A Continuous Approximation Model for the Vehicle Routing Problem for Emissions Minimization at the Strategic Level

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How to cite this paper?

Link to the published paper
http://ascelibrary.org/doi/abs/10.1061/(ASCE)TE.1943-5436.0000442
ABSTRACT

This paper presents a continuous approximation model for the Vehicle Routing Problem for Emissions Minimization (EVRP) and demonstrates the model’s applicability and usefulness with numerical studies based on implementations of the Solomon test instances. The EVRP is a variant of the Time Dependent Vehicle Routing Problem (TDVRP) in which minimizing emissions is an additional objective of the model. The continuous approximation model presented in this paper will facilitate strategic planning of one-to-many distribution systems and evaluate the effects of emissions costs. Furthermore, results from the continuous model can provide guidelines for constructing solutions for the discrete EVRP. Furthermore, results from a sensitivity analysis indicated that the optimal number of dispatches during the peak period is smaller than the optimal number of dispatches during the off-peak period when considering the temporal effects of congestion. Results revealed that the potential cost savings due to incorporating emissions in distribution problems are considerable. Incorporating emissions costs in distribution problems will contribute toward having more environmentally sustainable distribution systems. The proposed continuous approximation model can be extended for other specific conditions, e.g. when considering pipeline inventory, storage restrictions, different customers, random demand, discriminating strategies, etc.
INTRODUCTION

It is clearly evident that human activities have affected concentrations, distributions and life cycles of a number of radioactively active gases and aerosols. Transportation is a major human activity that contributes to climate change by emitting direct and indirect greenhouse gases (GHG). In 2008, transportation sources contributed approximately 27 percent of total U.S. GHG (EPA 2006). Transportation is also the largest end-use source of carbon dioxide (CO₂), which is the most prevalent greenhouse gas. In 2003, “heavy-duty” vehicles, which are used primarily in freight transportation and distribution systems, accounted for 19 percent of total transportation emissions (EPA 2006). In recent years, there has been a growing interest in reducing transportation GHG emissions not only for the general perception of a “green” transportation system but also for economic reasons. Under the cap and trade emissions initiative, it is likely that GHG emissions will have a monetary value in the near future. This paper presents a continuous approximation model for the Vehicle Routing Problem for Emissions Minimization (EVRP). In typical vehicle routing problems, it is assumed that carriers want to minimize the number of vehicles as a primary objective and distance travelled as a secondary objective. However, there have been very few studies that consider minimizing emissions as an additional objective. Incorporating emissions costs in distribution problems like the vehicle routing problem will contribute toward having more environmentally sustainable distribution systems.

For the first time, Figliozzi (2010a, 2010b) presented the EVRP based on the traditional Time Dependent Vehicle Routing Problem (TDVRP) with a heuristic solution method. Wygonik and Goodchild (2011) developed an ArcGIS model to assess the trade-offs between cost, service quality, and emissions of urban pickup and delivery systems. Gabali (2010) proposed a framework for modeling CO₂ emissions in a TDVRP context that considers limiting vehicle speed as part of the optimization. Due to the computational complexity of the problem, solutions to the discrete EVRP are limited by instance size. Alternatively, approximate solutions may be obtained with continuous approximation models. However, the use of continuous approximation models for the EVRP is limited, specifically when considering a time dependent problem. Daganzo (1984a, 1984b) presented distance formulas for certain non-detailed, near-optimal vehicle routing strategies using the continuous approximation approach. Burns et al. (1985) developed a strategy for routing and scheduling deliveries from a single origin to many destinations with similar demand. Hall (1985) built upon this strategy by generalizing it for problems with widely different customer sizes. Clarens and Hurdle (1975) showed how a detailed solution to a one-to-many problem can be obtained from continuous approximation guidelines. Janic (2007) developed a model for calculating combined internal and external costs of intermodal and road freight transport network based on Daganzo’s cost formulations. This paper presents an alternative approach for the EVRP where detailed data are replaced by concise summaries and numerical methods are replaced by analytical models using continuous approximation. Continuous approximation models use smooth functions, such as a demand density function that varies slowly in time and location, to describe the distribution system. Approximation models provide near optimal solutions and perform better on larger instances while numerical models provide more accurate solutions for smaller instances (Daganzo 2007). It is worth mentioning that continuous approximation and numerical optimization models are complementary to each other. Also, this paper provides a combinatorial perspective of the VRP problem. The proposed model is an outcome of the combination of (a) vehicle routing problem, (b) emissions estimation, and (c) Economic Order Quantity (EOQ) model. To the best knowledge of the authors, no published research has modeled EVRP and proposed a solution for it using continuous approximation.

The paper is organized as follows: The following section reviews the CO₂ emissions estimation models used in this paper. The third section reviews the discrete formulation of the EVRP from Figliozzi (2010a, 2010b) and introduces the continuous approximation model. The fourth section describes the solution method for the continuous approximation model. The fifth section discusses numerical studies, and the last section concludes with some insights and areas of future research.
CO\textsubscript{2} EMISSIONS ESTIMATION

An extensive number of methods are available for the estimation of vehicle emissions (Ropkins et al. 2009). Recent research results indicate that CO\textsubscript{2} is the predominant transportation GHG (Grant et al. 2008) that is emitted directly in proportion to fuel consumption. Moreover, congestion has a major impact on fuel efficiency and thus vehicle emissions. The volume of emissions is a function of the distance traveled and the speed profile. The British Transport Research Laboratory has developed a formulation as part of the project “Methodology for Calculating Transport Emissions and Energy Consumption” that relates travel speed to mass of emitted CO\textsubscript{2} for heavy duty vehicles (MEET 1999):

$$V_e^m = K + av + bv^2 + cv^3 + \frac{d}{v} + \frac{e}{v^2} + \frac{f}{v^3}$$

(1)

where \(v\) is travel speed in kmh, \(K=1576, a=-17.6, b=0, c=0.00117, d=0, e=36067, \) and \(f=0\) for heavy duty vehicles with gross weights from 32 to 40 tons. The output of equation (1) is CO\textsubscript{2} emissions in g/km.

Also a study by Frey et al. (2003) suggests that an average idling vehicle emits roughly 1.4 g of CO\textsubscript{2} per second. Therefore for CO\textsubscript{2} emissions estimation due to idling, the following expression will be used:

$$V_e^s = 1.4t_s$$

(2)

where \(t_s\) is stopping time in seconds. The output of equation (2) is CO\textsubscript{2} emissions in g/sec. Note that the above equation is for an average idling vehicle. Since heavy duty trucks tend to emit more when idling, equation 2 underestimates the CO\textsubscript{2} emissions.

MODELS OF THE VEHICLE ROUTING PROBLEM FOR EMISSIONS MINIMIZATION

The discrete formulation of the EVRP with hard time windows and time dependent speeds from Figliozi (2010a, 2010b) is presented in this section with our proposed continuous formulation.

Discrete Formulation of the EVRP

Figliozi (2010a, 2010b) proposed an EVRP based on the traditional TDVRP in which the objectives are minimizing the number of vehicles and the distance travelled. In the EVRP, the primary or secondary objective is the minimization of emissions. Figliozi (2010a, 2010b) proposed two formulations. The first formulation assumes a multi-objective function that includes the costs of vehicles, distance travelled, route durations, and emissions. The second formulation follows a more traditional, hierarchical approach and allows for a partial reduction of potential emissions where the primary and secondary objectives are the costs of vehicles and emissions and the tertiary objective is the minimization of distance traveled and route duration costs.

Figliozi (2010a, 2010b) proposed a solution for the partial EVRP which benefits from existing algorithms for the TDVRP. It first minimizes the number of vehicles using a TDVRP and then optimizes emissions subject to a fleet size constraint. However, the proposed model by Figliozi (2010a, 2010b) requires the identification of solutions in detail by means of numerous decision variables and using a computer to sort through this numerical maze which requires a large computational effort when the problem is not small.

Continuous Approximation of the Strategic EVRP

In this section, a continuous approximation model for the strategic EVRP is developed.
Continuous decision function and continuous data

In the continuous model, discrete variables and parameters are approximated with continuous functions that are assumed to be smooth and varying slowly over the service region $R$. Approximations replace exact data for customer locations and demand volumes. The objective is to obtain simple guidelines for the design of a set of routes and delivery schedules that will minimize the total cost per unit time including the emission costs for a region with several customers and one depot.

As in Daganzo (2007), the continuous approximation approach applies to “situations where a large number of customers $N$ is distributed over the region $R$ in a form that can be described by a slow varying continuous density function $\delta(x)$,” measured in customers per unit area. For a subregion $A$ of $R$, the number of customers in the subregion is $N(A) = \int_{x \in A} N \delta(x) dx$. It is assumed that the demand rate per customer $D'(t)$ is changing slowly over time. $D_n(t)$ is the demand of customer $n$ at time $t$. It is also assumed that vehicles are identical and capable of carrying $V_{\text{max}}$ items each. Vehicles are dispatched onto the service routes from the depot at successive time intervals and each vehicle visits a particular subset of customers. The objective is to obtain the set of dispatch times, as well as the delivery lot sizes, the specific customers served at each dispatch, and the routes that minimize the total cost. Note that it is also assumed throughout the paper that “the production process at the origin can be adjusted without a penalty to meet the scheduled shipment quantities.” (Daganzo, 2007)

Continuous cost model

Routing costs are based on the VRP approximations in Daganzo (2007). They are divided into line-haul transportation and emissions costs from the depot to the vicinity of the customers, detour transportation and emissions costs to visit individual customers, stopping costs, carrying costs, and inventory costs. Let $r(x)$ denote the distance from the depot to a point $x \in R$. The average distance between customers is approximated by the inverse of the square root of customer density $K\delta^{-1/2}(x)$ where $K$ is a metric-dependent constant (Daganzo, 2007). The cost per unit distance traveled is denoted as $c_d$; the cost per unit of emissions generated is denoted as $c_e$; the holding cost per item is denoted as $c_h$; the rent cost per item is denoted as $c_r$; the stopping cost per vehicle-stop is denoted as $c_s$; and the carrying cost per item is denoted as $c_c'$. The underlying assumptions here are that the customers are identical, the items are identical, the vehicle fleet consists of single type heavy-duty trucks, and the vehicle loads are fixed.

Emissions estimation can become complicated depending on the level of precision required. Since this study uses a continuous approximation method, it is reasonable to approximate emissions based on average travel speed only. Therefore, the MEET (1999) formulation seems to be a good fit. Moreover, a considerable amount of emissions are generated due to idling; hence they will be considered in this study as vehicles stop several times on their tours. The mass of emissions per distance due to vehicular movement is estimated by equation (1), and the mass of emissions due to stopping is estimated by equation (1). In this study, emissions costs are assumed to have three components: stopping/idling, line-haul transportation, and detour transportation.

First, a case is examined in which items are so cheap that most of the holding cost arises because of the rent paid to hold the items, $c_h \approx c_r$. The expression for the total logistic cost per item between $t = 0$ and $t = t_{\text{max}}$ is as follows (3). For details of the logistic cost function without emissions costs, see Daganzo (2007).
Min \[ z(x) = c_JN\left(\frac{D(t_{\max})}{v_{\max}} + L\right) \]

\[ + c_dKLNS^{-1/2} \]

\[ + c_d2r \frac{ND(t_{\max})}{v_{\max}} \]

\[ + c'ND(t_{\max}) \]

\[ + c't_{\max} \frac{ND(t_{\max})}{L} \]

\[ + c_e2r \frac{ND(t_{\max})}{v_{\max}}V^m_e \]

\[ + c_eKLNS^{-1/2}V^m_e \]

\[ + c_eN\left(\frac{D(t_{\max})}{v_{\max}} + L\right)V^m_e\tilde{t}_s \]

where

- \( L \): number of dispatches,
- \( N \): number of customers,
- \( t_{\max} \): end of desired time period (time unit), and
- \( v_{\max} \): vehicle capacity (items)
- \( \tilde{t}_s \): average stopping duration (time unit)

On the other hand, if the items are very expensive per unit volume, then most of the holding cost is inventory cost. In this case, the method proposed by Newell (1971) is used such that the holding cost should be proportional to the shaded area of Figure 1. Newell (1971) pointed out that the smallest shaded area cannot be expressed as a function of number of shipments alone, independent of \( D(t) \). Therefore, the search for \( \{t_i\} \) is replaced by a search for a continuous function such that \( D'(t) \) does not change rapidly.

Let \( I_i \) denote the \( i \)th interval between consecutive receiving times. The function \( H(t) \) is a smooth function replacing \( H_s(t) \) which is a step function such that \( H_s(t) = t_i - t_{i-1} \) that characterizes the headway between two consecutive dispatches. Then, divide the total cost during the study period into portions “\( COST_i \)” corresponding to each interval. That is, “\( COST_i \)” includes the cost of dispatching one shipment \( c_f \) plus the product of \( c_i \) and the shaded area for interval \( I_i \). For details, see Newell (1971) and Daganzo (2007).

\[ COST_i = c_f + c_i(area_i) \]

where

\[ c_f = N(c_s + c_eV^m_e) + KN\delta^{-1/2}(c_d + c_eV^m_e) \]

\[ (area_i) = \frac{1}{2}H^2(t)D'(t)N \]

In this case, the expression for the total logistic cost per item between \( t = 0 \) and \( t = t_{\max} \) is as follows:
(c_s + c_e t^m) N \frac{D(t_{max})}{v_{max}} + (c_d + c_e t^m) 2r \frac{ND(t_{max})}{v_{max}} + c_i' ND(t_{max}) + \sum COST_i \tag{4.d}

FIGURE 1 Selection of shipment times for least holding cost

Discussion of Models

Figliozzi (2010a, 2010b) developed a heuristic solution method for the discrete formulation of the operational EVRP based on the TDVRP formulation. Alternatively, the continuous model developed in this paper is an approximation of the discrete formulation at the strategic level. However, in the continuous model, the requirement for hard time windows is dropped in order to develop a simple problem that can be solved easily. Thus, the continuous model proposed in this paper is not suggested as a replacement for the discrete formulation; rather, it is a complementary model.

If any of the used parameters in the problem including the demand rate vary slowly over time and space, continuous approximation is capable of providing a time-dependent solution. However, continuous approximations may overestimate or underestimate the solution if the parameters change more abruptly. This is especially problematic for the EVRP since emissions are dependent on congestion, and it may not be very realistic to assume that congestion is changing slowly over time and space.

SOLUTION METHOD FOR THE CONTINUOUS APPROXIMATION MODEL

This section describes a solution method for the continuous approximation of the EVRP. Geographical decomposition is used to solve the problem. The spatial and temporal effects of congestion on emissions costs are also explained.

Geographical Decomposition

The problem is decomposed geographically into a set of subregions. A subregion A of R should be small enough such that \( \delta(x) \) is nearly constant over the subregion. This assumes that the values of all data
functions for points $x \in A$ are equal to the average value over the subregion. However, the subregion should also be large enough to contain at least one route.

Equation (3) depends on one decision variable, $L$. Thus, the optimal $L$ is the solution of an integer constrained Economic Order Quantity (EOQ) equation that balances the transportation plus emissions cost and the rent cost. Based on the proposed continuous approximation approach in Daganzo (2007), $L^*$ is obtained by taking the derivative of Equation (3) with respect to $L$ and equating it zero:

$$L^* = \sqrt{\frac{c_i t_{\text{max}} D(t_{\text{max}})}{c_s + c_e V_e^t + (c_d + c_e V_e^m) \delta^{-1/2}}}$$

(5.a)

Substituting Equation (5.a) into Equation (3), the optimal total logistic cost is:

$$2\sqrt{\frac{c_i t_{\text{max}} D(t_{\text{max}})}{c_s + c_e V_e^t + (c_d + c_e V_e^m) \delta^{-1/2}}} + N \frac{D(t_{\text{max}})}{v_{\text{max}}} (c_s + c_e V_e^t) \left(1 + \frac{ND(t_{\text{max}})}{v_{\text{max}}} (c_d + c_e V_e^m) + c_e ND(t_{\text{max}}) \right)$$

(5.b)

Equation (4.a) depends on one decision variable, $H(t)$. Thus, the optimal $H(t)$ is the solution of a constrained Economic Order Quantity (EOQ) equation that balances the transportation, stopping, and emissions cost together and the inventory cost. Based on the proposed continuous approximation approach in Daganzo (2007), $H(t)^*$ is obtained by taking the derivative of Equation (4.d) with respect to $H(t)$ and equating it zero:

$$H^*(t) = \sqrt{\frac{2N(c_s + c_e V_e^t) + 2KN\delta^{-1/2}(c_d + c_e V_e^m)}{c_i \times D'(t)N}}$$

(6)

**Spatial and Temporal Effects of Congestion**

Toward this end, the emission parameters are considered to be constant over time and space. If this assumption holds, the emission costs would just be additions to their corresponding transportation or stopping costs and serve to increase their relative importance in comparison to holding and carrying costs. However, in large regions and over a long period of time, emissions change due to variations in speed. Therefore, dispatchers are more likely to avoid congested time periods and congested areas in order to decrease their emissions costs. Hence, in this study, $V_e^m$ is treated as function of time and space, resulting in $V_e^m(t, x)$.

To capture the temporal aspect of congestion over a long time period, the dispatching period can be partitioned into multiple intervals, each having different constant average moving speeds, e.g. peak and off-peak periods. This introduces a trade-off between waiting to avoid the peak period versus dispatching as early as possible to avoid a high holding cost. Thus, an upper bound can be estimated by assuming that all of the dispatches are done during the peak period with the lowest average moving speed (highest emissions) and a lower bound can be estimated by assuming that all the dispatches are done during the off-peak period with the highest average moving speed (lowest emissions). Comparing the lower and upper bounds, results can show the impact of emissions cost on number of dispatches or headways.
To capture the spatial aspect of congestion in large regions, the study region can be partitioned into sub-regions containing routes with near constant customer density, constant distance from the depot, and constant average moving speed. Figure 2 shows a schematic representation of partitioning a region to incorporate the spatio-temporal effects of congestion. In the figure, the red colored area represents a highly congested subregion with very low average speed, the yellow colored areas represent moderately congested subregions with low average speed, the green colored areas represent uncongested subregions with high average speed, and the dots represent customers scattered throughout the region.

**FIGURE 2** Schematic representation of partitioning a region to incorporate the spatio-temporal effects of congestion

**COMPUTATIONAL STUDY**

In this section, numerical studies are presented to demonstrate the application of the proposed EVRP model in a strategic analysis. The experiments are conducted with the well-known Solomon problems (1987). The Solomon instances include distinct spatial customer distributions, customer demands, and customer time windows. The Solomon dataset contains 100 nodes distributed (R: randomly, C: clustered, or RC: mixed randomly and clustered) across a region. The nodes are visited over a single day period according to demand levels. It is assumed that the vehicle capacity is $v_{\text{max}} = 100$ items. In this study, the problem is solved for the R and C datasets.

**The R Dataset Solution**

To solve the problem instance with continuous approximation, the entire region of the R dataset is divided into four subregions such that the customer densities and demand rates are approximately uniform within each region (see Figure 3). For simplicity, pre-assumed average peak hour and off-peak hour speeds are randomly assigned to each subregion. All of the parameters are listed by subregion in Table 1. The subregions are large enough to contain at least one vehicle tour. To account for the travel distance to and from the depot, an average distance from the depot to a customer in each subregion is estimated using Euclidean distances.
The continuous approximation model is first applied to the R dataset without hard time windows to evaluate the routing cost results. Note that these results are not comparable with Figliozzi (2010a, 2010b) results because of dropping the requirement for time windows. The continuous approximation model yields optimum total costs of $28,697$ for the peak period and $24,497$ for the off-peak period when rent cost dominates. Table 2 shows the results of the continuous approximation for the R dataset when rent cost dominates. As can be seen, the optimal solution ($L^*$) for all the subregions, is higher during the off-peak period compared to the peak period. In this example, the optimal number of dispatches during the peak period is on average $11\%$ smaller than the optimal number of dispatches during the off-peak period.

Table 3 shows the results of the continuous approximation for the R dataset when inventory cost dominates. The continuous approximation model yields a solution of $40,665$ for the peak period and $35,071$ for the off-peak period when inventory cost dominates. Similarly, the optimal solution ($H^*$) is higher during the peak period compared to the off-peak period in all subregions. The optimal dispatching headway during the peak period is on average $13\%$ greater than the optimal dispatching headway during the off-peak period.
TABLE 2 Continuous approximation results for the R dataset when rent cost dominates

<table>
<thead>
<tr>
<th>Subregion</th>
<th>L*</th>
<th>Emissions Cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>Off-Peak</td>
<td>Peak</td>
</tr>
<tr>
<td>Subregion 1</td>
<td>1.48</td>
<td>1.73</td>
<td>5,216</td>
</tr>
<tr>
<td>Subregion 2</td>
<td>1.39</td>
<td>1.59</td>
<td>3,563</td>
</tr>
<tr>
<td>Subregion 3</td>
<td>1.37</td>
<td>1.53</td>
<td>3,994</td>
</tr>
<tr>
<td>Subregion 4</td>
<td>1.41</td>
<td>1.53</td>
<td>3,828</td>
</tr>
<tr>
<td>Total Logistic Cost</td>
<td>16,601</td>
<td>13,463</td>
<td>28,697</td>
</tr>
</tbody>
</table>

TABLE 3 Continuous approximation results for the R dataset when inventory cost dominates

<table>
<thead>
<tr>
<th>Subregion</th>
<th>H*</th>
<th>Emissions Cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>Off-Peak</td>
<td>Peak</td>
</tr>
<tr>
<td>Subregion 1</td>
<td>0.43</td>
<td>0.36</td>
<td>6,879</td>
</tr>
<tr>
<td>Subregion 2</td>
<td>0.46</td>
<td>0.40</td>
<td>4,733</td>
</tr>
<tr>
<td>Subregion 3</td>
<td>0.46</td>
<td>0.41</td>
<td>5,296</td>
</tr>
<tr>
<td>Subregion 4</td>
<td>0.45</td>
<td>0.41</td>
<td>5,064</td>
</tr>
<tr>
<td>Total Logistic Cost</td>
<td>21,972</td>
<td>18,057</td>
<td>40,665</td>
</tr>
</tbody>
</table>

Results show the trade-off between waiting to avoid the peak period versus dispatching as early as possible to avoid a high holding cost. Comparing the lower and upper bounds, results clearly show the impact of emissions cost on number of dispatches and headways.

The C Dataset Solution

To solve the problem instance with continuous approximation, the entire region of the C dataset is divided into 10 subregions such that the customer densities and demand rates are approximately uniform within each region (see Figure 4). To reduce complexity, pre-assumed average peak hour and off-peak hour speeds are randomly assigned to each subregion. However, the average speeds in the C dataset consistently follow the same spatial trend as the average speeds in the R dataset. All of the parameters are listed by subregion in Table 4. The subregions are large enough to contain at least one vehicle tour. To account for the travel distance to/from the depot, an average distance to/from the depot from customers in each subregion is estimated using Euclidean distances.
The continuous approximation model is applied to the C dataset without hard time windows to evaluate the routing cost results. The continuous approximation model yields a solution of 21,778 for the peak period and 18,714 for the off-peak period when rent cost dominates. Table 5 shows the results of the continuous approximation for the C dataset when rent cost dominates. Similar to the results from the R dataset, the optimal solution (L*) for all the subregions, is higher during the off-peak period compared to the peak period. In this example, the optimal number of dispatches during the peak period is on average 10% smaller than the optimal number of dispatches during the off-peak period.
In this section, we first examine the impact of emissions cost by incrementally changing the value of $c_e$, while keeping all of the other parameters constant, and monitor the changes in the total logistic cost and the optimal number of dispatches ($L^*$) in the first subregion during peak and off-peak periods for the case when rent cost dominates in the R dataset. Figures 5(a) shows the total cost during peak and off-peak periods along with the relative difference in the total costs between peak and off-peak periods when the
value of $c_e$ is incrementally changing from 0 to 0.01. As can be seen, the relative difference in the total costs between peak and off-peak periods increases as the value of $c_e$ increases. Similarly, Figure 5(b) shows the optimal L in the first subregion during peak and off-peak periods along with the relative difference in the optimal L between peak and off-peak periods. As shown, the relative difference in the optimal L between peak and off-peak periods decreases as the value of $c_e$ increases. However, the increase/decrease rates of the relative differences get smaller with higher values of $c_e$ in both figures. Also as expected, the optimal L during the peak period is smaller than the optimal L during the off-peak period. This is due to relatively higher cost of dispatching items because of the emissions costs during the peak period. A similar trend is observed for the case when inventory cost dominates and for the C dataset, as well as for the other subregions.

![Figure 5(a) Total Cost](image1)

![Figure 5(b) Optimal L for Subregion 1](image2)

**FIGURE 5 Sensitivity of (a) total cost and (b) optimal L to the value of $c_e$.**

For further examination, the impact of emissions cost on the amount of emissions produced is studied for the case when rent cost dominates in the R dataset. Figure 6 shows the amount of emissions that are produced (kg) when the value of $c_e$ is incrementally changing from 0 to 0.01 along with the percentage of reduction in emissions relative to the case when $c_e = 0$ for peak and off-peak periods. As can be seen, the amount of emissions produced decreases as the value of $c_e$ increases. However, the rate of the reduction in emissions gets smaller with higher values of $c_e$. Results indicate that higher values of $c_e$ could result in larger savings in emissions. Similar phenomena are observed when inventory cost dominates and also for the C dataset.
FIGURE 6 Amount of emissions (kg) produced versus the value of $c_e$.

Figure 7 illustrates the total logistic cost (including emissions cost) when minimized with and without considering emissions costs for different values of $c_e$ from 0 to 0.01 for the R dataset. The vertical difference between the two curves represents a potential cost saving if emissions cost is considered in the distribution problem. Results demonstrate that the potential cost saving due to incorporating emissions in distribution problems increases as $c_e$ increases.

FIGURE 7 Total logistic cost when minimized w/ and w/o considering emissions cost

Results confirm the findings of Figliozzi (2010a) that with a minimal increase in routing and/or renting costs, we can reduce CO$_2$ emissions. Also it is confirmed that there may be significant emissions savings, depending on the $c_e$ as shown in Figure 6. Overall, this section demonstrated the benefits of considering emission costs when solving a VRP problem and provided insights which agree with Figliozzi’s conclusion.
CONCLUSION

This paper presents a continuous approximation model for the Vehicle Routing Problem for Emissions Minimization (EVRP) and demonstrates the model’s applicability with numerical studies based on implementations of the Solomon test instances. The continuous approximation model presented in this paper facilitates strategic planning of one-to-many distribution systems and evaluates the effects of emissions costs. Furthermore, results of the continuous approximation model can be used to guide discrete solutions to determine exact vehicle tours. Approximation models provide near optimal solutions and perform better on larger instances while numerical models provide more accurate solutions for smaller instances (Daganzo, 2007). However, there is still a need to further validate the proposed model by comparing its results to the optimal or near-optimal solutions of other models. Figliozzi (2010a, 2010b) developed a heuristic solution method for the discrete formulation of the operational EVRP. Note that in the proposed continuous model, the requirement for hard time windows is dropped in order to develop a simple problem that can be solved easily. Therefore, the reported results are not comparable to the solutions from Figliozzi (2010a, 2010b). Further research is required to make a valid comparison. However, our results confirm the previous findings that with a minimal increase in routing and/or renting costs, we can reduce CO₂ emissions.

Results from a sensitivity analysis suggest that the optimal number of dispatches during the peak period is smaller than the optimal number of dispatches during the off-peak period when considering the temporal effects of congestion. Results reveal that the potential cost saving due to incorporating emissions in distribution problems is considerable. Incorporating emissions costs in distribution problems will contribute toward having more environmentally sustainable distribution systems. The proposed continuous approximation model can be extended for other specific conditions, e.g. when considering pipeline inventory, storage restrictions, different customers, random demand, discriminating strategies, etc.

ACKNOWLEDGMENTS

The authors gratefully thank Karen Smilowitz, Timothy Sweda, and Madison Fitzpatrick from Northwestern University for their review and feedback. Also, the authors would like to thank the anonymous reviewers for their valuable comments and suggestions to improve the quality of the paper.

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