Real-Time Enhancement of Reference Signals for Feedforward Control of Random Noise Due to Multiple Uncorrelated Sources

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Abstract—The problem envisaged in this paper is the optimization of the reference set for broadband feedforward control of sound and vibration in the presence of multiple, uncorrelated primary sources. A real-time algorithm is presented that generates a set of orthogonal virtual references out of a given set of reference signals. The algorithm is based on recursive singular value decomposition (SVD) techniques. The technique simplifies controller design in the sense that primarily, comparable control performance can be achieved using a reduced set of virtual references instead of the extensive original reference set. Hence, controller dimensions (and, thus, the memory requirements for implementation) and the computational burden reduce accordingly. Second, it is shown that convergence speed of the adaptive filtered-X LMS feedforward control algorithm is enhanced.

The technique is applied to active control of structure-borne road noise inside a car cabin. Time domain simulation of a control configuration, utilizing either an extensive set of original references or a reduced virtual reference set, shows that in the frequency ranges of interest, a set of four virtual references yields the same noise reductions as the original set of 12 references.

Index Terms—Adaptive control, adaptive signal processing, singular value decomposition.

I. INTRODUCTION

With the advent of powerful digital signal processors (DSP’s) at an affordable price, adaptive digital filtering systems have been applied to actively controlled sound fields in a wide variety of applications [18]. The most popular—if not the sole—adaptive control algorithm that is encountered in this kind of applications is the LMS algorithm, which was first introduced by Widrow and Hoff [1], and afterward in a modified form as the filtered-X LMS algorithm. Early references to this algorithm in its various forms are described in [2]–[4]. The filtered-X LMS algorithm offers robust control performance at the price of limited computational complexity.

Adaptive LMS feedforward control of random noise due to multiple uncorrelated sources involves a set of feedforward signals (references \( x \)) that are fed through a control filter \( W \) to generate a control signal \( y \) that is sent to the control sources (the control sources can be either loudspeakers or structural exciters), as shown in Fig. 1. A reference signal for feedforward control needs to be determined such that it characterizes the noise source as well as possible. In the particular case of multiple uncorrelated sources, one is faced with locating reference sensors that are most likely to sense the multiplicity of all noise sources. The control filter coefficients \( w_k \) are adapted purposely; the error \( e \) is fed back to drive the update such that \( E[e^2] \) is minimized. The adaptive process causes the system behavior during adaptation to differ significantly from that of a linear system. However, when the adaptive process converges and the weights settle to essentially fixed values with only minor random fluctuations about the equilibrium solution, the converged system exhibits essentially linear behavior.

The control performance after convergence is uniquely determined by the quality of the reference signals. In Section II, it is shown that the maximum reduction that can be achieved with a filtered-X LMS adaptive feedforward controller is determined by the multiple coherence between the set of references and the error signal. Thus, it is suggested that it is of prime importance to select a set of references that yields maximum multiple coherence. In the case of a control problem involving multiple uncorrelated sources, it is common practice to use a set of references that is, say, overdetermined. This is mainly done because it is often impossible to know the exact number of noise sources, and even if it were possible, it is hard to imagine that each of the noise sources could be measured directly. As a consequence, each of the references will, to some extent, carry information on each of the uncorrelated noise sources. However, with the hardware realization of the active control system in mind, it is also essential to reduce the number input channels (and thus relieve the computational burden for the DSP) as much as possible. In Section III, a new technique is introduced that aims at the enhancement of the information that is carried within the references such that the number of references that is used by the control algorithm can be reduced significantly. The basic idea is to generate a reduced set of virtual references out of an extended set of references such that duplication of information in the references is avoided.

An algorithm is presented that accomplishes this task in real
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Fig. 1. Filtered-X LMS adaptive feedforward control scheme.

time. In Section IV, it is shown that compressing information into a smaller set of references also enhances the convergence speed of the algorithm. In Section V, the technique of virtual references is applied to the practical problem of reducing the structure-borne road noise level inside a car cabin by means of active control.

II. MAXIMUM ACHIEVABLE REDUCTION FOR A GIVEN SET OF REFERENCES

The filtered-X LMS adaptive feedforward controller is configured to reduce the noise level $d$ due to multiple uncorrelated sources at a number of specific locations (the error microphones in ANC applications). The single-channel filtered-X LMS algorithm was independently derived by Widrow [2] in the context of adaptive control and Burgess [3] for ANC applications. The MIMO filtered-X LMS algorithm for ANC applications was introduced by Elliott et al. [4].

Extensive treatment of the algorithm is provided in the literature, e.g., by Kuo and Morgan [5], and some basic equations are recapitulated here. Assuming a control system with $N$ control sources, $M$ error sensors, and $P$ reference sensors, the sound pressure measured at $M$ error sensors is expressed in vector notation as

$$\mathbf{e}(k) = \mathbf{d}(k) + \mathbf{X}_f(k)^T \mathbf{w}(k)$$

where

$$\mathbf{e}(k) = [e_1(k), e_2(k), \ldots, e_M(k)]^T$$

$$\mathbf{d}(k) = [d_1(k), d_2(k), \ldots, d_M(k)]^T$$

is the contribution of the primary noise sources

$$\mathbf{X}_f(k) = [\mathbf{x}_f^1(k), \mathbf{x}_f^2(k), \ldots, \mathbf{x}_f^M(k)]$$

with

$$\mathbf{x}_f^{\text{mn}}(k) = [\mathbf{x}_f^{\text{m}1}(k)^T, \mathbf{x}_f^{\text{m}2}(k)^T, \ldots, \mathbf{x}_f^{\text{m}N}(k)^T]^T$$

in which

$$\mathbf{x}_f^{\text{mn}}(k) = \begin{bmatrix} x_p^{\text{mn}}(k) & x_p^{\text{mn}}(k-1) & \cdots & x_p^{\text{mn}}(k-L_{cf} + 1) \\
\end{bmatrix}^T$$

and

$$x_p^{\text{mn}}(k) = \sum_{i=0}^{L_{mn}} h_{mn,i} x_p(k-l-i)$$

for $k = k, k-1, \ldots, k - L_{cf} + 1$.

$\mathbf{w}(k) = [w_1(k)^T, w_2(k)^T, \ldots, w_N(k)^T]^T$ is the adaptive FIR control filter, with

$$w_n(k) = [w_{n1}(k)^T, w_{n2}(k)^T, \ldots, w_{nP}(k)^T]^T.$$

The FIR control filter $w_{np}(k)$ and the time vector $\mathbf{x}_p^{\text{mn}}(k)$ have length $L_{cf}$. $\mathbf{X}_f(k)$ is the filtered-$x$ reference signal matrix, $x_p(k)$ is the $p$th reference signal, and $h_{mn,i}$ is the $i$th coefficient of the FIR filter (length $L_{sp}$) that models the secondary path between the $n$th control actuator and the $m$th-error sensor. Secondary path modeling is performed in parallel with control and can be implemented both offline or online.

From (1), the control filter weights can be determined that minimize $E[e(k)^T e(k)]$ in a least squares sense. The optimum of this cost function is found for the combination of control filter weights $w_{np}(k)$ for which the gradient to $E[e(k)^T e(k)]$ equals zero. From (1), the gradient $\nabla E[e(k)^T e(k)]$ can easily be calculated as a function of $w_{np}(k)$, which yields the expression for the optimal control filter weights $\mathbf{w}^*_{opt}$ by setting the expression for the gradient to zero:

$$\mathbf{w}^*_{opt} = -[\mathbf{E} \{ \mathbf{x}_f(k) \mathbf{x}_f(k)^T \}]^{-1} \mathbf{E} \{ \mathbf{x}_f(k) \mathbf{d}(k) \}.$$ (2)

This is an exact least squares solution—the so-called Wiener optimal solution—which requires exact knowledge of the matrices

$$\mathbf{R} = \mathbf{E} \{ \mathbf{x}_f(k) \mathbf{x}_f(k)^T \},$$

the so-called input auto-correlation matrix, and

$$\mathbf{P} = \mathbf{E} \{ \mathbf{x}_f(k) \mathbf{d}(k) \},$$

the cross-correlation matrix

and is therefore somewhat complex from a computational point of view.

Simpler methods, such as the gradient search algorithm of Widrow and Hoff, adaptively search the optimum control filter weights $w_{np}(k)$ by updating the control filters inversely directional to the gradient of the error surface defined by $E[e(k)^T e(k)]$. An instantaneous estimate for the gradient that is found by differentiation of (1) with respect to $\mathbf{w}$ is used. This results in the recursive update rule

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu \mathbf{x}_f(k) e(k)$$

(3)

where a factor 2 is absorbed in $\mu$ and is the adaptation constant or convergence coefficient.

The behavior of the LMS algorithm and the filtered-X LMS algorithm has been analyzed extensively in the literature. Proving convergence with certain correlated and non-stationarity inputs is intricate. A rigorous mathematical treatment is
provided for the LMS algorithm in [6]. For the filtered-X LMS algorithm, it is shown in [7] that bounds can be determined for the convergence coefficient such that adaptive control filter converges to the Wiener optimal solution. The practical implication of this result is that any convergence coefficient that ensures stable convergence also assures convergence to the Wiener optimal solution (2). Assuming complete convergence, i.e., when \( w \) equals \( w^{opt} \), the reduction achieved at the error microphone can be determined. An expression, in decibels, for the maximum reduction achievable in the case of a random sound field generated by a number of independent noise sources has been derived in [8]:

\[
\Delta_{\text{max}} = -10 \log(1 - \gamma^2_{\text{ref}})
\]  

(4)

where \( \gamma^2_{\text{ref}} \) is the multiple coherence of all reference signals with respect to the error signal at the error microphone under consideration.

At this point, it is important to stress the fact that this expression is only exact for control filters that are not constrained to be causal, and therefore, it determines a theoretical upper bound that predicts a control performance that is typically better than that which could be achieved in practice. The desired control filter characteristics (magnitude and phase) to obtain the level of cancellation predicted by (4) are determined by the references used as well as by the dynamics of the system to be controlled. In broadband applications, this combination specifies a noncausal desired control filter impulse response. As the control filters implemented on the DSP are constrained to be causal, only an approximate optimal control filter response can be realized at the expense of the control performance.

System causality is affected not only by the selection of the references but also by the layout of the complete control configuration. A noncausal optimal filter specification cannot be made causal by merely moving the reference transducers “upstream” until some delay threshold is reached; it includes optimizing the location of control sources, error sensors, and reference sensors. It is outside the scope of this paper to consider the design issues that relate to complying with the causality constraints. Therefore, in the remainder of this paper, the multiple coherence of a reference set will be used to evaluate the quality of the reference set retained.

III. DATA ENHANCEMENT—VIRTUAL REFERENCE TECHNIQUE

With a set of references being overdetermined, one may expect to “cover” the ensemble of uncorrelated noise sources and thus maximize the multiple coherence between the reference set and the error signal, which in turn maximizes the maximum achievable noise reduction at the error sensor. At the same time, such an extended set of references seriously increases the computational burden for the DSP executing the filtered-X control algorithm, and it also deteriorates the convergence speed of the control algorithm.

For a control configuration involving \( P \) reference sensors, \( M \) control sources, \( N \) error sensors, \( L_{\text{sp}} \) secondary path filter taps, and \( L_{\text{cf}} \) control filter taps, the execution of the control action together with the control filter update according to (3) involves \( (P \times N) + (M \times N \times P) \times L_{\text{cf}} + (M \times N \times P) \times L_{\text{sp}} \) multiplications, \( (P \times N) (L_{\text{cf}} - 1) + (P \times N)(M)(L_{\text{sp}} - 1) \) additions, and \( P \times N \times M \) I/O conversions.

Hence, the computational burden is directly proportional to the number of references used. Furthermore, the memory requirements for the implementation of the filtered-X algorithm are also directly related to the number of references used \( (M \times N) \) arrays of filtered reference signals and \( M \times N \) control filters per reference signal, etc.). This paper proposes an estimation scheme that calculates from the original set of reference signals a reduced set of virtual references that carry the same information.

The idea of virtual references is not entirely new as it relates to similar techniques applied in signal analysis, namely, principal component analysis [9]. The application of these techniques to the active control of road noise inside automobiles has been presented in [21], where the number of independent sources (NIS), which is used to determine how many of the eight reference transducers under consideration are required to achieve significant noise reduction, is presented. Road noise was simulated by exciting a tire of a test vehicle using a shaker with broadband random noise. The number of independent sources occurring at the source, which directly relates to the minimal number of reference transducers, is determined by computation of the condition number (which is the ratio of the largest to the smallest singular value) of the matrix composed of the auto- and cross-spectra of the references. The best combination of references was determined using a condition number analysis and multiple coherence analysis for all possible sets of references. A fixed set of references was retained after the procedure.

However, due to the inherent nonlinear nature of the vibration energy propagation through a car’s suspension, the optimal set of reference transducers will depend on the operating conditions (pavement, smooth asphalt, etc.). As a result, although some upper limit to the number of independent sources—hence to the number of transducers—may be assumed, the physical transducers that best describe these sources may vary as a function of the operating conditions. The on-line estimation scheme proposed here uses a fixed, extensive, set of reference transducers. These responses are processed to a limited number of virtual reference signals. These virtual references may thus be different as a function of the operating conditions. The real-time implementation of the algorithm in order to generate virtual references for active control brings with it a number of critical issues that need detailed discussion. Therefore, although much of the underlying ideas are well-known to signal processing specialists, the remainder of this section reiterates some basic expressions in order to establish a comprehensive exposition of the technique.

A. Frequency Domain Data Enhancement

In [18], a frequency domain technique for enhancement of references is proposed based on the diagonalization of the
cross spectral density matrix $[S_{XX}]$ of the reference signals:

$$[S_{XX}(\omega)] = \text{E}\{[X(\omega)][X(\omega)]^H\}$$

$$= \text{E}\{[X_1(\omega)X_1(\omega)^H \cdots X_1(\omega)X_p(\omega)^H \cdots X_p(\omega)X_p(\omega)^H]\}$$

(5)

where $[X(\omega)]_{p \times 1} = [F\{x(k)\}_{p \times 1}] = [X_1(\omega) \cdots X_p(\omega)]^T$ are the spectra of the references ($F\{\cdot\}$ denotes the discrete Fourier transform).

The cross spectral density matrix $[S_{XX}]$ has the (real) power spectral densities of the references as diagonal elements and the (complex) cross spectral densities of the references as off-diagonal elements. Cross-diagonal elements are complex conjugate.

Calculating the singular value decomposition (SVD) of the cross spectral density matrix yields

$$[S_{XX}(\omega)]_{p \times p} = [U(\omega)]_{p \times p}[\Sigma(\omega)]_{p \times p}[V(\omega)]^H_{p \times p}.$$  

(6)

This expression holds for every frequency line. For readability, the explicit frequency dependence is dropped in the following arguments.

Since the power spectral density matrix is a complex conjugate matrix ($[S_{XX}] = [S_{XX}]^H$), left and right singular vectors are identical, or

$$[S_{XX}]_{p \times p} = [U]_{p \times p}^H[\Sigma]_{p \times p}[U]_{p \times p}^H.$$  

(7)

where

$[\Sigma]_{p \times p}$ diagonal matrix with the singular values (in descending order $\sigma_1 \geq \cdots \geq \sigma_p \geq 0$);

$[U]_{p \times p}$ orthonormal matrix ($[U][U]^H = [I]$) with the singular vectors as columns;

and the cross spectral density matrix can be written as a sum of independent terms, each corresponding to a singular value:

$$[S_{XX}]_{p \times p} \approx \sum_{i=1}^{p} (\sigma_i[U]_{p \times 1}[U]_{p \times 1}^H).$$

(8)

With $N_S$ singular values significantly larger than zero, $\sigma_1 \geq \cdots \geq \sigma_{N_S} > 0$, $\sigma_{N_S+1} \cdots \sigma_p \approx 0$, the sum can be truncated, and the cross spectral density matrix can be approximated by

$$[S_{XX}]_{p \times p} \approx \sum_{i=1}^{N_S} (\sigma_i[U]_{p \times 1}[U]_{p \times 1}^H)$$

$$= [U]_{p \times N_S}[\Sigma]_{N_S \times N_S}[U]_{N_S \times p}^H,$$

(9)

where $[\Sigma]_{N_S \times N_S}$ is the matrix with the $N_S$ largest singular values, and $[U]_{p \times N_S}$ is the matrix with the $N_S$ first singular vectors.

Thus, truncation is equivalent to keeping the $N_S$ largest singular values and the corresponding $N_S$ singular vectors. Hence, the rank of $[S_{XX}]$ is reduced from $p$ to $N_S$.

Thus, after some manipulations

$$[\Sigma]_{N_S \times N_S} \approx [U]_{N_S \times p}^H[S_{XX}]_{p \times p}[U]_{p \times N_S}.$$  

(10)

A set of $N_S$ independent virtual reference signals $[X^c]$ can now be obtained by defining

$$[X^c]_{N_S \times 1} = [U]_{N_S \times p}[X]_{p \times 1}. $$  

(11)

The cross spectral density matrix of these virtual reference signals yields

$$[S_{X^cX^c}]_{N_S \times N_S} = \text{E}\{[X^c(\omega)]_{N_S \times 1}[X^c(\omega)]^H_{N_S \times 1}\}$$

$$= [U]_{N_S \times p}^H[S_{XX}]_{p \times p}[U]_{p \times N_S}$$

$$\approx [\Sigma]_{N_S \times N_S}.$$  

(12)

Some physical considerations can be made at this point.

- The virtual reference signals obtained by (11) are independent since the cross spectral density function between every pair of virtual reference signals equals zero.
- The total power present in the original set of reference signals is conserved since the transformation matrix $[U]$ is unitary.
- The number of significant singular values $N_S$ is equal to the number of independent sources that adequately describe the set of original reference signals at a particular frequency line.

To summarize, it is possible to obtain a set of independent virtual reference signals that consists of only $N_S$ signals (instead of $p$). The required computation time for the control algorithm will thus decrease. At the same time, the set of virtual reference signals contains all the significant information that is present in the original reference signals. The theoretical maximum achievable reduction is thus unaffected.

However, the frequency domain virtual reference signals cannot be used for real-time control. Time-domain virtual references are obtained by inverse discrete Fourier transforming (11)

$$[x^c(k)]_{N_S \times 1} = F^{-1}\{[X^c(\omega)]_{N_S \times 1}\}$$

$$= F^{-1}\{[U(\omega)]_{N_S \times p}[X(\omega)]_{p \times 1}\}$$

$$= [u(k)]_{N_S \times p}^H * [x(k)]_{p \times 1}.$$  

(13)

$^1$The matrix $[U(\omega)]$ can be interpreted as the matrix of frequency response functions (FRF’s) between the original and the virtual references. The matrix $[u(k)]$ contains the corresponding impulse response functions (IRF’s). For computational reasons, these IRF’s are implemented as finite impulse response (FIR) filters, which are truncated impulse response functions.

Thus, each time domain virtual reference signal is obtained as a sum of convolutions of every time domain original reference signal and the corresponding FIR filters:

$$x_i^c(k) = \sum_{j=1}^{p} [u_{ij}(k) \otimes x_j(k)] \quad i = 1 \cdots N_S.$$  

(14)

This approach leads to severe problems in practical applications.

Since the SVD requires a lot of computations, it is calculated offline. The obtained transformation matrix is then transformed

$^1$denotes the convolution operator.
to the time domain and implemented as FIR filters. The off-line generated FIR filters are then used for the on-line control algorithm. This approach assumes stationary conditions that are not necessarily valid, especially not in the application at which this paper aims: the active control of road noise in vehicles. Furthermore, as the transformation matrix is obtained in the frequency domain, it is not constrained to be causal, which may imply that the transformation cannot be realized in the time domain.

Another problem is a cross-over effect that can occur when the frequency functions $U_{ij}(\omega)$ are transformed to the time domain. At a cross-over frequency, the order of importance of two independent sources changes. This is reflected by a crossing in the plot of the singular values. Since the SVD is calculated frequency line by frequency line, cross-over effects cause discontinuities in the calculated $\sigma_f(\omega)$ and $U_{ij}(\omega)$ functions. For a simple case with two singular values, the cross-over effect is shown in Fig. 2.

The cross-over effect can be detected by looking at the transformation matrices for two adjacent frequencies $[U(\omega_n)]$ and $[U(\omega_{n+1})] = [U(\omega_n + \Delta\omega)]$.

Each transformation matrix is orthonormal:

$$[U(\omega_n)][U(\omega_{n+1})]^H = [I]\quad \text{and}\quad [U(\omega_n + \Delta\omega)][U(\omega_{n+1} + \Delta\omega)]^H = [I].$$

When the functions $U_{ij}(\omega)$ are smooth functions of frequency, the cross product of two transformation matrices for adjacent frequency lines also approximates a unity matrix

$$[U(\omega_n)][U(\omega_{n+1})]^H \approx [I].$$

When a cross-over effect occurs between frequency lines $\omega_n$ and $\omega_{n+1}$, this is reflected in the matrix of singular values by two singular values that change position mutually. In the matrix of singular vectors, the two corresponding columns are switched:

$$[\Sigma(\omega_n)] = \begin{bmatrix} \sigma_1(\omega_n) & 0 \\ 0 & \sigma_2(\omega_n) \end{bmatrix}$$

$$[\Sigma(\omega_{n+1})] = \begin{bmatrix} \sigma_2(\omega_{n+1}) & 0 \\ 0 & \sigma_1(\omega_{n+1}) \end{bmatrix}$$

As a result of this, the cross product of the two transformation matrices approximates a unity matrix with two columns switched:

$$[U(\omega_n)][U(\omega_{n+1})]^H \approx \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$ (19)

B. Time Domain Data Enhancement

To overcome the problems of the frequency domain technique, an on-line time domain technique for data enhancement is now proposed. The aim is to find a transformation matrix $[T(k)]$ such that the virtual reference signals are defined as

$$[x^v(k)]_{N_S \times k} = [T(k)][x_1(k) \cdots x_p(k)]_{p \times k}$$

where $[x(k)]_{p \times k} = [x_1(k) \cdots x_p(k)]^T$ are the original reference signals until time step $k$; $[x^v(k)]_{N_S \times k} = [x_1^v(k) \cdots x_{N_S}^v(k)]^T$ are the virtual reference signals until time step $k$. $[x(k)]_{1 \times k} = [x_i(1) \cdots x_i(k)]$ is the $i$th original reference signal until time step $k$. $[x_i^v(k)]_{1 \times k} = [x_{i1}^v(1) \cdots x_{i1}^v(k)]$ is the $i$th virtual reference signal until time step $k$ and $[T(k)]$ is a transformation matrix that transforms the set of $P$ original reference signals into a set of $N_S$ independent virtual reference signals.

The independence of a set of time signals can be evaluated by looking at the covariance matrix. For the original reference signals, this covariance matrix yields

$$[\text{COV}_{xx}(k)]_{k \times k} = [x(k)]_{k \times k}[x(k)]^T = \begin{bmatrix} \sum_{n=0}^{k} [x_1^2(n)] & \cdots & \sum_{n=0}^{k} [x_1(n)x_P(n)] \\ \vdots & \ddots & \vdots \\ \sum_{n=0}^{k} [x_P(n)x_1(n)] & \cdots & \sum_{n=0}^{k} [x_P^2(n)] \end{bmatrix}$$

$$[\text{COV}_{xx}(k)]_{k \times k} = \begin{bmatrix} \sum_{n=0}^{k} [x_1^2(n)] & \cdots & \sum_{n=0}^{k} [x_1(n)x_P(n)] \\ \vdots & \ddots & \vdots \\ \sum_{n=0}^{k} [x_P(n)x_1(n)] & \cdots & \sum_{n=0}^{k} [x_P^2(n)] \end{bmatrix}$$

where $[\text{COV}_{xx}(k)]$ is a real symmetric matrix with variances of reference signals as diagonal elements and covariances between two reference signals as off-diagonal elements.

If the covariance matrix of a set of signals is diagonal, the signals are independent since the covariance function between every pair of signals is then equal to zero.

The covariance matrix for the virtual reference signals yields

$$[\text{COV}_{xx}(k)]_{N_S \times N_S} = [x^v(k)]_{N_S \times N_S}[x^v(k)]_{N_S \times N_S} = [T(k)]_{N_S \times P}[x(k)]_{P \times N_S}[T(k)]_{P \times N_S}$$

$$= [T(k)]_{N_S \times P}[\text{COV}_{xx}(k)]_{P \times P}[T(k)]_{P \times N_S}.$$ (22)

The covariance matrix of the reference signals can be diagonalized by means of a SVD. Since $[\text{COV}_{xx}(k)]$ is symmetric,
the left and right eigenvectors are equal:
\[
[\text{COV}_{xx}(k)]_{P\times P} = [uu(k)]_{P\times P}[\Sigma(k)]_{P\times P}[uu(k)]_{P\times P}^T
\]  \hfill (23)
where \([\Sigma(k)]_{P\times P}\) is the diagonal matrix with the singular values in descending order. \([uu(k)]_{P\times P}\) is the orthonormal matrix \(\{uu(k)\}uu(k)^T = [I]\) with singular vectors as columns.

With \(N_S\) significant singular values, \([\text{COV}_{xx}(k)]\) can be approximated by a truncated sum
\[
[\text{COV}_{xx}(k)]_{P\times P} \approx \sum_{i=1}^{N_S} \{\sigma_i(k)[uu(k)]_{P\times 1}[uu(k)]_{P\times P}^T\}
\]  \hfill (24)

If the matrix of singular vectors is now taken as the transformation matrix in (20)
\[
[T(k)] = [uu(k)]
\]  \hfill (25)
the covariance matrix of these virtual reference signals becomes
\[
[\text{COV}_{xx}\cdot}(k)]_{N_S \times N_S} = [x^e(k)]_{N_S \times x}[x^e(k)]_{N_S \times N_S}^T
\]  \hfill (26)
\[
= [uu(k)]_{N_S \times P}[\text{COV}_{xx}(k)]_{P\times P}[uu(k)]_{N_S \times N_S}^T
\]  \hfill (27)
\[
\approx [\Sigma(k)]_{N_S \times N_S}.
\]  \hfill (28)
Thus, virtual reference signals obtained in this way are independent.

So far, a technique has been proposed to transform a set of \(k\) time samples of references in a set of \(k\) time samples of virtual references. The transformation is determined such that it orthogonalizes all reference vectors until time instant \(k\) and is therefore guaranteed to be causal. However, it does not guarantee orthogonality at time instant \((k+1)\), and therefore, it must be determined at each time instant, after which the virtual references can be calculated. For real-time control applications, an on-line technique is needed that directly transforms each new arriving sample of the references to a new sample of virtual reference signals.

Suppose a new time sample of reference signals \([u(k+1)]\) arrives at time step \((k+1)\). The new set of reference signals is
\[
[x(k+1)]_{P\times (k+1)} = [x(k) \ u(k+1)]
\]  \hfill (29)
where \([u(k+1)]_{P\times 1} = [x_1(k+1) \ldots x_P(k+1)]^T\) is the new sample of original reference signals.

The new covariance matrix of the reference signals is
\[
[\text{COV}_{xx}(k+1)]_{P\times P} = [x(k+1)]_{P\times (k+1)}[x(k+1)]_{P\times (k+1)}^T
\]  \hfill (30)
\[
= [\text{COV}_{xx}(k)]_{P\times P} + [u(k+1)]_{P\times 1}[u(k+1)]_{P\times P}^T.
\]  \hfill (31)
A preliminary covariance matrix \([C]\) of the virtual reference signals can be defined, analogous to (26), as
\[
[C(k+1)]_{N_S \times N_S} = [uu(k)]_{P\times P}[\text{COV}_{xx}(k+1)]_{P\times P}[uu(k)]_{P\times N_S}
\]  \hfill (32)
\[
= [\text{COV}_{xx}(k+1)]_{N_S \times N_S} + [u(k+1)]_{P\times 1}[u(k+1)]_{P\times N_S}^T
\]  \hfill (33)
\[
\approx [\Sigma(k)]_{N_S \times N_S} + [u(k+1)]_{P\times 1}[u(k+1)]_{P\times N_S}^T
\]  \hfill (34)
\[
\approx [\Sigma(k)]_{N_S \times N_S} + [u(k+1)]_{P\times 1}[u(k+1)]_{P\times N_S}^T
\]  \hfill (35)
where \([C(k+1)]\) is not necessarily diagonal because of the extra term that corresponds to the new sample. It is partially diagonalized by a Givens-like [19] rotation \([R(k+1)]\) that makes the off-diagonal element \(C_{pq}\) with the largest absolute value of \([C(k+1)]\) equal to zero:
\[
C_{pq}(k+1) = \frac{\max_{j\neq p-q} \{ |C_{ij}(k+1)| \}}{\max_{j\neq p-q} \{ |C_{ij}(k+1)| \}} \cdot |C_{pq}(k+1)|
\]  \hfill (36)
\[
[R(k+1)]\) realizes a rotation of the \((p, q)\) plane over an angle \(\alpha\) obtained by
\[
\tan(2\alpha) = \frac{2C_{pq}(k+1)}{C_{pp}(k+1) - C_{qq}(k+1)}
\]  \hfill (37)
\[
\Rightarrow \tan(\alpha) = \frac{-1 \pm \sqrt{1 + \tan^2(2\alpha)}}{\tan(2\alpha)}.
\]  \hfill (38)
By choosing the “+” sign, \(\alpha\) is constrained to \(-45^\circ \leq \alpha \leq 45^\circ\), and it converges to \(\alpha = 0\). Choosing the “−” sign yields \(45^\circ \leq \alpha \leq 90^\circ\) or \(-90^\circ \leq \alpha \leq -45^\circ\), and it converges to \(\alpha = \pm 90^\circ\), which results in a continuous permutation of two columns of the covariance matrix.

The new, partially diagonalized covariance matrix of the virtual reference signals yields
\[
[\text{COV}_{xx}\cdot}(k+1)]_{N_S \times N_S} = [R(k+1)]_{N_S \times N_S}[C(k+1)]_{N_S \times N_S}[R(k+1)]_{N_S \times N_S}^T
\]  \hfill (39)
\[
= [R(k+1)]_{N_S \times N_S}[uu(k)]_{P\times N_S}[\text{COV}_{xx}(k+1)]_{P\times P}[uu(k)]_{N_S \times N_S}^T
\]  \hfill (40)
\[
\approx [\Sigma(k)]_{N_S \times N_S} + [u(k+1)]_{P\times 1}[u(k+1)]_{P\times N_S}^T
\]  \hfill (41)
Where the rotation transformation is incorporated in the new transformation matrix
\[
[u(k+1)]_{P\times N_S} = [uu(k)]_{P\times N_S}[R(k+1)]_{N_S \times N_S}^T
\]  \hfill (42)
To conclude, a technique has been proposed to generate real-time independent virtual references by taking linear combinations of the original references. The transformation matrix is updated every time step. The update consists of a postmultiplication with a Givens-like rotation matrix that is chosen such that it zeros the largest off-diagonal element of the covariance matrix.

In order to start with a set of independent virtual references, the transformation matrix should, in theory, be initialized as the matrix with the first \( N_S \) singular vectors, and the covariance matrix as the matrix with the first \( N_S \) singular values, obtained from an exact SVD of a certain time block of the original references.

Experiments have shown (see Section V) that this exact SVD is not necessary and that it is possible to start from scratch, i.e., to initialize the transformation matrix as a unity matrix and the covariance matrix as a zero matrix. Care should be taken to extract the singular vectors corresponding to the largest \( N_S \) diagonal elements of the covariance matrix as they appear after a certain number of recursion steps. These are not necessarily the first singular values in this case.

For a discussion of the convergence properties of recursive SVD algorithms, refer to [17]. Reference [20] discusses the error propagation due to successive Givens rotations.

To comply with the notations used in Section II on the control algorithm, the vector of the virtual references used for control is defined as

\[
[X^o(k)] = [X^o_1(k)^T \ X^o_2(k)^T \ \cdots \ X^o_{N_S}(k)^T]^T
\]

where

\[
[X^o_n(k)] = [x^o_n(k) \ x^o_n(k-1) \ \cdots \ x^o_n(k-L_{cf}-1)]^T
\]

for \( n = 1, 2, \cdots, N_S \).

In the remainder of this paper, (35) is referred to as “the virtual reference signals.”

IV. ENHANCEMENT OF THE CONVERGENCE SPEED DUE TO VIRTUAL REFERENCE TECHNIQUE

The (conventional) LMS algorithm is sometimes associated with a certain deterioration in performance for problems with a large spread between the eigenvalues of the input autocorrelation matrix \( \mathbf{R} \) [11], [12]. In general, it can be shown [13] that for stationary input and sufficiently small \( \mu \), the speed of convergence of the algorithm is dependent on the ratio of the maximum to minimum eigenvalues of the matrix \( \mathbf{R} \).

When plotting the mean square error as a function of the iteration number, “modes of convergence” will appear that decay exponentially. The decay rate of the slowest mode of convergence is determined by \( \lambda_{\text{min}} \), which is the smallest eigenvalue of \( \mathbf{R} \). On the other hand, the maximum convergence rate \( \mu \) is limited by \( 1/\lambda_{\text{max}} \) [13]. The eigenvalues of \( \mathbf{R} \) are determined by the spectral content of the reference signals. Whereas a reference signal with a flat energy spectrum (white noise signal) indicates fast convergence, a reference spectrum with a large dynamic range will lead to slow convergence.

In order to eliminate the potential deficiency of the Widrow–Hoff algorithm, several solutions have been presented [11], [12]. The reference signal is basically transformed in such a way that it has uniform eigenvalue spread. Although this process is often referred to as orthogonalization, a uniform eigenvalue spread can only be achieved by prewhitening the reference signal such that it gets decorrelated in time (note that such a procedure implicitly requires perfect knowledge of \( \mathbf{R} \)).

The problem that is envisaged here implies two major complications in the sense that multiple references are involved and that the filtered-X LMS algorithm is used. The filtered-X LMS algorithm differs from the conventional LMS algorithm in that the reference signals (here, virtual references) are prefiltered by a model of the secondary paths (the transfer function from the controller output to the error sensor) in order to ensure convergence of the algorithm in the presence of secondary paths. Unlike the single-channel LMS algorithm, the convergence behavior of the multi-channel filtered-X LMS algorithm for an arbitrary wideband signal is difficult to analyze. However, it has been shown [14], [15] that the convergence rate is determined i) by the delays caused by the physical separation of secondary sources and error microphones and ii) by the eigenvalue spread of the input autocorrelation matrix of the filtered reference signals. In order to avoid slow modes of convergence, it is of prime importance to get rid of the extremely small eigenvalues of the input autocorrelation matrix of the filtered reference signals.

The technique of virtual references presented here orthogonalizes the reference signals and removes linear dependent signals from the virtual reference set. The virtual references are decorrelated in space but not in time. Removing \( (P - N_S) \) virtual references that yield zero or near-zero diagonal elements in \( \text{COV}_{X^oX^o} \) implies that \( (P - N_S) \times L_{cf} \) near-zero eigenvalues are removed from the autocorrelation matrix of the virtual references \( \mathbf{E}(\mathbf{X}^o(k)\mathbf{X}^o(k)^T) \) (note that it does not change the eigenvalue spread of the virtual references that are retained). Similarly, \( (P - N_S) \times L_{cf} \times M \times N \) near-zero eigenvalues are removed from the input autocorrelation matrix of the filtered virtual references (which are defined in analogy with \( \mathbf{R} = \mathbf{E}(\mathbf{X}^f(k)\mathbf{X}^f(k)^T) \)). Thus, it may be anticipated that the convergence speed is enhanced by removing the extremely slow modes of convergence of the algorithm.

However, secondary path filters often give much spectral coloration to the references, and thus, some degree of correlation is introduced between the filtered virtual references. Compared with \( \mathbf{E}(\mathbf{X}^o(k)\mathbf{X}^o(k)^T) \), the input autocorrelation matrix of the filtered virtual references will typically have a range of significantly smaller eigenvalues (more and smaller eigenvalues are introduced in the case of lightly damped secondary paths compared with more heavily damped systems). Hence, although filtering the references is necessary in the presence of secondary paths, it may slow down the convergence rate of the filtered-X LMS algorithm. In the application presented here (Section V), it was observed that this latter effect did not completely deteriorate the increase in convergence speed achieved by using virtual references instead of the original reference set.

V. APPLICATION TO ACTIVE CONTROL OF STRUCTURE-BORNE ROAD NOISE IN A CAR CABIN

Structure-borne road noise is generated by road induced forces that originate at the tire-road contact and act as multiple,
uncorrelated, noise sources. The design of an active control system to control structure-borne road noise by means of feedforward control is discussed extensively in [16] and [22]. The control sources that are used to control the sound field inside the car cabin can either be inertia shakers connected to the car body or loudspeakers located in the passenger compartment. The set of reference signals needs to be determined such that it characterizes the road excitation as well as possible. The acceleration measured at the wheel centers of the car was used as the reference set.

Two series of experiments were performed on a Volkswagen Passat station wagon with a VR6 petrol engine.

A. Validation on Laboratory Test Data

A laboratory test was performed, generating a data set on which the virtual references technique could be validated. Four shakers were mounted to the four wheel hubs, exciting the car wheels vertically. As the car was excited while the car was standing on the laboratory floor, the tires were left in place, and thus, a certain degree of nonlinearity was introduced.

All four shakers were driven with the same white noise signal (bandlimited white noise in a frequency range 0–400 Hz). The original reference set was constituted of four vertical accelerations measured at the four wheels. The sound field inside the car cabin was evaluated using Bruel and Kjær 0.5-in condenser microphones (type 4188) installed at four locations on the car roof right above the passenger’s seats. The location of these microphones was determined such that they could serve as error microphones for the active control experiments described in [16]. A time domain data set was acquired at a sample frequency of 1024 Hz.

Not surprisingly, application of the recursive algorithm, which was described in Section III, revealed that only one virtual reference was satisfactory to describe the noise source. The singular values of the virtual references’ covariance matrix are shown as a function of time in Fig. 3. In Fig. 4, the multiple coherence between the original references and the rear left microphone is compared with the multiple coherence between the virtual reference that was retained and the rear left microphone. Neither curve exactly matches due to system nonlinearities. However, it is clear that almost identical information is carried within one single virtual reference.

In Fig. 5, the largest off-diagonal element of the virtual references’ covariance matrix is shown as a function of time. The figure illustrates that the covariance matrix is nearly diagonal (note the different scaling of the y axis). Fig. 6 presents the evolution of the Givens rotation angle during the first 300 iteration steps. Relatively large corrections are applied in the first few iteration steps (near to 45°, which is the maximum rotation angle), after which the rotation angle rapidly decreases and converges to zero.

B. Road Test Data, Including Simulation of Active Control of Structure Borne Road Noise

A second set of time domain data was generated by driving the test car at 90 km/hr over a rough road. An extended set of reference signals, consisting of the acceleration measured in three directions at each wheel, was recorded on a 16-channel DAT tape together with four microphone signals measured at locations specified in the laboratory test. The data set was then analyzed using the recursive algorithm.
Fig. 7. Evolution of the two largest diagonal elements of $\text{COV}_{\mathbf{X}_t\mathbf{X}_0}$ for an initial COV equal to (a) $\Sigma_{12\times 12}$ and (b) $\Sigma_{12\times 12}$.

In Fig. 7, the two largest diagonal elements of the (diagonalized) covariance matrix of the virtual references are shown as a function of time for two initial covariance matrices: a) the matrix of singular values obtained from an exact SVD of the original reference signals and b) the $12 \times 12$ null matrix. The figure demonstrates that the result obtained with initial condition b) converges to condition a). In contrast with the laboratory tests that were performed in stationary conditions, the diagonal elements of the covariance matrix vary as a function of time.

Fig. 8 graphically represents the order of magnitude of the covariance matrix obtained after 80,000 iteration steps. Although it is clear that the first two virtual references need to be retained, it is not obvious which of the remaining diagonal elements need to be considered as being significant or not. In order to determine the number of virtual references that needs to be retained, both the cumulated power of the virtual references and the multiple coherence between the virtual reference set and the rear left microphone were calculated. The cumulated power of the $r$th virtual reference was calculated as the ratio between the sum of the $r$ largest diagonal elements and the sum of all diagonal elements of the covariance matrix. From Fig. 9, it is clear that more than 75% of the total cumulated power is carried within the first four virtual references.

Fig. 10 presents the multiple coherence between the reference set and the rear left microphone for a) 12 original references, b) four main virtual references, and c) an arbitrary selection of four of the original references. The coherence obtained with set c) indicates that such limited reference sets are not useful for active control purposes. However, due to the imperfect match of the curves for set a) and b), the usefulness of the virtual reference set may be questionable as well. Therefore, both the extended original reference set a) and the four main virtual references b) were used to simulate the noise reduction achieved with a control configuration aimed at the reduction of road noise in a car that is driven on rough asphalt at 90 km/h. The simulation tool is described in [16]. A control configuration with four control loudspeakers and four error microphones is simulated. The synthesized sound pressure level (SPL) at the rear left microphone obtained after 80,000 iteration steps with and without control is presented in Fig. 11. Both reference sets yield nearly identical noise reductions (an average of 8 to 9 dB noise reduction in the frequency band 75–105 Hz). This is explained by the fact that the sound spectrum inside the car cabin of the test car is dominated by two “booms,” viz., in the frequency ranges 75–105 and 225–275 Hz. As these booms dominate $\beta f \mathbb{E}(\epsilon^2)$, the adaptive controller converges to a setting that primarily maximizes the noise reduction in these frequency bands. Therefore, it is most important to maximize the multiple coherence of the reference set at “booming” frequencies. The selected virtual reference set yields multiple coherence values comparable with the complete original reference set in the frequency bands of interest (compare Figs. 10 and 11).

VI. DISCUSSION

1) With a set of the four “best” original references, a maximum reduction of 6 dB could achieved in the same frequency band. This set was selected by simulating the control performance for all possible sets of four references, which is an involving procedure that will need to be repeated regularly in the case where the “best” set of physical transducers would be a function of the operating conditions. The real-time virtual reference estimation scheme performs a similar task automatically. Moreover, the algorithm will always converge to the best solution when the operating conditions would change.

For reasons of confidentiality, no calibrated sound pressure levels are used in this paper. Sound pressure levels, which are presented in the figure in decibels, are actually dBV values (voltage measured at the microphone) without mentioning the calibration factor.
A real-time algorithm is presented that generates a set of orthogonal virtual references out of a given set of reference signals. The technique simplifies controller design in the sense that primarily, a reduced set of virtual references is sufficient to yield comparable control performance. Hence, controller dimensions (and, thus, memory requirements for implementation) and the computational burden are reduced accordingly. Second, it was shown that convergence speed of the adaptive filtered-X LMS algorithm is enhanced.

Confirmation is given by means of a numerical simulation of an active control configuration aimed at reducing structure-borne road noise in a car cabin. A reduced set of references yields nearly identical noise reductions as the original extended reference set.

The increase of the computational burden due to the integration of the on-line generation of virtual references into the control algorithm is not analyzed in this paper. As the recursive procedure lends itself well to parallel implementation, extremely high throughput can be provided by implementation of the algorithm on customized systolic computing systems [17].

VII. CONCLUSIONS

This paper deals with the optimization of the reference set for broadband feedforward control of sound and vibration in the presence of multiple, uncorrelated primary sources. A real-time algorithm is presented that generates a set of orthogonal virtual references out of a given set of reference signals. The technique simplifies controller design in the sense that primarily, a reduced set of virtual references is sufficient to yield comparable control performance. Hence, controller dimensions (and, thus, memory requirements for implementation) and the computational burden are reduced accordingly. Second, it was shown that convergence speed of the adaptive filtered-X LMS algorithm is enhanced.

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REFERENCES


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