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Choosing Strings for Plucked Musical Instruments

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Summary
The various factors constraining the choice of synthetic polymer and natural gut strings for musical instruments are discussed, including a newly-formulated constraint based on the internal damping of the string material. It is shown that all these constraints can be summarised graphically in a design chart, and calibrated versions of the chart are presented for monofilament strings of nylon, fluorocarbon and natural gut. Based on these charts, detailed case studies are presented for the stringing of a lute and a harp. An explanation is suggested for why harpists continue to favour gut over nylon.

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1. Introduction

A perennial problem for players and designers of stringed musical instruments is how to choose the best set of strings for a given instrument. This question has long been discussed, and the basic science underlying this question is not new: see for example [1, 2, 3]. However, recent experimental measurements highlight a phenomenon which may be familiar to some musicians, but which has not previously attracted scientific attention. If a plain nylon or gut string is fitted to a plucked-string instrument and tensioned just enough to begin to make a musical note, the sound is muffled and unsatisfactory. As the string is tuned upwards, the sound first becomes musically acceptable but rather mellow, and then becomes progressively brighter until the point where the string breaks. It will be demonstrated that this perception of increasing brightness is largely the result of a material damping effect: there is a roll-off frequency above which the string overtones progressively become so highly damped that they are no longer "musical", and this roll-off frequency rises dramatically as the string is tightened.

In this paper the relevant results are reviewed, and a design chart is constructed that encapsulates the damping roll-off as well as other aspects of practical stringing choices. The main focus will be on plucked-string instruments like the harp, guitar and lute. The issue of string choice can be particularly pressing for period instruments like the lute or vihuela, because the musician is not presented with a ready-made choice in the form of sets of strings selected by instrument makers or string manufacturers, in contrast to the situation for a modern guitar or harp. The clearest results will be obtained for sets of monofilament strings, such as plain nylon or gut strings for a harp or lute. The rationale for over-wrapped strings will become clear in the course of the discussion, but detailed constructional choices for such strings will not be explored.

When choosing strings, a musician is likely to be interested in questions of "loudness", "tone quality" and "feel". All three concepts are decidedly slippery from the perspective of a scientist, but it is easy to identify physical correlates that capture at least part of what each means. All will play a role in the design charts to be presented here, and in detailed case studies for the lute and the harp to be presented in Sections 4.2 and 4.3.

2. Background theory

2.1. Frequency and impedance

The linear theory of free vibration of a stretched string is quite familiar, but key results needed in the later discussion will be summarised here. Consider a string of circular cross-section of diameter d and length L, under tension T and made of material with Young's modulus E and density ρ. The nth natural frequency $\omega_n = 2\pi f_n$ satisfies

$$\omega_n^2 \approx \frac{T}{m}\left[\frac{n\pi}{L}\right]^2 + \frac{EI}{m}\left[\frac{n\pi}{L}\right]^4, \tag{1}$$

where $m = \pi d^2\rho/4$ is the mass per unit length, and the second moment of area $I = \pi d^4/64$. This result follows

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from the Rayleigh quotient (see for example [1]), and the approximation sign arises because it has been assumed that the corresponding mode shape is

$$u_n(x) = \sin \frac{n\pi x}{L}, \tag{2}$$

where $0 \le x \le L$ is the position variable along the length of the string. A real string will deviate slightly from this assumption because of end effects: coupling to a non-rigid structure at both ends, and other effects of evanescent fields arising from the detailed end boundary conditions. Nevertheless, it has been shown previously [4, 5] that Equation (1) holds up very well for real strings, especially at higher values of n which will be important in this work.

The second term on the right-hand side of Equation (1) arises from the non-zero bending stiffness of the string. For realistic musical strings, the bending stiffness effect is relatively weak so that the fundamental frequency (i.e. $n = 1$) is always well approximated by neglecting this term,

$$f_1^2 = \frac{\omega_1^2}{4\pi^2} \approx \frac{T}{m}\left[\frac{1}{2L}\right]^2. \tag{3}$$

Two different musically-important effects are governed by the influence of the bending term: inharmonicity of natural frequencies, as shown by Equation (1), and the damping behaviour as a function of frequency, to be discussed in the next subsection. It is useful to introduce a non-dimensional parameter to express the proportion of the potential energy associated with this effect,

$$\lambda = \frac{EI\left[\frac{n\pi}{L}\right]^4}{T\left[\frac{n\pi}{L}\right]^2 + EI\left[\frac{n\pi}{L}\right]^4} \approx \frac{E\pi^2 d^2 n^2}{64\rho L^4 f_1^2 + E\pi^2 d^2 n^2}, \tag{4}$$

where the final expression makes use of Equation (3).

Using Equations (3) and (4) in Equation (1), the natural frequencies can be expressed in the form

$$f_n^2 \approx \frac{n^2 f_1^2}{1-\lambda}. \tag{5}$$

For sufficiently high mode numbers n the bending term ceases to be a small perturbation. However, it will be argued shortly that, for musically-relevant natural frequencies of strings of the kind to be discussed here, λ is always quite small. It follows that

$$f_n \approx n f_1(1+\lambda/2), \tag{6}$$

with

$$\lambda \approx \frac{E\pi^2 d^2 n^2}{64\rho L^4 f_1^2}. \tag{7}$$

It has been shown [6, 7] that the mechanical properties of a nylon string depend significantly on the stress state and history. In particular, the Young's modulus increases by roughly a factor of 3 between the unstressed state before it is fitted to the instrument and the state just before it breaks. Similar results have recently been shown for fluorocarbon strings [8], which are becoming increasingly popular with musicians. Natural gut strings are also still popular, especially with harpists, and for these it has been found that no such "strain stiffening" effect occurs: within the limits of accuracy of the measurements, a single value of Young's modulus is consistent with the results over the entire stress/strain range [8]. However, all these strings, including the gut strings, show significant frequency dependence of Young's modulus, as is normal for viscoelastic materials. The value of E relevant to the equations stated above is the high-frequency value called E_B in the previous work [7, 8].

Two other parameters for vibrating strings should be mentioned. First is the wave impedance

$$Z_0 = \sqrt{Tm} \approx \pi d^2 \rho L f_1/2, \tag{8}$$

which is an important contributory factor to the strength of coupling between the string and the body of the instrument, and hence to the loudness of the played note. A second quantity has been proposed by Firth [9, 10, 11] as important to harpists: what he called "feel", although no doubt this simple quantity only captures part of what a musician may understand by that term. Firth's quantity is defined as the plucking force necessary to produce a given initial displacement. For a mid-point pluck, to produce a small displacement δ the required force F_p is given by a simple static equilibrium calculation as

$$F_p \approx \frac{4T\delta}{L}, \tag{9}$$

so that the "feel" γ is

$$\gamma = F_p/\delta \approx 4T/L. \tag{10}$$

2.2. Damping model

The other key ingredient needed for this study concerns damping. There are three main physical mechanisms of energy loss in a vibrating string: loss by coupling to the instrument body, viscous dissipation due to the surrounding air, and viscoelastic loss within the material of the string. All three effects can be estimated, leading to a model that has been shown previously to give a good fit to measurements [3, 4, 5]. That model will be fine-tuned in the light of more extensive measurements now available, and then an important conclusion will be drawn relating to the influence of damping on "brightness" of plucked strings and hence on string selection.

Energy loss via the bridge to the instrument body will vary strongly with frequency, especially at lower frequencies, depending on the proximity of individual body resonances [12]. However, a simple approximation to the energy loss at higher frequencies can be obtained using Statistical Energy Analysis, giving a loss factor η_{body} [4]. Substituting typical numerical values for the high-frequency behaviour of musical instrument bodies reveals that this loss mechanism is usually insignificant compared to the other two mechanisms in the frequency range that will be of interest here [4]. Examples of its effect will be shown in Figures 2 and 3.

Energy loss due to viscosity in the surrounding air can be estimated using a classical analysis going back to Stokes. The associated loss factor is given by Fletcher and Rossing [2] in the form

$$\eta_{\mathrm{air}} \approx \frac{\rho_a}{\rho}\frac{2\sqrt{2}M+1}{M^2}, \tag{11}$$

where ρ_a is the density of air, and

$$M = \frac{d}{4}\sqrt{\frac{2\pi f_n}{\eta_a}}, \tag{12}$$

where η_a is the kinematic viscosity of air. Textbook values will be used: $\rho_a = 1.2\ \mathrm{kg/m^3}$ and $\eta_a = 1.5 \times 10^{-5}\ \mathrm{m^2/s}$. In the light of tests on many musical strings covering a wide range of string gauges, it has been found that this formula does not quite capture the variation with diameter to best accuracy: it slightly underestimates the damping of thick strings and overestimates that of thin strings. It has been found that the measurements can all be approximated well enough for the present purpose by applying an *ad hoc* correction factor $(d + 0.2)$ to η_{air}, with the string diameter d expressed in mm. Some examples will be shown in Section 3, but it is not possible to reproduce the entire set of results here: between them, the previous studies covered 20 strings of various materials, each tested at up to 8 different tuned frequencies [4, 5, 7, 8, 13, 14].

To see more clearly how this damping contribution varies with frequency, it may be noted that for all the strings and tuning frequencies under consideration here, the value of M is fairly large so that

$$\eta_{\mathrm{air}} \approx (d+0.2)\frac{\rho_a}{\rho}\frac{2\sqrt{2}}{M} = (d+0.2)\frac{8\sqrt{2}\rho_a}{\rho d}\sqrt{\frac{\eta_a}{2\pi f_n}}. \tag{13}$$

The third damping mechanism will prove to be the most important for the purposes of this investigation. Energy loss from viscoelasticity in the string arises from the influence of bending stiffness. Young's modulus becomes a complex value $E(1 + i\eta_E)$. An argument based on Rayleigh's principle [15] can be used to yield an expression for the associated loss factor of the nth string mode, which takes a very simple form in terms of the parameter λ introduced earlier,

$$\eta_{\mathrm{bend}} \approx \lambda\eta_E. \tag{14}$$

The total modal damping factor can finally be estimated by

$$\eta_n \approx \eta_{\mathrm{body}} + \eta_{\mathrm{air}} + \eta_{\mathrm{bend}}. \tag{15}$$

As described above, the contribution from η_{body} is small and can largely be ignored. From Equation (13), η_{air} varies inversely with the square root of frequency, so it is dominant at low frequency. On the other hand, from Equation (7), $\lambda \propto n^2$ so that η_{bend} dominates at sufficiently high frequency. The result is a minimum of damping at an intermediate frequency, above which η_n increases rapidly with frequency. It will be argued that this increase accounts for the change in the sound of strings as they are tuned to different frequencies, as described earlier. The damping model will first be confirmed and illustrated with measurements.

3. Measured mechanical behaviour of strings

Data collected in the course of previous projects [4, 7, 8] can be combined to give a fairly comprehensive picture of the mechanical behaviour of monofilament strings of synthetic polymer or natural gut. Those earlier papers give full details of the experimental methodology, including the confidence limits associated with measurement errors. Most of the strings were tested using a purpose-designed measurement rig that included provision for control of temperature and humidity, and an automated system for monitoring and maintaining the string's tuning. These tests were conducted over a sufficiently long period that the string's plastic response at any given tuning level had time to level off to a state that a musician would describe as "settled". The tests also took full account of the slight reduction in the string's diameter under tension, but for the purposes of the present work this effect is small enough to be insignificant, and it will be ignored throughout this paper.

It should be emphasised that all the tested strings were regular commercial strings marketed for musical instruments: the authors were not able to obtain detailed information about the chemical composition despite asking the manufacturers. So, for example, for strings described here as "nylon" we do not know exactly which members of the family of nylons were involved: indeed, evidence was found that strings marketed as a homogeneous set for use on the harp were not all made of the same precise material. However, for the present purpose these uncertainties were sufficiently small that they will turn out not to be significant in the context of the "broad-brush" design charts to be presented later. The behaviour of Young's modulus at high frequencies (E_B in the earlier work) was found to be very similar for all the nylon strings tested, and similar uniformity was also found for the gut and fluorocarbon strings tested.

Damping behaviour can be extracted from the string response to wire-pluck excitation [4]. The string data collected in connection with [4] is restricted to a small number of strings, but it has high quality because the response was collected under laboratory conditions, from an accelerometer on the bridge of the guitar to which the tested strings were attached. The accelerometer gave excellent high-frequency response data. The data associated with [7, 8] covers a far wider range of strings. It was collected using a pair of purpose-built test rigs, described in detail

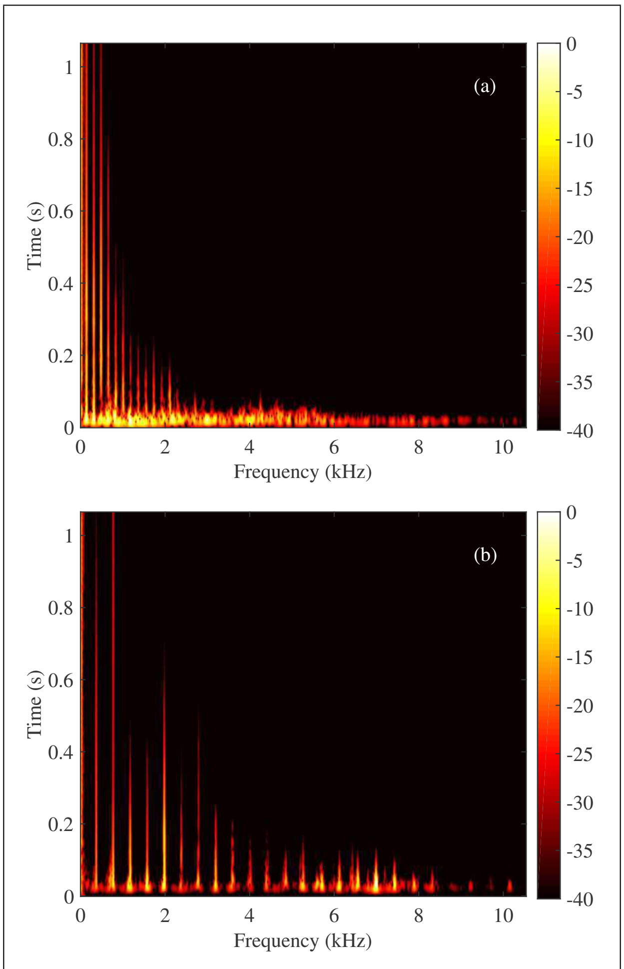

Figure 1. (Colour online) Spectrograms of pluck response of a nylon string of length 0.5 m and unstretched diameter 1.20 mm, tuned to (a) 174 Hz and (b) 403 Hz. Levels are plotted in dB relative to the maximum in each case, to a depth of 40 dB. This string was labelled "string 5" in the earlier study [7].

in [7]. The data quality is not as good because the pluck responses were only collected as secondary material in the course of tests primarily devoted to the mechanical and thermal response of strings over a long timescale, but it is sufficient for a thorough test of the proposed damping model.

The first use of these results is to illustrate the underlying phenomenon described earlier, in which the brightness of the sound of a given string depends on its state of tuning. Figure 1 shows spectrograms of the pluck response of a nylon string of diameter 1.20 mm and vibrating length 0.5 m, which was labelled "string 5" in the earlier study [7]. The plots show this same nylon string tuned to two different frequencies, the lowest and highest used in the testing sequence. At the higher tuning frequency, it can clearly be seen that the decay rate at any given frequency is much slower, and that the bandwidth containing identifiable strong string overtone lines is much wider. It is no surprise that the sound of this string was perceived as getting progressively brighter as it was tuned higher.

Next, it is useful to show results to support the damping model presented in Section 2.2. Figure 2 shows results for damping factors of three different nylon strings fitted to a guitar. Two of them are the thinnest and thickest strings used in an earlier study (called "string 1" and "string 4" there) [4]. The third string shown here is a section of very thick harp string from the same original piece as "string 23a" in a different earlier study [7]. The two thinner strings were tuned to the same frequency, but the thickest string was tuned to a lower frequency than the thinner strings to reinforce the main point to be learnt from these results (and because the structure of the guitar would not tolerate tuning such a heavy string to the same pitch).

The discrete symbols in the plot show the measured points, while the various lines indicate the theoretical comparison. The separate contributions to the total damping η_n are shown: values for the loss factor η_{body} associated with energy transfer to the guitar body are included in this plot, calculated as explained in earlier work [4]. The plot confirms the earlier statement that this contribution is small enough that it can be ignored for the purposes of this study.

To calculate η_{bend}, a value for the loss factor η_E was needed. As has already been remarked, the real part of the Young's modulus E for nylon strings has been shown to vary significantly with both frequency and stress state. However, careful examination of the damping results suggested that a good fit was given to all measurements by assuming that the imaginary part remained constant, independent of stress. Furthermore, the frequency dependence can be ignored at the high frequencies relevant here, in the kHz range: as can be seen in Figures 2 and 3, a satisfactory fit to the measurements was given by this approximation. In terms of the usual terminology of the theory of viscoelasticity (see for example [16]), $E = E' + iE''$ where E' varies with stress but E'' does not. A simple fit to the results for E_B in Figure 10 of [7] gives

$$E' \approx 4.5 + 39\sigma \text{ GPa}, \qquad \text{(nylon)}, \tag{16}$$

where σ is the stress expressed in GPa, while the damping results are consistent with a value $E'' = 0.25$ Gpa. The result is that $\eta_E = E''/E'$ varies with stress, falling from approximately 0.04 to 0.02 as stress increases over the range tested here.

The solid curves in Figure 2 show the combined loss factor η_n for each string predicted according to the analysis of Section 2.2, and it is immediately clear that each curve follows the trend of the measured data points remarkably well. This confirms the damping model developed in Section 2.2, and also illustrates the pattern implied by that model. Damping is relatively high at low frequency, falls to a minimum around the frequency where η_{air} and η_{bend} are equal, then rises rapidly at higher frequencies so that there is an effective roll-off frequency above which string modes have damping that is too high to sound "musical". The thinnest string has this roll-off around 13 kHz, the middle string has it around 7 kHz, but for the thick string all modes have a higher loss factor than the plotted points for the thinner strings, and it was only possible to determine values up to about 1.5 kHz. It will come as no surprise that this thick string produced a thoroughly unsatis-

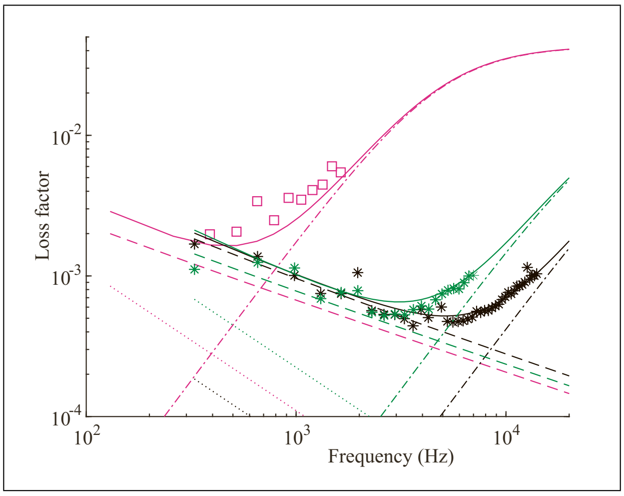

Figure 2. (Colour online) Loss factor as a function of frequency for three different nylon strings of vibrating length 0.65 m fitted to the same guitar body. Black: string of diameter 0.50 mm tuned to 327.5 Hz; green: string of diameter 0.96 mm tuned to 327.5 Hz; magenta: string of diameter 1.68 mm tuned to 131 Hz. Discrete points: measured results; dotted lines: loss factor η_{body}; dashed lines: loss factor η_{air}; dash-dot lines: loss factor η_{bend}; solid lines: combined loss factor η_n.

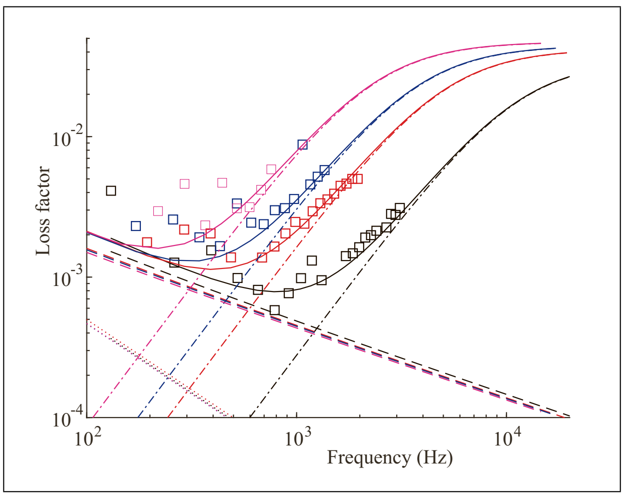

Figure 3. (Colour online) Loss factor as a function of frequency for four fluorocarbon strings of vibrating length 0.625 m, mounted on a lute. Black: string of diameter 0.88 mm tuned to 130 Hz; red: string of diameter 1.22 mm tuned to 97 Hz; blue: string of diameter 1.33 mm tuned to 86 Hz; magenta: string of diameter 1.57 mm tuned to 73 Hz. These strings will be discussed in Section 4.2. The format is similar to Figure 2. Discrete points: measured values; dotted lines: loss factor η_{body}; dashed lines: loss factor η_{air}; dash-dot lines: loss factor η_{bend}; solid lines: combined loss factor η_n.

factory sound on the guitar: more a muffled thud than a ringing guitar-like note.

For the two thinner strings, which produced acceptable musical sounds, it can be seen that the data points run out at roughly the same value of $\eta_n \approx 10^{-3}$. In the light of Equation (14), this suggests that the useful bandwidth of a given plucked string might be associated with a threshold value of λ, since the material loss factor η_E is fixed for a given string and tuning. Of course there is not a crisply-defined threshold for damping, but for the purposes of a design criterion with the right order of magnitude, a threshold value $\lambda \approx 0.05$ will be used in the subsequent development: the appropriateness of this choice will be confirmed later in the light of the two case studies in Sections 4.2 and 4.3. The chosen value justifies a statement made earlier in connection with Equations (6) and (7), that for musically-relevant string resonances the value of λ is always small.

The full set of results for other nylon strings and for fluorocarbon and gut strings cannot be reproduced here because of space constraints, but plots similar to Figure 2 have been examined for every tested case. A suitably calibrated version of the proposed damping model was found to give a satisfactory fit for each material, over the full range of string diameters and tensions. Four examples for fluorocarbon strings are shown in Figure 3: these strings were fitted to a lute and tuned to their normal playing pitches, and will form part of a case study of lute stringing in Section 4.2. Note that for the present purpose, the important aspect of these results is the high-frequency damping trend, leading to the effective roll-off frequency. The three thinnest strings give an excellent fit in this region. The thickest string gives a less clear scatter of points with higher damping, somewhat similar to what was seen in Figure 2. This plot was produced using fitted values for the Young's modulus of fluorocarbon strings as a function of stress similar to the results for nylon shown in Equation (16),

$$E' \approx 3.2 + 41\sigma \text{ GPa}, \qquad \text{(fluorocarbon)}. \tag{17}$$

For the imaginary part of Young's modulus, as was found for nylon, a constant value gives a satisfactory fit: $E'' = 0.18$ Gpa.

For gut strings the corresponding fitted model is simpler: fixed values $E' = 6$ Gpa, $\eta_E = 0.04$ are appropriate. It may be remarked that the gut strings did not generally give measured results as clean as those shown here for nylon and fluorocarbon strings. The trend is always clear, but the individual points usually show more scatter relative to the predicted curves. This scatter is probably a direct consequence of the construction of gut strings. The twisting and polishing processes are carried out by hand, on each length of string individually. Some variation of detailed structure along the length of the string is bound to occur, and this may lead to spatial variation in the complex Young's modulus. That variation will interact with the different mode shapes of the string overtones, and the two different polarizations of vibration in each mode, to produce variation in the modal loss factors. Indeed, if the detailed distribution of Young's modulus was known, the Rayleigh's principle argument could be applied mode by mode to predict these variations in damping, in a similar way to earlier predictions of modal variations in damping factor: see for example the results for composite plates [17].

Table I summarises the values of E' and E'' obtained for the three materials, together with the corresponding bulk density values used here. The earlier studies of harp strings found differences between the bulk densities of the thinner and thicker monofilament nylon strings tested [7, 13], and between the thinner (monofilament) and thicker (wound) fluorocarbon strings [8, 14]. The bulk density values shown in Table I are representative of the measured values for the thinner nylon and fluorocarbon harp strings, and were selected as being appropriate for the string diameters common to other instruments. For the construction of the string design charts to be shown here, however, the exact values of the material bulk densities are not particularly important; what matters is the relative difference between the densities of the three materials, with fluorocarbon having a significantly higher density than natural gut or nylon.

For all three materials, at low stress levels where the damping effect will turn out to be most important, η_E is around 0.04. This coincidence of values is very convenient, because it means that if the same threshold value of λ is used for all three materials, that will correspond to essentially the same values of modal damping factor for the string modes. This will allow directly comparable string design charts to be presented for the three materials, in the next section. Furthermore, it was noted earlier that λ also governs the degree of inharmonicity of a string. This means that design guidelines based on a threshold value of λ will set a limit on inharmonicity as well as damping. Both damping and inharmonicity have been associated in earlier literature with the perceptual discrimination of “warmth” versus “brightness”. The damping roll-off affects the spectral centroid, and there is a well-established correlation of perceived brightness with variation in spectral centroid [18]. Quite separately, the perceptual consequences of inharmonicity have been investigated in literature extending back at least as far as the classic work of Fletcher *et al.* [19]. The fact that both effects are governed by the same parameter may give a new perspective on relevant perceptual questions, as will be discussed further in Section 5.

4. A design chart for string selection

4.1. Development of the chart

The criterion of a threshold value of λ can be expressed in graphical form. From Equation (4), it can be seen that the value of λ for a given material depends on three parameters relevant to string choice: d, f_1 and L. However, the expression for λ can be written in terms of two combinations of them,

$$\alpha = L f_1, \quad \beta = d/L. \tag{18}$$

The parameter α is a natural one to bring in, since if bending stiffness is ignored, α remains constant as a given string on a guitar, say, is fingered in different positions. Using Equation (3), $\alpha \approx c/2$ where c is the wave speed on

Table I. Best-fitted material properties for the three string materials. For calculating E' for nylon and fluorocarbon, the value of σ should be expressed in GPa.

Material	E' (GPa)	E'' (GPa)	ρ (kg/m^3)
Nylon	$4.5 + 39\sigma$	0.25	1070
Fluorocarbon	$3.2 + 41\sigma$	0.18	1800
Natural Gut	6.0	0.24	1320

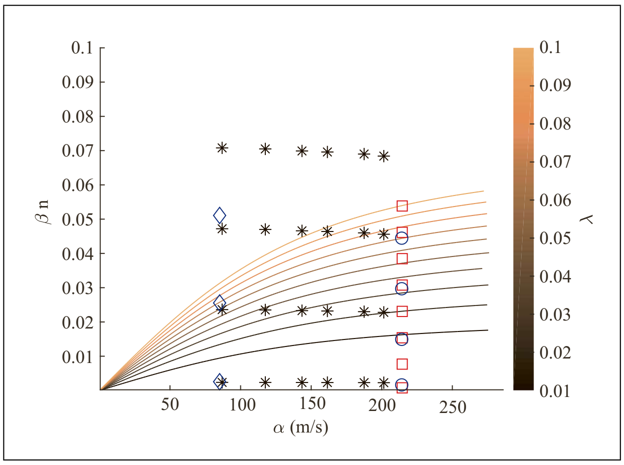

Figure 4. (Colour online) Contours of λ for nylon strings, plotted at intervals of 0.01 up to 0.1. Discrete symbols show results for strings previously mentioned: for each column of points the lowest symbol shows the value for $n = 1$, then above it values for $n = 10, 20, 30 \ldots$ Squares: string of diameter 0.50 mm tuned to 327.5 Hz, plotted in black in Figure 2; circles: string of diameter 0.96 mm tuned to 327.5 Hz, plotted in green in Figure 2; diamonds: string of diameter 1.68 mm tuned to 131 Hz, plotted in magenta in Figure 2; stars: string of diameter 1.20 mm as in Figure 1, tuned to a range of frequencies between 174 Hz and 403 Hz.

the string. For material of a given density, its value determines the stress:

$$\sigma = 4\rho\alpha^2. \tag{19}$$

It is straightforward to draw a contour map of λ in the $(\alpha, n\beta)$ plane, noting that for nylon or fluorocarbon, E is a function of α through Equations (16), (17) and (19). An example is shown in Figure 4, for nylon strings. Contours of λ have been plotted at intervals of 0.01 up to the value 0.1. Beyond that value the string overtones will surely be too highly damped to be of interest: recall that the suggested threshold value is 0.05, in the middle of the plotted range.

Points corresponding to particular strings can be calculated and added to the plot, but because of the presence of n in the quantity plotted on the y-axis, a given string gives a point for every relevant overtone. These overtones all have the same value of α, so they make a regular vertical column in the plot. Points are included here for the four nylon strings for which results have already been shown. The open symbols show the three strings from Figure 2, while the stars show the string from Figure 1, at all the tunings tested. The two cases plotted in Figure 1 were the

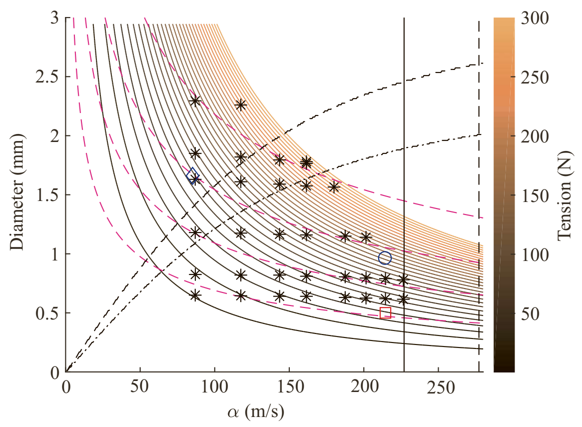

Figure 5. (Colour online) Design chart for nylon strings. Solid curves: contours of equal tension in steps of 10 N up to 300 N; dashed magenta curves: contours of impedance Z_0 at values 0.08, 0.2, 0.4, 0.8 Ns/m; black curves: damping threshold for $L = 0.65$ m (dashed) and $L = 0.5$ m (dash-dot); vertical lines: indications of breaking stress, see text. Discrete points mark all tested nylon strings. Square, circle, diamond: strings from Figure 2 as shown in Figure 4; stars: all strings and tested frequencies from [7, 13].

leftmost and rightmost of the set. For each string, the lowest plotted symbol shows the fundamental $n = 1$, and then to indicate the pattern without cluttering the plot with too many points, symbols are plotted above it for $n = 10$, 20, 30…

To interpret the plot, consider first the thinnest string of the set in Figure 2, indicated by square symbols towards the right-hand side of Figure 4. Locating the contour corresponding to $\lambda = 0.05$, it can be seen that the closest square symbol to that contour marks the value $n = 50$, so the prediction is that this string should have about 50 overtones with damping lower than the chosen threshold. The middle string from Figure 2 is indicated by circular symbols, and because this string had the same length and the same tuning as the thinnest string, they appear at the same value of α. However, the circular symbols are wider apart, and the $\lambda = 0.05$ contour passes between the two symbols marking $n = 20$ and 30. So for this string, roughly 25 overtones should have damping below the threshold. Comparing the two, the prediction is that the bandwidth of lightly-damped string modes should be roughly twice as big for the thinner string. Looking at where the plotted points run out in Figure 2, this prediction matches the observations quite well.

The thickest string from Figure 2 is indicated in Figure 4 by diamond symbols, towards the left-hand side. For this string, even the symbol corresponding to $n = 10$ lies above the $\lambda = 0.05$ contour, so the prediction is that this string should have very few lightly-damped overtones. Recall that the criterion underlying this plot captures only the damping due to viscoelasticity: for a very thick string like this, the damping due to air viscosity takes over at low frequency while the viscoelastic loss is still quite high, so that in fact the model predicts that this string should have no modes at all with low damping. That is exactly what the measurements in Figure 2 revealed.

This criterion based on damping can now be incorporated into a design chart for string selection. For a given string on a given instrument, the desired values of L and f_1 will be known, and the task is to select a string material and gauge d. It is possible to summarise all the constraints on string selection into a single chart with $\alpha = Lf_1$ on the horizontal axis and d on the vertical axis. The string tension $T = \pi d^2\sigma/4 = \pi\rho d^2\alpha^2$. This means that for a given material, contours of equal tension can be plotted in the chart, as illustrated for nylon in Figure 5. Note that the values of tension shown here will be somewhat inaccurate because they are based on the unstretched linear density of the string, whereas the actual linear density of the stretched string is a little lower [7, 8]. However, this is a small effect and it makes no significant difference to the broad-brush argument underlying the design chart presented here.

From Equation (8), the string impedance can be expressed as $Z_0 = \pi\rho d^2\alpha/2$ and so impedance can also be indicated on this chart. The set of magenta dashed curves, falling towards the right, shows some selected contours of equal impedance. If one wished to select a set of strings with constant impedance, in the interests of equal loudness, these lines indicate the trend that should be followed.

Discrete symbols have been added to Figure 5 to indicate previously-studied nylon strings. Stars show the strings from [7], each appearing as an approximately horizontal row of stars showing the different tested tunings. Open symbols correspond to those in Figure 4, for the three strings whose properties were shown in Figure 2. The vertical lines give an indication of the ultimate breaking stress of nylon strings. The solid line marks the highest value of α for which a string survived the sequence of testing described in [7] without breaking. But nylon does not break immediately if this threshold is exceeded: what happens instead is that the string never stops creeping (and thus requiring to be re-tuned), and eventually it will fail. But many musical instruments are fitted with nylon strings requiring a higher value of α: the most extreme the authors have been able to find for an instrument in regular professional use is the top string of the 8-string "Brahms guitar" developed by luthier David Rubio in collaboration with guitarist Paul Galbraith [20, 21]: the original version of this guitar has a top string of length 630 mm, tuned to 440 Hz (A_4), giving the value of α shown by the dashed vertical line.

Now the damping criterion can be added. Because the vertical axis depends on d rather than on β as in Figure 4, the length L will make a difference. For a given value of L, it is easy to take each value of α and use the expression for λ to calculate the threshold value of d for which a string of that length would have a specified number of overtones with damping lower than the chosen value. For the purpose of plotting something that gives a good indication of practical limits, the lines shown here correspond to requiring 10 overtones with $\lambda < 0.05$. This leads to the rising curving lines in Figure 5. The dashed line is for

$L = 0.65$ m, the length of the guitar strings shown by the open symbols, while the dash-dot line is for $L = 0.5$ m, the string length for the rig in which all the strings with star symbols were tested. It is reassuring to see that the two earlier examples of strings giving unsatisfactory sound both lie above the relevant lines in this plot: the diamond corresponding to the thickest string shown in Figure 2 lies above the $L = 0.65$ m line, and the left-most star for the string with diameter 1.2 mm, corresponding to Figure 1a, lies just above the $L = 0.5$ m line.

To understand what this chart shows, it is helpful to look at the schematic version plotted in Figure 6. For any given instrument, the mechanical structure will impose an upper limit on string tension. Practical considerations of playing any plucked-string instrument enforce a lower limit on tension. The string needs to be below its yield stress, or at least to be not too far above that limit so that it survives for long enough to be useful. Finally, the damping criterion must be satisfied. The result is that the string needs to be chosen from within a region of the plot like the one shown shaded here, bounded by these various limiting conditions. For a given value of α, varying the gauge results in moving along a vertical line within this region. Moving upwards will increase the tension and the impedance. However, especially if the value of α is fairly low, it also results in moving closer to the curve giving the trend of the damping limit. So, in very broad terms, a thicker string will tend to be louder but less bright. Conversely, a thinner string will have lower tension and impedance, and so be quieter but brighter-sounding.

Corresponding charts can be plotted for the other string materials for which mechanical properties are available. Figure 7 shows the pair of charts for fluorocarbon strings. These are qualitatively similar to the charts for nylon, but with subtle differences in the shapes and positions of the curves that will be shown to have musical consequences in Sections 4.2 and 4.3. Figure 8 shows corresponding plots for plain gut strings, and now the curves are quite different. The lines in Figure 8a are straight rather than curved, and this results in the damping threshold lines in Figure 8b also being straight. The explanation lies in the "strain stiffening" effect discussed earlier. The curvature in the lines for the two synthetic polymer materials is a direct result of the variation of E with α. Gut has a strain-independent value of E, and for that case it can be deduced directly from Equation (4) or (7) that the contours of λ are straight radial lines as seen in Figure 8a. That in turn produces straight lines in Figure 8b.

Both sets of plots include points for individual strings similar to those shown for the case of nylon. Figure 7b includes points for all fluorocarbon strings and tunings tested in [8]. One of these strings was selected to include in 7a, the one with closest match of d to the 1.20 mm string chosen for Figure 4. The vertical line in Figure 7b marks the highest value of α for which a fluorocarbon string survived the test programme described in [8] without breaking. The authors have no information about the use of such strings on instruments with higher values of α: but this is not surprising since the plot reveals that fluorocarbon breaks at a lower value of α than nylon, so for instruments requiring extreme values of α, nylon is likely to be chosen in preference to fluorocarbon. Similarly, Figure 8b shows stars for all the gut strings and tunings tested in [8], and

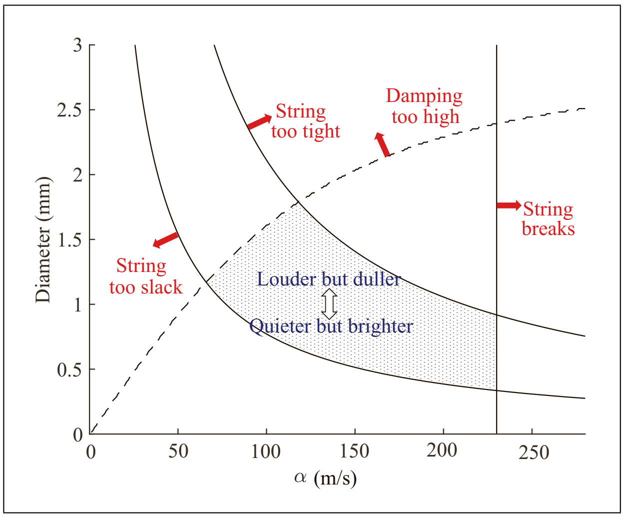

Figure 6. (Colour online) Schematic version of design chart for nylon strings as in Figure 5.

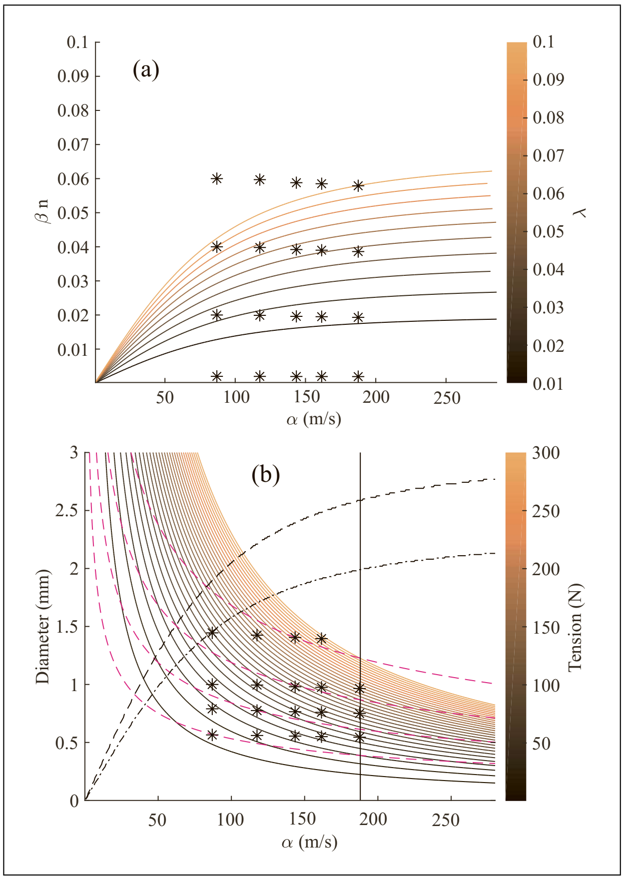

Figure 7. (Colour online) Variation of λ and design chart for fluorocarbon strings, in the same format as Figures 4 and 5 and with identical scales to those figures. Stars: all fluorocarbon strings and tested frequencies from [8, 14].

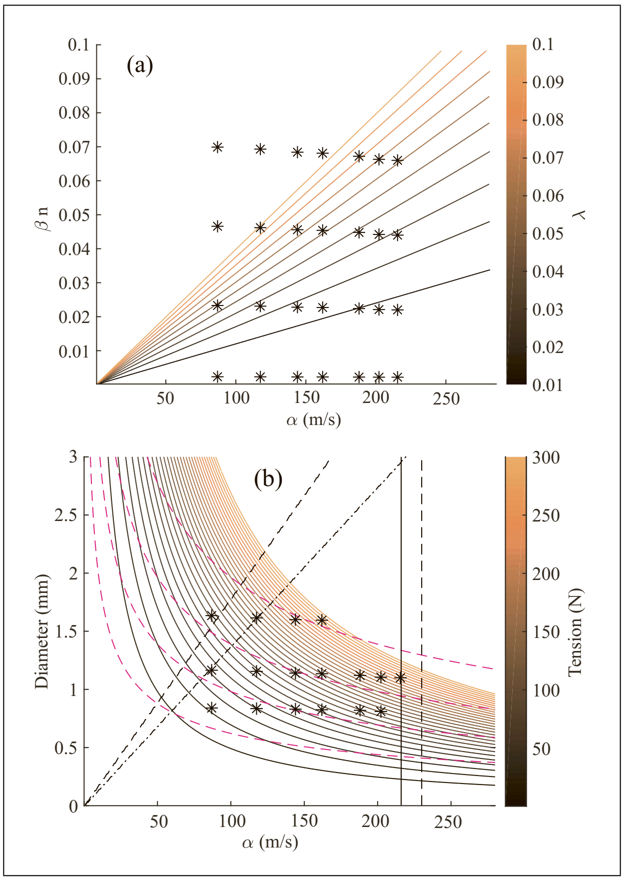

Figure 8. (Colour online) Variation of λ and design chart for gut strings, in the same format as Figures 4 and 5 and with identical scales to those figures. Stars: all gut strings and tested frequencies from [8, 14].

the string with the closest d value to 1.20 mm has been selected to show in Figure 8a. The solid vertical line in Figure 8b again marks the highest value of α for which a string survived the test programme, and the dashed vertical line marks the value corresponding to the highest string of the harp which will be studied shortly. To unpack the significance of all these charts, two case studies of practical string selection will now be presented.

4.2. Case study: the lute

String choice can be a particularly tricky issue for period instruments. There is some guidance from historical sources about the choice of stringing, but this typically dates from before the availability of precise measuring equipment, and so the guidance is purely qualitative. The lute is a case in point. Lutes began to appear in European music and art around about the 15th century, and the instrument continued to be played until about the time of Bach, gradually evolving in form: see for example [22]. The instrument has undergone a revival in the last 50 years or so. A case study to "reverse engineer" a particular choice of stringing for an 8-course lute will be presented here. This style of lute is typical of the late 16th century, the time of Dowland. A lute of this kind would originally have been strung with plain gut strings, but the set examined here consists of synthetic polymer strings, intended to produce a similar sound and feel to gut.

Table II. The string materials and nominal diameters (in mm) for the 8-course lute string set, tuned in G based on A 440 Hz, and with string length 0.625 m. 'N': Nylon string; 'F': Fluorocarbon string.

Course	Pitch (Hz)	String 1	String 2
1	392.0	0.475 N	Not present
2	293.7	0.52 N	As string 1
3	220.0	0.725 N	As string 1
4	174.6	0.900 N	As string 1
5	130.8	0.91 F	As string 1
6	98.0	1.25 F	0.775 N (196.0 Hz)
7	87.3	1.40 F	0.875 N (174.6 Hz)
8	73.4	1.60 F	0.925 N (146.8 Hz)

The particular set studied here is offered commercially for this purpose [23], without any explanation of the rationale behind the choices. The details of string length, gauge, tuning and material are given in Table II. The strings are a mixture of nylon and fluorocarbon, and at first glance the underlying logic is not obvious. However, by plotting them on the design charts for nylon and fluorocarbon it will emerge that the choice is entirely rational.

Figure 9a shows the nylon design chart as in Figure 5, with various relevant points added. The three open symbols are the same as in the earlier plots, for orientation. The three star symbols show the top three strings of a typical modern classical guitar, which are usually of plain nylon. The remaining symbols, plotted as filled squares of various kinds for nylon and filled circles of various kinds for fluorocarbon, correspond to the set of lute strings. The full set has been plotted here, including the ones that are in fact fluorocarbon. In a similar way, Figure 9b shows the design chart for fluorocarbon as in Figure 7b, with the same full set of points for the lute strings. In both plots, the damping threshold curve has been plotted for two values of L: 625 mm, the open string length of the particular lute tested (solid lines), and 312.5 mm, the length at the octave (dashed lines). Lutes of this period usually have no more than 12 frets, so the dashed lines mark the shortest relevant length. Armed with all this information, the string choices will be discussed, starting from the highest string.

The top string of the lute lies to the right of the vertical line in Figure 9a, whereas the top string of the classical guitar lies to the left of it. It is indeed the case that guitar top strings rarely break, whereas the top string of the lute has a finite lifetime before it breaks through progressive creep and eventual necking to failure. This top string is traditionally a single string, but all the remaining strings of the lute come in pairs called "courses". It can be seen in the plot that the 2nd, 3rd and 4th courses (all in nylon) have essentially the same tension, around 30 N. There is some historic evidence for choosing strings of equal tension in instruments of this period (for example it is recommended by Mersenne in *Harmonie Universelle* [24]), so this choice is obviously deliberate. Note that in an instru-

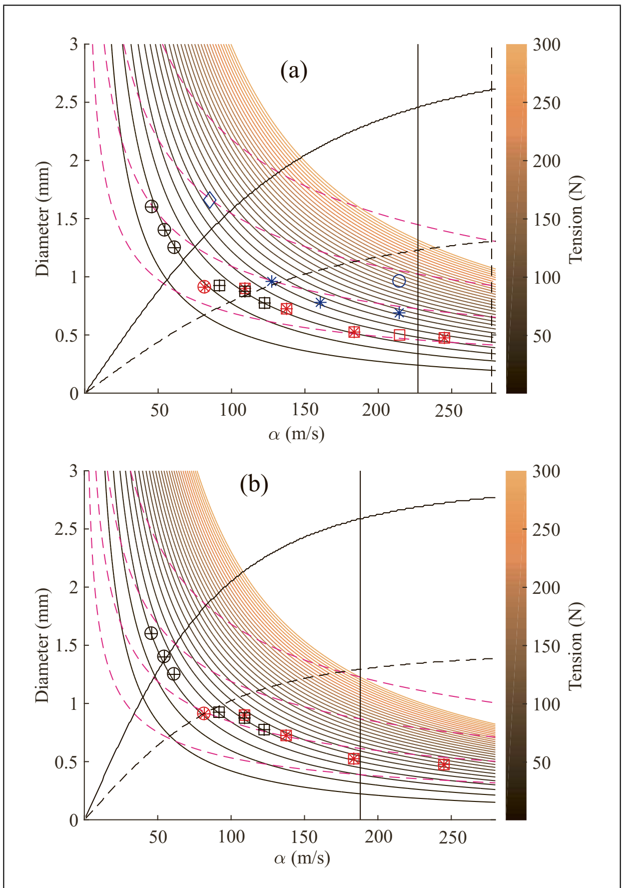

Figure 9. (Colour online) Design charts for (a) nylon; (b) fluorocarbon as in Figures 5 and 7, annotated to illustrate case study of lute stringing. Black curves: damping threshold for $L = 0.625$ m (solid), $L = 0.3125$ m (dashed). Empty square, circle, diamond: as in Figure 5; stars: top three strings of a typical set of classical guitar strings; star-in-square: nylon strings for top four courses of lute; star-in-circle: fluorocarbon strings for 5th course of lute; plus-in-circle: fluorocarbon strings for courses 6, 7, 8 of lute; plus-in-square: nylon strings for octave strings of courses 6, 7, 8 of lute.

ment like the lute where all strings have equal length, equal tension automatically means "equal feel" from Equation (10). The top string has somewhat higher tension, perhaps to raise its impedance to compensate somewhat for the reduction in loudness as a result of being single, or perhaps to increase the feel for a similar reason. The contour lines for impedance show that the first and second courses have similar values of impedance. As an aside, the top three strings of the classical guitar (stars) show a rather similar pattern in this chart to the top three courses of the lute.

The 5th course, shown as a star-in-circle symbol, presents a problem for nylon strings. If a nylon string at this value of α was chosen with a gauge to give it the same tension as courses 2, 3 and 4, it would lie close to the solid curve indicating the damping roll-off for the open string length. This would lead to a very unsatisfactory sound. But the designer of the set of strings investigated here has taken the sensible decision to switch to fluorocarbon for this course. The corresponding symbol in Figure 9b falls on the dashed curve, but lies well short of the solid curve. This has been made possible by the subtle difference of shape between the limit curves for nylon and fluorocarbon, having its origin in the higher density and slightly different Young's modulus behaviour of fluorocarbon (see Table I). The chosen gauge of the fluorocarbon strings for the 5th course has resulted in essentially the same tension as the nylon strings for the higher-frequency courses, as can be seen by counting the contour lines in the two plots. However, the charts reveal that the impedance of this fluorocarbon string is rather greater than that of the higher nylon courses.

For the three bass courses of the lute, even fluorocarbon strings suffer from the problem of high damping. The three strings are plotted as plus-in-circle symbols: all three lie near or beyond the solid curve in the fluorocarbon diagram, although it can be seen that the gauges have been chosen to continue the constant-tension pattern. To deal with this problem, standard lute stringing uses a trick. Instead of having two strings in unison, as was the case for the other courses, the second string of each pair is tuned an octave higher: these octave strings are indicated by the plus-in-square symbols, and the string set specifies nylon for these strings at approximately the same tension as the other strings. The player plucks the two strings simultaneously, with the result that the bass string gives the desired fundamental frequency component, while the octave string (which lies below the solid curve) is able to give acceptable brightness to the tone by contributing a spread of higher overtones.

In summary, the designer of this string set has done an excellent job by mixing the two different string materials. The only detail one might question is whether the three octave bass strings might have been specified in fluorocarbon (with suitably modified gauges) rather than nylon, to give them a little extra brightness. But even here, the decision may have been deliberate: a player probably does not want the octave string in a bass course to sound too prominently, because that might impair the illusion of a combined string sound with both low frequency components and brightness at higher frequencies. As a final comment on this case study, it should be noted that the damping roll-off predictions based on the chosen threshold value $\lambda = 0.05$ are in very good general agreement with the subjective impressions of a player.

4.3. Case study: the harp

A sharply contrasting case study in string choice is given by the harp. Again, this discussion is an exercise in reverse engineering: the information to be shown here is based on the chosen set of gauges for either nylon or gut strings on a particular pedal harp. There is an interesting issue to be explained. Many harpists, especially in Europe and North America, prefer gut strings, on grounds of "sounding better". They tend to regard nylon strings as only suitable for beginners. But virtually all classical guitarists have switched from gut to nylon strings, the only exceptions being those specifically choosing to play period instruments in an authentic style. A possible explanation for this difference of opinion will be suggested in this section.

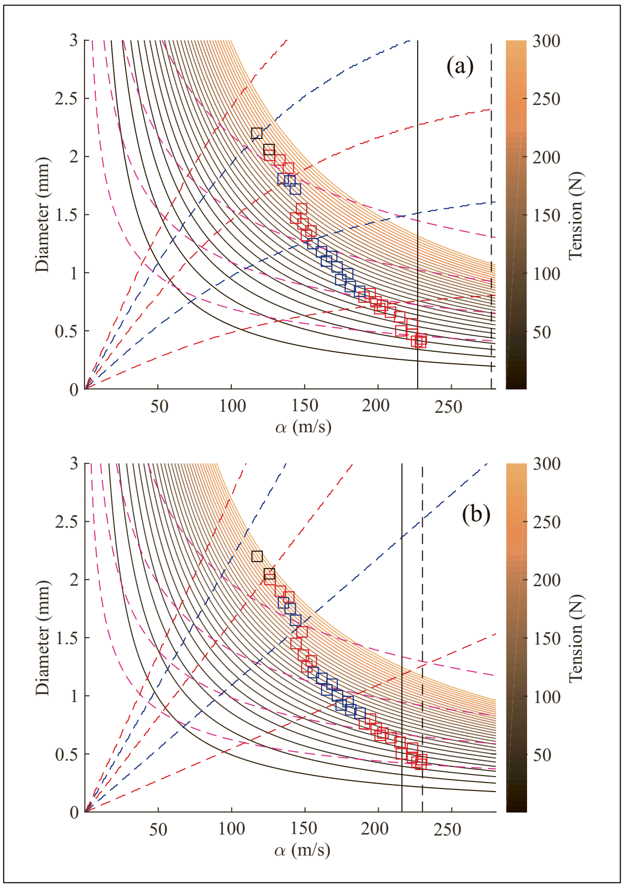

Figure 10. (Colour online) Design charts for (a) nylon; (b) gut as in Figures 5 and 8, annotated to illustrate the case study of harp stringing. Dashed curves: damping threshold for $L = 0.2, 0.4, 0.6, 0.8, 1.0$ m, in alternating colours. Squares: selected stringing for a particular harp, with colours that alternate in the same pattern as the damping threshold curves (see text for details).

Figure 10 shows versions of the nylon and gut design charts, with points marked for the top 35 strings of the chosen harp (a Russian-made Elysian Cecilia 46 pedal harp). The string lengths and gauges used are shown in Table III. The lower notes of this harp used metal overwound strings, outside the scope of these design charts. A distinctive feature of the harp, of course, is that each string has a different length. This means that each string strictly needs it own version of the damping roll-off curve. In an attempt to show the pattern sufficiently clearly, a set of curves has been plotted for string lengths 0.2, 0.4, 0.6, 0.8 and 1.0 m, covering the range of the actual string lengths. The curves are plotted in alternating colours, and the discrete symbols for the individual strings switch colour in the same pattern, as the string length passes the values corresponding to these plotted curves. (The two strings longer than 1 m are shown with black symbols.) What is revealed in both plots is that the string choice has stuck rather close to the damping threshold used in these calculations, over the entire range: the symbol colours change in a manner that is approximately synchronised with the plotted curves.

The other conspicuous feature of these plots is that the string tension increases steadily from treble to bass over the entire range of the instrument, reaching tensions far higher than those seen earlier for the guitar, let alone for the lute. The tensions approach the 300 N limit of the contours plotted here. It may have been noted in the earlier plots, Figures 5, 7b and 8b, that the test points for the earlier studies [7, 8] ceased at about the same tension. This is not a coincidence: the test rig for those earlier studies was designed with harp strings in mind, with a load cell set up for a force measurement limit of 300 N.

As a result of these two features of the stringing choices, the harp explores the region of the design charts where

Table III. String lengths and diameters for the harp string sets. Strings are numbered according to the harp convention, and the corresponding notes on the piano scale are given. The gut string gauges are from Bowbrand's "Pedal Light" range [25]. The nylon string gauges are based on Bowbrand's "Pedal Nylon" range, with scaling adjustments to provide a better comparison with the gut "Pedal Light" range, which uses slightly thinner strings for some notes than the "Pedal Standard" gut range. Fluorocarbon ("Carbon") gauges are suggested, based on the scaling approach described in the text.

String	Note	Length (mm)	Gut (mm)	Nylon (mm)	Carbon (mm)
00	G7	73	0.41	0.40	0.35
0	F7	81	0.43	0.41	0.37
1	E7	87	0.45	0.43	0.39
2	D7	95	0.47	0.47	0.40
3	C7	103	0.50	0.50	0.43
4	B6	113	0.55	0.56	0.47
5	A6	122	0.60	0.62	0.51
6	G6	133	0.64	0.66	0.55
7	F6	144	0.66	0.69	0.57
8	E6	154	0.68	0.72	0.58
9	D6	169	0.72	0.75	0.62
10	C6	182	0.76	0.79	0.65
11	B5	197	0.80	0.82	0.69
12	A5	213	0.85	0.84	0.73
13	G5	231	0.88	0.88	0.75
14	F5	250	0.92	0.94	0.79
15	E5	272	0.95	0.99	0.81
16	D5	293	1.00	1.05	0.86
17	C5	315	1.05	1.10	0.90
18	B4	341	1.10	1.14	0.94
19	A4	367	1.15	1.18	0.98
20	G4	397	1.20	1.25	1.03
21	F4	433	1.25	1.32	1.07
22	E4	468	1.30	1.36	1.11
23	D4	507	1.35	1.42	1.16
24	C4	550	1.45	1.47	1.24
25	B3	599	1.55	1.55	1.33
26	A3	653	1.65	1.72	1.41
27	G3	714	1.75	1.79	1.50
28	F3	775	1.80	1.81	1.54
29	E3	843	1.85	1.90	1.58
30	D3	907	1.90	1.97	1.63
31	C3	965	2.00	2.01	1.71
32	B2	1018	2.05	2.06	1.76
33	A2	1067	2.20	2.20	1.88

the nylon and gut results are most different. The straight lines of the gut chart give a little more "headroom" than the curves of the nylon chart, and this may be the key to harpists' preference for gut strings. Figure 11 gives direct comparisons for several important quantities between the nylon and gut stringings, shown as the two solid lines in each plot. The first subplot shows the string impedance, as a function of string number (harp strings are numbered from highest to lowest, starting with the highest strings being labelled 00 and 0, then 1, 2, 3 etc.). The values for nylon and gut follow a similar trend, but the nylon strings have consistently lower values across the whole range. The string impedance rises dramatically towards the bass end of the instrument, since both mass per unit length and tension have been chosen to increase as the strings get longer. Harp strings are certainly not selected with an eye to constant impedance, to result in constant loudness: as is shown by the contour lines of impedance in Figure 10, that would require tension to decrease, not increase, for the longer strings.

The clue to this apparently perverse choice may come in the second subplot of Figure 11: the increasing tension for longer strings means that the "feel" only varies rather slowly across the range of the instrument. From Equation (10), longer strings require higher tension to maintain "feel" at similar values. As with the impedance, the gut and nylon strings follow similar trends in this plot, with nylon having consistently lower values.

The remaining subplots show two different views of the predicted damping roll-off frequency, according to the criterion used in all calculations here. The first of these plots shows the number of string overtones passing the threshold test, while the second turns this number into a frequency bandwidth by using the relevant fundamental frequencies. A clear difference is seen between nylon and gut: the gut strings have a significantly wider bandwidth than the nylon strings, across the entire range. The reason is the increased "headroom" mentioned above. The attempt to limit the variation of "feel" has forced the use of very high tensions, and this inevitably runs the danger of approaching uncomfortably close to the damping roll-off threshold. Nylon suffers from this problem to a greater extent than gut, and so these results strongly suggest that gut strings will sound brighter than nylon strings over the entire range of the harp. This is true even though the gauges of the nylon strings have been chosen to give lower tension than the gut strings: a compromise has been struck between loudness and brightness.

This observation seems a strong candidate for explaining players' continued preference for gut strings, despite the disadvantages of higher cost and higher sensitivity to changing environmental conditions [8]. Figure 9a shows why guitarists can afford to make a different choice. The plain nylon strings of a classical guitar do not fall in the region of the design chart where there is such a big difference between gut and nylon, and so they opt for the convenience and practicality of synthetic strings. They sometimes opt for fluorocarbon rather than nylon, especially for the 3rd string, just as the charts suggest.

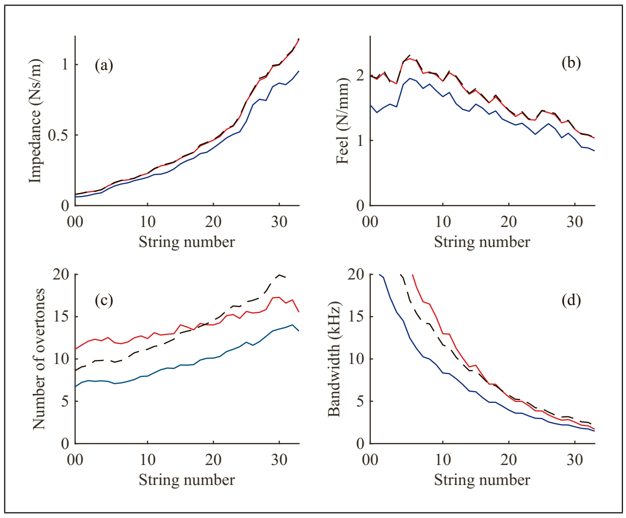

Figure 11. (Colour online) For the harp stringing as in Figure 10, variation with string number of (a) impedance Z_0; (b) feel γ; (c) number of overtones with damping lower than chosen threshold; (d) damping threshold as in (c), expressed as bandwidth. Blue: nylon strings; red: gut strings; dashed black: fluorocarbon strings, with diameters scaled from gut as described in text.

Harpists are also showing a growing enthusiasm for fluorocarbon strings, saying that they "sound more like gut". The analysis presented here sheds light on this claim. This choice of string material is sufficiently new that there is not yet a well-established choice of string gauges for the harp. Results are shown here for a particular choice. Starting from the specification of the gut strings, string diameters d for fluorocarbon can be chosen by scaling the gut values by a factor 0.86, which is the square root of the density ratio of the two materials. That has the result of giving the two sets of strings the same mass per unit length, and hence the same tension. It follows that the impedance and the feel will be identical (at least in this limited sense of the word "feel").

The dashed lines in Figure 11 show the results of this choice. The lower two subplots show that the fluorocarbon strings consistently beat nylon strings in terms of damping roll-off. The values fall quite close to those for the gut strings, and for the lower strings they even beat them. It seems that fluorocarbon strings, with this choice of gauges, should indeed sound "more like gut", and for the lower-frequency strings they should actually sound brighter than gut. Of course, one would not in fact want to use fluorocarbon for the highest strings of the harp: nylon would still be used on grounds of strength. However, there is no problem with brightness of nylon for the highest strings: all three curves show a bandwidth which exceeds the limits of human hearing, so that in practice the bandwidth will be determined by the details of the player's pluck gesture. There might even be advantages in the higher damping of nylon if there is a danger of harshness in these highest notes.

5. Discussion and Conclusions

The choice of string materials and gauges for musical instrument use is influenced by several factors. There are limits on tension arising from practical considerations of strength and playability, and there is an upper limit on stress so that the string does not break. These are very familiar, but it has been shown here that there is also a less familiar limit arising from the influence of material damping. For the particular case of monofilament strings of a given material, it has been shown that all these factors can be represented in a single design chart. Such charts give a synoptic view, shedding new light on the selection process. It must be emphasised again that all the "limits" shown in these charts should be regarded as approximate. The intention is to show trends to guide the process of selecting a coherent set of strings for an instrument, not to show ultra-precise numerical values. Ultimately, the musician's ears are still the most important item of test equipment.

Examples of the design charts have been shown for nylon, fluorocarbon and natural gut strings and, based on these charts, detailed case studies for the stringing of a lute and a harp have been presented. In both cases, the selection process has been shown to be significantly influenced by the new limit associated with damping. This has given possible explanations for two observations: why many harpists still prefer gut over nylon whereas classical guitarists have almost universally switched from gut to nylon, and why fluorocarbon strings are gaining in popularity over nylon in certain contexts because they are said to sound more like gut.

The damping limit has been expressed in terms of a threshold value of the parameter λ, defined in Equation (4), quantifying the influence of the string's bending stiffness. Interestingly, the same parameter governs the extent of inharmonicity in the string's overtones, also associated with the effect of bending stiffness. The fact that these two effects are directly connected in this way may have perceptual significance, and it may shed new light on some previously published perceptual experiments relating to inharmonicity. Järveläinen *et al.* [26] used synthesised tones to establish perceptual thresholds for the timbral effects of inharmonicity. They concluded that the inharmonicity effect should be clearly audible in standard classical guitar strings, especially the 3rd and 6th strings. But a later study by the same group [27] used manipulated sounds based on "authentic guitar sounds", and reached a different conclusion: they state that inharmonicity should be barely audible in classical guitar sounds.

This second study differed from the first in how the damping of the string overtones was modelled. The reference does not give very much detail, but it seems clear that the authors were not aware of the direct linkage between inharmonicity and damping behaviour. The results presented here suggest that it might be worthwhile to carry out a new perceptual test employing synthesised guitar-like or harp-like tones for strings of different materials and different gauges. The object of the test would be to establish the threshold of perception for a change in the value of λ, taking into account both the inharmonicity and the damping effect. Such a test would yield results of direct applicability to musical strings of the kind discussed here. Rather than being restricted to strings on a particular instrument by manipulating measured sounds, the results should generalise to strings of a given type in any musical context.

The detailed results presented in this work have all related to monofilament strings of synthetic polymer or natural gut. But a few comments can be added in relation to the other main types of musical string: monofilament metal strings, and metal-overwrapped strings with either metal or polymer cores. The methods used here would carry across directly to monofilament metal strings, but the loss factor η_E for relevant metals such as piano wire usually has a value at least an order of magnitude smaller than those seen earlier for polymeric materials. Valette has presented measurements on metal strings [3] which demonstrate that the damping model still works well, but such strings do not exhibit a "damping roll-off" in the sense explored in this work: the total loss factor η_n at high frequency never falls below η_E whatever happens, and so modal damping is always low enough to be "musically acceptable".

Overwrapped strings are more complicated, and detailed analysis lies outside the scope of this article. The rationale of overwrapped strings is to increase the mass per unit length while limiting the effects of bending stiffness. Various details of the construction of such strings contribute to the sound they make: the choice of core material and of wrapping material(s), the relative thicknesses of core and wrapping, and the details of the wrapping procedure all play a role. Valette [3] has shown direct evidence that the tightness of the windings of a metal-wrapped string can have a large effect on the damping behaviour, because tight windings introduce a new damping mechanism associated with dry friction, whereas an open winding eliminates this effect.

Initially tight windings will loosen a little as the core stretches under tension, particularly with polymer-cored strings. This suggests that the sound of the string will change a little as the string stretches and settles. However, this is not the only mechanism for the sound of wrapped strings changing over time. It is very familiar to guitarists that new strings lose their "twang" as they age. Convincing evidence has been shown that suggests a mechanism for this change: dirt and grease from the player's fingers gradually penetrates between the windings of the string, changing the damping behaviour [28]. It has been shown that this effect can be represented within a damping model of the kind used here by an increase in the effective value of η_E, exactly as one would guess from the mechanism just described [29]. That brings the string-ageing phenomenon into contact with the earlier discussion in this article: the change in sound of an ageing guitar string results from a gradual decrease in the damping roll-off frequency, not because λ is changing but because the effective threshold value of λ falls as η_E increases.

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Supplementary material

The file 'v105n01_woodhouse_lynch_supplementary_files.zip', containing versions of the three design charts suitable for printing and using for practical string selection can be downloaded from

http://aaua-material.com/t_ON6111

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