condition (4.12). The three generation chiral massless spectrum is shown in table 4.

matter	$SU(3) \times SU(2) \times U(1)^3$	$U(1)_Y$	$U(1)_{B-L}$
$(Q_L)_i$	$(\mathbf{3}, \mathbf{2})_{(1,1,0)}$	$\frac{1}{3}$	$\frac{1}{3}$
$(u_L^c)_i$	$(\bar{\mathbf{3}}, \mathbf{1})_{(2,0,0)}$	$-\frac{4}{3}$	$-\frac{1}{3}$
$(d_L^c)_i$	$(\bar{\mathbf{3}}, \mathbf{1})_{(-1,0,1)}$	$\frac{2}{3}$	$-\frac{1}{3}$
$(l_L)_i$	$(\mathbf{1}, \mathbf{2})_{(0,-1,1)}$	-1	-1
$(e_L^+)_i$	$(\mathbf{1}, \mathbf{1})_{(0,2,0)}$	2	1
$(\nu_L^c)_i$	$(\mathbf{1}, \mathbf{1})_{(0,0,-2)}$	0	1

Table 4: *Left-handed fermions for the 3 generation model.*

Of course, we now assume that the non-chiral fermions have paired up and decoupled as was described earlier. Note, that the right-handed leptons are realized as open strings in the antisymmetric representation of $U(2)$ and the right-handed neutrinos as open strings in the symmetric representation $\bar{\mathbf{S}}$ of the $U(1)$ living on the third stack of D6-branes. As expected from the general analyses of the $U(1)$ anomalies there is one anomalous $U(1)$ gauge symmetry

$$U(1)_{\text{mass}} = 3U(1)_1 + 2U(1)_2 + U(1)_3 \quad (5.2)$$

and two anomaly free ones which can be chosen to be $U(1)_Y$ and $U(1)_{B-L}$

$$\begin{aligned} U(1)_Y &= -\frac{2}{3}U(1)_1 + U(1)_2, \\ U(1)_{B-L} &= -\frac{1}{6}(U(1)_1 - 3U(1)_2 + 3U(1)_3). \end{aligned} \quad (5.3)$$

Analogous to [9,11,12], since the one-loop consistency of the string model requires the formal cancellation of the $U(2)$ and $U(1)$ (non-abelian) gauge anomalies, the possible models are fairly constrained and require the introduction of right-handed neutrinos. Because the lepton number is not a global symmetry of the model, there exists the possibility to obtain

Majorana mass terms and invoke the see-saw mechanism for the neutrinos mass hierarchy. Since, after introducing the right-handed neutrino into the Standard Model, the $U(1)_{B-L}$ symmetry becomes anomaly-free, it is not too surprising that in the string theory this symmetry is gauged. After the Green-Schwarz mechanism the anomalous $U(1)_{\text{mass}}$ decouples, but survives as a global symmetry. Thus, the possible Yukawa couplings are more constrained than in the Standard Model.

It is straightforward but extremely tedious to find realizations of the (Y_a, Z_a) given above in terms of actual winding numbers $[(n_a^I, m_a^I)]$. We have performed a systematic computer search and identified 36 solutions for each stack $[a]$, the number being independent of which of the four possible types of the torus had been chosen. Surprisingly, all winding numbers range between -3 and 3, and only for the **BBB** torus from -5 and 5. The actual number of inequivalent string models with the above mentioned Standard Model like features then is $4 \cdot 36^3$.

5.2. Stability and Higgs scalars

As mentioned already, the primary motivation to study the present class of $\Omega\mathcal{R}$ orientifolds of type IIA is their stability. While the closed string sector does not suffer from any massless disc tadpole apart from the dilaton, and thus all moduli sit at extrema of their potential, the open string sector contains tachyonic scalars which indicate an instability. Assuming the closed string moduli to be qualitatively unaffected by the condensation, the endpoint of this will presumably be a new vacuum, where the isolated cycles themselves are general supersymmetric ones but still intersects each other in a non-supersymmetric way. Still everything will be stable with respect to tachyons. There are two different patterns of gauge symmetry breaking which arise when any two branes condense via this mechanism. When the two branes are of different class $[a]$ and $[b]$, the tachyonic Higgs field is in the bifundamental representation of the $U(N_a) \times U(N_b)$ gauge group and the condensation resembles the Higgs mechanism of electroweak symmetry breaking. On the contrary, when the two branes are elements of the same orbit $[a]$, the Higgs field will be in the antisymmetric, symmetric or adjoint representation of the $U(N_a)$ and thus affect only this factor.

The version of the Standard Model extended by a gauged $B - L$ symmetry together with right-handed neutrinos requires a two step gauge symmetry breaking (For the probably first appearance of such models see [54]). In order to avoid conflicts with various experimental facts a hierarchy of Higgs vacuum expectation values is required. First the

$U(1)_{B-L}$ has to be broken at a scale at least some 10^{4-6} above the electroweak scale. This requires a Higgs field charged under this group but a singlet otherwise, which can be met with a tachyon from a sector of strings stretching between two branes in the orbit that supports the $U(1)_3$. The second step is the familiar electroweak symmetry breaking which needs a bifundamental Higgs doublet.

We have therefore performed a study among all the $4 \cdot 36^3$ models looking for such a suitable tachyon spectrum. In any sector of open strings stretching between two D6-branes a and b the lightest physical state has a mass given by (4.26). By expressing the angle variables in terms of winding numbers, one can set up a computer program to do the search for models with a Higgs scalar in the $(\mathbf{2}, \mathbf{1})$ and/or another one in the ‘symmetric’ representation of $U(1)_{B-L}$. All other open string sectors need to be free of tachyons. The results are the following: For the **AAA** and the **BBB** type tori one can get D6-brane configurations that display only tachyons charged under $U(1)_{B-L}$, but none of these models does have a suitable Higgs in the $(\mathbf{2}, \mathbf{1})$. Vice versa, the **AAB** and **ABB** models do have Higgs fields in the $(\mathbf{2}, \mathbf{1})$ but no singlets charged under $U(1)_{B-L}$.¹ Actually, we find a couple of hundred models having either a Higgs in $(\mathbf{2}, \mathbf{1})$ or a Higgs in the ‘symmetric’ representation of $U(1)_{B-L}$. But no model contains both Higgs fields. This looks discouraging at first sight. Regarding the necessity to have a hierarchy of a high scale breaking of $U(1)_{B-L}$ and low scale electroweak Higgs mechanism, we are forced to choose a model with a singlet Higgs condensing at the string scale but without Higgs field in the $(\mathbf{2}, \mathbf{1})$ and favour an alternative mechanism for electroweak symmetry breaking. An explicit realization is for example given by

$$\begin{aligned} [(n_1^I, m_1^I)] &= [(-3, 2), (0, 1), (0, -1)], \\ [(n_2^I, m_2^I)] &= [(-3, 2), (0, 1), (0, -1)], \\ [(n_3^I, m_3^I)] &= [(-3, 2), (1, -1), (-1, 0)]. \end{aligned} \tag{5.4}$$

This model has precisely 3 Higgs singlets

$$h_i : \quad (\mathbf{1}, \mathbf{1})_{(0,0,-2)} \tag{5.5}$$

which carry only $B - L$ but no hypercharge. They are former ‘superpartners’ of the right-handed neutrinos. Interestingly, it turns out that all solutions to the tadpole conditions

¹ For all tori except the **AAA** type, one can even set up D6-brane configurations without tachyons at all.

which display the Higgs singlet charged under $U(1)_{B-L}$ and no tachyons otherwise result from a model with gauge group $SU(5) \times U(1)$ deformed by giving a vacuum expectation value to a scalar in the adjoint of $SU(5)$. Geometrically this is evident in the fact that the stacks of branes that support the $SU(3)$ and $SU(2)_L$ are always parallel, thus their displacement is a marginal deformation at tree level. Of course, we have to expect that quantum corrections will generate a potential for the respective adjoint scalar.

5.3. An $SU(5) \times U(1)$ GUT model

In this section we reinterpret the above direct realization of the extended Standard Model as a GUT scenario. The unified model basically consists in moving the two stacks for the $U(2)$ and $U(3)$ sector on top of each other, thus tuning the adjoint Higgs **24** to a vanishing vacuum expectation value. The common GUT gauge group $SU(5)$ is extended by a single gauged $U(1)$ symmetry. On two stacks of branes with $N_5 = 5$ and $N_1 = 1$ the model is realized by picking again

$$(Y_5, Z_5) = \left(3, \frac{1}{2}\right), \quad (Y_1, Z_1) = \left(3, -\frac{1}{2}\right). \quad (5.6)$$

The task of expressing these effective winding numbers in terms of $[(n_a^I, m_a^I)]$ quantum numbers is identical to that for the previously discussed extended Standard Model. The number of solutions is again 36 per stack, i.e. the total set consists of $4 \cdot 36^2$ inequivalent models. The resulting spectrum of net chiral fermions is featured in table 4.

Number	$SU(5) \times U(1)^2$	$U(1)_{\text{free}}$
3	$(\bar{\mathbf{5}}, \mathbf{1})_{(-1,1)}$	$-\frac{6}{5}$
3	$(\mathbf{10}, \mathbf{1})_{(2,0)}$	$\frac{2}{5}$
3	$(\mathbf{1}, \mathbf{1})_{(0,-2)}$	-2

Table 4: *Left-handed fermions for the 3 generation $SU(5) \times U(1)$ model.*

The anomalous $U(1)$ is given by

$$U(1)_{\text{mass}} = 5U(1)_5 + U(1)_1, \quad (5.7)$$

in accord with (4.25), and the anomaly-free one is

$$U(1)_{\text{free}} = \frac{1}{5}U(1)_5 - U(1)_1. \quad (5.8)$$

This is the desired field content of a grand unified Standard Model with extra right-handed neutrinos, which then also fits into $SO(10)$ representations. The usual minimal Higgs sector consists of the adjoint $\mathbf{24}$ to break $SU(5)$ to $SU(3) \times SU(2)_L \times U(1)_Y$ and a $(\mathbf{5}, \mathbf{1})$ which produces the electroweak breaking. In addition we now also need to have a singlet to break the extra $U(1)_{\text{free}}$ gauge factor. The adjoint scalar is present as part of the vectormultiplet of the formerly $\mathcal{N} = 4$ supersymmetric sector of strings starting and ending on identical branes within the stack [5]. Turning on vacuum expectation values in the supersymmetric theory means moving on the Coulomb branch of the moduli space, which geometrically translates to separating the 5 D6₅-branes into parallel stacks of 2 plus 3. Actually, the form of the potential generated for this modulus after supersymmetry breaking is not known, and the existence of a negative mass term as required for the spontaneous condensation remains speculative.

Having identified the $SU(5)$ GUT as a Standard Model where two stacks of branes are pushed upon each other, we can refer to the former analysis of the scalar spectrum for the other two Higgs fields needed. The results of our search for Higgs singlets and bifundamentals done for the $SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ model in the previous chapter apply without modification as the two stacks for $SU(3) \times SU(2)_L$ are parallel in all cases.

5.4. Yukawa couplings

There is another deviation from the Standard Model which also supports a replacing of the fundamental Higgs scalar by an alternative composite operator. Namely, due to the additional global symmetries, an appropriate Yukawa coupling giving a mass to the (u, c, t) -quarks is absent. This can be seen from the quantum numbers of the Higgs field \tilde{H} resulting from of the relevant Yukawa coupling

$$\tilde{H} \bar{Q}_L u_R. \quad (5.9)$$

Thus, \tilde{H} is forced to have the quantum numbers

$$\tilde{H} : \quad (\mathbf{1}, \mathbf{2})_{(3,1,0)}. \quad (5.10)$$

Apparently, no microscopic open string state can transform in the singlet representation of $U(3)$ and nevertheless having $U(1)$ charge $q = 3$. Note, that for the (d, s, b) quarks and the leptons the relevant Yukawa coupling is

$$H \bar{Q}_L d_R, \quad H \bar{l}_L e_R, \quad H^* \bar{l}_L \nu_R \quad (5.11)$$

leading to the quantum numbers $(\mathbf{1}, \mathbf{2})_{(0,1,1)}$ for the Higgs fields H , which at least is not in contradiction to the open string origin of the model. We conclude, that in open string models where the (u, c, t) -quarks arise from open strings in the antisymmetric representation of $U(3)$, there appears a problem with the usual Higgs mechanism.

In a very similar fashion, in the $SU(5) \times U(1)$ GUT model the $U(1)_{\text{mass}}$ does not allow Yukawa couplings of the type $\mathbf{10} \cdot \mathbf{10} \cdot \mathbf{5}$ so that the standard mass generation mechanism does not work. Again we are drawn towards a more exotic version of gauge symmetry breaking and mass generation. The only resolution to this obstacle is to propose that the Higgs fields are not fundamental fields but composite objects with the quantum numbers given in (5.10). This possibility is further supported by the analysis in sections 5.5 and 5.6.

Finally, let us discuss the generation of neutrino masses. A scalar h in the ‘symmetric’ representation of the $U(1)_3$ can break the $U(1)_{B-L}$ symmetry via the Higgs mechanism, but does not directly lead to a Yukawa coupling of Majorana type for the right moving neutrinos. However, the dimension five coupling

$$\frac{1}{M_s} (h^*)^2 (\bar{\nu}^c)_L \nu_R \quad (5.12)$$

is invariant under all global symmetries and leads to a Majorana mass for the right moving neutrinos. Together with the above mentioned (to be found) composite Higgs mechanism for the standard Higgs field this in principle allows the realization of the see-saw mechanism to generate small neutrino masses. This is in contrast to the neutrino sector as found in the toroidal models [12] where only neutrino masses of Dirac type could be generated due to the conservation of the lepton number.

5.5. Gauge couplings

We now comment on the patterns of gauge coupling unification. By dimensional reduction the $U(N_a)$ gauge couplings are given by

$$\frac{4\pi^2}{g_a^2} = \frac{M_s}{g_s} \prod_{I=1}^3 L_a^I \quad (5.13)$$

where g_s is the string coupling. Using for the abelian subgroups $U(1)_a \subset U(N_a)$ the usual normalization $\text{tr}(Q_a^2) = 1/2$, the gauge coupling for the hypercharge

$$Q_Y = \sum_a c_a Q_a \quad (5.14)$$

is given by

$$\frac{1}{g_Y^2} = \sum_a \frac{1}{4} \frac{c_a}{g_a^2}. \quad (5.15)$$

Thus, in our case we get with

$$Q_Y = -\frac{2}{3}U(1)_1 + U(1)_2 = -\frac{2}{3}\sqrt{6}\widetilde{U(1)}_1 + 2\widetilde{U(1)}_2 \quad (5.16)$$

for the Weinberg angle

$$\sin^2 \vartheta_W = \frac{3}{6 + 2\frac{g_2}{g_1}}. \quad (5.17)$$

The tilded $U(1)$ s in (5.16) denote the correctly normalized ones. Since in all interesting cases the $U(3)$ branes have the same internal volumes than the $U(2)$ branes, (5.17) reduces to the prediction $\sin^2 \vartheta_W = 3/8$, which is precisely the $SU(5)$ GUT result. Note, that in contrast to the toroidal intersecting brane world scenario, here the Weinberg angle is completely fixed by the wrapping numbers of the D6-branes. Thus, these models are more predictive and, of course, easier to falsify.

As usual, in order for the gauge couplings and the string coupling to be of order one at the string scale, the sizes of the tori are forced to be of order the string scale. In principle, by blowing up the 27 orbifold fixed points we can realize a large extra dimension scenario with arbitrary string scale. The gauge couplings remain of order one as the D6-branes can avoid the blown up \mathbb{P}^1 s, whereas the Planck scale gets large due to the large overall volume of the compactification manifold. However, the above result for the Weinberg angle rather suggests that the string scale is close to the GUT scale.

In order to compare the gauge couplings to their experimental values at the weak scale, one has to include in the beta-function all states with masses between the weak and the string scale. For a detailed analysis we would need the precise masses of all fields. Some of these masses are due to loop correction as for instance for the superpartners in the $\mathcal{N} = 4$ vectormultiplet, other masses are already there at tree level like for the lowest energy scalars in the non-supersymmetric open string NS sectors.

Thus, the Standard Model gauge couplings run up to the string scale in the same way as in the non-supersymmetric $SU(5)$ GUT model. However, around 1 or 2 orders of magnitude below the string scale a lot of new states will begin to contribute to the beta-function and change the running considerably. Since all the states from the $\mathcal{N} = 4$ vectormultiplets might contribute we expect the one-loop beta-function even to change sign. Thus, at least in principle it is not excluded that the non-supersymmetric $SU(5)$ model will feature gauge coupling unification.

5.6. Proton decay

In the three generation models in [12] the decay of the proton was prohibited, as the baryon and lepton numbers survived the Green-Schwarz mechanism as separate global symmetries. In our orbifold models only the combination $B - L$ appears as a symmetry, so that there are potential problems with the stability of the proton.

In [10] it was argued that perturbatively the proton is stable in intersecting brane models, as effective couplings with three quarks are forbidden as long as the quark fields appear in bifundamental representations of the stringy gauge group. Apparently, also this argument does not directly apply to our case. Indeed the disc diagram in figure 5 generates a dimension six coupling

$$\mathcal{L} \sim \frac{1}{M_s^2} (\bar{u}_L^c u_L) (\bar{e}_L^+ d_L), \quad (5.18)$$

which preserves $B - L$ but violates baryon and lepton numbers separately. The numbers at the boundary indicate the $D6_a$ -brane to which the boundary of the disc is attached.

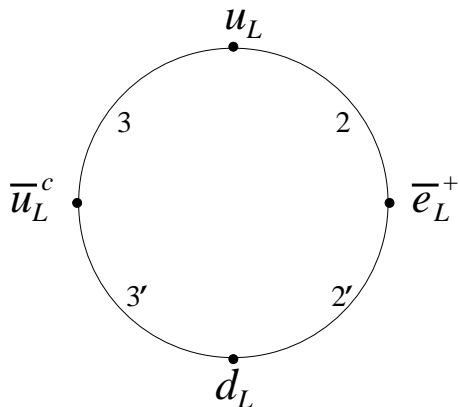


Figure 5

Thus we conclude, that, as long as we want to work in the large extra dimension scenario with $M_s \ll 10^{16}\text{GeV}$, these models do have serious problems with proton decay. Said differently, also the issue of proton decay leads one to chose the string scale rather at the GUT scale than in the TeV region.

6. Conclusions

In this paper we have discussed the issue of stability of toroidal intersecting brane worlds to the next to leading order in string perturbation theory. The arising instability for

the geometric parameters of the internal torus was cured in the case of specific orbifold models where the complex structure is completely frozen. We have studied such a partly stabilized \mathbb{Z}_3 orbifold model in great detail and focused on the derivation of the Standard Model. It was possible in this, compared to the toroidal case, fairly constrained set-up to construct a three generation Standard Model extended by right-handed neutrinos and a gauged $U(1)_{B-L}$ symmetry. A detailed study of the tachyon spectra of these models revealed that it was impossible to realize the entire minimal set of Higgs scalars as open string tachyons. Thus we had to propose an alternative composite operator instead of the fundamental Higgs field to achieve the electroweak symmetry breaking. Furthermore, it turned out that all the models with the required Higgs to break the $U(1)_{B-L}$ at the string scale are obtained from an $SU(5) \times U(1)$ GUT model by the condensation of an adjoint Higgs which is massless at tree level. Thus, the present class of models appears to be naturally unified in terms of the common $SU(5)$ scenario.

The analysis of more detailed phenomenological issues uncovered some important deviations from the ordinary Standard Model due to extra global symmetries, remnants of a larger gauge symmetry after a Green-Schwarz anomaly cancellation mechanism. The most serious one is surely the absence of appropriate Yukawa couplings for the (u, c, t) -quarks to generate masses via a fundamental Higgs condensation. It appears to be a generic feature that whenever the Standard Model or GUT matter fields do not exist in the string spectrum as bifundamental fields exclusively, global symmetries forbid some of the Yukawa couplings required in the standard mass generation process. Therefore, the only resolution seems to consist of a model with composite Higgs which presumably circumvents the gauge hierarchy problem simultaneously. Unfortunately, there is no direct way to prove the existence of such a composite operator with a condensate at the electroweak scale.

Moreover, without the string scale being of the same order as the GUT scale there would appear problems with proton-decay and gauge coupling unification. Thus, it appears that the natural scale for this intersecting brane model on an orbifold is not the TeV scale but the GUT scale.

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Appendix A. Effective wrapping numbers

In this appendix we present the precise form of the effective wrapping numbers, $Y_{[a]}$ and $Z_{[a]}$, in terms of the fundamental wrapping numbers, (n_a^I, m_a^I) .

AAA torus

$$\begin{aligned}
Z_{[a]} &= n_a^1 n_a^2 n_a^3 + \frac{1}{2} m_a^1 n_a^2 n_a^3 + \frac{1}{2} n_a^1 m_a^2 n_a^3 + \frac{1}{2} n_a^1 n_a^2 m_a^3 - \frac{1}{2} m_a^1 m_a^2 n_a^3 \\
&\quad - \frac{1}{2} m_a^1 n_a^2 m_a^3 - \frac{1}{2} n_a^1 m_a^2 m_a^3 - m_a^1 m_a^2 m_a^3 \\
Y_{[a]} &= n_a^1 m_a^2 m_a^3 + m_a^1 n_a^2 m_a^3 + m_a^1 m_a^2 n_a^3 + n_a^1 n_a^2 m_a^3 + n_a^1 m_a^2 n_a^3 + m_a^1 n_a^2 n_a^3
\end{aligned} \tag{A.1}$$

AAB torus

$$\begin{aligned}
Z_{[a]} &= n_a^1 n_a^2 n_a^3 + \frac{1}{2} m_a^1 n_a^2 n_a^3 + \frac{1}{2} n_a^1 m_a^2 n_a^3 + \frac{1}{2} n_a^1 n_a^2 m_a^3 - \frac{1}{2} m_a^1 m_a^2 n_a^3 - m_a^1 m_a^2 m_a^3 \\
Y_{[a]} &= m_a^1 m_a^2 m_a^3 + 2 n_a^1 m_a^2 m_a^3 + 2 m_a^1 n_a^2 m_a^3 + n_a^1 n_a^2 m_a^3 + 3 m_a^1 m_a^2 n_a^3 \\
&\quad + 3 n_a^1 m_a^2 n_a^3 + 3 m_a^1 n_a^2 n_a^3
\end{aligned} \tag{A.2}$$

ABB torus

$$\begin{aligned}
Z_{[a]} &= n_a^1 n_a^2 n_a^3 + \frac{1}{2} n_a^1 n_a^2 m_a^3 + \frac{1}{2} n_a^1 m_a^2 n_a^3 + \frac{1}{2} m_a^1 n_a^2 n_a^3 + \frac{1}{6} n_a^1 m_a^2 m_a^3 - \frac{1}{6} m_a^1 m_a^2 m_a^3 \\
Y_{[a]} &= 3 \left(m_a^1 m_a^2 m_a^3 + n_a^1 m_a^2 m_a^3 + 2 m_a^1 n_a^2 m_a^3 + 2 m_a^1 m_a^2 n_a^3 + n_a^1 n_a^2 m_a^3 \right. \\
&\quad \left. + n_a^1 m_a^2 n_a^3 + 3 m_a^1 n_a^2 n_a^3 \right)
\end{aligned} \tag{A.3}$$

BBB torus

$$\begin{aligned}
Z_{[a]} &= n_a^1 n_a^2 n_a^3 + \frac{1}{2} n_a^1 n_a^2 m_a^3 + \frac{1}{2} n_a^1 m_a^2 n_a^3 + \frac{1}{2} m_a^1 n_a^2 n_a^3 + \frac{1}{6} n_a^1 m_a^2 m_a^3 \\
&\quad + \frac{1}{6} m_a^1 n_a^2 m_a^3 + \frac{1}{6} m_a^1 m_a^2 n_a^3 \\
Y_{[a]} &= 3 \left(2 m_a^1 m_a^2 m_a^3 + 3 n_a^1 m_a^2 m_a^3 + 3 m_a^1 n_a^2 m_a^3 + 3 m_a^1 m_a^2 n_a^3 + 3 n_a^1 n_a^2 m_a^3 \right. \\
&\quad \left. + 3 n_a^1 m_a^2 n_a^3 + 3 m_a^1 n_a^2 n_a^3 \right)
\end{aligned} \tag{A.4}$$

Appendix B. Intersecting branes on the 6D \mathbb{Z}_3 orbifold

In this appendix we summarize the results for the tadpole cancellation conditions and the massless spectra for the six-dimensional \mathbb{Z}_3 orbifolds. The orbifold action on two complex coordinates is

$$Z_1 \rightarrow e^{2\pi i/3} Z_1, \quad Z_2 \rightarrow e^{-2\pi i/3} Z_2. \quad (\text{B.1})$$

As in the four-dimensional case we can distinguish between the two differently oriented tori, **A** and **B**, so that in this case we get the three different models, **AA**, **BB** and **AB**. For the six-dimensional closed string spectrum with $\mathcal{N} = (0, 1)$ supersymmetry one gets besides the supergravity multiplet

$$\begin{aligned} \mathbf{AA} &: 8 \times \text{tensors} + 13 \times \text{hypers}, \\ \mathbf{AB} &: 6 \times \text{tensors} + 15 \times \text{hypers}, \\ \mathbf{BB} &: 21 \times \text{hypers}. \end{aligned} \quad (\text{B.2})$$

Similar to the four-dimensional case we can define the following quantity

$$Z_{[a]} = \frac{1}{3} \sum_{(n_b^I, m_b^I) \in [a]} \prod_{I=1}^2 \left(n_b^I + \frac{1}{2} m_b^I \right) \quad (\text{B.3})$$

which for the three different tori read

$$\begin{aligned} \mathbf{AA} &: Z_{[a]} = n_a^1 n_a^2 + \frac{1}{2} n_a^1 m_a^2 + \frac{1}{2} m_a^1 n_a^2 - \frac{1}{2} m_a^1 m_a^2, \\ \mathbf{AB} &: Z_{[a]} = n_a^1 n_a^2 + \frac{1}{2} n_a^1 m_a^2 + \frac{1}{2} m_a^1 n_a^2 + \frac{1}{6} m_a^1 m_a^2, \\ \mathbf{BB} &: Z_{[a]} = n_a^1 n_a^2 + \frac{1}{2} n_a^1 m_a^2 + \frac{1}{2} m_a^1 n_a^2. \end{aligned} \quad (\text{B.4})$$

Then, the RR-tadpole cancellation condition can be expressed as

$$\sum_a N_a Z_{[a]} = 4. \quad (\text{B.5})$$

Moreover, we define

$$\begin{aligned} \mathbf{AA}, \mathbf{BB} &: Y_{[a]} = n_a^1 m_a^2 + m_a^1 n_a^2 + m_a^1 m_a^2, \\ \mathbf{AB} &: Y_{[a]} = \frac{1}{2} n_a^1 m_a^2 + \frac{3}{2} m_a^1 n_a^2 + m_a^1 m_a^2 \end{aligned} \quad (\text{B.6})$$

and the L_a^I as in equation (4.14). Then the chiral massless spectra in the $(\mathbf{1}, \mathbf{2})$ representation of the little group $SO(4) = SU(2) \times SU(2)$ for the three different four-dimensional tori read:

AA torus

Rep.	Number
$(N_a, \overline{N}_b) + c.c.$	$2Z_{[a]} Z_{[b]} + \frac{3}{2}Y_{[a]} Y_{[b]}$
$(N_a, N_b) + c.c.$	$2Z_{[a]} Z_{[b]} - \frac{3}{2}Y_{[a]} Y_{[b]}$
$A_a + c.c.$	$2Z_a$
$A_a + S_a + c.c.$	$2Z_{[a]}^2 - Z_{[a]} - \prod_I (L_a^I)^2$
$\text{Adj}_a + c.c.$	$\prod_I (L_a^I)^2$

Table B1: *Chiral fermions for the AA torus .*

AB torus

Rep.	Number
$(N_a, \overline{N}_b) + c.c.$	$6Z_{[a]} Z_{[b]} + 2Y_{[a]} Y_{[b]}$
$(N_a, N_b) + c.c.$	$6Z_{[a]} Z_{[b]} - 2Y_{[a]} Y_{[b]}$
$A_a + c.c.$	$6Z_{[a]}$
$A_a + S_a + c.c.$	$6Z_{[a]}^2 - 3Z_{[a]} - 3 \prod_I (L_a^I)^2$
$\text{Adj}_a + c.c.$	$3 \prod_I (L_a^I)^2$

Table B2: *Chiral fermions for the AB torus .*

BB torus

Rep.	Number
$(N_a, \overline{N}_b) + c.c.$	$18Z_{[a]} Z_{[b]} + \frac{3}{2}Y_{[a]} Y_{[b]}$
$(N_a, N_b) + c.c.$	$18Z_{[a]} Z_{[b]} - \frac{3}{2}Y_{[a]} Y_{[b]}$
$A_a + c.c.$	$18Z_{[a]}$
$A_a + S_a + c.c.$	$18Z_{[a]}^2 - 9Z_{[a]} - 9 \prod_I (L_a^I)^2$
$\text{Adj}_a + c.c.$	$9 \prod_I (L_a^I)^2$

Table B3: *Chiral fermions for the BB torus .*

Let us check explicitly the cancellation of the F^4 and R^4 anomaly to provide an additional check of the consistency of the construction. For the F^4 anomaly of the $U(N_a)$ gauge group we get

$$\begin{aligned} & \mathfrak{3}^{\mathbf{n}_b} \left(\sum_{b \neq a} 4 N_b Z_{[a]} Z_{[b]} + 2(N_a - 8)Z_{[a]} + 2 N_a (2 Z_{[a]}^2 - Z_{[a]}) \right) = \\ & \mathfrak{3}^{\mathbf{n}_b} \left(4 Z_{[a]} (4 - N_a Z_{[a]}) - 16 Z_{[a]} + 4 N_a Z_{[a]}^2 \right) = 0. \end{aligned} \quad (\text{B.7})$$

The R^4 anomaly reads

$$\begin{aligned} & \mathfrak{3}^{\mathbf{n}_b} \left(\frac{1}{2} \sum_{b \neq a} 4 N_a N_b Z_{[a]} Z_{[b]} + \sum_a \frac{N_a(N_a - 1)}{2} 2 Z_{[a]} + \sum_a N_a^2 (2 Z_{[a]}^2 - Z_{[a]}) \right) = \\ & \mathfrak{3}^{\mathbf{n}_b} \left(\frac{1}{2} \sum_a 4 N_a Z_{[a]} (4 - N_a Z_{[a]}) - \sum_a N_a Z_{[a]} + \sum_a 2 N_a^2 Z_{[a]}^2 \right) = 28 \cdot \mathfrak{3}^{\mathbf{n}_b}, \end{aligned} \quad (\text{B.8})$$

which is precisely what one needs to cancel the R^4 anomaly resulting from the closed string spectrum in (B.2).

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