Pilot-Aided Channel Estimation for WiMAX 802.16e Downlink Partial Usage of Subchannel System Using Least Squares Line Fitting

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SUMMARY This paper presents a pilot-aided channel estimation method which is particularly suitable for mobile WiMAX 802.16e Downlink Partial Usage of Subchannel mode. Based on this mode, several commonly used channel estimation methods are studied and the method of least squares line fitting is proposed. As data of users are distributed onto permuted clusters of subcarriers in the transmitted OFDMA symbol, the proposed channel estimation method utilizes these advantages to provide better performance than conventional approaches while offering remarkably low complexity in practical implementation. Simulation results with different ITU-channels for mobile environments show that depending on situations, enhancement of 5 dB or more in term of SNR can be achieved.

key words: channel estimation, IEEE 802.16e, OFDMA, downlink PUSC, least squares line fitting

1. Introduction

Worldwide interoperability for microwave access (WiMAX) is a promising technology that was predicted to be the great leap toward broadband metropolitan area wireless network. This technology is specified in IEEE Std. 802.16\texttextsuperscript{TM}, in which the orthogonal frequency division multiplexing (OFDM) and orthogonal frequency division multiplexing multiple access (OFDMA) techniques are used as modulation methods. There are two different standards, IEEE Std. 802.16d\texttextsuperscript{2004} and IEEE Std. 802.16e\texttextsuperscript{2005} for fixed and mobile environments respectively [1], [2].

OFDM is a modulation technique that has been proved to be very effective in utilizing the transmission bandwidth and superior to other techniques for mitigating the impairment of severely frequency-selective channel [3]. DAB and DVB are two significant systems that have adopted OFDM [4]. As shown in Fig. 1, OFDMA is an OFDM-based multiple access technique which provides several users with clusters of subcarriers and scatters them on different locations over the OFDM symbol so that each user has a low probability to be damaged rigorously by the fading channel. Besides, in IEEE Std. 802.16e, adaptive modulation scheme is introduced, providing a maximized adaptable transmission rate according to the quality of the wireless channel.

Signal propagating through wireless channel is usually influenced by multi-path effect causing different fluctuation on frequency components of the transmitted OFDM symbol. In mobile environment, the channel condition also changes proportionally to the moving speed. In order to recover the original signal at the receiver, transfer function of the channel (CTF) has to be estimated. Channel estimation can be carried out by exploiting known data called pilots which are scattered at predefined locations inside OFDM symbols. Figure 2 shows a typical OFDMA transmission system.

Several pilot-aided channel estimations have been proposed and some of them are commonly used in practice [5]–[10]. The principle is to estimate the CTF at pilot locations and then perform some interpolation technique to get the whole CTF for all data locations. These studies showed a trade-off between performance and computational complexity for implementation. In [5], the piecewise constant and piecewise linear interpolation approaches were suggested which give fair performance and low complexity. Other methods [5], [6] combined linear interpolation and averag-
ing to realize higher performance while maintaining a low computational requirement. The optimum solution, minimum mean squared error (MMSE) estimator, was also studied in [7]–[10]. This approach theoretically can achieve the best performance but requires extremely heavy computation. Some parameters in MMSE are adequately assumed to be known in advance or need measuring and tracking continuously in order to guarantee a converged result. For IEEE Std. 802.16e, there are some studies to prove that fixed-coefficient MMSE method offers a good result and slightly decreases the complexity [9], or maximum likelihood (ML) is preferable since it has relatively comparable performance as MMSE but simpler implementation [10]. However, those works were interested in preamble symbol, which carries only pilots and is usually used for synchronization, rather than data symbols.

This study focuses on the downlink partial usage of subchannels (DL-PUSC) physical layer of IEEE 802.16e standard in order to propose a suitable pilot-aided channel estimation method for data-carrying symbols. The next parts are presented as follows. In Sect. 2, OFDMA signal model is explained in detail. Conventional channel estimation techniques are reviewed in Sect. 3. Section 4 proposes the least squares line fitting (LSLF) based channel estimation technique. Simulation results are shown in Sect. 5 and implementation complexity is discussed in Sect. 6. Finally, Sect. 7 summarizes and concludes the paper.

2. Signal Model

Figure 3 shows the pattern of DL-PUSC mode using 1024-subcarrier OFDM symbol in which 14 consecutive subcarriers form a cluster and every 2 successive clusters are grouped to make a subchannel which is assigned for a specific user. Depending on the number of users that require service at a particular period, each of them can be allocated onto one or more subchannels. Before transmitting, clusters are permuted among themselves so that information from each user is spread over the OFDM symbol, avoiding data loss whenever the channel is deeply faded. The technique is the reason why, at the receiver, the channel estimation and equalization should be performed for each cluster-size data block separately.

The pilot pattern in DL-PUSC mode is shown in Fig. 4 where their positions are different depending on which OFDM symbol they belong to (even or odd). The first data-carrying OFDM is considered as symbol 0.

Assume from base station to mobile station, $M$ OFDM symbols are transmitted: $[x_m]$, $0 \leq m \leq M - 1$ in which $x_m = [x_n]_m$, $0 \leq n \leq N - 1$ is the symbol at time $m$, $N$ is the number of subcarriers, and $[.]^T$ denotes vector transpose. The multi-path channel impulse response (CIR) at symbol time $m$ is expressed as follows:

$$g_m(t) = \sum_{l=0}^{L-1} \alpha_{l,m} \delta(t - \tau_{l,m})$$  \hspace{1cm} (1)

$L$ is the number of paths, $\alpha_l$, and $\tau_l$ are the amplitude and delay of the $l$th path respectively. $\delta$ is the delta Dirac function. In order to avoid inter symbol interference (ISI), the maximum delay spread $\tau_{L-1}$ should not exceed the guard interval. The CIR after sampling at system frequency and zero-padding is given by:

$$f_m = [g_n]_m^T, 0 \leq n \leq N - 1$$  \hspace{1cm} (2)

The CTF or the channel frequency response is obtained by performing the discrete Fourier transform (DFT) of this sampled CIR $h_m = Fg_m$ where $F$ is $N \times N$ DFT matrix with entries $f_{nk} = \frac{1}{\sqrt{N}} e^{-j2\pi nk/N}$, $0 \leq k, n \leq N - 1$.

At the receiver, suppose that ISI is negligible, the $m$th OFDM symbol is obtained as follows:

$$y_m = X_m h_m + w_m$$  \hspace{1cm} (3)

where $X_m$ is the $N \times N$ diagonal matrix whose values are $x_m$, and $w_m$ is the additive white Gaussian noise.

To recover $x_m$ from $y_m$, the CTF $h_m$ has to be estimated. This task can be performed by calculating the least squared channel values at pilot positions $n_p$.
This time interpolation can be expressed by:

\[
\hat{h}_{nm} = \frac{[y_{nm}]_m}{[n_{nm}]_m}
\]  

(4)

and then other values corresponding to data subcarriers can be derived by using some interpolating techniques. Our goal is to obtain an estimated version \( \hat{h}_m \) as close as possible to \( h_m \). Note that for this particular DL-PUSC system, channel estimation must be carried-out in cluster-size data block. Hence, in (4) \( n_p = [0, 12] \) at even symbol and \( [4, 8] \) at odd symbol. Since data of a user is spread over non-connected clusters and each cluster has only 14 subcarriers including 2 pilots, an idea can be drawn out that the CTF in a cluster-long interval can be considered as a line although it can be very frequency-selective in a whole OFDM symbol. Therefore, some simple interpolation techniques can be utilized whereas the MMSE estimator [7], [9]–[11] appears inadequate in term of performance enhancement and complexity.

3. Conventional Channel Estimation

Since clusters belonging to a specific user remain the same locations inside the stream of OFDM symbols, this helps achieve a time-frequency channel estimation which can be done either in 2-dimension form or in two successive 1-dimension forms. In this paper, the latter approach is used, time interpolation is performed first and frequency interpolation follows after. As soon as the channel values at pilot positions are obtained from (4), some conventional techniques such as linear [5], [6], [12] or cubic-spline [13], [14] interpolation can be utilized to estimate channel values at data subcarriers.

3.1 Linear Interpolation

Figure 5 demonstrates an example of the linear interpolation technique in time direction for the same-located clusters in a stream of an even number of OFDM symbols. Depending on which OFDM symbol it belongs to, besides its own pilots, each cluster can have two more channel values at the positions where the pilots of the adjacent clusters are located. This time interpolation can be expressed by:

\[
\hat{h}_{(0,12),m} = \begin{cases} 
\hat{h}_{(0,12),0} , & m = 0 \\
\frac{\hat{h}_{(0,12),m-1} + \hat{h}_{(0,12),m+1}}{2}, & m = 2, 4, \ldots, M-2 
\end{cases} 
\]  

(5)

\[
\hat{h}_{(4,8),m} = \begin{cases} 
\hat{h}_{(4,8),1} , & m = 0 \\
\frac{\hat{h}_{(4,8),m-1} + \hat{h}_{(4,8),m+1}}{2}, & m = 2, 4, \ldots, M-2 
\end{cases} 
\]  

(6)

Then, interpolation in frequency direction can be done by evaluating:

\[
\hat{h}_{np} = \frac{k}{4} \hat{h}_{np,m} + \frac{4-k}{4} \hat{h}_{np+1,m}
\]

\[k = n_p + 1, \ldots, n_{p+1} - 1; \]

\[n_p = [0, 4, 8, 12]; \quad p = 0, \ldots, 3 \]  

(7)

where \( k \) denotes the positions of channel values inside the interval of two consecutive pilots.

3.2 Cubic-Spline Interpolation

This method interpolates the channel values by using a third-order function and requires a sufficient number of pilots. For this reason, time interpolation must be performed first with a stream of at least 8 OFDM symbols; afterward, frequency interpolation can be carried out. As indicated in [13]–[15], from a group of 4 pilots, a set of parameters is calculated to form a third-order equation which is used to estimate the channel values at data positions. After this step, each cluster will have 4 channel values, enough to repeat the same procedure in frequency direction.

4. LSLF Channel Estimation

In this section, a new method based on LSLF technique is proposed to estimate the partial CTF. Despite the fact that the channel can be very frequency-selective, since a cluster size is small, the CTF over a cluster can be considered as a ‘line.’ Therefore, this ‘line’ can be estimated by a least squares line going through all channel values at pilot positions. Figure 6 illustrates an example where a line needs to be reconstructed from a given set of noise-added points (representing pilots in this case) and LSLF technique highly outperforms others.

4.1 Interpolation in Time

Suppose there is a stream of an even number of OFDM symbols indexed from 0 to \( M-1 \). In time direction, there are four sub-streams (which associate to subcarrier indexes: 0, 4, 8, and 12) containing channel values at pilot positions. Denote \( \mathbf{c} = [c_m]^T \) and \( \mathbf{p} = [p_m]^T \) are the channel values vector at pilot positions and pilot positions vector respectively, where \( m = 1, 2, \ldots, M/2 \) and \( \mathbf{p} = [0, 2, \ldots, M-2]^T \) for stream 4, 8 and \( \mathbf{p} = [1, 3, \ldots, M-1]^T \) for stream 0, 12.

LSLF technique is used to find the pair of coefficients \( \omega = [a, b]^T \) to create a line containing the set of points \( z \)
= \lfloor z_m \rfloor; \quad z_m = ap_m + b \) so that the least squares error:
\[ s = \sum_{m=1}^{M/2} (c_m - z_m)^2 = \sum_{m=1}^{M/2} (c_m - ap_m - b)^2 \] is minimized. That means it is necessary to find two coefficients \( a, b \) in order to make \( s \) minimum, leading to vanishing the partial derivatives \( \frac{\partial s}{\partial a} \) and \( \frac{\partial s}{\partial b} \). Therefore, from [16], this problem turns into solving the following system of equations:
\[ \begin{align*}
\frac{\partial s}{\partial a} &= 0 \\
\frac{\partial s}{\partial b} &= 0
\end{align*} \]
\[ (8) \]
\[ \Rightarrow \]
\[ \begin{align*}
a &= \frac{M}{2} \sum_{m=1}^{M/2} p_m c_m - \sum_{m=1}^{M/2} p_m \sum_{m=1}^{M/2} c_m \\
b &= \sum_{m=1}^{M/2} p_m \sum_{m=1}^{M/2} c_m - \sum_{m=1}^{M/2} p_m \sum_{m=1}^{M/2} c_m \\
&= \frac{M}{2} \sum_{m=1}^{M/2} p_m - \left( \sum_{m=1}^{M/2} p_m \right)^2
\end{align*} \]
\[ (9) \]
From [17], \( \omega = [a, b]^T \) also can be derived in matrix-form by solving the equation below:
\[ \omega = (A^T A)^{-1} A^T c \]
\[ (10) \]
including data and pilots in stream 0, 4, 8 and 12 will be calculated by applying \( h_m' = ap_m' + b \). Channel values at all locations, \( M \) as shown in Fig. 7. At this point, it is crucial to examine the maximum number of OFDM symbols \( M \) so that the fitting by using a ‘line’ is favorable. By the fact that the fading channel will change in time with a coherent time \( T_c \), it is clear to see the limit should be \( MT_c \text{symbol} < T_c \) whereas \( T_c \sim 1/f_D \) [18] with \( f_D \) is the Doppler frequency shift that depends on the relative moving speed between base station and mobile station. So a rough limit range for \( M \) can be: \( 4 \leq M < T_c/T_{\text{symbol}} \) and for convenient \( M \) should be an even number.

4.2 Interpolation in Frequency

The same procedure used in time interpolation can be applied for frequency estimation since now there is a block of \( M \) clusters in which each of them contains 4 channel values that have been estimated from the previous stage. A difference in this step is that all clusters have ‘pilots’ (estimated channel values) at the same indexes; as a result, complexity is less.

The limitation of using LSLF for frequency interpolation is the coherent bandwidth \( B_c \) of the multi-path fading channel. From [18], \( B_c \) is proportional to \( 1/\sigma_r \) in which \( \sigma_r \) denotes the root mean squared delay spread of the channel. For LSLF method working properly, the frequency bandwidth covering 14 subcarriers of a cluster should be less than \( B_c \).

5. Simulation Result

The main simulation parameters for the IEEE 802.16e DL-PUSC mode are given in Table 1. Transmitted and received symbol streams are assumed synchronized perfectly.

Performance analysis is studied in term of system Bit Error Rate (BER) and estimated channel Mean Squared Error (MSE) versus Signal to Noise Ratio (SNR). The chosen multi-path fading channels are the well known ITU (International Telecommunications Union) [19] channels recommended for mobile environments. Case 1 is the ITU-Pedestrian B (Ped.B) model with limited speed but strongly affected by multi-path phenomenon as longer delay spread whereas case 2 is the ITU-Vehicular A (Veh.A) model with much higher speed and less delay. These channel models represent the typical situations might happen in practice. In literatures, CTF is usually assumed unchanged within a symbol time since the symbol period is considerably less than the channel coherent time. However, in this paper, all the channels are modeled as if they changed continuously within the OFDM symbol. This gives not so much difference from the former but is closer to the real channel effect.
### Table 1  Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>FFT size</td>
<td>1024</td>
</tr>
<tr>
<td>Number of used subcarriers</td>
<td>840</td>
</tr>
<tr>
<td>OFDM symbol time $T_s$</td>
<td>115.2 $\mu$s</td>
</tr>
<tr>
<td>Useful symbol time $T_b$</td>
<td>102.4 $\mu$s</td>
</tr>
<tr>
<td>Subcarrier spacing</td>
<td>9.765625 KHz</td>
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<tr>
<td>Cyclic Prefix</td>
<td>1/8</td>
</tr>
<tr>
<td>Number of symbol</td>
<td>48</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>2.3 GHz</td>
</tr>
<tr>
<td>Symbol BW</td>
<td>8.75 MHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>$8.75 \times \frac{8}{7} = 10$ MHz</td>
</tr>
<tr>
<td>Modulation mode</td>
<td>QPSK, 16-QAM and 64-QAM</td>
</tr>
</tbody>
</table>

### Table 2  Channel profiles used in simulations.

#### Case 1 (speed 12 km/h)

<table>
<thead>
<tr>
<th>Path Power(dB)</th>
<th>-3.9</th>
<th>-4.8</th>
<th>-8.8</th>
<th>-11.9</th>
<th>-11.7</th>
<th>-27.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Delay($\mu$s)</td>
<td>0</td>
<td>0.2</td>
<td>0.8</td>
<td>1.2</td>
<td>2.3</td>
<td>3.7</td>
</tr>
</tbody>
</table>

#### Case 2 (speed 30, 120, 150 km/h)

<table>
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<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Path Delay($\mu$s)</td>
<td>0</td>
<td>0.31</td>
<td>0.71</td>
<td>1.09</td>
<td>1.73</td>
<td>2.51</td>
</tr>
</tbody>
</table>

Parameters of these channels are shown in Table 2.

At first glance, LSLF approach clearly surpassed the other 2 conventional methods. For instance, with 16-QAM modulation mode, the gain is about 3 dB at BER = 10^{-3} dB and can be up to 5 dB at BER = 10^{-5} dB in all typical channels as shown in Fig. 8, Fig. 10, and Fig. 12 whereas it is very much better in the case of 16-QAM Veh.A 150 km/h (Fig. 14) since the conventional methods’ performances tend to saturate. MSEs figures again confirmed this with noticeable improvement in all modulation modes and channel models. The results showed that the proposed method performs very well and robustly in various channel conditions.

Figures 8, 9 show the results for Ped.B channel. According to [18], the coherent bandwidth is roughly calculated to be 530 KHz. In this particular DL-PUSC system, a cluster just covers a bandwidth of 137 KHz. Therefore, there is no performance limitation in frequency interpolation. Besides, since the moving speed is quite slow, the coherent time of the channel is much greater than the symbol stream time, as a result, no problem for time interpolation as well. LSLF shows at least 3 dB performance improvement toward other methods.

Figures 10–15 show results for Veh.A channel with different moving speed. Also from [18], the Doppler frequency shift at 120 km/h is about 255 Hz (suppose 2.3 GHz carrier is used), corresponding to a channel coherent time of 0.78 ms whereas a block of 8 OFDM symbol used for channel estimation in the simulation lasts about 0.92 ms. Thus, the performance slightly degrades in the case of 120 km/h compared to 30 km/h. The clearer degradation can be seen in the case of 150 km/h. However, one interesting note is that the LSLF method always greatly outperforms other methods. In some high SNR cases, it can reach the performance of the ideally-estimated channel. In the case of 150 km/h, performance of LSLF still keeps the downward tendency whereas others are already saturated.
Note that in all cases, cubic-spline method gives the worst performance. This fact intuitively proves the premise of seeing the CTF over each cluster as ‘line.’

One can notice that the performance of LSLF method is much better than conventional methods, but compared to the ideal case, it degrades when increasing the modulation modes. Apparently, the higher modulation mode is always suffered more from the channel estimation error if the transmit power is kept the same. The channels used in these
simulations are specially considered representing the practical mobile channels where their characteristics limit a lot the performance of pilot-aided channel estimation schemes. Hence, in those conditions, not 64-QAM but QPSK and 16-QAM should be seen as the typical used modulation methods. The results showed that LSLF method can reach (with QPSK) or stay very near (with 16-QAM) to the ideal case.

Figure 16 illustrates different modulation schemes with channel coding (Convolutional code (171,133) rate 1/2) in Veh.A at 120 km/h to give a view of a practical system performance. It can be seen that the performance of 64-QAM is not favorable though LSLF still beats other methods at least 5 dB, it is still more than 10 dB far from the ideal case. However, with 16-QAM, the BER of LSLF is truly superior with 10 dB greater than others and less than 1 dB apart from the ideal case.

6. Complexity Analysis

For complexity analysis and comparison, the linear interpolation and LSLF methods are preferable candidates whereas the cubic-spline technique is eliminated since it performs worst but requires rather high computational resource.

From the equations that perform the estimation task, there is no complex multiplication, just only some complex additions used in linear interpolation method. For convenient, the complex operation can be split into 2 real operations equivalently. Complexity examination is done for 1, and then the complete computation is obtained easily by multiplying the result by 2.

6.1 Complexity of Linear Interpolation Approach

Apply (7) for calculating all channel values for data locations, it shows that \(MN + N_p\) multiplications and the same amount of additions are needed. For the system used in the simulation above, each OFDM symbol requires \(M\times 720\) real multiplications and additions.

6.2 Complexity for Proposed Method Using LSLF

Tables 3, 4 show the computational complexity when LSLF is used with different formulas for a cluster in a block of \(M\) OFDM symbols. Table 5 lists the average computational requirement for performing channel estimation for one OFDM symbol in the system described above by using linear and LSLF interpolation methods.

It is worth making a complexity comparison between the proposed method and the MMSE approach analyzed in [9]. Although the given complexity in [9] is for preamble symbol, it can be inferred easily for data symbols by changing the number of pilot subcarriers in the formula. Considering a complex multiplication is equivalent to 4 real multiplications and 2 real additions, the MMSE scheme requires 403200 real multiplications and 201600 real additions for 1 data symbol. These numbers are extremely higher than both the conventional and the LSLF methods, making the MMSE estimator very difficult for realization.

LSLF approach requires a computation about 4 times, if using (9), more than linear interpolation, but still remarkably small. Thus, the significant performance gain achieved with LSLF makes it favorable for practical implementation. (10) is the matrix-form calculation which is useful for simulation [20] but (9) is preferable for hardware realization as it is simpler and has some reusable terms.

7. Conclusion

This paper proposed a channel estimation scheme using the LSLF technique for DL-PUSC mode of WiMAX 802.16e system. Simulation results showed that the new channel estimator is much better than conventional methods such as linear and cubic-spline interpolation methods for this particular system. Channel estimation performance enhancement is about 5 dB or more in some cases. This approach is also very robust with various standardized channel models for mobile environments. Complexity analysis showed that the computation requirement is about 4 times compared to linear interpolation method but it is still considerable low, yielding a high potential for practical implementation.

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