RESEARCH ARTICLE

Narrow-band interference suppression in wavelet packets based multicarrier multicode CDMA overlay system

Maryam M. Akho-Zahieh and Nasser Abdellatif

Department of Electrical and Computer Engineering, Applied Science Private University, Amman 11931, Jordan

ABSTRACT

The wavelet packets based multicarrier (MC) multicode (MCD) code-division multiple-access (CDMA) transceiver consists of the MCD part, which ensures the transmission for high speed and flexible data rate; the MC part contributing to robustness to frequency-selective fading and flexibility for handling multiple data rates; and wavelet packets (WPs) modulation technique, which contributes to the mitigation of the interference problems. As WPs have lower sidelobes compared with sinusoidal carriers, this system is very effective in reducing the problem of inter-carrier interference. Of course, like any CDMA system, the system can suppress a given amount of interference. This paper considers an interference suppression scheme which will enhance the performance of the system. The receiver employs suppression filters to mitigate the effect of narrow-band jammer interference. The framework for the system and the performance evaluation are presented in terms of the bit error rate and the outage probability over a Nakagami fading channel. Also, we investigate how the performance is influenced by various parameters, such as the number of taps of the suppression filter and the ratio of narrow-band interference bandwidth to the spread-spectrum bandwidth. Finally, the performance of the system is compared with the performance of sinusoidal based MC/MCD-CDMA system denoted Sin-MC/MCD-CDMA. Copyright © 2012 John Wiley & Sons, Ltd.

KEYWORDS
suppression filter; wavelet packet; multicarrier; multicode; CDMA

1. INTRODUCTION

The multicarrier (MC) multicode (MCD) code-division multiple-access (CDMA) system achieves the advantages of both MC-CDMA and MCD-CDMA systems, which are (i) variable data rates without interference scaling and (ii) enhanced robustness to mitigating multipath fading. By combining the properties of MCD and MC techniques, a MC/MCD-CDMA system that uses wavelet packet (WP) as subcarrier is proposed in this paper. The system is denoted as WP-MC/MCD-CDMA.

Previous studies [1,2] have shown that the problems posed by sinusoidal carriers can be solved by WPs. Unlike sinusoids, WPs have many attractive properties including:

1. Much lower sidelobes with negligible sidelobe energy leakage compared with sinusoid carriers.
   This property is effective in suppressing inter-carrier interference (ICI) and multiple-access interference.
2. Naturally orthogonal and well localized in both time and frequency domains, which relaxes the requirement of frequency or time guard between different user signals. In fact, orthogonality is maintained for overlapped WPs in both time and frequency domains. This is an advantage of using WPs to model communication channels that are characterized not only by frequency selectivity but also by time variation.

The WP-MC/MCD-CDMA [1] uses WP as subcarrier instead of sinusoidal function, because of that, it could mitigate the problem of ICI associated with using sinusoidal carrier and suppress interferences caused by multipath fading. Various schemes of MC-CDMA systems have been proposed in recent years, and they can be
classified into two groups. The first group is orthogonal MC in which different adjacent subcarriers are orthogonally overlapped. The second group is disjoint MCs in which all subcarriers are not overlapped. The second group contains two subtypes: in the first type, different data streams modulate different carriers, whereas in the second subtype, all carriers are modulated by the same data stream.

The system proposed in this paper belongs to the second subtype, all carriers are modulated by the same data stream. The system under consideration consists of K active users transmitting data simultaneously to the base station.

As it is well known, the inherent processing gain of CDMA system will, in many cases, provide the system with a sufficient degree of narrow-band interference rejection capability. But, if the interference signal is powerful enough, the conventional receiver is ineffective in mitigating this problem. Such instances of narrow-band interference may be intentional, as in various military applications, or may be unintentional, as in a commercial CDMA overlay scheme. An interference suppression filter (SF) can be employed to reject the narrow-band interference. A wiener-type filter, described in [3–5], employs a tapped delay line structure to first predict and then subtract out the narrow-band interference.

The aim of this paper is to study the effect of narrow-band interference binary phase shift keying (BPSK) waveform on the WP-MC/MCD-CDMA system and employs an SF to reject it, then compare the performance of the two systems WP-MC/MCD-CDMA and Sin-MC/MCD-CDMA with and without SF.

2. SYSTEM MODEL AND DESCRIPTION

The system model under consideration consists of K active users transmitting data simultaneously to the base station. If \( k \) denotes the \( k \)th user, \( j \) denotes the \( j \)th code substream, and \( h \) denotes the \( h \)th WPs superstreams, the model of our system for the \( k \)th user is shown in Figure 1. The data for the \( k \)th user are given by

\[
d_k(t) = d_k^I(t) - jd_k^Q(t) = \sum_{i=-\infty}^{\infty} d_k^I(t) \Pi_{T/JH}(t - i T/JH)
\]

where \( \Pi_{T/JH} \) is a rectangular pulse of duration \( T/JH \). These data have a bit rate of \( H/T \). The \( j \)th data substream corresponding to the \( k \)th user signal is given by

\[
d_{kj}(t) = \sum_{i=-\infty}^{\infty} d_{kj}^I(t) \Pi_{T/JH}(t - i T/JH)
\]

After S/P conversion, the substreams are coded by a set of orthogonal signals \( a_j(t) \), which has a chip rate of \( T_c = HN_c \), and given by

\[
a_j(\tau) = \sum_{i=0}^{N_c-1} a_j^i \Pi_{T_c}(\tau - iT_c)
\]

where \( N_c \) is the code length, and \( T_c \) is the code pulse duration. Note that \( a_j^i \) and \( d_{kj}^i \) belong to \( \{\pm 1\} \) with probabilities \( P(1) = P(-1) = 0.5 \). In order to maintain orthogonality of the coding signals, the maximum number of substreams \( J \) is limited to \( N_c = \frac{T}{T_c} \). The coded

![Figure 1. WP-MC/MCD-CDMA system model.](image-url)
substreams are added, and the resulting signal \( s_k(t) = \sum_{j=1}^{J} a_j(t) d_{kj}(t) \) is again S/P converted into \( H \) superstreams \( b_{kj}(t) \). As \( b_k(t) \) has a bit rate of \( \frac{B}{T} \), \( b_{kj}(t) \) will have a bit rate of \( \frac{B}{T} \).

The next step is the modulation by the \( k \)th user pseudo-random noise (PN) signature sequence

\[
c_k(t) = \sum_{i=0}^{N_n-1} c_k^i \prod_{n=1}^{T_n} (t - iT_n) \quad (4)
\]

where \( N_n \) is the length of the PN code sequence, \( c_k^i \in \{\pm 1\} \) is the \( i \)th bit value with probabilities \( P(1) = P(-1) = 0.5 \), and \( T_n = \frac{T}{N_n} \) is the chip duration. Each of the spreading superstreams will be used to modulate a WPs \( w_{ph}(t) \) given by

\[
w_{ph}(t) = \sum_i w_i(t - iT_n) \quad (5)
\]

Assuming identical power for all users, the transmitted signal \( s_k(t) \) can be written as

\[
s_k(t) = \sqrt{2P} \sum_{h=1}^{H} \sum_{j=1}^{J} \hat{a}(t - \tau_{kl}) c_k(t) w_{ph}(t) \exp(j\omega_0 t) \quad (6)
\]

where \( d_{kj}(t) \) with period \( T \) is the data symbol of \( k \)th user, \( r \)th is the substream of the \( t \)th superstream, \( \exp(j\omega_0 t) \) is the carrier signal, and \( P \) is the users’ power.

The equivalent impulse response for the channel used in this paper can be written as

\[
h(t) = \sum_{l=1}^{L} \beta_{kl} e^{j\Phi_{kl}} \delta(t - \tau_{kl}) \quad (7)
\]

where \( L \) is the number of propagation paths; \( \beta_{kl}, \Phi_{kl}, \) and \( \tau_{kl} \) are respectively the path gain, the phase delay, and the time delay of \( l \)th path of the \( k \)th user. The phase \( \Phi_{kl} \) is assumed to be uniformly distributed over \([0, 2\pi]\). The path gain model and distribution function depend on the nature of the channel and the propagation environment. We assume that the channel path gain \( \beta_{kl} \) is Nakagami distributed [6].

The output of the channel for the \( k \)th user is given by

\[
y_k(t) = \sqrt{2P} \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{l=1}^{L} \hat{a}(t - \tau_{kl}) c_k(t - \tau_{kl}) w_{ph}(t - \tau_{kl}) \times \beta_{kl} e^{j\Phi_{kl}} \times \left[ d_{kjh}^I(t - \tau_{kl}) \cos(\omega_0 t - \Phi_{kl}) + d_{kjh}^Q(t - \tau_{kl}) \sin(\omega_0 t - \Phi_{kl}) \right] \quad (8)
\]

where the \( khl \) subscripts denote the \( k \)th user, \( j \)th code, \( h \)th wavelet, and \( l \)th path, respectively, and \( d_{kjh}^I(t) = d_{kjh}^I(t - jd_{kjh}^Q(t)) \). The preceding analysis is carried out assuming quadrature phase shift keying (QPSK); if BPSK modulation is used, then the baseband data and the carrier term will be real, such that \( d_{kjh}(t) \) is written as \( d_{kjh}^I(t) \) and \( \exp(j\omega_0 t) \) is replaced with \( \cos(\omega_0 t) \).

The receiver shown in Figure 1 is assumed to be synchronous, designed to detect the first substream of the first user’s first WP propagating via the first path. It consists of a bandpass filter (BPF), tap SF, and correlators. The received signal is first passed through the BPF having bandwidth \( B_r \) equal to the spread spectrum bandwidth = 2\( T_n^{-1} \), which removes the out-of-band noise and lets the desired signal and inferences pass without distortion.

It can be shown that the received signal, \( r(t) \), can be written as

\[
r(t) = \sum_{k=1}^{K} y_k(t) + n(t) + 3(t)
\]

\[
= \sqrt{2P} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{l=1}^{L} \hat{a}(t - \tau_{kl}) c_k(t - \tau_{kl}) w_{ph}(t - \tau_{kl}) \times \beta_{kl} e^{j\Phi_{kl}} \times \left[ d_{kjh}^I(t - \tau_{kl}) \cos(\omega_0 t - \Phi_{kl}) + d_{kjh}^Q(t - \tau_{kl}) \sin(\omega_0 t - \Phi_{kl}) \right] + n(t) + 3(t) \quad (9)
\]

where \( \theta_{kl} = \phi_{kl} - \omega_0 \tau_{kl}, n(t) \) is the AWGN, and \( 3(t) \) is BPSK narrow-band interference jammer given by

\[
3(t) = \sqrt{2\pi} f_j(t) \cos(2\pi(f_0 + \Delta)t + \psi) \quad (10)
\]

having power 3, offset interference carrier with respect to signal carrier \( \Delta \), and phase \( \psi \). The information sequence \( j(t) \) has a bit rate \( 1/T_j \), where \( T_j \) denotes the duration of one bit. The interference bandwidth is \( B_j = 2/T_j \), and we assume \( B_j < B_r \). An important quantity is the ratio of the interference band to system bandwidth \( p = B_j/B_r = T_n/T_j \). The filtered signal is then passed through an SF. The impulse response of the filter may be written as

\[
h_s(t) = \sum_{m=-M_1}^{M_2} \alpha_m \delta(t - mT_n) \quad (11)
\]

where \( \alpha_0 = 1 \) and \( M_1 \geq 0 \) and \( M_2 \geq 0 \) represent the number of taps of the filter on the left and right of the center tap [3]. For each tap, the output of the filter is given by
\[
    r_\omega(t) = \left( \sum_{k=1}^{K} y_k(t) + n(t) + \hat{z}(t) \right) \ast h_c(t)
\]

\[
    = \sum_{m=M_1}^{M_2} \left\{ \sqrt{2P} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{l=1}^{L} \alpha_m \beta_{kl} a_j(t - \tau_{kl} - mT_n) c_k(t - \tau_{kl} - mT_n) \right. \\
    \times w_p(t - \tau_{kl} - mT_n) \left[ d_{1, kjh}^I(t - \tau_{kl} - mT_n) \cos(\omega_0 t + \theta_{kl}^m) + d_{1, kjh}^Q(t - \tau_{kl} - mT_n) \sin(\omega_0 t + \theta_{kl}^m) \right] \\
    + \alpha_m \hat{n}(t - mT_n) + \alpha_m \sqrt{2} \Im \left( t - mT_n \right) \cos[2\pi(f_0 + \Delta)(t - mT_n) + \psi]. \right\} \tag{12}
\]

where \( \theta_{kl}^m = \theta_{kl} - mT_n \omega_0 \) and \( \hat{n}(t - mT_n) \) is the filtered AWGN.

The output of the filter, \( r_\omega(t) \), is demodulated by a local oscillator, despread by a user specific code sequence, multiplied by the WPs, and correlated over a period \( T \) to recover the super bit stream. The resulting data are then parallel-to-serial (P/S) converted again and despread by the MCD to recover the \( J \) parallel data substreams. Finally, the correlated outputs from \( J \) paths is P/S converted to recover the original data bit.

Observe that the output of the first correlator in the WP part denoted as \( x_1 \) can be written as

\[
x_1 = \int_{0}^{T} r_\omega(t) c_1(t) w_p(t) \cos(\omega_0 t) - j \sin(\omega_0 t) dt \\
= x_{1, DS}^1 + x_{1, DS}^I + x_{1, MPI} + x_{1, MCDI} + x_{1, WPI} \\
+ x_{1, MUI} + n^1 + \hat{\Xi}^1 \tag{13}
\]

where \( x_{1, DS}^1 \) is the desired signal, \( x_{1, DS}^I \) is the self-interference, \( x_{1, MPI} \) is the multipath interference, \( x_{1, MCDI} \) is the MCD interference, \( x_{1, WPI} \) is the WPs interference, \( x_{1, MUI} \) is the multisuser interference, \( n^1 \) is suppressed correlated AWGN component, and \( \hat{\Xi}^1(t) \) is the suppressed narrow-band interference. The output of first P/S converter for the first user’s signal may be written as

\[
    \hat{b} = x_{1, DS}^1 + x_{1, DS}^I + \sum_{h=1}^{H} \left[ x_{1, MPI}^h + x_{1, MCDI}^h + x_{1, WPI}^h + x_{1, MUI}^h + n^h + \hat{\Xi}^h \right] \tag{14}
\]

The output for the first correlator \( z_1 \) in the MCD part is given by

\[
z_1 = \int_{0}^{T} \hat{b} \times a_1(t) dt \\
= x_{1, DS}^1 + x_{1, DS}^I + x_{1, MPI} + x_{1, MCDI} + x_{1, WPI} \\
+ x_{1, MUI} + \hat{n}^1 + \hat{\Xi}^1 \tag{15}
\]

After evaluation, it can be shown that

\[
z_{1, DS}^1 = \sqrt{\frac{P}{2}} \beta_{111} N_n T \left[ d_{1, 111}^I(t) - j d_{1, 111}^Q(t) \right] \tag{16}
\]

and

\[
z_{1, DSI}^1 = \beta_{11} T \sqrt{\frac{P}{2}} \sum_{m=-M_1}^{M_2} \alpha_m \left[ \left\{ \cos(\theta_{111}^m) + j \sin(\theta_{111}^m) \right\} T_1^I(m_1) \\
+ \left\{ \sin(\theta_{111}^m) - j \cos(\theta_{111}^m) \right\} T_1^Q(m_1) \right] \\
+ \sum_{m_2=1}^{M_2} \left[ \left\{ \cos(\theta_{111}^m) + j \sin(\theta_{111}^m) \right\} T_1^O(m_2) \\
+ \left\{ \sin(\theta_{111}^m) - j \cos(\theta_{111}^m) \right\} T_1^O(m_2) \right] \right] \tag{17}
\]

where

\[
T_1^K(m_1) = d_{111, 0}^K F_{111}(m_1) + d_{111, 1}^K F_{111}(N_n + m_1) \\
T_1^K(m_2) = d_{111, 0}^K F_{111}(m_2 - N_n) + d_{111, 0}^K F_{111}(m_2)
\]

where \( d_{111, 1}^K \) are the previous data bit and current data bit, respectively, with \( k = I \) for inphase part and \( k = Q \) for quadrature part. \( F_{k}(l) \) is the discrete-time aperiodic cross-correlation function defined in [7] and is given by

\[
F_{k,j}(l) = \left\{ \begin{array}{ll}
N_n - l - 1, & 0 \leq l \leq N_n - 1 \\
N_n - l + 1, & 1 - N_n \leq l \leq 0
\end{array} \right.
\]

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The value of the four components $z_{\text{MPI}}^1$, $z_{\text{MCDI}}^1$, $z_{\text{WPI}}^1$ and $z_{\text{MUI}}^1$ can be written as given in [1, Chapter 4] as follows:

$$
z_{\text{MPI}}^1 = \sqrt{\frac{P}{2}} \sum_{m=M_1}^{M_2} \sum_{l=2}^{L} \alpha_m \beta_{1l} \times \left[ \cos(\theta_{1l}^m) + j \sin(\theta_{1l}^m) \right] x_1^1 + \left[ \sin(\theta_{1l}^m) - j \cos(\theta_{1l}^m) \right] x_1^0 \right] \tag{18}
$$

$$
z_{\text{MCDI}}^1 = \sqrt{\frac{P}{2}} \sum_{m=M_1}^{M_2} \sum_{l=1}^{J} \sum_{h=2}^{H} \sum_{l=1}^{L} \alpha_m \beta_{1l} \times \left[ \cos(\theta_{1l}^m) + j \sin(\theta_{1l}^m) \right] x_{1j} + \left[ \sin(\theta_{1l}^m) - j \cos(\theta_{1l}^m) \right] x_{1j}^0 \right] \tag{19}
$$

$$
z_{\text{WPI}}^1 = \sqrt{\frac{P}{2}} \sum_{m=-M_1}^{M_2} \sum_{k=2}^{K} \sum_{h=2}^{H} \sum_{l=1}^{L} \alpha_m \beta_{kl} \times \left[ \cos(\theta_{kl}^m) + j \sin(\theta_{kl}^m) \right] x_{kj} + \left[ \sin(\theta_{kl}^m) - j \cos(\theta_{kl}^m) \right] x_{kj}^0 \right] \tag{20}
$$

$$
z_{\text{MUI}}^1 = \sqrt{\frac{P}{2}} \sum_{m=-M_1}^{M_2} \sum_{k=2}^{K} \sum_{h=1}^{H} \sum_{l=1}^{L} \alpha_m \beta_{kl} \times \left[ \cos(\theta_{kl}^m) + j \sin(\theta_{kl}^m) \right] x_{kjh} + \left[ \sin(\theta_{kl}^m) - j \cos(\theta_{kl}^m) \right] x_{kjh}^0 \right] \tag{21}
$$

where $R_{h,n,xy}[\tau_{kl}(m)]$, $\hat{R}_{h,n,xy}[\tau_{kl}(m)]$ are the partial cross-correlation functions; $d_{kj,h,-1}^1$, $d_{kj,h,0}^1$ are the inphase part of previous data bit and current data bit, respectively; $d_{kj,h,-1}^0$, $d_{kj,h,0}^0$ are the quadrature component of previous data bit and current data bit, respectively. Following the definition in [7,8], the partial cross-correlation functions $R_{h,n,xy}(\tau_{kl})$ and $\hat{R}_{h,n,xy}(\tau_{kl})$ can be written as follows:

$$
R_{h,n,xy}(\tau_{kl}) = \int_{0}^{\tau_{kl}} c_h(t) c_n(t - \tau_{kl} + T) w_p(x(t)) w_p(y(t)) \times (t - \tau_{kl} + T) dt
$$

$$
\hat{R}_{h,n,xy}(\tau_{kl}) = \int_{\tau_{kl}}^{T} c_h(t) c_n(t - \tau_{kl} + T) w_p(x(t)) w_p(y(t)) \times (t - \tau_{kl}) dt
$$

Also, $G_{xy}(\cdot)$ and $\hat{G}_{xy}(\cdot)$ are the partial cross-correlation functions defined as in [7] and are given by

$$
G_{xy}(\tau_{kl}) = \int_{0}^{\tau_{kl}} a_x(t) a_y(t - \tau_{kl} + T) dt
$$

$$
\hat{G}_{xy}(\tau_{kl}) = \int_{\tau_{kl}}^{T} a_x(t) a_y(t - \tau_{kl}) dt
$$

The suppressed correlated AWGN component $\tilde{n}_1^1$ is given by

$$
\tilde{n}_1^1 = \sum_{m=-M_1}^{M_2} \alpha_m \int_{0}^{T} a_1(t) \times \left[ \int_{0}^{T} \left\{ \tilde{n}(t - mT_n) [\cos(\omega_c t) - j \sin(\omega_c t)] \right\} \times \sum_{h=1}^{H} \sum_{i=0}^{N_h-1} c_{1h}^i w_{h'}(t - i T_n) \right] dt \right] \tag{22}
$$
and the suppressed narrow-band interference \( \tilde{z} \) is given by

\[
\tilde{z} = \sum_{m=-M_1}^{M_2} a_m \sqrt{\frac{\delta}{2}} \int_0^T a_1(t) \times \left[ \int_0^T \left( f(t-mT) [\cos(2\pi \Delta(t-mT) + \psi) + j \sin(2\pi \Delta(t-mT) + \psi)] \times \sum_{h'}^{H} + \sum_{i=0}^{N-1} c_i^w h'(t-iT) \right) dt \right] \quad (23)
\]

3. DETERMINATION OF SUPPRESSION FILTER COEFFICIENTS

It is shown in [3,5] that the coefficients of the SF can be determined using

\[
\sum_{m=-M_1}^{M_2} a_m \rho((n-m)T_n) + \rho(nT_n) = 0
\quad (24)
\]

\( \rho(vT_n) \) is a low-pass autocorrelation function consisting of three components

\[
\rho(vT_n) = \rho_s(vT_n) + \rho_n(vT_n) + \rho_j(vT_n) \quad (25)
\]

where \( \rho_s(vT_n) \) is the lowpass version of the desired signal, \( \rho_n(vT_n) \) is due to noise, and \( \rho_j(T_n) \) is due to narrow-band interference. The \( \rho_s(vT_n) \) is given by

\[
\rho_s(vT_n) = E \left\{ \sqrt{2P} \sum_{k_1=1}^{K} \sum_{h_1=1}^{H} \sum_{j_1=1}^{J} \sum_{l_1=1}^{L} \beta_{k_1,l_1} d_{k_1,j_1,h_1} \times (t-\tau_{k_1,l_1}) \times a_{j_1} (t-\tau_{k_1,l_1}) \sum_{i=0}^{N-1} c_i^k h_1 (t-iT_n-\tau_{k_1,l_1}) \times w_{h_1} \times \sqrt{2P} \sum_{k_2=1}^{K} \sum_{h_2=1}^{H} \sum_{j_2=1}^{J} \sum_{l_2=1}^{L} \beta_{k_2,l_2} d_{k_2,j_2,h_2} \times (t+vT_n-\tau_{k_2,l_2}) \times a_{j_2} (t+vT_n-\tau_{k_2,l_2}) \sum_{i=0}^{N-1} c_i^k h_2 \right\}
\quad (26)
\]

As \( E \left[ a_{j_1} (t-\tau_{k_1,l_1}) a_{j_2} (t+vT_n-\tau_{k_2,l_2}) \right] = 0 \) when \( k_1 \neq k_2 \) or \( l_1 \neq l_2 \). Furthermore,

\[
E \left[ a_{j_1} (t-\tau_{k_1,l_1}) a_{j_2} (t+vT_n-\tau_{k_2,l_2}) \right] = \begin{cases} 1, & v = 0 \\ 0, & v \neq 0 \end{cases}
\]

and

\[
N_{n-1} \sum_{i=0}^{N-1} E \left[ \left( c_i^k w h (t-iT_n-\tau_{k_1,l_1}) \right)^2 \right] = 1
\]

then

\[
\rho_s(vT_n) = 2P \sum_{k_1=1}^{K} \sum_{h_1=1}^{H} \sum_{j_1=1}^{J} \sum_{l_1=1}^{L} E \left[ \beta_{k_1,l_1}^2 \right] E \left[ d_{k_1,j_1,h_1}^2 \right] \times (t-\tau_{k_1,l_1}) \times a_{j_1} (t-\tau_{k_1,l_1}) \sum_{i=0}^{N-1} c_i^k h_1 \times \sqrt{2P} \sum_{k_2=1}^{K} \sum_{h_2=1}^{H} \sum_{j_2=1}^{J} \sum_{l_2=1}^{L} \beta_{k_2,l_2} d_{k_2,j_2,h_2} \times (t+vT_n-\tau_{k_2,l_2}) \times a_{j_2} (t+vT_n-\tau_{k_2,l_2}) \sum_{i=0}^{N-1} c_i^k h_2 \right\}
\quad (27)
\]

The \( \rho_n(vT_n) \) and \( \rho_j(vT_n) \) are given in [3] by

\[
\rho_n(vT_n) = \begin{cases} \frac{2N_0}{P} v = 0 \\ 0, & v \neq 0 \end{cases}
\quad (28)
\]

\[
\rho_j(vT_n) = \begin{cases} \frac{1}{2} (1-|v|) \cos(2\pi v q), & |v| \leq \text{int} \left[ \frac{1}{p} \right] \\ 0, & |v| > \text{int} \left[ \frac{1}{p} \right] \end{cases}
\quad (29)
\]
From (27)–(29), one obtains
\[
\rho(vT_n) = 2 P \Omega \begin{cases} 
K L H J + 2N_n [E_s/N_0]^{-1} + 3/S, & v = 0 \\
(3/S) (1 - |v|) p \cos(2\pi vq), & |v| \leq \text{int}[1/p] \\
0, & |v| > \text{int}[1/p]
\end{cases}
\]

\[
\text{var} [\tau_{kl}], \text{var} [\sigma_k], \text{var} [\rho_{kl}], \text{var} [\rho_k], \text{var} [\sigma], \text{var} [\rho_0], \text{var} [\rho_1], \text{var} [\rho_2]
\]

Note that the SF can be double sided (DS, \(M_1 \neq 0\) and \(M_2 \neq 0\)) or single sided (SS, \(M_1 \neq 0\) and \(M_2 = 0\), or \(M_1 = 0\) and \(M_2 \neq 0\)).

### 4. SIGNAL-TO-NOISE PLUS INTERFERENCE RATIO

To find the signal-to-noise plus interference ratio (SNIR), we need to find the desired signal power and the noise plus interference variances. The process of computing the variances for the different interference terms is quite involved. Fortunately, the variances \(\sigma_{\text{MPI}}^2, \sigma_{\text{MCID}}^2, \sigma_{\text{WPI}}^2, \sigma_{\text{MU}}^2\), and \(\sigma_{\text{SI}}^2\) for a system without SF, have been computed in [1, Chapter 4]. Without loss of generality, we invoke the results in [1,3] for the preceding variances with SF, which is given by

\[
\sigma_\text{T}^2 = \sigma_{\text{MPI}}^2 + \sigma_{\text{MCID}}^2 + \sigma_{\text{WPI}}^2 + \sigma_{\text{MU}}^2 + \sigma_{\text{SI}}^2
\]

\[
\sigma_\text{T}^2 = \frac{P (T N_n)^2}{2} (M_1 + N_1)
\]

where

\[
M_1 = \sum_{m=-M_1}^{M_2} \sum_{m=-M_1}^{M_2} \sigma_m \sigma_{m+1}
\]

\[
N_1 = \frac{H \Omega}{N_n (E_s/N_0)} \sum_{m=-M_1}^{M_2} \sigma_m^2
\]

with \(x^m = 6\) for QPSK and \(x^m = 12\) for BPSK. \(\Omega = \text{var}[\beta_{k1}], Q\) represents the sum of amplitude levels of all multipath components, and \(E_s = 2 P \Omega T\) is the mean received symbol energy. Note that in the derivation of the variances, it is assumed that the variation in \(\tau_{kl}\) and \(\rho_{kl}\) is very small and can be ignored for all \(K\) users and \(L\) paths, and that the variances are independent of \(k, h, j\). The variance for the inphase self-interference \(\sigma_{\text{SI}}^2\) is given by

\[
\sigma_{\text{SI}}^2 = \text{var} [\tau_{\text{SI}}^2]
\]

\[
= \text{var} \left\{ \beta_{11} T \sqrt{\frac{P}{2}} \int_{-M_1}^{-1} \int_{-M_1}^{-1} \alpha_{m_1} \alpha_{m_2} \left[ \cos (\theta_{m_1}^{11}) \cos (\theta_{m_2}^{11}) \right] \right\}
\]

Assuming that \(\theta_{m_1}^{11}\) is a zero mean random variable uniformly distributed in \([0, 2\pi]\)

\[
E \left[ \cos (\theta_{m_1}^{11}) \sin (\theta_{m_1}^{11}) \right]
\]

\[
= \int_0^{2\pi} \int_0^{2\pi} \cos (\theta_{m_1}^{11}) \sin (\theta_{m_1}^{11}) d \theta_{m_1}^{11} = \frac{1}{2}, \quad m_1 = \tilde{m}_1, \quad m_1 \neq \tilde{m}_1
\]

Similarly,

\[
E \left[ \cos (\theta_{m_1}^{11}) \cos (\theta_{m_1}^{11}) \right]
\]

\[
= \int_0^{2\pi} \int_0^{2\pi} \cos (\theta_{m_1}^{11}) \cos (\theta_{m_1}^{11}) d \theta_{m_1}^{11} = \frac{1}{2}, \quad m_1 = \tilde{m}_1, \quad m_1 \neq \tilde{m}_1
\]
Also, it is evident that
\[
\begin{align*}
(d_{111,1}^l)^2 &= (d_{111,0}^l)^2 = (d_{111,-1}^l)^2 = (d_{111,1}^Q)^2 \\
&= (d_{111,0}^Q)^2 = (d_{111,-1}^Q)^2 = 1
\end{align*}
\]

And from [3]
\[
E[F_{k,i}(l)F_{k,i}(m)] = \begin{cases} N_n - |l|, & l = |m| \\ 0, & l \neq |m| \end{cases}
\]

According to this, (35) becomes
\[
\sigma_{DSI}^2 = \frac{PT^2 E[\beta_{111}^2]}{2} \times \left[ \sum_{m_1=-M_1}^{M_1} a_{m_1}^2 N_n + \sum_{m_2=-1}^{M_2} a_{m_2}^2 N_n \right] = \frac{PN_n T^2 \Omega}{2y^m} \sum_{m_1=-M_1, m \neq 0}^{M_2} a_m^2 
\]

where \(y^m = 1\) for QPSK and \(y^m = 2\) for BPSK. The reason that \(y^m\) and \(x^m\) in the case of BPSK are double that for QPSK is as follows: as indicated previously, instead of \(d_k(t)\), we use \(d_k^2(t)\), and \(\exp(j\omega_0 t)\) is replaced by \(\cos(\omega_c t)\) in BPSK modulation. Hence the interference variances are calculated from (17)–(21) by considering only the cosine part of the inphase signal.

From (23), the inphase component of narrow jamming signal is given by
\[
\tilde{\zeta}^1 = \sum_{m=-M_1}^{M_2} a_m \sqrt{\frac{\beta}{2}} \int_0^T a_1(t) \times \left[ \int_0^T f(t-mT) \cos[2\pi \Delta(t-mT_n) + \psi] \times \frac{H}{N_n-1} \sum_{h'=1}^{N_n-1} c_1 h_h'(t-iT_n)dt \right] dt
\]

The variance can be evaluated as follows:
\[
\text{var}[\tilde{\zeta}^1] = E\left\{ \left[ \sum_{m=-M_1}^{M_2} a_m \sqrt{\frac{\beta}{2}} \int_0^T a_1(t) \times \left[ \int_0^T f(t-mT) \cos[2\pi \Delta(t-mT_n) + \psi] \times \frac{H}{N_n-1} \sum_{h'=1}^{N_n-1} c_1 h_h'(t-iT_n)dt \right] dt \right]^2 \right\} = \frac{3}{2} E\left[ \int_0^T \int_0^T a_1(t) a_1(\lambda) \times \left[ \int_0^T f(t-mT_n) f(\lambda-mT_n) \times \cos[2\pi \Delta(t-mT_n) + \psi] \times \cos[2\pi \Delta(\lambda-mT_n) + \psi] \times \frac{H}{N_n-1} \sum_{h'=1}^{N_n-1} c_1 h_h'(t-iT_n) \sum_{h'=1}^{N_n-1} c_1 h_h'(\lambda-iT_n) \right] d\lambda d\lambda d\lambda \right]\]

where \(\sigma^2_{\tilde{\zeta}^1} = \frac{3 HT^2}{4} \sum_{m_1=-M_1}^{M_2} \sum_{m_2=-M_1}^{M_2} a_{m_1} a_{m_2} \sigma_j^2(m_1, m_2)
\]

Using [3, Eq. 18], also given \(c_1^2 = 1\) and because
\[
\left\langle w_h'(t-l (\frac{T_n}{\Psi})) w_h'(t-n (\frac{T_n}{\Psi})) \right\rangle = \delta_{ln}, \text{ then (37) can be written as}
\]
\[
\sigma^2_{\tilde{\zeta}^1} = \frac{3 HT^2}{4} \sum_{m_1=-M_1}^{M_2} \sum_{m_2=-M_1}^{M_2} a_{m_1} a_{m_2} \sigma_j^2(m_1, m_2)
\]
is given by

\[
\sigma_f^2 = \frac{P(TN_n)^2}{2} (M_1 + M_2 + NI + JI)
\]

\[
= \frac{P(TN_n)^2}{2} (MI + NI + JI)
\]  

(40)

where

\[
M_1 = \frac{\Omega y^m N_n}{m^2} \sum_{m=1, m \neq 0} a_m^2
\]

\[
JI = \frac{H \Omega (3/\bar{S})}{N_n^2} \sum_{m_1=-M_1, m_2=-M_1} M_2 a_{m_1} a_{m_2} a_f^2 (m_1, m_2)
\]  

(42)

and \( \bar{S} = 2P \Omega \). Therefore, the output SNIR, \( \gamma \), can be written as

\[
\gamma = \frac{S}{\sigma_f^2} = \frac{P}{2} \left( \beta_{11} T N_n \right)^2 = \beta_{11}^2 \gamma_1
\]  

(43)

with

\[
\gamma_1 = [MI + NI + JI]^{-1}
\]  

(44)

5. BIT ERROR RATE

A common method used to measure the performance for communication systems is the average bit error rate (BER), \( \bar{P}_e \). It is obtained by averaging the instantaneous BER of \( \gamma \) over the channel fading functions. As the channel path gain \( \beta_{11} \) is assumed to be Nakagami distributed random variable (RV) with parameter \( (m, \Omega) \), then using variable transformation, the probability density function (pdf) of \( \gamma \) is obtained as follows:

\[
f_\gamma(\gamma) = f_\beta(\beta) \frac{d\beta}{d\gamma} \bigg|_{\beta^2=\frac{\gamma}{m}} = \frac{1}{\Gamma(m)} \left( \frac{m \Omega^2}{m} \right)^m \gamma^{m-1} \exp \left( -\frac{m \gamma}{\Omega^2 m} \right)
\]  

(45)

where the subscripts 11 are omitted for simplicity and \( \Gamma(m) \) is the gamma function. Thus \( \bar{P}_e \) is given by

\[
\bar{P}_e = \int_0^\infty f_\gamma(\gamma) P_e(\gamma)d\gamma
\]

\[
= \int_0^\infty \left[ \frac{1}{\Gamma(m)} \left( \frac{m \Omega^2}{m} \right)^m \gamma^{m-1} \exp \left( -\frac{m \gamma}{\Omega^2 m} \right) \right] \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{\gamma}{2}} \right) d\gamma
\]  

(46)

where \( P_e(\gamma) = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{\gamma}{2}} \right) \) is the instantaneous BER of \( \gamma \). To calculate \( \bar{P}_e \), the \( \gamma \) given in (43) is first calculated, then its instantaneous BER and pdf are used to evaluate the \( \bar{P}_e \).

6. OUTAGE PROBABILITY PERFORMANCE

Because of the variable nature of the channel, which is caused by the Doppler effect, multipath fading, interference, and noise, the instantaneous signal power is not always strong enough to achieve satisfactory reception. Thus the receiver may experience outages even when the mean signal strength is adequate [9,10]. As an indication of a minimum requirement on the grade of service, outage probability represents the probability of unsatisfactory reception over the intended coverage area. In practical applications, the received signal is required to meet a minimum signal power threshold, or a minimum error probability for satisfactory reception. In this paper, we consider the outage probability, defined as the probability of failing to achieve an SNIR sufficient enough to give satisfactory signal reception.

Let \( \gamma_{th} \) be the preset threshold SNIR, which depends on the quality of service and system design parameters such as modulation, number of users, power control features, WPs, and number of code substreams. Outage occurs whenever the SNIR is less than \( \gamma_{th} \), that is,

\[
P_{out} = Pr(\gamma < \gamma_{th}) = \int_0^{\gamma_{th}} f_\gamma(\gamma)d\gamma
\]

\[
= \frac{1}{\Gamma(m)} \int_0^{m \gamma_{th}/\Omega^2 m} x^{m-1} \exp(-x)dx = \tilde{G} \left( \frac{m \gamma_{th}}{\Omega^2 m}, m \right)
\]  

(47)

where \( \gamma_{th} \) is the threshold value for SNIR, \( x = \frac{m \gamma}{\Omega^2 m} \) and \( \tilde{G} \left( \frac{m \gamma_{th}}{\Omega^2 m}, m \right) \) is the incomplete gamma function. For the different systems considered, the SNIR is first calculated, and then (47) is used to evaluate the outage probability for specific value of \( \gamma_{th} \).

7. RESULTS AND DISCUSSIONS

In this section, using the preceding analytical results, the BER and outage performances of the system in the presence of narrow-band interference are presented, as well as the effects of changing the system parameters such as the number of taps of the SF, and the ratio of narrow-band interference bandwidth to the spread-spectrum bandwidth. Also, the outage probability performance of the system in the presence of SF is compared with that without the presence of SF for different values of \( (J/S) \). Finally, the BER performances of the system and MC/MCD-CDMA system are compared. In this section, unless otherwise mentioned, BPSK modulation and the parameters listed in Table 1 are used to evaluate the BER (46) and \( P_{out} \) (47) by means of the MATLAB program.

7.1. Effect of filter type and number of taps

Figure 2 shows the BER performance of the system with DS-SF, SS-SF, and without SF as a function of \( E_b/N_0 \). The
Table I. Measure system parameters used in simulation.

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelet packets, Daubechies 3</td>
<td>db3</td>
</tr>
<tr>
<td>Number of wavelet packets superstreams</td>
<td>$H = 4$</td>
</tr>
<tr>
<td>The chip duration for PN code</td>
<td>$T_n = 3 \times 10^{-8}$ s</td>
</tr>
<tr>
<td>The delay time</td>
<td>$\rho = T_n/10$</td>
</tr>
<tr>
<td>Local mean power</td>
<td>$\Omega = 10$ dB</td>
</tr>
<tr>
<td>The mean energy-to-noise power</td>
<td>$E_s/N_0 = 10$ dB</td>
</tr>
<tr>
<td>Number of multipath</td>
<td>$L = 3$</td>
</tr>
<tr>
<td>Number of multicode substreams</td>
<td>$J = 3$</td>
</tr>
<tr>
<td>MIP $Q = L$</td>
<td>$Q = L = 3$</td>
</tr>
<tr>
<td>Ratio of $B_f$ to $B_s$</td>
<td>$p = 0.1$</td>
</tr>
<tr>
<td>Ratio of $\Delta$ to $B_s$</td>
<td>$q = 0$</td>
</tr>
<tr>
<td>Nakagami parameter (channel fading)</td>
<td>$m = 1$</td>
</tr>
</tbody>
</table>

Figure 2. BER as a function $E_s/N_0$ with and without SF.

Figure 3. BER as a function $E_s/N_0$ for QPSK and BPSK modulation.

7.2. Effect of type of modulation

Figure 3 represents the BER performance of the system with a DS-SF as a function of $E_s/N_0$, The system parameters are $K = 35$, $E_s/N_0 = 40$ dB, and $J/S = 30$ dB. The DS-SF has three taps and is symmetric. As expected, the BER performance is improved by increasing $N_n$. When $p = 1$, the BERs are the same for systems with and without SF. This is because $\rho_j(vT_n) = 0$ for $v \neq 0$; thus $\alpha_m = 0$ for $m \neq 0$. Accordingly, the SF will be with only one tap, and the system will act as the system without SF. The effects of other values of $p$ on the BER can be explained as follows:

(i) For the case without SF, because the filter coefficients $[\alpha_j \alpha_0 \alpha_1] = [0 \ 1 \ 0]$ are constant and do not depend on $p$, the only effect of $p$ on the BER

7.3. Effect of interference bandwidth to spread-spectrum bandwidth

Figure 4 shows the BER performance of the system with a DS-SF and without SF as a function of $p$ for two values of $N_n$. The system parameters are $K = 35$, $E_s/N_0 = 40$ dB, and $J/S = 30$ dB. The DS-SF has three taps and is symmetric. As expected, the BER performance is improved by increasing $N_n$. When $p = 1$, the BERs are the same for systems with and without SF. This is because $\rho_j(vT_n) = 0$ for $v \neq 0$; thus $\alpha_m = 0$ for $m \neq 0$. Accordingly, the SF will be with only one tap, and the system will act as the system without SF. The effects of other values of $p$ on the BER can be explained as follows:

(i) For the case without SF, because the filter coefficients $[\alpha_j \alpha_0 \alpha_1] = [0 \ 1 \ 0]$ are constant and do not depend on $p$, the only effect of $p$ on the BER

Figure 4. BER as a function $p$ with and without SF for different values of $N_n$.
is from $\sigma^2(m_1, m_2)$, given by (39), which decreases as $M$ increases. Accordingly, $J_I$ (42) decreases, and BER performance of the system decreases.

(ii) For the case with SF, the BER as function of $M$ is affected by $\rho_j (vT_n)$, given by (30), which is used to calculate $\alpha_m$, given by (24), and $\sigma^2(m_1, m_2)$. For the effect from $\rho_j (vT_n)$, because as $M$ increased, both $\rho_j (vT_n)$ and $\alpha_m^2$ decreased; as a result, $MI_1$ and $MI_2$, which are given by (34) and (41), respectively, decreased as well. $MI_1$, given by (33), is dependent on the term $\frac{\sum_{m=-M_1}^{M_2} \alpha_m^2}{\sum_{m=-M_1}^{M_1} \alpha_m}$, because the product $\sum_{m=-M_1}^{M_2} \alpha_m \alpha_{m+1}$ has positive values at some times and negative values at others, the total effect is that $MI_1$ decreased for $M < 0.3$ and then increased for values of $M > 0.3$. However, $J_I$ is dependent on $\sum_{m=-M_1}^{M_2} \sum_{m=-M_1}^{M_1} \alpha_m \alpha_{m+1}$, which decreased for values of $M < 0.5$ and increased for values of $M > 0.55$ and accordingly did $J_I$. Overall, the variance $\sigma^2$, given by (40), decreased for $M < 0.5$ and increased for $M > 0.55$, hence the BER improved and degraded accordingly.

### 7.4. Outage performance

Figure 5 shows the outage performance of the system with a DS filter and without SF as a function of $\gamma_{th}$ for three values of $J/S$. The system parameters are $K = 10$, $E_s/N_0 = 50$ dB, and $N_n = 512$. The DS filter has three taps and is symmetric. As expected, the outage performance is degraded by increasing $J/S$. When $J/S = 8$ dB, outages are the same for systems with and without SF. This is because the system is capable of removing small power jammer signals. When $J/S = 40$ dB, the system without SF degrades significantly, whereas the system is much more tolerant of interference when SF is present.

![Figure 5. Outage performance as a function of $\gamma_{th}$, for different values of $J/S$.](image)

### 7.5. Performance comparison

Figure 6 shows the comparison of our system, WP-MC/MCD-CDMA, with Sin-MC/MCD-CDMA. The system’s parameters are $K = 10$, $J/S = 40$ dB, and $N_n = 512$. The DS filter has three taps and is symmetric. To calculate the BER for Sin-MC/MCD-CDMA, we use the results in [11, Chapter 2, eq 2.59 and 2.61], [4], and [5].

As seen from the figure, performances of the two systems without SF are almost the same, with our system being better because WPs have much lower sidelobes than windowed sinusoids and the resulting sidelobe energy leakage is negligible. As expected, the BER performance can be improved by using SF, but the improvement in BER of our system is much more than that of the MC/MCD-CDMA system. This means that our system uses the SF to suppress the narrow-band interference more efficiently than does the MC/MCD-CDMA system.

### 8. CONCLUSION

The BER and outage performance of a WP-MC/MCD system overlaying a narrow-band BPSK waveform and employing SF in the receiver have been evaluated. It is found that the performance is improved by using SF. The DS-SF is superior to the SS-SF for the same number of total taps. For $M = 1$, the SF is unable to eliminate the undesired band because the jammer signal is no longer a narrow-band signal; rather it completely overlaps the SS bandwidth. For small value of $J/S$ ($8$ dB or less), there is no need to use the SF, because the performance with and without SF is the same. This is because the system itself can eliminate the small jamming signal. Our system uses the SF to suppress the narrow-band interference more efficiently than does the Sin-MC/MCD-CDMA system.
REFERENCES


AUTHORS’ BIOGRAPHIES

Maryam M. Akho-Zahieh (M’07) was born in Kuwait. She received her BSc, and MSc degrees in Electrical Engineering from Kuwait University in 1984 and 1989, respectively, and her PhD degree from The University of Akron, Akron, OH, in 2006. In 1984–1989, she was a Teaching Assistant with Kuwait University. From 1992 to 2003, she was a Faculty Member with the Department of Electrical and Computer Engineering, Applied Science University, Amman, Jordan, where she is currently an Assistant Professor. Her areas of interests include signal processing, multicarrier modulation, and multiple-access communication techniques. Dr. Akho-Zahieh was the recipient of the Graduate Teaching Assistant of the Year Award from the Department of Electrical and Computer Engineering, The University of Akron, in 2005. Dr. Akho-Zahieh is a member of several IEEE societies.

Nasser Abdellatif has earned a BEEE and MEEE in 1995 and 1996, respectively, from the City College of the City University of New York and a PhD in Electrical Engineering 1991 from the City University of New York. He is has been employed as a professor and dean of students at Applied Science University in Amman Jordan since 2005. He was employed by CSC Mobile Comm. and CSC Telecomm. from 1989 to 1991. He served at Bronx Community College from 1991 to 2005. Dr. Abdellatif served as the chairperson of the Department of Physics and Technology from 1995 to 2005, and the Science, Mathematics and Technology Division coordinator from 1997 to 2005. Over his career, Dr. Abdellatif has published extensively and has made numerous presentations at local, regional, and national conferences. Over the last 10years, one of his major interests has been teaching methodology and student-centered learning. He has conducted many faculty training workshops on integrating skills into the curriculum. He has been an advocate for faculty training and has trained over 300 faculty worldwide.